Statistical Properties of Voltage Unbalance Factor in Three-Phase Power Systems

Diego Bellan

Department of Electronics, Information and Bioengineering, Politecnico di Milano,
Piazza Leonardo da Vinci 32, 20133 Milan, Italy.
E-mail: diego.bellan@polimi.it

Abstract

Three-phase power systems under sinusoidal steady-state condition are commonly analyzed by means of the well-known symmetrical component transformation. The main assumption underlying such a transformation is symmetry of the phases of a power system. In modern power systems, however, phase symmetry cannot be guaranteed for several reasons, e.g., large single-phase loads including high-speed railway systems. Nonsymmetrical phase condition results in negative-sequence voltages even if the three-phase source is balanced. This phenomenon is called voltage unbalance emission. Therefore, the ratio between the negative-sequence voltage and the positive-sequence voltage (i.e., the so called voltage unbalance factor, VUF) is defined to provide a quantitative measure of phase asymmetry effects. VUF usually takes small values (e.g., 1-2%) and therefore its measurement can be significantly affected by additive noise. In this work, the statistical properties of VUF are derived in closed form as functions of input noise level in a measurement system based on analog-to-digital conversion of voltage waveforms and spectrum evaluation through the discrete Fourier transform. In particular, it is shown that the magnitude of VUF can be regarded as a random variable with Rician distribution. Numerical simulations are performed in order to validate the probability density function, the cumulative distribution function, the mean and the standard deviation of VUF.

Keywords: Additive noise, Voltage unbalance emission, Discrete Fourier transform, Power quality, Statistical analysis.

INTRODUCTION

Analysis of three-phase power systems under sinusoidal steady-state conditions is commonly performed through the well-known symmetrical component transformation [1]. Such a transformation is a special case of the more general theory of modal analysis. Under the assumption of a power system with symmetrical phases, such an approach leads to three uncoupled modal circuits, i.e., the positive, negative, and zero sequence circuits. Since the analysis of uncoupled circuits is much easier than the analysis of the actual power system with mutually coupled phases, the symmetrical component transformation is the standard tool for three-phase power system analysis (e.g., [2]-[3]). The basic assumption of symmetrical phases, however, is often not met in modern power systems. The main reason for phase asymmetry is usually related to large single-phase loads, e.g., high-speed railway lines [4]-[13]. Asymmetrical phases result in coupling of modal circuits. In particular, when three-wire power systems are considered, phase asymmetry results in coupling between positive and negative sequence circuits. Thus, even in the case of balanced source, currents and voltages appear in the negative sequence circuit. From this viewpoint, phase asymmetry can be treated as a source of voltage unbalance emission [4]-[11], [14]. Quantification of voltage unbalance emission is usually provided by the so-called voltage unbalance factor (VUF) defined as the ratio between the negative sequence voltage and the positive sequence voltage [4]-[11]. In normal operating conditions, reasonable values of such a factor can be of the order of 1-2 %. As a consequence, accuracy of VUF measurement can be significantly affected by measurement noise. In fact, the actual measurement system consists in the measurement of the three phase voltages in the time domain through analog-to-digital (A/D) conversion, transformation into the frequency domain through the discrete Fourier transform (DFT) to estimate the phasors of the phase voltages, and finally the use of the symmetrical component transformation to evaluate positive and negative sequence voltages. Real measurements are always affected by additive noise, coming from instrumentation and from the measured waveforms. Effects of additive noise can be certainly neglected in the evaluation of the positive sequence voltage (i.e., the denominator in the definition of VUF) since in normal operating condition such voltage component is much greater in magnitude with respect to noise level. When the negative sequence voltage (i.e., the numerator in the definition of VUF) is considered, however, the contribution of additive noise cannot be neglected since in normal operating condition the negative sequence voltage is much smaller than the positive sequence voltage. Propagation of additive noise through A/D conversion and DFT was thoroughly investigated in many previous papers (e.g., [15]-[24]). To the Author's knowledge, however, investigation of noise effects through the symmetrical component transformation is not available in the existing literature. In this work, the statistical properties of the negative sequence voltage due to additive input noise in the measurement system are derived in closed form. In particular, it is shown that the magnitude of the negative sequence voltage is described by a Rician random variable with scale parameter related to the noise level and to the sampling conditions (i.e., sampling frequency, number of samples, and window against spectral leakage). Analytical results are validated through numerical simulation of the whole measurement process.

BACKGROUND: THE SYMMETRICAL COMPONENT TRANSFORMATION

The transformation matrix of the symmetrical component transformation, in its rational form, is defined as [13]-[14]

$$\mathbf{S} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \tag{1}$$

where

$$\alpha = e^{j\frac{2}{3}\pi} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{2}$$

and $\alpha^2 = \alpha^*$. The transformation matrix is a Hermitian (or self-adjoint) matrix, i.e., $\mathbf{S}^{-1} = \mathbf{S}^{T*}$. This is a crucial property with respect to the issue of conservation of power. On the contrary, in its classical form the transformation (1) does not hold the power conservation property.

The symmetrical components transformation when applied to phasor voltages provides

$$\begin{bmatrix} V_+ \\ V_- \\ V_0 \end{bmatrix} = \mathbf{S} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{3}$$

where V_+, V_- and V_0 are the positive, negative, and zero sequence voltages, respectively. Of course, the same transformation applies to phasor currents.

Symmetrical three-phase passive components (i.e., lines and loads) can be described in terms of an impedance matrix with the following structure

$$\mathbf{Z} = \begin{bmatrix} Z & Z_m & Z_m \\ Z_m & Z & Z_m \\ Z_m & Z_m & Z \end{bmatrix} \tag{4}$$

By defining the column vectors

$$\mathbf{V}_s = \begin{bmatrix} V_+ \\ V_- \\ V_0 \end{bmatrix}, \quad \mathbf{I}_s = \begin{bmatrix} I_+ \\ I_- \\ I_0 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}, \quad \mathbf{I} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{5}$$

The transformed current/voltage relationship for a symmetrical passive component can be written

$$\mathbf{V}_s = \mathbf{S}\mathbf{Z}\mathbf{S}^{-1}\mathbf{I}_s = \mathbf{Z}_s\mathbf{I}_s \tag{6}$$

where

$$\mathbf{Z}_s = \begin{bmatrix} Z_+ & 0 & 0 \\ 0 & Z_- & 0 \\ 0 & 0 & Z_0 \end{bmatrix} \tag{7}$$

and

$$Z_+ = Z_- = Z - Z_m \tag{8}$$

$$Z_0 = Z + 2Z_m \tag{9}$$

The diagonal form of the sequence impedance matrix (7) leads to the above-mentioned uncoupled sequence circuits when the transformation is applied to the whole three-phase system.

VOLTAGE UNBALANCE FACTOR

Voltage unbalance factor across a specific load in a three-phase power system is defined as [4]-[11]:

$$VUF = \left| \frac{V_-}{V_+} \right| \tag{10}$$

From (3) the explicit expression of the negative sequence voltage V_- can readily obtained:

$$V_- = \frac{1}{\sqrt{3}}(V_a + \alpha^2 V_b + \alpha V_c) \tag{11}$$

Each phasor can be written as a complex number in rectangular form by putting into evidence its real and imaginary parts as $V = V_r + jV_i$. Moreover, by taking into account (2) the phasor (11) can be rewritten as:

$$V_- = \frac{1}{\sqrt{3}}\left\{ \left[V_{ar} - \frac{1}{2}(V_{br} + V_{cr}) + \frac{\sqrt{3}}{2}(V_{br} - V_{cr}) \right] + \right.$$
$$\left. + j\left[V_{ai} - \frac{1}{2}(V_{bi} + V_{ci}) - \frac{\sqrt{3}}{2}(V_{bi} - V_{ci}) \right] \right\} \tag{12}$$

From (12) it is clear that the statistical properties of V_- must be derived from the statistical properties of the real and imaginary parts of the measured phase voltages V_a, V_b, V_c. Notice that, as mentioned above, in (10) only V_- is treated as a random variable, whereas V_+ is treated as a given parameter insensitive to noise effects.

STATISTICAL PROPERTIES OF VUF

Time-domain phase voltages $v_a(t), v_b(t),$ and $v_c(t)$ are digitized through A/D conversion with sampling frequency f_s and number of samples N_s. Each voltage is supposed to be corrupted by additive zero-mean white noise with variance σ_n^2. Then, the three digitized noisy waveforms are transformed into the frequency domain through DFT with weighting function w characterized by specific equivalent noise bandwidth (ENBW). As an example, for the well-known Hann window $ENBW = 1.50$. The DFT values at the fundamental frequency, properly weighted, provide the estimates of the three phasors V_a, V_b, V_c. Of course, due to the presence of additive noise, such phasors must be treated as random variables. A well-known result states that the real and the imaginary parts of each rms phasor estimate can be approximated by an unbiased Gaussian random variable with variance [15]-[24]:

$$\sigma^2 = ENBW \frac{1}{N_s} \sigma_n^2 \tag{13}$$

This result can be used in (12) for each of the real and imaginary parts of the three phasors involved. Notice that by rewriting (12) as:

$$V_- = \frac{1}{\sqrt{3}}(A + jB) \tag{14}$$

We can observe that both A and B are unbiased Gaussian random variables with variance [25]:

$$\sigma_A^2 = \sigma_B^2 = \left(1 + 2 \times \frac{1}{4} + 2 \times \frac{3}{4}\right)\sigma^2 = 3\sigma^2 \tag{15}$$

It follows that the random variable

$$|V_-| = \sqrt{\frac{A^2 + B^2}{3}} \tag{16}$$

is a Rician random variable with noncentrality parameter equal to the zero-noise value of $|V_-|$, i.e., $|V_{-0}|$, and scale parameter equal to σ [26]. Thus, by taking into account (10), and by denoting as U the random variable VUF, the probability density function (PDF) can be expressed as:

$$f_U(U) = |V_+|^2 \frac{U}{\sigma^2} exp\left(-\frac{|V_+|^2 U^2 + |V_{-0}|^2}{2\sigma^2}\right) I_0\left(\frac{|V_+||U||V_{-0}|}{\sigma^2}\right)$$

(17)

where $I_0(\cdot)$ is the modified Bessel function of the first kind.

The corresponding cumulative distribution function (CDF) is given by:

$$F_U(U) = 1 - Q_1\left(\frac{|V_{-0}|}{\sigma}, \frac{|V_+|U}{\sigma}\right)$$

(18)

where Q_1 is the first-order Marcum Q-function.

Finally, the moments of U are given by:

$$E\{U^k\} = (2\sigma^2)^{k/2}\Gamma\left(1+\frac{k}{2}\right)L_{k/2}\left(-\frac{|V_{-0}|^2}{2\sigma^2}\right)/|V_+|^k$$

(19)

where $L_{k/2}$ denotes a Laguerre polynomial. Notice that the mean value μ_U of U can be obtained from (19) with $k=1$, whereas the variance is given by $\sigma_U^2 = E\{U^2\} - \mu_U^2$.

NUMERICAL VALIDATION

Numerical simulation of the whole measurement process were performed in order to validate the analytical results (17)-(19). A positive sequence voltage $V_+ = 100\ V$ and a noise-free negative sequence voltage with $|V_{-0}| = 1\ V$ and random phase were selected. From the inverse of the transformation (3) the phasors corresponding to the phase voltages were evaluated, and the corresponding sinusoidal waveforms in the time domain were obtained. A signal-to-noise ratio $|V_{-0}|/\sigma$ was selected, and from (13) the corresponding time-domain noise level σ_n was derived. Gaussian noise with zero mean and such standard deviation was added to the three waveforms in the time domain. Sampling was performed by taking $N_s = 2^{12}$ samples in 10 fundamental periods of each waveform. The fundamental frequency was 50 Hz, and therefore the resulting sampling frequency was 20.48 kHz. The window against spectral leakage was the Hann window. Then the DFT of the three waveforms were evaluated, and the spectral lines corresponding to the fundamental frequency were selected. Such spectral lines, providing the noisy estimates of the phasors V_a, V_b, V_c were used in (3) to obtain the estimate of the negative sequence voltage V_-. Such procedure was repeated 10^4 times in order to obtain numerical estimation of the PDF and the CDF of the voltage unbalance factor (10). Figures 1 and 2 show the comparison between the numerical estimates (dashed lines) and the analytical results (solid lines) for the PDF and the CDF of the VUF for two different values of signal-to-noise ratio, i.e., 10 and 20. A very good agreement between analytical and numerical results is clearly apparent in the figures.

Figures 3 and 4 refer to the mean value and the standard deviation of the VUF. The simulation procedure outlined above was repeated for several values of signal-to-noise ratio ranging from 10 to 100. In particular, Figure 3 shows a very low bias only for small values of signal-to-noise ratio. In Figure 4 two different values for the noise-free negative

sequence voltage $|V_{-0}|$ were selected, i.e., 0.01 and 0.02. A large difference in the standard deviations can be observed for low signal-to-noise ratios, whereas the curves are closer as the signal-to-noise ratio increases. Notice that dashed and solid lines cannot be distinguished because of the good agreement between analytical and numerical results.
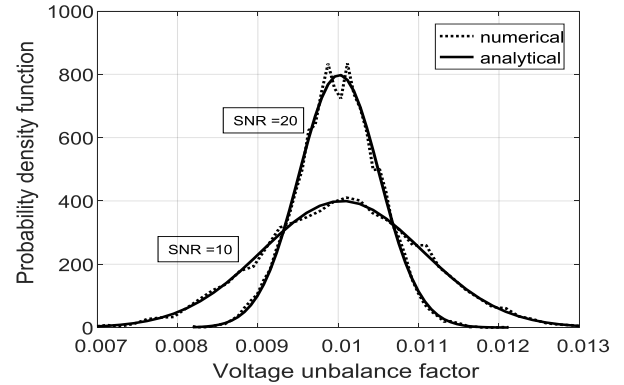

Figure 1: Probability density function of the voltage unbalance factor assuming a noise-free value equal to 0.01, and two different noise levels.
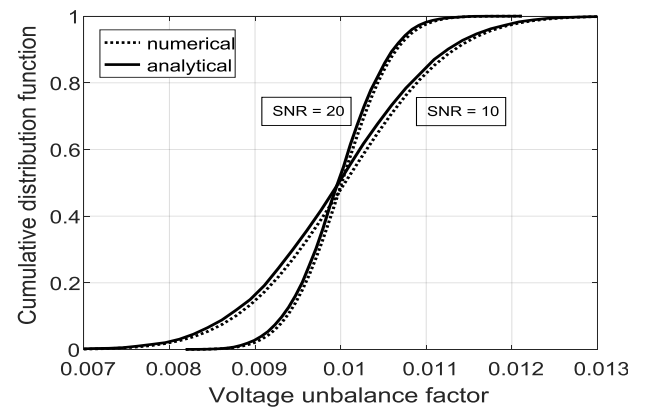

Figure 2: Cumulative distribution function of the voltage unbalance factor assuming a noise-free value equal to 0.01, and two different noise levels.
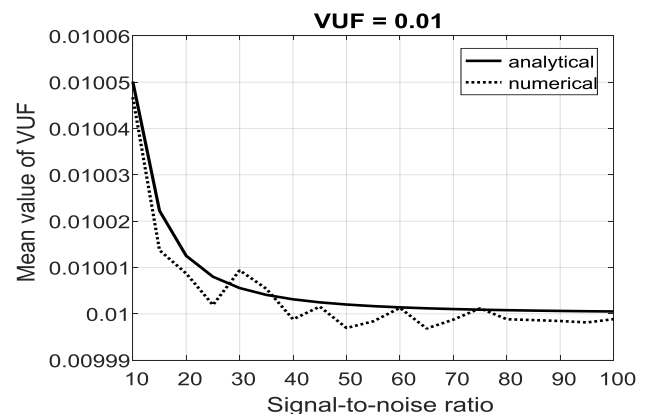

Figure 3: Mean value of the voltage unbalance factor, assuming a noise-free value equal to 0.01, as a function of the noise level.
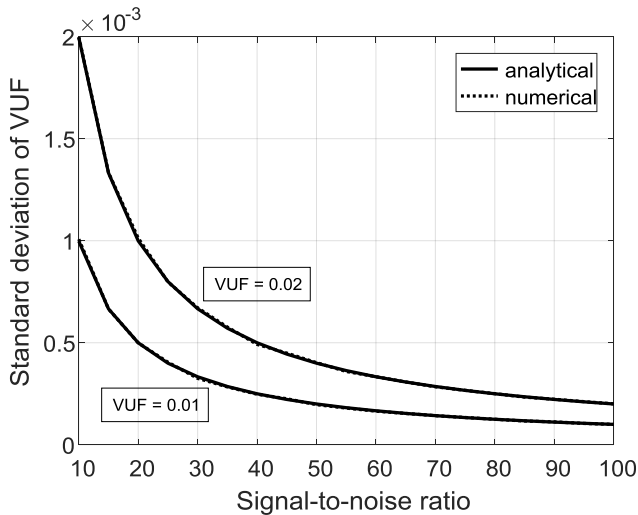
Figure 4: Standard deviation of the voltage unbalance factor, assuming two different noise-free values, i.e., 0.01 and 0.02, as a function of the noise level.

CONCLUSION

The complete statistical characterization of the voltage unbalance factor in a three-phase power system was provided. It was shown that additive noise in the time-domain waveforms results in a VUF characterized by a Rician distribution. Both the PDF and the CDF were successfully validated against numerical simulations. The mean value of VUF has shown very small bias, whereas the behavior of the standard deviation of VUF has shown that it must be taken into account when uncertainty evaluation of the whole measurement process is required.

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