NON-LINEAR DIGITAL IMPLEMENTATION OF A PARAMETRIC ANALOG TUBE GROUND CATHODE AMPLIFIER

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ABSTRACT

In this paper we propose a digital simulation of an analog amplifier circuit based on a grounded-cathode amplifier with parametric tube model. The time-domain solution enables the online valve model substitution and zero-latency changes in polarization parameters. The implementation also allows the user to match various types of tube processing features.

1. INTRODUCTION

Valves are today mainly limited to musical analog processors such as stomp boxes, equalizers, dynamic processors, power amplifiers, etc. As a matter of fact they are physically used in commercial devices because their performance and sound are generally quite difficult to match with digital processing systems [1]. Although in recent years digital processors have gained more and more respect in this field, musicians are still reluctant to give up the "warmth" and the "added dirt" that make the tube sound so characteristic. Our DSP solution for digital digital tube simulation in ground cathode configuration is based on Koren's phenomenological tube model [2], which has the ability to match different harmonic distributions and dynamic behavior. This solution was chosen over other triode tube models for its flexibility and its intuitive parametrization.

The common cathode circuit was analyzed and split between polarization circuit and small signal circuit even if the solution is calculated on the large signal. In fact, the tube has a "built-in feedback", as a large signal on the grid affects the gain of the circuit: the tube polarization that sets the desired voltage gain is, in fact, affected by the same amplified input signal that is present on the plate. More recently a real-time wave digital solution was presented [3], which discusses a different time-domain technique to solve the same problem.

In this paper we discuss the reference analog circuit, the polarization and the small signal solution, and how they are combined with the nonlinear resistance. After examining the dynamic properties of the stage, we finally present the adopted solution.

2. GROUNDED CATHODE TUBE AMPLIFIER

The circuit consists of three voltage generators: V_1 is the DC power supply, V_2 is the AC or DC heater supply used for warming the tube up, V_{in} is the input signal. The 12AX7 tube polarization is set by three resistances: R_1 is the anode resistor, R_2 is the cathode resistor, R_3 is the load resistor. V_{in} is directly connected to the grid. While increasing the grid voltage, the current in the tube that flows from the anode (plate) to the cathode increases, which means

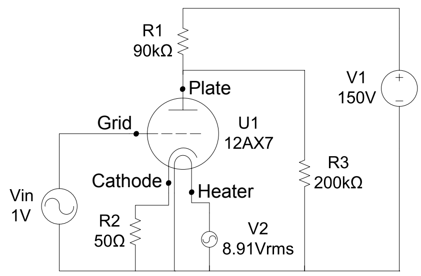

Figure 1: *Simplified grounded cathode triode amplifier schematic.*

that the voltage on R_1 increases as well. As V_1 is fixed, the voltage on R_3 decreases, and so does the corresponding current (which is small compared to R_3 polarization current). The situation when we decrease the grid voltage is completely dual, except for the fact that the relationship between grid voltage and plate current is non linear.

3. PHENOMENOLOGICAL VALVE MODEL

According to [4], there are three aspects to consider in modeling and simulating the nonlinear behavior of valves:

- the stage is expected to work at audio frequency, therefore secondary effects such as Miller capacity are less than relevant;
- the heater does not need to be included in the model as it is independent from the grid, plate and cathode signals;
- the model is expected to be as general as possible, so that different tube models can be accommodated within its parameter space.

The tube input (grid) will be here considered as an ideal voltage reader (infinite impedance port), which means that the polarization can be passed to the model as a parameter. Any voltage offsets, due to the fact that a small current is flowing into the grid (which has a finite impedance) will be compensated by another parameter. It is thus quite reasonable to model only the nonlinear current generator in the output section. This allows us to focus our attention on the dynamic plate-cathode resistor. The phenomenological model turns out to be particularly well-suited for describing the real nonlinear function of the triode and for matching different types of tube models.

3.1. Koren's Triode Model

Koren [2] proposes a phenomenological model that is very close to the expected triode behavior on a wide range of plate currents and voltage values. This model is based on the triode 3/2 power law and it consists in the two following equations:

$$E_1 = \frac{V_{pk}}{K_p} \cdot \log\left(1 + \exp\left(K_p \cdot (\frac{1}{\mu} + \frac{V_{gk} + V_{ct}}{\sqrt{K_{vb} + V_{pk}^2}})\right)\right), \quad (1)$$

$$I_p = \frac{E_1^{Ex}}{K_{g1}} \cdot \big(1 + sgn(E_1)\big), \quad (2)$$

where V_{pk} is the plate cathode voltage, V_{gk} is the grid cathode voltage and I_p is the plate current. An extensive description of the model and of its parameters μ, K_{g1}, Ex, K_p, V_{ct}, K_{vb} can be found in [5] and [6]. V_{ct} is the contact potential between grid and cathode, which can be seen as an offset on the grid voltage:

$$V'_{gk} = V_{ct} + V_{gk}, \quad (3)$$

Eq. (3) enables a more accurate matching of the plate I-V characteristic, with the result of improving the accuracy of the parametrization. Ex and K_{g1} can be optimized to obtain a good matching with the experimental data for low grid voltage. K_p models the behavior for large negative grid values while K_{vb} is related to the location of the "knee" of the plate curve.

TUBE	μ	K_{g1}	K_p	K_{vb}	V_{ct}	Ex
12AX7	100.8	1890	828	72	0.612	1.4979
ECC88	32.92	155.625	225	4492	0.248	1.204
300B	3.99	2715	51	3.9375	2.16	1.526

Table 1: *Parameters of different tubes.*

4. CIRCUIT CONSIDERATIONS AND SOLUTION

The system is modeled in a rather traditional fashion: the circuit is split into a linear polarization circuit and a small-signal nonlinear circuit.

4.1. Polarization Circuit

The plate current I_p is computed with a fixed E_p (linearization around the selected working point). There are four resistors: the anode resistor R_a, the cathode resistor R_k, the load resistor R_l, and the static plate-cathode estimated resistance. R_a, R_k, V_{aa} and E_p are user-controlled parameters. R_l exists if:

$$V_{aa} < E_p + I_p \cdot (R_k + R_a). \quad (4)$$

R_l and R can be computed as in eqs. (5) and (6):

$$R = \frac{E_p}{I_p}, \quad (5)$$

$$R_l = \frac{E_p + I_p \cdot R_k}{\frac{V_{aa} - E_p + I_p \cdot R_k}{R_a} - I_p}, \quad (6)$$

which preserves the validity of eqs. (1), (2). This circuit is rather flexible, as it computes the adapted load on a desired polarization and decouples input from output. The input port is an ideal voltage reader, with a bias that sets the desired working point of the triode. Other currents are computed with current partitioning resistor-sets.

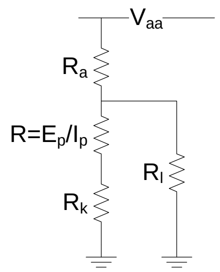

Figure 2: *Polarization circuit schematic.*

4.2. Small-Signal Circuit

The small-signal circuit in Fig. 3 is composed of three static resistors, a non linear resistor, and an ideal nonlinear voltage-controlled current generator. R_a, R_k and R_l are the same as those of the polarization circuit, while i_p and r must be considered in a different way. Notice that Koren's model is a large signal model therefore every current computation is based on the sum of small signal and a polarization voltage.

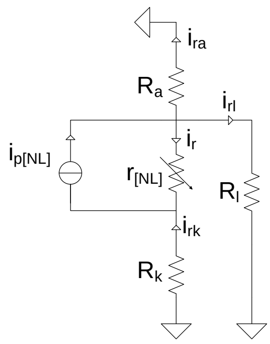

Figure 3: *Small-signal circuit schematic. Orientation of currents for a negative input,* $v_g < 0$

The current generator injects a negative current into the circuit when a positive voltage v_g is applied to the grid. The tube gain is defined by the model parameters but also by the voltage between anode and cathode U_p and by the large-signal voltage that drives the grid U_g. The current is split in the node on the right of the generator by a current divider: i_r controlled by i_p is the small-signal current in the dynamic resistor between anode and cathode. As a consequence we have

$$i_r = i_r\big(i_p, r(i_p)\big), \quad (7)$$

$$v_p = i_r \cdot r = i_r\big(i_p, r(i_p)\big) \cdot r(i_p), \quad (8)$$

where v_p is the small-signal contribution to the plate polarization. The current divider equations allow us to compute all voltages in the circuit.

4.3. Non Linear Dynamic Resistor

On the basis of experimental data taken from a 12AX7 tube datasheet we found that with a plate cathode voltage of $250V$ and plate current of $1.2mA$ the plate resistance is close to $210k\Omega$: this value is quite different from the $62.5k\Omega$ that appears on the datasheet.

$$r = a \cdot exp(b \cdot J_p) + c \cdot exp(d \cdot J_p) \tag{9}$$

Using a suitable model and curve fitting it is possible to compute the dynamic resistance as a function of the plate's large signal J_p.

coefficient	estimated value	95% confidence interval
a	$3.268 \cdot 10^5$	$(3.164 \cdot 10^5, 3.373 \cdot 10^5)$
b	-5238	$(-5347, -5129)$
c	$9.174 \cdot 10^4$	$(9.078 \cdot 10^5, 9.27 \cdot 10^5)$
d	-315.3	$(-322.7, -307.9)$

Table 2: *Values that model the nonlinear current variable plate resistance of a 12AX7 tube with* $250V$ *on the plate.*

4.4. Large-Signal Operation

Large signals are computed with the sign convention that current is positive flowing top to bottom in Fig. 3. Eqs. (1) and (2) can be used for computing J_p as a function of U_g and U_p.

$$U_p = E_p + v_p, \tag{10}$$

With a fixed grid voltage large signal value U_g we have

$$U_p = U_p(i_p, r) = U_p\Big(i_p(U_p), r\big(i_p(U_p)\big)\Big). \tag{11}$$

Eq. (9) will use J_p as variable which is the maximum current which flows in the tube. With reference to eqs. (1) and (2) it is possible to notice a certain asymmetry with respect to the working point.

$$i_p\big(U_p, v_g\big) > \big|i_p\big(U_p, v_g'\big)\big| \qquad \forall v_g > 0, \forall v_g' < 0, \tag{12}$$

i_r is computed by a current divider between i_p and r to simulate an approximation of the nonlinear behavior of r in parallel configuration with a voltage-controlled current generator. We found that a small-signal voltage v_p between plate and cathode was large enough to affect E_p, which plays a significant role in the built-in tube compression.

4.5. Working Conditions

One of the aspects that make the valve behavior musically interesting is their "gentle" dynamic compression. The input signal is transferred to the output with a modified amplitude ratio: small voltages are more amplified than big values and, taking this to the extreme, signal may be affected first by soft clipping and then by hard clipping when the load reaches the voltage imposed by the power supply. Unlike what expected from a "smooth" sounding device, even triode amplifiers are capable of hard clippings [7].

With the current signs of Fig. 3 a negative input enables a larger current flow than a zero input because of the contribution of $i_r \cdot r$ to the plate voltage, which increases the tube gain. The input signal turns out to be more expanded until the magnitude of the negative input becomes large enough. On the other hand reducing the input too much blocks the flow of currents. Soft clipping depends on the shapes of the tube transfer function, as negative inputs are subject to gain expansion and then gradually to heavier compression.

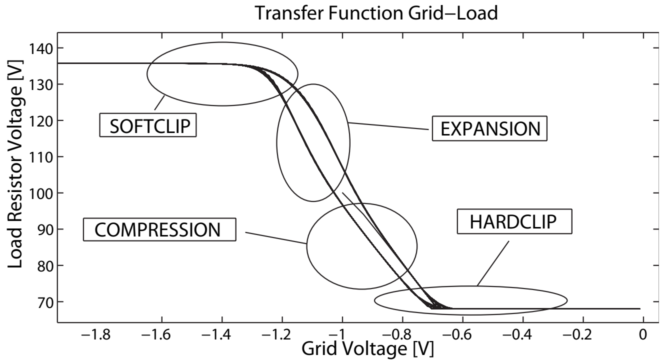

Figure 4: *Cyclic behavior of transfer function's working conditions, caused by a 2V sine wave input*

As far as compression is concerned, it is useful to think of currents in Fig. 3 with inverted signs, which allows us to deal with positive signal with respect to grid polarization. Voltage contribution $i_r \cdot r$ is negative and is lowering the plate cathode voltage giving rise to gain reduction. If the input value gets too high (when i_{rk} gets equal to the polarization current flowing in R_k), the system starts saturating until the large-signal voltage on R_a goes to zero and the system begins hard clipping the output to a voltage value that is fixed by the power supply.

5. NUMERICAL SOLUTION

The algorithm consists basically of two parts: polarization and small-signal output computation. First the following polarization variables are set: E_g, E_p, R_a, R_k and V_{aa}. On the basis of these parameters plate current $I_p\big(E_g, E_p\big)$, static resistance R and dynamic resistance for null input $r_0(I_p)$ are computed. Then the load resistor $R_l\big(E_p, I_p, R_a, R_k\big)$ (if it exists with the desired values) is computed with an imposed plate cathode voltage. In the small-signal solution the voltage grid input is updated:

$$U_g = E_g + \frac{v_g}{C}, \tag{13}$$

where C is a scaling factor. The plate polarization E_p is updated with the small-signal voltage contribution of the previous sample: this will be clearer with eq. (17). Large signal $J_p\big(U_g, U_p\big)$ is computed by using the plate polarization state of the previous sample.

$$U_p(n) = E_p + v_p(n-1), \tag{14}$$

This voltage influences the gain of the stage, therefore the dynamic resistor for that plate current is computed. The saturation takes place when the small signal i_p cancels the large signal on R_k and then on R_a. In both cases the growth of $|i_p|$ is stopped and a zero voltage ends up being forced on both resistors: these conditions depend on polarization currents. If i_p turns out to be saturated, the large signal J_{pS} and the dynamic resistor $r_{1S}(J_{pS})$ are recomputed. A good estimation of plate current and dynamic resistance based on that polarization are determined. A new estimated current $i_r(i_p)$ can be obtained from the value of J_p computed in the previous step. In fact, i_r multiplied by r_1 or r_{1S} returns a good estimation of the plate polarization.

$$\begin{aligned} U_{p_1}(n) &= E_p + i_r(n) \cdot r_1(n) \\ &= E_p + i_r\Big(i_p\Big(v_g(n), v_p(n-1)\Big), r_1(n)\Big) \cdot r_1\Big(J_p(n)\Big). \end{aligned} \quad (15)$$

The value of U_{p_1} is kept as a safe estimate of the tube's gain while a new "final" estimation of both plate current $J_{p_X}(U_g, U_{p_1})$ and dynamic resistance $r_2(J_{p_X})$ are computed. Through a new evaluation of the working conditions we can find whether the tube is saturating and fix the current at $i_{p_X S}$ and the dynamic resistor at $r_{2S}(i_{p_X S})$. With the second estimation of the plate current it is possible to calculate the output voltage:

$$v_{out} = V_{aa} - U_{ra} - I_{rl} \cdot R_l. \quad (16)$$

As a final step, the plate polarization based on J_{p_X} is sent to the next sample

$$v_{pF}(n) = v_p(n+1) = r_2(i_{p_X}) \cdot i_r(i_{p_X}, r_2). \quad (17)$$

The plate's small signal contribution is recursively used for the computation of the next sample.

6. RESULTS

Building a digital model of an analog circuits enables to have different dynamic and frequency response even from the same tube, or better, from the same nonlinearity. The filter is working at $192kHz$ to avoid aliasing problems.

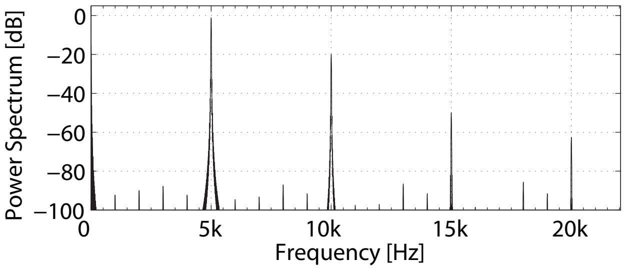

Figure 5: *12AX7 soft driven with 5kHz sine input*

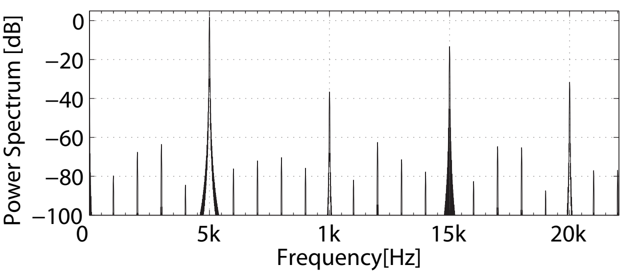

Figure 6: *12AX7 hard driven with 5kHz sine input*

In Fig. 5 input gain and R_a are smaller than in Figure 6 but the tube is the same. The aliased components are slightly greater than in [3] but this stage implements hardclipping. Different effects can be produced by reducing the power supply voltage as output

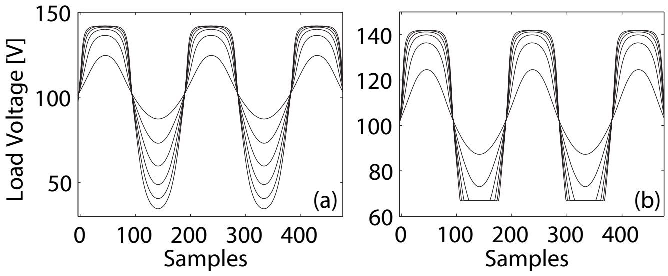

Figure 7: *12AX7 output for 1kHz sinewave input: voltage ranges from 0.25V to 1.5V in 0.25V steps. (a) hard clipping routine disabled, (b) hard clipping routine enabled*

waveform is pushed down in the negative half because amplifier dynamic is reduced. By changing the working point, E_g and E_p, the model produces different distributions of harmonics.

Fig. 7 shows a comparison with the stage distortion in [3]. The aliasing is still acceptable even when causing the stage to go into hard clipping. Reducing the sampling rate at $44.1kHz$ allows only soft clip operation with an acceptable $15dB$ signal degradation: hardclipping involves harsh sounding alias. The adopted model does not account for capacitive behavior and grid input model.

A correct parametrization for large positive grid voltage is yet to be found. Real-time parameter adjustments would improve the performance and the ease of use.

7. CONCLUSIONS

We proposed a versatile nonlinear processing stage for tube simulation that allows the user to account for a variety of distortions, from mild even harmonics to heavier odd harmonics. A specific class of tube models was tested and proved to provide a dynamic spectral response.

8. REFERENCES

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