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# A new cuckoo search and its application of Spread Spectrum Radar Polly Phase Code Design

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***Abstract*— The Lévy flight was used in cuckoo search to achieve good optimization performance. Although the optimization performance of cuckoo search algorithm is good based on the Lévy flight some complicated mathematical operations should be used to realize the Lévy flight such as trigonometric function, gamma function and exponential functions, which limited the application of the cuckoo search algorithm especially there is high requirement about computation complex like in the embedded systems. The Lévy flight is replaced by a simple uniform distribution function based on the randomly chosen dimension and a local search method is applied to improve the optimization performance although the general structure of cuckoo search algorithm is not changed. The proposed cuckoo search algorithm is applied to several benchmark functions and the simulation results show the simplified cuckoo search algorithm can achieve better optimization performance than the original cuckoo search algorithm. Finally the simplified cuckoo search algorithm is applied to the spread spectrum radar Polly phase code design.**

***Keywords—cuckoo search; Lévy flight; Radar; optimization performance.***

## I. INTRODUCTION

Optimization problems arise in almost all areas of science, engineering and technology such as Economics [1, 2], Physics [3], telecommunication [4], Petroleum engineering [5] and network [6]. The optimization problems are becoming more complicated since more real conditions such as constraints are considered in the real world optimization problems and it is difficult to use the classical mathematical optimization algorithms to solve them. Hence some intelligent or nature inspired optimization have been proposed.

Nature inspired optimization algorithms are those intelligent optimization algorithms that are derived from the study of natural systems. Candidate solutions to the optimization problem play the role of individuals in a population, and the fitness function determines the quality of the solutions [7, 8]. The popular used nature inspired algorithms are Genetic algorithm, Hopfield Neural network, Ant colony optimization, Particle swarm optimization, Bee Colony Optimization, Differential Evolution, and so on. One of the recently developed nature inspired optimizations is cuckoo search [7] and it has shown its efficiency by the simulations and applications [9, 10].

In cuckoo search the Lévy flight was used to achieve good optimization performance [7-9]. Although the optimization performance of cuckoo search algorithm is good based on the Lévy flight some complicated mathematical operations should be used to realize the Lévy flight such as trigonometric function, gamma function and exponential functions, which limited the application of the cuckoo search algorithm especially there is high requirement about computation complex like the realization in the embedded systems. Hence the Lévy flight function can be replaced and some other techniques can be used to improve the optimization performance.

The remainder of this paper is structured as follows. Section 2 describes the preliminary knowledge about cuckoo search algorithm. The simplified cuckoo search algorithm is presented in Section 3. Section 4 furnishes a description of the experimental settings for the benchmarks and simulation results. The application of the proposed cuckoo search is presented in Section 5. Finally, Section 6 presents some concluding remarks.

## II. BRIEF DESCRIPTION OF CUCKOO SEARCH ALGORITHM

Cuckoo search (CS) is a powerful search algorithm that it inspired by the breeding behaviour of cuckoos which was developed by X Yang and S Deb in 2009 [7]. It was inspired by the obligate brood parasitism of some cuckoo species by laying their eggs in the nests of other host birds (of other species). Some host birds can engage direct conflict with the intruding cuckoos. For example, if a host bird discovers the eggs are not their own, it will either throw these alien eggs away or simply abandon its nest and build a new nest elsewhere. Some cuckoo species such as the New World brood-parasitic Tapera have evolved in such a way that female parasitic cuckoos are often very specialized in the mimicry in colors and pattern of the eggs of a few chosen host species [7, 11]. Cuckoo search idealized such breeding behavior, and thus can be applied for various optimization problems. It seems that it can outperform other metaheuristic algorithms in applications [7, 12].

Cuckoo search (CS) uses the following representations:

Each egg in a nest represents a solution, and a cuckoo egg represents a new solution. The aim is to use the new and potentially better solutions (cuckoos) to replace a not-so-good solution in the nests. In the simplest form, each nest has one egg. The algorithm can be extended to more complicated cases in which each nest has multiple eggs representing a set of solutions. [7]

CS is based on three idealized rules:

1) Each cuckoo lays one egg at a time, and dumps its egg in a randomly chosen nest;
2) The best nests with high quality of eggs will carry over to the next generation;
3) The number of available hosts nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a probability $p_a \in (0,1)$. Discovering operate on some set of worst nests, and discovered solutions dumped from farther calculations.

The pseudo-code can be summarized as:

```
Begin
  Generation t = 1;
  Initialized with random vector values,
  and initialize parameters;
  Evaluate fitness for every individual
  and determine the best (minimization)
  individual with the best objective
  value;
  While (stopping criterion is not met)
    Get a Cuckoo randomly by lévy flights
    Evaluate fitness for the cuckoo F
    Choose a nest among all the nests
    (here it is j) randomly
    If (Fi < Fj)
      Replace j by the new solution i;
    End if
    A fraction (pa) of worse nests are
    abandoned and new ones are built;
    Keep the best solution;
    Rank the solutions and find the
    current best.
    Update the generation number t = t +
    1
  End while
End.
```

Figure 1. The framework of the original CS.

The follows of the section are the relevant formula to realize the CS. In cuckoo search, each egg can be regarded as a solution and it is updated according to

$$x_i^{t+1} = x_i^t + \alpha \oplus Le'vy(\lambda) \tag{1}$$

where $\alpha > 0$ is the step size, $\oplus$ denotes entry-wise multiplications. A Lévy flight is a random walk where the step-lengths are distributed according to a heavy-tailed probability distribution:

$$Le'vy \sim u = t^{-\lambda}, (1 < \lambda < 3) \tag{2}$$

The implementation the Levy flights is realized by Mantegna's algorithm [19]:

$$Le'vy(\lambda) = \frac{\phi\mu}{|v|^{1/\beta}} \tag{3}$$

where $\beta$ ( $1 < \beta < 2$ ) is a constant; $\mu$ and $v$ are normally distributed pseudorandom numbers; and

$$\phi = \left( \frac{\Gamma(1+\beta) \times \sin(\frac{\pi\beta}{2})}{\Gamma\left(\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\frac{\beta-1}{2}}\right)} \right)^{1/\beta} . \tag{4}$$

Hence the update formula is

$$x_i^{t+1} = x_i^t + \alpha_0 \frac{\phi\mu}{|v|^{1/\beta}} \left( x_i^t - x_{best}^t \right) \tag{5}$$

where $x_{best}^t$ is the best solution at time t; $\alpha_0$ is a constant and usually chosen as 0.01.

The other part of cuckoo search is to create some nests by constructing a new solution shown as follows:

$$u_i = \begin{cases} x_i^t + rand(x_{rand1}^t - x_{rand2}^t), rand < p_a \\ x_i^t \qquad \text{otherwise} \end{cases} \tag{6}$$

After the new solution $u_i$ is created, $u_i$ will be evaluated and compared to $x_i^t$. If the objective value related to $u_i$ is better than the objective value related to $x_i^t$, the new solution $u_i$ will replace $x_i^t$ otherwise $x_i^t$ will be retained.

## III. SIMPLIFIED CUCKOO SEARCH ALGORITHM

As can be seen from (3)-(6), there are some complicated mathematical operations such as trigonometric function, gamma function and exponential functions, which limited the application of the cuckoo search algorithm especially there is high requirement about computation complex like the realization in the embedded systems. To reduce the computation complex, it is necessary to simplify the formula (3) or (5) while other formula are kept as the standard CS to keep the characteristics of the standard CS and it means the following stepsize should be considered.

$$stepsize = \alpha_0 \frac{\phi\mu}{|v|^{1/\beta}} \left( x_i^t - x_{best}^t \right) \tag{7}$$

Since $\frac{\phi\mu}{|v|^{1/\beta}}$ is a random number, it is logical to use a uniformly distributed number. However, a uniformly distribute number may cause the solutions to converge to a local optimum point which is called premature. But if a rand vector rand(D) is used, it can introduce too much random effect and reduce the effect of $\left(x_i^t - x_{best}^t\right)$ which affect the optimization performance (a lot of simulations have been done to get this conclusion). Hence the following formula is proposed:

$$stepsize = \text{rand}(1,1).\times \text{ceil}(\text{rand}(1,\text{D})\text{-}p_d).\times\left(x_i^t - x_{best}^t\right) \quad (8)$$

Here, rand(M,N) is a uniformly distribute M×N matrix, D is the dimension of the problem or the number of the problem variables, $p_d$ ( $0 < p_d \le 1$ )is a dimension chosen probability (here it is chosen a 0.2), ceil(A) rounds the elements of A to the nearest integers greater than or equal to A.

Similar with most of the intelligent optimization algorithms, CS cannot achieve a good local research performance at the beginning of the search process, which results in the particles being trapped into the local minima, particularly for the optimization problems with a large problem space and a large number of minima. It is possible to improve the optimization performance if some local searching techniques are used to search the local area around the current solutions during the whole searching process.

According to (5), CS works by iteratively searching in a region which has the tendency to the best previous success of solutions. Hence, it is more possible that the fitness value related to the current position is better than the random search. Here the golden ratio is used to determine the local search position, because the golden ratio has been widely used or demonstrated in many fields such as architecture [13,14], aesthetics [15], music [16], industrial design [17], and so forth. Moreover, the golden ratio is also used in optimization [18] and achieves good efficiency. Here, only one position is selected based on the golden ratio between the two solutions as shown in Fig. 2.

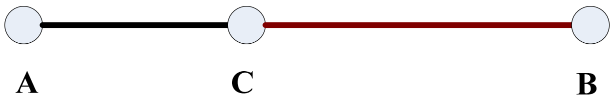


Figure 2. Local position determined by the golden ratio

In Fig. 2, A and B represent new solution and current solution. To realise the local search for current solution, position C is determined by the new solution and the golden ratio. Here,

$$\frac{\left|\overrightarrow{AC}\right|}{\left|\overrightarrow{CB}\right|} = 0.618\,. \quad (9)$$

Here, $|\bullet|$ is the vector norm function. To extend the search space, the opposition based method is used. The opposition concept has been used in evolution optimization algorithm and a good optimization performance was obtained [20]. In this paper, the method of using opposition concept is rotating C with $180^o$.

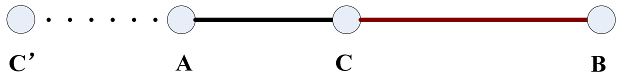


Figure 2 Local positions with opposition

To simplify the complexity of the proposed method, only two extra positions, which are C and C', are checked.

## IV. Comparisons of the simplified CS and the original CS

To demonstrate the efficiency of the proposed technique, several well-known benchmarks are used to compare the proposed method and standard CS (Matlab version) [19]. The parameters are set as standard CS (Matlab version) [19] and the CS was run for 50 trials per function with a maximum iteration of 1000 for every trial. The benchmarks are listed in Table I. Five well-known benchmark functions are chosen with big search range.

The statistical results of the standard CS and the proposed method are given in Table II. In Table II, it is evident that the proposed method can achieve better optimization performance than the standard CS.

TABLE I. FUNCTIONS USED TO TEST THE EFFECTS OF THE PROPOSED METHOD

| **Function Name** | **Formulation** | **Dimension** | **Variable Range** |
|---|---|---|---|
| Sphere function | $f_1(X) = \sum_{i=1}^{n} x_i^2$ | 20 | ±500 |
| Rastrigin function | $f_2(X) = \sum_{i=1}^{n}\left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$ | 20 | ±500 |
| Step function | $f_2(X) = \sum_{i=1}^{D}\lfloor x_i + 0.5\rfloor^2$ | 20 | ±500 |

| | | | |
|---|---|---|---|
| Alpine function | $f_4(x)=\sum_{i=1}^{n}\text{abs}\left[x_i * \sin(x_i)+0.1x_i\right]x_i^2$ | 20 | ±500 |
| Griewank function | $f_5(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2+\prod_{i=1}^{n}\cos(\frac{x_i}{\sqrt{i}})+1$ | 20 | ±500 |

TABLE II. RESULTS USING THE ORIGINAL CS AND THE PROPOSED METHOD

| Problem | Method | best | Mean | Worst | Std.dev |
|---|---|---|---|---|---|
| Sphere function $f_1$ | Original CS | 1.4688e-005 | 4.4824e-004 | 0.0163 | 0.0016 |
| | Proposed CS | **1.2116e-017** | **1.8867e-015** | **9.3211e-014** | **9.3901e-015** |
| Rastrigin function $f_2$ | Original CS | 51.0554 | 92.1159 | 199.5587 | 25.4259 |
| | Proposed CS | **13.7512** | **37.6850** | **65.8089** | **12.2080** |
| Step function | Original CS | **0** | 0.0400 | 1 | 0.1969 |
| | Proposed CS | **0** | **0** | **0** | **0** |
| Alpine function | Original CS | 86.7648 | 227.9363 | 565.9721 | 75.5206 |
| | Proposed CS | **9.7689** | **77.8136** | **201.5621** | **46.8437** |
| Griewank function | Original CS | 4.5566e-005 | 0.0263 | 0.1364 | 0.0251 |
| | Proposed CS | **0** | **0.0020** | **0.0153** | **0.0039** |

## V. SIMPLIFIED CUCKOO SEARCH BASED SPREAD SPECTRUM RADAR POLLY PHASE CODE DESIGN

When designing a radar-system that uses pulse compression, great attention must be given to the choice of the appropriate waveform. Many methods of radar pulse modulation that make pulse compression possible are known. Poly phase codes are attractive as the other lower side-lobes in the compressed signal and easier implementation of digital processing techniques. Later Dukic and Do-brosavljevic [21, 22] proposed a new method for poly phase pulse compression code synthesis, which is based on the properties of the aperiodic autocorrelation function and the assumption of coherent radar pulse processing in the receiver. The problem under consideration is modelled as a min–max nonlinear non-convex optimization problem in continuous variables and with numerous local optima. It can be expressed as follows [22]:

$$\text{global} \ \min_{x\in X} f(x)=\max\{\phi_1(x),\ldots,\phi_{2m}(x)\}$$

$$X=\{(x_1,\ldots,x_n)\in R^n \mid 0\le x_j\le 2\pi, j=1,\ldots,n\}$$

where $m = 2n$-1 and

$$\phi_{2i-1}(x)=\sum_{j=i}^{n}\cos\left(\sum_{k=|2i-j-1|+1}^{j}x_k\right),\quad i=1,\ldots,n$$

$$\phi_{2i}(x)=0.5+\sum_{j=i+1}^{n}\cos\left(\sum_{k=|2i-j|+1}^{j}x_k\right),\quad i=1,\ldots,n-1$$

$$\phi_{m+i}(x)=-\phi_i(x),\quad i=1,\ldots,m$$

Here the objective is to minimize the module of the biggest among the samples of the so called auto-correlation function which is related to the complex envelope of the compressed radar pulse at the optimal receiver output, while the variables represent symmetrized phase differences. This problem is NP-hard and the objective function is piecewise smooth. The statistical results of the standard CS and the proposed method are given in Table III. In Table III, it can be found that the proposed method can achieve better optimization performance than the standard CS.

TABLE III. RESULTS USING THE ORIGINAL CS AND THE PROPOSED METHOD

| Problem | Method | best | Mean | Worst | Std.dev |
|---|---|---|---|---|---|

| | | | | | |
|---|---|---|---|---|---|
| | | | | | |
| Application in radar-system | Original CS | 1.1393 | 1.3646 | 1.5918 | 0.0973 |
| | Proposed CS | **0.9789** | **1.1009** | **1.3583** | **0.0755** |

## VI. CONCLUSION

This study presented a simplified cuckoo search algorithm. There are no trigonometric function, gamma function and exponential functions in the simplified cuckoo search algorithm, which can reduces the computation complexity. To improve the local search ability, the golden ratio based technique was applied to the simplified CS. The simulation results showed that the simplified cuckoo search could achieve good optimization performance based on several famous benchmark optimization functions. Finally the proposed cuckoo search algorithm was applied to the spread spectrum radar Polly phase code design and showed the efficiency of the simplified cuckoo search algorithm.

## ACKNOWLEDGMENT

This work was supported by South African National Research Foundation Incentive Grants (No. 81705 and 95687). The author thanks the Matlab version codes of the standard CS by XS Yang [19].

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