International Journal of Pattern Recognition and Artificial Intelligence © World Scientific Publishing Company

Dynamic Small World Network Topology for Particle Swarm Optimization

Qingxue Liu

College of Electrical Engineering and Automation, Shandong University of Science and Technology, Qingdao, 266590, China Department of Electrical Engineering, Tshwane University of Technology, Pretoria, 0001, South Africa hmxue2000@163.com

Barend Jacobus van Wyk

Faculty of Engineering and Built Environment, Tshwane University of Technology, Pretoria, 0001, South Africa vanwykb@tut.ac.za

Shengzhi Du *

Department of Mechanical Engineering, Mechatronics, and Industrial Design, Tshwane University of Technology, Pretoria, 0001, South Africa dushengzhi@gmail.com

Yanxia Sun

Department of Electrical and Electronic Engineering Science, University of Johannesburg, Johannesburg, 2006, South Africa sunyanxia@qmail.com

A new particle optimization algorithm with dynamic topology is proposed based on a small world network. The technique imitates the dissemination of information in a small world network by dynamically updating the neighborhood topology of the particle swarm optimization(PSO). In comparison with other four classic topologies and two PSO algorithms based on small world network, the proposed dynamic neighborhood strategy is more effective in coordinating the exploration and exploitation ability of PSO. Simulations demonstrated that the convergence of the swarms is faster than its competitors. Meanwhile, the proposed method maintains population diversity and enhances the global search ability for a series of benchmark problems.

Keywords: Particle swarm optimization1; small world network2; dynamic neighborhood topology3; local model4

*Corresponding Author

1. Introduction

Intelligent optimization algorithms such as genetic algorithms, ant colony optimization and more recently Particle Swarm Optimization (PSO) algorithm became topical issue due to their robustness and efficiency^{25,39,40,42}.PSO algorithm, inspired by the behavior of birds flocking and fish schooling, is one form of artificial intelligence algorithm for finding optimal solutions to hard numerical functions. In a particle swarm optimizer, each individual (called particle) is representing a potential solution to the optimization problem and flies towards the optimal region of the high dimensional solution space by adjusting its trajectory^{1,2,3,4,12}. PSO techniques have been successfully applied in many science and engineering areas such as pattern recognition, signal processing, robot control, data clustering and so forth.

However, the basic PSO algorithm has some shortcomings in dealing complex problems. For instance, it is not sensitive to the change of environment, and premature convergence phenomenon often occurs because of the influences of the 'gbest' and 'pbest'. So many improved PSO algorithms were proposed by researchers. In 2002, an artificial immune network technique was first proposed to handle multimodal function optimization problems²⁵. Jiang and Bompmard proposed a hybrid chaotic PSO based on combining with linear interior point, which was applied to reactive power optimization areas²⁶. In the same year, Adaptive PSO was proposed and applied to solve the typical nonconves $optimization problems^{27}$. An efficient GA/PSO-hybrid method was proposed, the technique improved the diversity of PSO algorithm and decreased the computational costs of genetic algorithms²⁸. A niching PSO algorithm based on a ring neighborhood topology was proposed, which did not need any parameters²⁹. In recent years, hybrid intelligent algorithms became popular³², and a hybrid PSO was applied to estimate the Muskingum model parameters³⁰. After a while, a novel hybrid migration algorithm was used to resolve the multi-objective $problems^{31}$.

In the PSO, every particle has a number of neighbors, affecting each other⁷ by sharing specific information. Neighborhood topology reflects the mode of sharing information among particles, so the topology plays a critical role in the performance of the algorithm. In the early stages of PSO research, the topology commonly considered was the 'global best version' ('gbest')^{1,2}. Then the 'local best version' ('lbest')^{5,6,7,8} was proposed to deal with more complex engineering problems. The effects of various neighborhood topologies in the performance of PSO were investigated, such as ring topology, wheel topology, von Neumann topology, and so on. Suganthan proposed a number of improvements, such as gradually increasing the local neighborhood, time varying random walk, inertia weight values, and two alternative schemes for determining the local optimal solution for an individual⁹. However, this method introduced additional time consumption when calculating the distance.

Based on the 'Six Degrees of Separation' phenomenon^{14,21}, Watts and Strogatz¹⁰ proposed the concept of classic small world network model. It was shown that the

characteristics of the network connections influence the velocity of the information flowing^{6,11}. Small world networks have been observed in many real-world problems, such as data clustering, optimization of oil and gas field development planning, linear programming, reactive power optimization, computer science, networks of brain neurons, telecommunications, mechanics, and social influence networks^{17,18,19,20}.

In addition, Network-structured PSO combined with small world network(NSPSO) was applied to various benchmark functionsIn^{33,35,36}. In 2013, an improved PSO algorithm with adaptive small world topology (ASWPSO) was introduced, in which the parameters of topology was adaptively adjusted with the increasing of iteration³⁴. In [6] and [8], the impact of Von Neumann topology structure on PSO performance was considered. Based on this topology structure, an improved algorithm (SWPSO) was proposed by combining with the concept of small world^{37,38}. For the topology structure of SWPSO, in addition to four immediate neighbors, each particle has two random neighbors which were chosen from the population excluding the particles selected previously. However, the aforementioned small world topology algorithms were all suffering from a serious problem they take much time because of a large number of comparisons and computations among dimensions, especially for multi-dimensional functions.

Since the PSO was inspired by nature, this paper postulates that it is possible to enhance optimization performance if the small world network topology is considered in the particle swarm optimization process. A Dynamic Small World Network Topology PSO (DTSWPSO) is proposed, which imitates information disseminating in small world networks by dynamically adjusting the neighborhood topology. Moreover, a time varying neighborhood strategy can effectively coordinate the exploration and exploitation ability of the algorithm. The DTSWPSO is compared with classical topology versions and two dynamic small world neighborhood structure PSO algorithms.

The rest of the paper is organized as follows: In Section 2 and Section 3, the neighborhood topology and small world network are described. Section 4 discusses the proposed method and in Section 5 the experimental scheme is given. In Section 6, the comparative numerical simulation results are given, which are also discussed and analyzed. Finally, the conclusive remarks are given in Section 7.

2. Neighborhood Topology of PSO Algorithms

In PSO algorithms, each individual defines its trajectory according to its previous best solution and the optimal solution of some specific neighbors^{6,7,13}. Each individual considers the success of its neighbors to be a source of influence and ignores the others. So the neighborhood topology structure of the particle swarm plays an important role in the optimization performance, and the size of the neighborhood directly affects the performance of the algorithm as well.

At present, PSO has been studied in two general types of neighborhood structures, that is, global best (called 'gbest') and local best (called 'lbest')^{7,13}. Fig. 1(a)

is the 'gbest' neighborhood also known as 'all' topology, where any two individuals in the entire community are connected. This neighborhood topology structure is equivalent to a fully connected topology, in which each particle is attracted to the best position found by all the other members. All members of the whole can share information, and each individual chooses a new point for the next iteration according to the best success of the entire population⁵.

However, in the local best network, each particle is allowed to be influenced by only a small number of adjacent members. Kennedy and Mendes⁶ constructed and tested typical neighborhood configurations depicted in Fig. 1. The classic 'lbest' neighborhood topology is a ring lattice as shown in Fig. 1(b). In this local version PSO, each particle is connected to its two immediate neighbors only, located respectively on its left and right sides in the topological structure. In Fig. 1, (c) is the 'Star' (also called 'Wheel') neighborhood topology in which all particles are connected to the central individual⁵. Fig. 1(d) is the 'Pyramid' neighborhood topology, which is in fact a triangular wire-frame structure 13 . The 'Von Neumann' (also called 'square') neighborhood topology is showed in Fig. 1(e)), where each particle is connected to four neighbors, which makes it a three-dimensional torus. 'Four clusters' topology (Fig. 1(f)) has four cliques for a population of 20 particles, and in every clique, there are 5 fully connected individuals¹³. It was discovered that PSO with the 'Von Neumann' neighborhood topology performed better than other ones for a suite of standard test functions, such as Sphere function, Schwefel 2.21 and 2.22 functions, Rosenbrock function, Griewank function, Ackley function, Rstrigin function, and Shaffer's f6 function⁷.



Fig. 1. Neighborhood topologies: (a) Global best (All), (b) Ring, (c) Star (Wheel), (d) Pyramid, (e) Von Neumann (Square), (f) Four clusters

3. Small World Network

In the real world, the relationship between most things can be described by a network topology structure, such as biological, social and computer networks. However, in these networks, the connection patterns between individuals are neither purely regular nor purely random^{23,24}, which reveals non-trivial topological features. Complex networks were defined to model these relationships. The small world network is a commonly recognized complex network.

Inspired by the 'Six Degrees of Separation' phenomena¹⁴, the small world network¹⁰ was gradually noticed and became a hot research topic in complex system and complexity theory. Small world network is based upon the relationships in human society and is an intermediate form between a regular and a random network. The small world network has a small characteristic path length of the random lattice and relative highly clustering coefficient of the regular lattice¹⁰. In extreme cases, when p = 0, the original is the 'lbest' topology graph, and p = 1 for the 'gbest' topology graph. Four realizations of small world networks^{10,11,14} are shown in Fig. 2, where, in order to facilitate the description of the problem, the number of vertices is set to 12. The construction of a small world network is summarized as follows^{10,11}:

- 1. Start with a ring structure of n nodes as shown in Fig. 2(a).
- 2. Each node is connected to its nearest k neighbors by undirected edges, for instance, k = 4 in Fig. 2(b). On its left and right, there are two(k/2) directly connected neighbors, respectively.
- 3. Choose a node in a clockwise direction, with probability p, connect this vertex to a node uniformly chosen over the entire ring. Repeat this randomly choosing and connecting process for k times. Duplicate edges are forbidden, where another one is re-chosen(Fig. 2(c)).
- 4. Repeat the process 3 by moving clockwise around the ring. Consider each node in turn until the whole lap is completed (Fig. 2(d)).



Fig. 2. Construction process of small world network with p probability in sequence of (a)–(b)–(c)–(d)

4. Dynamic small world topology for PSO Algorithms

In PSO algorithms, each particle changes its velocity and direction according to the nodes (called neighborhoods) connected to it and decides which one in its neighborhood found the optimal solution so far⁵. Therefore, the neighborhood topology structure of population determines the breadth and extent of influences among the particles, and the quantity of the particle's neighbors determines the speed of information dissemination.

The 'gbest' neighborhood topology was discovered to converge fast⁶ because of the high interconnectedness. It has a strong global searching ability, yet it is easy to fall into a local optimum. On the other hand, in the PSO with the 'lbest' neighborhood topology, each particle explores its own searching space, so their information disseminates slowly along the neighborhood topology, and the information of success takes a long time to spread throughout the entire population¹³. So it has a strong local search ability, it also maintains the diversity of the population to a certain extent, and it is not easily trapped in a local optima.

According to the characteristics of the small world network^{10,11,15}, the randomness of the construction process determines the diversity of population, and the connections in an otherwise orderly network ensure the propagation of information throughout the entire neighborhood.

From the above discussion, it is not hard to imagine that if we introduce some characteristics of small world network to the PSO as the neighborhood topology, that the global search ability and convergence speed can be improved. The global search ability comes from the population diversity due the randomness, and convergence is ensured by the connections designed to avoid premature convergence.

On the other hand, the essential characteristics of the small world network indicate a high clustering coefficient and a small average path length^{10,11,14,15}. The clustering coefficient reflects the degree of aggregation of the nodes in a graph, so a high clustering coefficient of the small world network enables different neighbors to search only in their own exclusive neighborhood, which helps to enhance the local searching ability of the PSO algorithm. Path length is defined as the shortest path between two nodes, which is a measure of the efficiency of information transport in a network¹⁰, therefore, a small average path length increases the speed of information transmission between particles. Meanwhile, the efficiency of small world network neighborhood topology can improve the global search ability of the proposed PSO algorithm.

To balance the exploration and exploitation abilities of the PSO algorithm, a Dynamic Topology Small World network (DTSWPSO) PSO is proposed. In the proposed small world topology, each node represents one particle respectively, and all particles are connected according to the small world topology. The neighborhood topology of this algorithm changes gradually by adjusting the probability p for randomly building edges between two particles after every fixed number of iterations⁴¹. The specific adjustment procedure of this dynamic topology is shown in Table 1. In

Table 1, the 'max_iteration' represents the maximum number of iterations to be considered. The probability p is decreased linearly after certain iterations. In the early stage of iteration, the neighborhood topology resembles the 'gbest' structure, and then it is gradually decreased, so that at last, it becomes the 'lgbest' structure (i.e. a 'ring' topology). The detailed procedure of the proposed algorithm is summarized as follows:

Table 1. The value of p on different stage of evolution, where 'max_iteration' stands for the maximum number of iteration.

Iteration numbers	The value of p
$[1, max_iteration/10]$	0.9
$[(max_iteration/10) + 1, \ 2max_iteration/10]$	0.8
$[8(max_iteration/10) + 1, 9max_iteration/10]$	0.1
$[9(max_iteration/10) + 1, 1]$	0

- 1. Generate the adjacency matrix of the small world topology of DTSWPSO according to the construction mode of small world in section 3. The adjacency matrix is a square matrix which indicates the connection relationship of the small world topology structure, it is able to indicate whether pairs of nodes are adjacent or not in the topology.
- 2. Randomly initialize the population X_i and the velocity V_i of each particle particle i.
- 3. According to the iterative formulas (1) and (2), update the velocity V_i and the position X_i . For each particle x_i and its velocity V_i , if they exceed the boundary of the variable range, their values are reset to the boundary.
- 4. Calculate and evaluate the fitness of each particles according to the optimization function.
- 5. Evaluate the P_i (Individual optimal), P_l (Local optimal) and P_g (Global optimal) according to the generated adjacency matrix, and use the adjacency matrix to calculate the position of the corresponding particle which are 'pbest', 'lbest' and 'gbest'.
- 6. Update the adjacency matrix. Decrease the edges in the small world with the probability p after a certain number of iterations. The detailed regulation of p with the increase iteration is shown in Table 1.
- 7. Repeat the process 3), 4), 5), and 6) until 'max_iteration' is reached or the solution is mature.

5. Numerical Simulation

The simulation experiments were performed using an Intel Corei5 platform with 8GB memory. The computational model was implemented using Windows10 and

Mtlab R2014a.

In order to test the optimization performance of the proposed method, we compared with other four typical topologies: 'star (wheel)', 'ring', 'Von Neumann (square)', as well as 'four clusters' and other two samll world topology algorithms. Fifteen commonly used benchmark optimization problems were considered in this comparison. These test functions include Sphere function (f_1) , Schwefel 1.2 function (f_2) , Schwefel 2.21 function (f_3) , Schwefel 2.22 function (f_4) , Step function (f_5) , Quartic function with noise in fitness (f_6) , Rosenbrock function (f_7) , Griewank function (f_8) , Ackley function (f_9) , Rastrigin function (f_{10}) , Stretched V sine wave function (f_{11}) , Schaffer's f6 function (f_{12}) , Six-Hump Camel-Back function (f_{13}) , Branin (f_{14}) , and Goldstein-Price function (f_{15}) , where f_1 - f_7 are unimodal functions, the rest are multimodal functions. For the multimodal functions, they have a great number of local optima which are usually difficult to search. In these test functions, all functions were run in 30 dimensions, except for the last four functions $(f_{12}-f_{15})$ which are 2 dimensions, and 10 dimensions were considered for Griewank function, since it is harder to find the global optimum with a decrease of dimension for this function. The parameters and criteria of the eight standard test functions are given in Table 2.

Table 2. Parameters and criteria for the eight test functions conditions.

Function	Dimensions	Domain	Optimum	Criterion	Number of iteration
Sphere (f_1)	30	[-100, 100]	0	0.01	1000
Schwefel $1.2(f_2)$	30	[-100, 100]	0	100	8000
Schwefel $2.21(f_3)$	30	[-100, 100]	0	0.1	5000
Schwefel $2.22(f_4)$	30	[-10, 10]	0	0.1	2000
$\operatorname{Step}(f_5)$	30	[-100, 100]	0	0.1	2000
Quartic with noise in fitness (f_6)	30	[-1.28, 1.28]	0	0.1	1000
$\operatorname{Rosenbrock}(f_7)$	30	[-30, 30]	0	100	2000
$\operatorname{Griewank}(f_8)$	10	[-600, 600]	0	0.05	1000
$\operatorname{Griewank}(f_8)$	30	[-600, 600]	0	0.05	1000
$Ackley(f_9)$	30	[-30, 30]	0	100	1000
$\operatorname{Rastrigin}(f_{10})$	30	[-5.12, 5.12]	0	100	1000
Stretched V sine wave (f_{11})	30	[-10, 10]	0	100	1000
Schaffer's $f6(f_{12})$	2	[-100, 100]	0	0.00001	1000
Six-Hump Camel-Back (f_{13})	2	[-5, 5]	-1.0316285	0.00001	500
$\operatorname{Branin}(f_{14})$	2	$[-5, 10] \times [0, 15]$	0.398	0.00001	500
Goldstein-Price (f_{15})	2	[-2, 2]	3	0.00001	500

Since the fifteen benchmark functions and the four topologies are standard^{22,23,24}, we haven't included more details which are available in the literature¹⁶. The four typical topologies for PSO algorithms, the two small world topology algorithms^{33,34} and the proposed algorithm are applied with a population size of 20, and the maximum number of iterations on each benchmark problem is listed in Table 2. Every algorithm runs 20 times. The position and the velocity

of each particle are initialized with random values. The iterative formulas are as follows:

$$V_{id}^{t+1} = \omega V_{id}^t + c_1 r_1 (p_{id} - X_{id}^t) + c_2 r_2 (p_{ld} - X_{id}^t) + c_3 r_3 (p_{gd} - X_{id}^t), \quad (1)$$

$$X_{id}^{t+1} = X_{id}^t + V_{id}^{t+1}, (2)$$

Here, V_{id} and X_{id} represent the velocity vector and the position vector (solution to the optimization problem) respectively of particle *i* in the d^{th} dimension. The index *t* is the iteration number. P_{id} is the best position of particle *i* found by itself in the searching space. P_{ld} refers to the best solution found so far by its neighbors, and P_{gd} is the best solution among all particles. ω is the inertia weight by which the memory of previous velocities can be retained. c_1 , c_2 and c_3 are the positive constant parameters. r_1 , r_2 and r_3 are three uniform random weights ranging on [0, 1] ensuring the random particle trajectory.

In the simulation experiments, for each function, the inertia weight $\omega = 0.729^{12}$. The parameter values $\{c_1, c_2, c_3\}$ were set to $\{1, 1, 1\}$ for the functions f_1 , f_2 and f_7 , $\{1.5, 1.5, 0\}$ for the functions f_3 - f_6 and f_9 - f_{15} , and $\{1.5, 1, 0.5\}$ for the function f_8 .

The parameter settings of the two small world topology algorithms are the same to the corresponding references [29] and [30].

6. Results and Discussion

Table 3-18 shows a comparison of algorithms using seven neighborhood topologies on each benchmark problem, where 'Best', 'Mean Best' and 'Worst' represent best values, mean best values and worst values searched over 20 runs respectively. 'Std Dev' indicates the standard deviation, and 'Time (s)' denotes the computation time in seconds. The solutions obtained by the proposed method(DTSWPSO) are better than the other six algorithms. However, the DTSWPSO is computationally more expensive than the other four classic topology algorithms as the adjacency matrix of population needs to be dynamically adjusted in the searching process. On the other hand, in contrast to the other two small world topology algorithms, our proposed method took the least time in the same number of iterations since it had no complex computation of dimensions.

As can be seen from Table 3-18, the DTSWPSO technique gives comparable or better performance than the other six algorithms. Specifically, for Sphere function, Schwefel 2.21 function, Step function, Quartic function with noise in fitness, Griewank function with 10 dimensions, Rastrigin function, and Stretched V sine wave function, the proposed technique performed better than other six algorithms with different topologies. However, the 'NSPSO' algorithm is better than the other six algorithms on Schwefel 2.22 function and Rosenbrock function. The 'Ring' and the 'Von Neumann' topology perform better than other five topologies on Schwefel

Table 3. Comparison of seven neighborhood topologies algorithms on f_1 .

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	9.37E - 5	2.50E + 3	4.33E + 3	1.00E + 4	16.88
Ring	1.33E - 6	9.92E - 6	1.02E - 6	5.81E - 5	17.41
Von Neumann	2.22E - 9	3.77E - 8	4.03E - 8	1.77E - 7	20.14
Four Clusters	4.02E - 9	1.20E - 8	2.11E - 8	9.31E - 8	20.28
ASWPSO	9.13E - 1	2.85E + 0	1.61E + 0	7.02E + 0	112.20
NSPSO	5.27E - 15	5.67E - 10	1.67E - 9	7.60E - 9	67.19
DTSW	$1.31\mathrm{E}-17$	$\mathbf{3.91E}-11$	1.03 E-10	$\mathbf{4.48E}-10$	27.83

Table 4. Comparison of seven neighborhood topologies algorithms on f_2 .

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	2.62E - 5	3.25E + 3	4.26E + 3	1.50E + 4	145.09
Ring	5.79E - 4	$5.27\mathrm{E}-3$	$5.61\mathrm{E}-3$	$\mathbf{2.03E} - 2$	183.94
Von Neumann	2.01E - 8	2.00E + 3	4.30E + 3	1.50E + 3	156.57
Four Clusters	3.94E - 7	1.75E + 3	2.94E + 3	1.00E + 4	151.05
ASWPSO	4.56E + 2	4.37E + 3	3.03E + 3	1.49E + 4	1076.86
NSPSO	$\mathbf{2.25E} - 25$	1.08E + 3	2.85E + 3	5.17E + 3	552.81
DTSW	2.89E - 20	2.08E + 3	3.02E + 3	1.00E + 4	162.92

Table 5. Comparison of seven neighborhood topologies algorithms on f_3 .

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	5.68E - 1	6.04E + 0	3.44E + 0	1.36E + 1	101.14
Ring	5.01E - 2	2.33E - 1	4.97E + 3	1.58E - 1	92.16
Von Neumann	1.71E - 3	1.14E - 2	1.19E - 2	4.40E - 2	71.88
Four Clusters	5.52E - 3	5.83E - 2	8.58E - 2	3.78E - 1	64.19
ASWPSO	1.49E + 1	2.35E + 1	5.19E + 0	3.21E + 1	528.94
NSPSO	1.98E + 0	6.16E + 0	4.09E + 0	1.94E + 1	333.52
DTSW	$7.05\mathrm{E}-4$	$1.10\mathrm{E}-2$	$9.14\mathrm{E}-3$	$3.14\mathrm{E}-2$	83.49

1.2, Griewank function, and Ackley function. For the Rosenbrock function, there is a narrow valley between the local optimum and the global optimum. It is difficult to distinguish the global optimum for algorithms with larger neighborhood size. For the Griewank function, with increase of the dimension, the range of local optimum is getting narrower, which makes it easier to find the global optimum. The 'ring' and the 'Von Neumann' topologies obviously show good performance because of their smaller neighborhood sizes. The DTSWPSO algorithm has a relatively large neighborhood in the early searching stage, so its performance is not always optimal. For Schaffer's f6 function, Six-Hump Camel-Back function, Branin function, and Goldstein-Price function, there is no statistically significant difference among all seven topologies. The main reason is that the four optimization functions are simple 2D functions, and it is easy to find the optimal solutions by any the seven

Topology	Best	Mean Best	Std Dev	Worst	$\operatorname{Time}(s)$
Star	8.50E - 5	7.11E + 0	7.06E + 0	2.00E + 1	37.88
Ring	5.51E - 9	3.00E + 0	5.57E + 0	2.00E + 1	46.51
Von Neumann	3.63E - 12	2.00E + 0	6.78E + 0	3.00E + 1	43.28
Four Clusters	3.55E - 12	2.50E + 0	4.33E + 0	2.00E + 1	48.49
ASWPSO	2.60E - 4	$1.50\mathrm{E}-3$	1.23E - 3	4.33E - 3	202.36
NSPSO	8.62E - 8	3.20E + 1	4.35E + 1	1.19E + 2	116.95
DTSW	$1.18\mathrm{E}-13$	3.50E + 0	5.72E + 0	2.00E + 1	45.94

Table 6. Comparison of seven neighborhood topologies algorithms on f_4 .

Table 7. Comparison of seven neighborhood topologies algorithms on f_5 .

Topology	Best	Mean Best	Std Dev	Worst	$\operatorname{Time}(s)$
Star	0	1.07E + 3	3.99E + 3	1.01E + 4	45.27
Ring	0	$\mathbf{2.50E} - 1$	8.87E - 1	4.00E + 0	44.91
Von Neumann	0	1.75E + 1	5.41E + 1	2.50E + 1	51.38
Four Clusters	3.94E - 7	3.30E + 1	1.42E + 1	6.50E + 1	51.13
ASWPSO	0	8.00E - 1	1.23E + 0	4.00E + 0	209.29
NSPSO	6.00E + 0	5.96E + 1	6.34E + 1	2.46E + 2	117.02
DTSW	0	6.60E - 1	$\mathbf{1.20E} + 0$	$\mathbf{4.00E} + 0$	43.27

Table 8. Comparison of seven neighborhood topologies algorithms on f_6 .

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	2.38E + 1	3.67E + 1	8.13E + 0	5.50E + 1	47.90
Ring	1.33E + 1	1.98E + 1	6.16E + 0	3.86E + 1	46.62
Von Neumann	7.60E + 0	1.73E + 1	7.64E + 0	3.35E + 1	48.40
Four Clusters	8.05E + 0	2.08E + 1	9.27E + 0	4.46E + 1	42.29
ASWPSO	6.96E + 0	1.79E + 1	7.21E + 0	3.16E + 1	146.01
NSPSO	1.37E - 1	3.74E - 1	2.09E - 1	1.13E + 0	70.46
DTSW	$\mathbf{2.21E} - 2$	$4.17 \mathrm{E} - 2$	$1.44\mathrm{E}-2$	$6.96\mathrm{E}-2$	48.84

algorithms.

Success rate is one the most important parameters to evaluate the performance of an algorithm. It denotes the number of successful optimal hits for an algorithm in 100 runs, which is used to measure global searching ability of an algorithm, and the success rates of different neighborhood topologies on fifteen benchmark functions over 20 runs are shown in table 19. For the functions f_1 , f_3 , f_6 , f_7 , and f_{12} , the success rate of our proposed algorithm is higher than other six algorithms. All the seven algorithms are able to achieve one hundred percent success rate for the function f_9 , f_{11} , f_{13} - f_{15} . For f_2 , f_5 , and f_8 , the 'ring' and 'Von Neumann' topologies perform slightly better than other five topologies.

In order to further compare these algorithms, the evolutionary curves were presented on each test function as shown in Fig. 3-5. It can be seen that the DTSWPSO

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Topology	Best	Mean Best	Std Dev	Worst	$\operatorname{Time}(s)$
Star	1.71E + 1	4.18E + 2	8.95E + 2	3.16E + 3	57.12
Ring	6.54E - 1	8.28E + 1	6.10E + 1	3.02E + 2	47.67
Von Neumann	6.39E + 1	1.05E + 2	1.11E + 2	5.55E + 2	51.82
Four Clusters	6.26E + 1	3.89E + 2	8.95E + 2	3.09E + 3	43.86
ASWPSO	6.21E + 1	6.36E + 2	1.60E + 2	6.48E + 2	199.89
NSPSO	3.01E - 1	$\mathbf{2.33E} + 1$	$\mathbf{2.69E} + 1$	$1.09 \mathbf{E} + 2$	121.65
DTSW	$7.80\mathrm{E}-3$	4.55E + 3	1.96E + 4	9.00E + 4	51.24

Table 10. Comparison of seven neighborhood topologies algorithms on f_8 with 10 dimensions.

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	2.46E - 2	1.03E - 1	4.86E - 2	2.34E - 1	25.18
Ring	1.19E - 2	$4.77 \mathrm{E} - 2$	2.86E - 2	1.53E - 1	22.60
Von Neumann	1.23E - 2	5.06E - 2	$2.66 \mathrm{E} - 2$	1.68E - 1	26.59
Four Clusters	1.72E - 2	7.96E - 2	4.50E - 2	1.77E - 1	23.86
ASWPSO	2.41E - 2	1.79E - 1	1.19E - 1	4.76E - 1	91.34
NSPSO	4.92E - 2	1.11E - 1	8.29E - 2	4.13E - 1	68.77
DTSW	$1.13\mathrm{E}-2$	7.66E - 2	3.25E - 2	$1.50\mathrm{E}-1$	28.74

Table 11. Comparison of seven neighborhood topologies algorithms on f_8 with 30 dimensions.

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	4.34E - 3	4.74E + 0	1.96E + 1	9.02E + 2	29.01
Ring	6.15E - 6	$1.07\mathrm{E}-2$	1.33E-2	$5.26\mathrm{E}-2$	26.41
Von Neumann	1.15E - 8	1.30E - 2	2.48E - 2	1.10E - 1	19.25
Four Clusters	2.89E - 8	1.57E - 2	2.43E - 2	9.11E - 2	20.22
ASWPSO	8.42E - 1	9.93E - 1	6.16E - 2	1.08E + 0	124.17
NSPSO	1.70E - 11	4.46E - 2	4.67E - 2	1.88E - 1	68.17
DTSW	0	7.86E - 2	1.42E - 1	6.31E - 1	29.90

algorithm converges faster than the other six algorithms, especially on multimodal functions. Because it is easy to find the optimum solutions for unimodal function, the more simple a topology structure is, the faster it converges.

In summary, during the earlier searching stage of DTSWPSO, the value of p is large, and the neighborhood population is relatively large, hence it behaves more like the 'gbest' topology version, and has a high searching speed. Moreover, due to the randomness and the rapid information dissemination ability by dynamically adjusting the probabiligy p for updating the small world network, the proposed method maintains the diversity of the population and improves the local searching ability.

Std Dev Topology Best Mean Best Worst Time(s) Star 1.90E + 08.13E + 04.44E + 01.49E + 121.75 $\mathbf{2.39E} - \mathbf{3}$ Ring 3.19E - 47.69E - 31.71E - 324.104.07E - 51.28E + 01.40E + 1Von Neumann 2.98E + 023.15Four Clusters 3.78E - 51.79E + 04.08E + 01.41E + 127.49ASWPSO 1.10E + 02.32E + 05.99E - 13.30E + 0137.65 NSPSO 1.90E + 04.53E + 06.61E + 07.00E + 095.42DTSW 3.63E - 54.33E + 05.67E + 01.45E + 024.98

Table 12. Comparison of seven neighborhood topologies algorithms on f_9 .

Table 13. Comparison of seven neighborhood topologies algorithms on f_{10} .

Topology	Best	Mean Best	Std Dev	Worst	$\operatorname{Time}(s)$
Star	7.56E + 1	1.34E + 2	3.22E + 1	1.81E + 2	24.17
Ring	8.86E + 1	1.36E + 2	2.54E + 1	1.75E + 2	19.29
Von Neumann	6.87E + 1	1.22E + 2	3.13E + 1	1.89E + 2	18.82
Four Clusters	6.77E + 1	1.32E + 2	3.23E + 1	1.92E + 2	17.23
ASWPSO	8.05E + 1	1.53E + 2	4.98E + 1	2.42E + 2	125.80
NSPSO	3.98E + 1	7.37E + 1	$1.95 \mathrm{E} + 1$	1.04E + 2	78.88
DTSW	$\mathbf{3.48E} + 1$	$\mathbf{5.89E} + 1$	2.26E + 1	$\mathbf{1.03E+2}$	24.66

Table 14. Comparison of seven neighborhood topologies algorithms on f_{11} .

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	2.38E + 1	3.67E + 1	8.13E + 1	5.50E + 1	47.90
Ring	1.33E + 1	1.97E + 1	6.16E + 0	3.86E + 1	46.62
Von Neumann	7.60E + 0	1.73E + 1	7.64E + 0	$\mathbf{3.35E} + 1$	48.40
Four Clusters	8.05E + 0	2.08E + 1	9.27E + 10	4.46E + 1	42.29
ASWPSO	2.55E + 1	3.39E + 1	6.53E + 1	4.53E + 1	121.60
NSPSO	2.68E + 1	4.02E + 1	9.63E + 1	6.06E + 1	79.07
DTSW	$7.57 \mathrm{E} + 0$	$1.49\mathrm{E} + 1$	$6.08 \mathrm{E} + 0$	3.54E + 1	50.37

7. Conclusion

This paper proposed a new dynamic neighborhood topology structure for PSO algorithms. According to the way the small world network is generated, the local neighborhood topology decreases gradually by adjusting the probability p with increasing iterations. The simulation results of fifteen typical test functions demonstrated that the new method maintains the diversity of population, balances the exploration and exploitation ability, and ensures the convergence of the particle swarm. Consequently, the proposed technique improves the practicality and effectiveness of PSO. Because of the randomness of the small world network, the dynamic neighborhood topologies should have many adjustment modes, which will be studied in future work.

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	0	3.89E - 3	4.76E - 3	9.72E - 3	22.52
Ring	0	5.35E - 3	4.83E - 3	9.72E - 3	23.51
Von Neumann	0	1.46E - 3	4.47E - 3	9.72E - 3	23.12
Four Clusters	0	3.11E - 3	4.40E - 3	9.72E - 3	20.62
ASWPSO	0	1.00E - 3	2.91E - 3	9.72E - 3	38.31
NSPSO	0	7.77E - 3	3.89E - 3	9.72E - 3	60.31
DTSW	0	$4.85 \mathrm{E} - 4$	$\mathbf{2.23E} - 3$	$9.72 \mathrm{E} - 3$	21.86

Table 15. Comparison of seven neighborhood topologies algorithms on f_{12} .

Table 16. Comparison of seven neighborhood topologies algorithms on f_{13} .

Topology	Best	Mean Best	Std Dev	Worst	$\operatorname{Time}(s)$
Star	-1.03E + 0	-1.03E + 0	2.11E - 16	-1.03E + 0	9.86
Ring	-1.03E + 0	-1.03E + 0	2.11E - 16	-1.03E + 0	9.27
Von Neumann	-1.03E + 0	-1.03E + 0	2.11E - 16	-1.03E + 0	9.65
Four Clusters	-1.03E + 0	-1.03E + 0	2.11E - 16	-1.03E + 0	9.25
ASWPSO	-1.03E + 0	-1.03E + 0	7.60E - 10	-1.03E + 0	14.39
NSPSO	-1.03E + 0	-1.03E + 0	1.98E - 16	-1.03E + 0	39.21
DTSW	$-\mathbf{1.03E}+0$	$-1.03\mathbf{E} + 0$	$1.98\mathrm{E}-16$	$-\mathbf{1.03E}+0$	11.66

Table 17. Comparison of seven neighborhood topologies algorithms on f_{14} .

Topology	Best	Mean Best	Std Dev	Worst	Time(s)
Star	3.98E - 1	3.98E - 1	0	3.98E - 1	9.18
Ring	3.98E - 1	3.98E - 1	0	3.98E - 1	10.82
Von Neumann	3.98E - 1	3.98E - 1	0	3.98E - 1	10.88
Four Clusters	3.98E - 1	3.98E - 1	0	3.98E - 1	10.85
ASWPSO	3.98E - 1	3.98E - 1	1.97E - 9	3.98E - 1	16.21
NSPSO	3.98E - 1	3.98E - 1	0	3.98E - 1	45.66
DTSW	$3.98\mathrm{E}-1$	$3.98\mathrm{E}-1$	0	$3.98\mathrm{E}-1$	12.39

Acknowledgments

This work was supported by the Natural Science Foundation of Shandong Province, China (Grant no.ER2012EM021), and the Promotive Research Fund for Excellent Young and Middle-aged Scientists of Shandong Province, China (Grant no.BS2015DX010).

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Topology	Best	Mean Best	Std Dev	Worst	$\operatorname{Time}(s)$
Star	3.00E + 0	3.00E + 0	1.43E - 15	3.00E + 0	10.38
Ring	3.00E + 0	3.00E + 0	1.80E - 15	3.00E + 0	9.95
Von Neumann	3.00E + 0	3.00E + 0	1.41E - 15	3.00E + 0	11.64
Four Clusters	3.00E + 0	3.00E + 0	1.43E - 15	3.00E + 0	10.38
ASWPSO	3.00E + 0	3.00E + 0	1.43E - 15	3.00E + 0	13.94
NSPSO	3.00E + 0	3.00E + 0	1.54E - 15	3.00E + 0	30.64
DTSW	$\mathbf{3.00E} + 0$	$\mathbf{3.00E} + 0$	$1.40\mathrm{E}-15$	$\mathbf{3.00E} + 0$	13.69

Table 18. Comparison of seven neighborhood topologies algorithms on f_{15} .

Table 19. Success rate of different neighborhood topologies on fifteen benchmark functions.

Function	Star	Ring	Von Neumann	Four clusters	ASWPSO	NSPSO	DTSW
$Sphere(f_1)$	55	100	100	100	100	100	100
Schwefel $1.2(f_2)$	55	100	80	70	0	85	95
Schwefel $2.21(f_3)$	0	15	100	80	0	0	100
Schwefel $2.22(f_4)$	40	80	80	75	100	40	80
$\operatorname{Step}(f_5)$	5	90	60	90	50	0	70
Quartic with noise in fitness (f_6)	10	100	95	95	0	5	100
$\operatorname{Rosenbrock}(f_7)$	50	80	75	75	25	90	90
$\operatorname{Griewank}(10)(f_8)$	15	35	45	30	10	10	35
$\operatorname{Griewank}(30)(f_8)$	35	90	95	85	0	65	70
$\operatorname{Ackley}(f_9)$	100	100	100	100	100	100	100
$\operatorname{Rastrigin}(f_{10})$	20	15	35	15	10	95	85
Stretched V sine wave (f_{11})	100	100	100	100	100	100	100
Schaffer's $f6(f_{12})$	60	40	85	65	85	20	95
Six-Hump Camel-Back (f_{13})	100	100	100	100	100	100	100
$\operatorname{Branin}(f_{14})$	100	100	100	100	100	100	100
Goldstein-Price (f_{15})	100	100	100	100	100	100	100

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Fig. 3. Time evaluation of seven neighborhood topologies on f_1 - f_4

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Fig. 4. Time evaluation of seven neighborhood topologies on f_5 - f_9

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Fig. 5. Time evaluation of seven neighborhood topologies on f_{10} - f_{15}

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Biographical Sketch and Photo



Qingxue Liu obtained a B.Sc. degree in Automatic Engineering from Naval Aviation Engineering Academy, China, in 2002, and a M.Sc. degree in Control Theory and Control Engineering from Xi'an University of Technology,

China, in 2005. Currently, he is pursuing his D-Tech degree in Electrical Engineering, Tshwane University of Technology, South Africa. His research interests include evolutionary optimization, complex network and electric power system.



Barend Jacobus van Wyk is the executive dean of the Faculty of Engineering and the Built Environment at the Tshwane University of Technology (TUT). He obtained a PhD in Electrical and Information Engineering from Witwatersrand is regis

the University of the Witwatersrand, is registered as a professional engineer, is a member of the IEEE, and a council member of Engineering Council of South Africa (ECSA). He has a passion for making technology accessible and understandable. He has supervised and co-supervised more than 30 post graduate students, is a C2 rated researcher and has published more than a 100 peer reviewed conference/journal publications. His research interests include engineering education, image processing, machine intelligence, control and signal processing.



Shengzhi Du is a professor of the Mechanical Engineering, Mechatronics, and Industrial Design Department, Tshwane University of Technology (TUT). He obtained a PhD in Control Theory and Control Engineering from

Nankai University, China. His research interests include Pattern Recognition, Image Processing, Machine Intelligence, Control Systems, Network Behaviour Analysis, Brain Computer Interfaces.



Yanxia Sun got her D-Tech in Electrical Engineering, Tshwane University of Technology, South Africa and PhD in Computer Science, University Paris-EST, France in 2012. Currently she is serving University of Johannes-

burg as an Associate Professor, Johannesburg, South Africa. Her research interests are evolutionary optimization, artificial intelligence and nonlinear dynamics.