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Modelling of Thermo-Acoustic Refrigerators Using General Algebraic Modelling System

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ABSTRACT

While thermo-acoustic refrigeration is new among emerging technology, it shows strong potential towards the development of sustainable and renewable energy systems by utilising a sound wave to remove the heat. This work discusses a new mathematical programming approach that provides fast initial engineering estimates to initial design calculations, describing the optimal geometry of thermo-acoustic refrigerators. Three different criteria describing their performances were taken into account: maximum cooling, best coefficient of performance and acoustic power loss. As the stack has been identified as the heart of the device where heat transfer takes place, this new approach aims to optimise its geometrical parameters: namely, the stack position, the stack length, the blockage ratio and the stack pore sizes. Hence, the optimisation task is formulated as a three-criterion nonlinear programming problem with discontinuous derivatives. This approach was implemented in the General Algebraic Modelling Systems. The unique characteristic of this research is the computation of all efficient optimal solutions, allowing the decision maker to identify the most efficient solution. The proposed modelling approach was investigated experimentally to evaluate its ability to predict the best parameters describing the geometry of the stack. Similar trends were obtained to support the use of the proposed approach in the design of thermo-acoustic refrigerators.

KEYWORDS

Thermo-acoustic refrigerator, optimisation, modelling, stack geometry, GAMS, AUGMENCON

INTRODUCTION

In order to address current environmental problems [1], thermo-acoustic refrigerators (TARs) have been proposed as a solution to the current search for alternative refrigerants and alternative technologies for refrigeration (such as absorption refrigeration, thermoelectric refrigeration and pulse-tube refrigeration). Thermo-acoustic refrigerators (see Figure 1) consist mainly of a vibrating diaphragm (a loudspeaker) attached to a resonator filled with gas, a stack usually constructed of thin parallel plates, and two heat exchangers placed at either side of the stack. The heat pumping process occurs within the stack which forms the heart of the refrigerator, and thus is an important element determining the performance of the refrigerator [2].

In a typical standing wave thermo-acoustic refrigerator, an acoustic wave is generated using a sound source such as a loudspeaker to make the gas resonant. The gas within the resonator tube oscillates. This standing sound wave creates a temperature difference along the length of the stack. This temperature change is a result of the compression and expansion the gas undergoes because of the sound pressure and thermal interaction between the surface of the plate and the oscillating gas. Heat is exchanged with the surrounding through the hot and cold heat exchangers located on each side of the stack [3]. The basic mechanics behind thermo-acoustics are already well-understood. Detailed studies on the working of thermo-acoustic coolers have been conducted by Swift [3] and Wheatly *et al*. [4]. Recent research focuses on improving the performance of the designing of the devices to allow thermo-acoustic coolers to compete with commercial refrigerators.

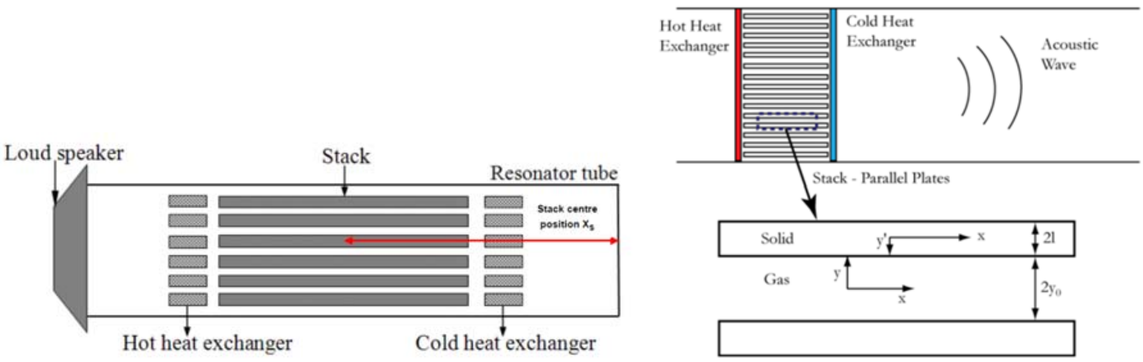

Figure 1: Schematic diagram of a typical thermo-acoustic refrigerator

OPTIMISATION OF THERMO-ACOUSTIC REFRIGERATORS

Mathematical optimisation, defined as the selection of the best element among some set of available alternatives with regard to some criteria, consists of maximising or minimising a function of one or more variables. *Engineering optimisation,* using optimisation techniques to achieve design goals, can be described as the process of searching for the conditions that give maximum or minimum value of a function [5]. The optimisation criterion are normally formulated in mathematical form, as an objective function. This paper addresses the application of mathematical programming techniques suitable for the solution of engineering design problems.

Previous studies have highlighted various parameters affecting the performance of the thermo-acoustic refrigerators (TARs). For example, geometrical parameters and fluid properties are considered in the study reported by Minner *et al.* [6] in order to optimise the coefficient of performance of a TAR. A locally optimal solution is obtained with the Nelder-Mead simplex method. The optimisation of the inertance sections of thermo-acoustic devices is demonstrated by Zoontjens *et al*. [7]. Both Minner *et al*. [6] and Zoontjens *et al*. [7] have in common models that rely extensively upon DELTAE, a black box simulation tool based on linear acoustic theory, initially developed by Swift *et al.* [3]. The evaluation of certain engine parameters affecting the pressures amplitude was offered by Ueda *et al.* [8]. Tijani *et al.* [9] have investigated the influence of the stack spacing with DELTAE. Herman and Travnicek [10] have discussed a systematic design approach that provides fast engineering estimates for initial design calculations of TARs, with results suggesting that sets of parameters leading to two seemingly similar outcomes – maximum efficiency and maximum cooling – are not the same.

More recently, experimental studies conducted by Tartibu [11] have provided evidence of the difference between design for maximum cooling and maximum efficiency for thermo-acoustic refrigerators. With the exception of studies by Minner *et al*. [6], previous works vary no more than a single parameter, holding all others constant. The solution approaches that are applied guarantee only a locally optimal solution, which may potentially be significantly inferior to a globally optimal solution. In addition, parameters like frequency, stack position, stack length, and plate spacing involved in designing TARs were optimised by Hariharan N. *et al.* [12] using Response Surface Methodology (RSM), with results showing that geometrical variables chosen for their investigation are interdependent. While this is by no means a comprehensive list of the ‘optimisation’ of refrigerator components, it is certainly a thorough overview of optimisation targets.

MOTIVATION

Most of the previous optimisation efforts have in common heavy reliance on the effect of a single design parameter on device performance. In all likelihood, each optimal design is a local optimum as the solution obtained is optimal (either maximal or minimal) within a neighbouring set of candidate solutions. This is in contrast to the global optimum proposed in this particular study, which provides a global optimal solution among *all* possible solutions in a specific domain, not simply those in a particular neighbourhood of variables.

In other words, a novel mathematical programming approach to handling design and choice between maximum cooling and maximum coefficient of performance of thermo-acoustic refrigerators (TARs) is presented in this paper. In addition, the blockage ratio, stack spacing, stack length and position of the stack have been identified as design parameters, so their interdependency is taken into account while computing the optimal set describing optimal performance of TARs, unlike previous studies.

The remainder of this paper is organised in the following fashion: Section 4 presents the model development. The fundamental parameters and equations in our mathematical models characterising the standing wave thermo-acoustic refrigerators are presented. Section 5 describes the proposed optimisation approach using the General Algebraic Modelling System (GAMS). Sections 6 and 7 discuss the valuable contributions of this work.

MODEL DEVELOPMENT

Figure 1 shows the geometry used to derive and discuss the thermo-acoustic equations. The model proposed in this paper does not consider any effect of the stack material and the interdependency between the coefficient of performance of thermo-acoustic core, the effectiveness of heat exchangers and the acoustic power efficiency. In addition, no attempt is made to derive TAR equations, as detailed derivations of the equations are available in Tijani's [13] thesis.

Design parameters of the thermo-acoustic core

The basic design of thermo-acoustic refrigerator requires the following [14]:

(a) to supply the desired cooling load; and
(b) to simultaneously achieve the prescribed cooling temperature.

The coefficient of performance of a thermo-acoustic core COP is dependent on 19 independent design parameters [14]. Herman and Travnicek [10] have collapsed the number of parameters to six normalised parameter spaces. The resultant normalised operation parameters are presented in Table 1 and Table 2. The number of parameters can be reduced by making a choice of some normalised parameters.

Objectives functions

The performance of the thermo-acoustic stack depends on three main stack design parameters: the centre position, the length and the cross-section area of the stack. The normalised heat flow (Φ_H) and acoustic power (Φ_W) neglecting axial conduction in the working fluid as well as in the stack plates are given as follows [15]:

$$\Phi_H = -\left[\frac{\delta_{kn} DR^2 \sin(2X_{Sn})}{8\gamma(1+\sigma)\left(1-\sqrt{\sigma}\delta_{kn}+\frac{1}{2}\sigma\delta_{kn}^2\right)}\right]\times\left[\frac{\Delta T_{mn}\tan(X_{Sn})}{(\gamma-1)BR\;L_{Sn}}\times\frac{\left(1+\sqrt{\sigma}+\sigma\right)}{1+\sqrt{\sigma}}-\left(1+\sqrt{\sigma}-\sqrt{\sigma}\delta_{kn}\right)\right] \qquad (1)$$

$$\Phi_W = \left[\frac{\delta_{kn} DR^2 L_{Sn}(\gamma-1)BR\;\cos^2(X_{Sn})}{4\gamma}\right]\times\left[\frac{\Delta T_{mn}\tan(X_{Sn})}{BR\;L_{Sn}(\gamma-1)\left(1+\sqrt{\sigma}\right)\left(1-\sqrt{\sigma}\delta_{kn}+\frac{1}{2}\sigma\delta_{kn}^2\right)}-1\right]$$
$$-\left[\frac{\delta_{kn} L_{Sn} DR^2}{4\gamma}\times\frac{\sqrt{\sigma}\sin^2(X_{Sn})}{BR\left(1-\sqrt{\sigma}\delta_{kn}+\frac{1}{2}\sigma\delta_{kn}^2\right)}\right] \qquad (2)$$

Table 1: Normalised cooling load and acoustic power

Operation parameters	
Normalised cooling power	$\Phi_H = \frac{\dot{Q}_c}{p_m aA}$
Normalised acoustic power	$\Phi_W = \frac{\dot{W}}{p_m aA}$

Table 2: TAR parameters

Operation parameters	
Drive Ratio (DR)	$DR = \frac{p_0}{p_m}$ where p_0 and p_m are, respectively, the dynamic and mean pressure
Normalised temperature difference	$\theta = \Delta T_{mn} = \frac{\Delta T_m}{T_m}$ where ΔT_m and T_m are, respectively, the desired temperature span and the mean temperature span
Gas parameters	
Normalised thermal penetration depth	$\delta_{kn} = \frac{\delta_k}{y_0}$ where $2y_0$ is the plate spacing
Stack geometry parameters	
Normalised stack length	$L_{Sn} = \frac{2\pi f}{a} L_S$ where L_S the stack length
Normalised stack position	$X_{Sn} = \frac{2\pi f}{a} X_S$ where f, a and X_S are, respectively, the resonant frequency, the speed of sound and the stack centre position
Blockage ratio or porosity	$BR = \frac{y_0}{(y_0 + l)}$ where $2l$ is the plate thickness

The normalised cooling load (Φ_C) and the coefficient of performance of the thermo-acoustic core COP and COPR are obtained, respectively, as follows [14]:

$$\Phi_C = \Phi_H - \Phi_W \tag{3}$$

$$COP = \frac{\Phi_H - \Phi_W}{\Phi_W} \tag{4}$$

$$COPR = \frac{COP}{COP_C} = \frac{\left(|\Phi_H| - |\Phi_W|\right)/|\Phi_W|}{(2-\theta)/(2\theta)} \tag{5}$$

The cooling load Φ_C is function of eight non-dimensional parameters [10]:

$$\Phi_C = F\left(\sigma,\ \gamma,\ \varepsilon_S,\ T_{mn},\ L_{Sn},\ X_{Sn},\ BR,\ \delta_{kn}\right) \tag{6}$$

Where σ, γ, ε_S and T_{mn} represent, respectively, the Prandtl number, the polytropic coefficient, the stack heat capacity correction factor and the normalised temperature difference. The influence of the working fluid on the gas is exerted through parameters σ, γ and ε_S.

In the boundary layer approximation, the acoustic power loss per unit area of the resonator is given as follows [15]:

$$\overset{o}{W}_2 = \frac{dW_2}{dS} = \frac{\delta_{kn} DR^2 L_{Sn} (\gamma - 1) BR \ \cos^2(X_{Sn})}{4\gamma} + \frac{\delta_{kn} L_{Sn} DR^2}{4\gamma} \times \frac{\sqrt{\sigma} \sin^2(X_{Sn})}{BR \ 1 \ \ \sqrt{\sigma}\delta_{kn} + \frac{1}{2}\sigma\delta_{kn}^2} \tag{7}$$

Where the first term on the right-hand side is the kinetic energy dissipated by viscous shear. The second term is the energy dissipated by thermal relaxation. Details description of the model objectives and the constraints used are available in reference [10], [13] and [14].

PROPOSED MMP SOLUTION APPROACH TO EMPHASISE ALL OBJECTIVE COMPONENTS

Most of the equations involved in the mathematical problem formulation (MPF: multi-objective mathematical programming problem) have been presented in the previous section. The three objective components – namely the cooling load (Φ_C), the coefficient of performance (COP) and the acoustic power lost ($\overset{o}{W}_2$) – are the three distinct objective components to optimise simultaneously. Detailed descriptions on the single objective optimisation analysis for each objective function are reported by Tartibu *et al*. [16] revealing that these three objective functions are conflicting and thus suitable for a multi-objective optimisation approach. The optimisation task is formulated as a three-criterion nonlinear programming problem with discontinuous derivatives (DNLP) that simultaneously maximise the magnitude of the cooling load (Φ_C), maximise the coefficient of performance (COP), and minimise acoustic power lost ($\overset{o}{W}_2$).

$$(\text{MPF}) \quad \max_{L_{Sn}, X_{Sn}, BR, \delta_{kn}} \xi = \Phi_C(L_{Sn}, X_{Sn}, BR, \delta_{kn}), COP(L_{Sn}, X_{Sn}, BR, \delta_{kn}), \overset{o}{X}_2(L_{Sn}, X_{Sn}, BR, \delta_{kn}) \tag{8}$$

Subject to bound limits $\Phi_{C\max}$, $\Phi_{C\min}$, COP_C (details available in studies by Tartibu *et al.* [16]) and the following constraint:

$$\Phi_C = \Phi_H - \Phi_W > 0 \tag{9}$$

The four different parameters describing the geometry of the stack $(L_{Sn}, X_{Sn}, BR, \delta_{kn})$ (Equation 8) are the four variables to optimise. The constraint described by Equation 9 means that a negative cooling load does not have any physical meaning (and thus the solutions for which Equation 9 is not satisfactory have been eliminated). There is no single optimal solution to simultaneously optimise all the three objectives functions. In these cases, the decision makers are interested in the ‘preferable’ solution.

In order to yield *only* Pareto optimal solutions, avoiding the generation of weak, non-efficient solutions [17], the lexicographic optimisation (Augmented ε-constraint method) for each objective function to construct the payoff table for the multi-objective mathematical programming models (MPF) is proposed and implemented in the software GAMS (Figure 2). The advantages of the augmented ε-constraint method (or AUGMENCON) over traditional weighted methods and ordinary ε-constraint methods can be found in research by Mavrotas [17] and Tartibu *et al*. [18]. A flowchart of AUGMENCON method is presented in Figure 3, with “p” representing the number of objectives function; “q” the number of intervals; and “ni” the index of objective function.

The code describing the AUGMENCON method is available in the GAMS library (http://www.gams.com/modlib/libhtml/epscm.htm) with an example. While the part of the code that has to do with the example (the specific objective functions and constraints), as well as the parameters of AUGMENCON that have been modified in the current modelling of TAR, the part of the code that performs the calculation of payoff table with lexicographic optimisation and the production of the Pareto optimal solutions, is fully parameterised to be ready to use (detailed description of the code is available in Tartibu’s doctoral thesis [19]).

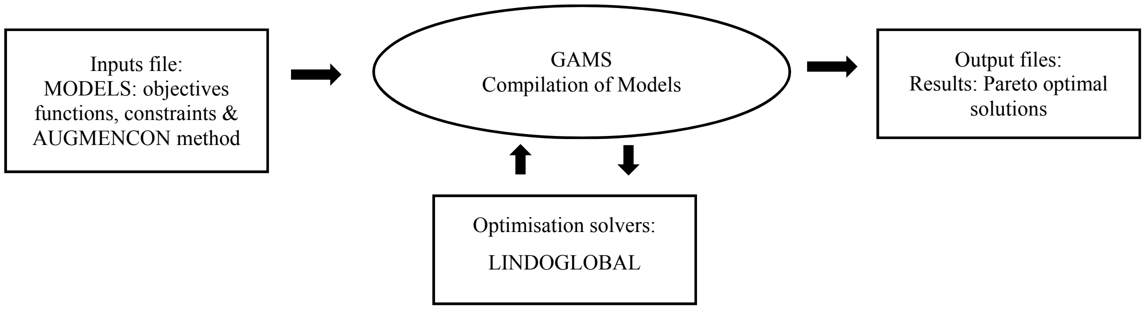

Figure 2: GAMS process illustration

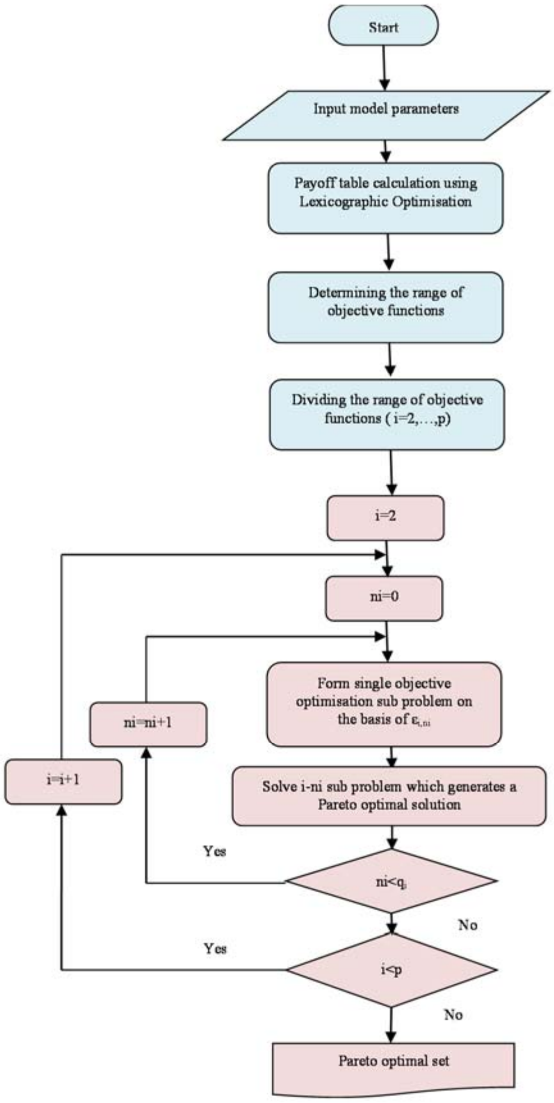

Figure 3: Flowchart of AUGMENCON method for optimisation

Formulation of the problem is a significant aspect of this study. The choice of the most important function or the primary objective function depends on the decision maker. Most of the time, this choice is dependent on problem information and results in partial representation of Pareto optimal sets due to the tendency of the solution to cluster toward the maximum of the primary objective function. Therefore, the preferences and specific limits on objective functions are articulated rather than relying on relative importance of objectives, as suggested by Marler [20], to identify the best problem formulation. Guidance on the best problem formulation is provided by Tartibu *et al.* [16]. Subsequently, the augmented ε-constraint method for solving the model (Equation 8) has been formulated as follows:

$$\max\left(\mathrm{COP}_{(L_{Sn},X_{Sn},BR,\delta_{kn})}+\mathrm{dir}_1\ r_1\times\left(\frac{s_2}{r_2}+\frac{s_3}{r_3}+\frac{s_4}{r_4}+\frac{s_5}{r_5}\right)\right) \tag{10}$$

Subject to

$$\Phi_{C(L_{Sn},X_{Sn},BR,\delta_{kn})}-\mathrm{dir}_2\ s_2=\varepsilon_2$$

$$\overset{o}{W}_{2(L_{Sn},X_{Sn},BR,\delta_{kn})}-\mathrm{dir}_3\ s_3=\varepsilon_3$$

$$s_i\in\Re^{+}$$

Where dir_i is the direction of the ith objective function, which is equal to -1 when the ith function should be minimised, and equal to +1 when it should be maximised. Efficient solutions to the problem are obtained by parametrical iterative variations in the ε_i; s_i represents the introduced surplus variables for the constraints of the MP problem; and $r_1 \; s_i / r_i$ are used in the second term of the objective function to avoid scaling problems.

RESULTS AND DISCUSSIONS

The cordierite honeycomb ceramic stacks (Figure 3) are considered. These stacks have different pore sizes (64 CPSI [Cells per Square Inch], 100 CPSI, 230 CPSI and 300 CPSI) and lengths (100 mm, 70 mm, 48 mm and 26 mm). These stacks were successively positioned at six different locations from the closed end: 100 mm, 200 mm, 300 mm, 400 mm, 500 mm and 600 mm. The experimental results reported by Tartibu [11] are considered for evaluating the ability of the models developed in this research to compute optimal solutions describing the stack geometries.

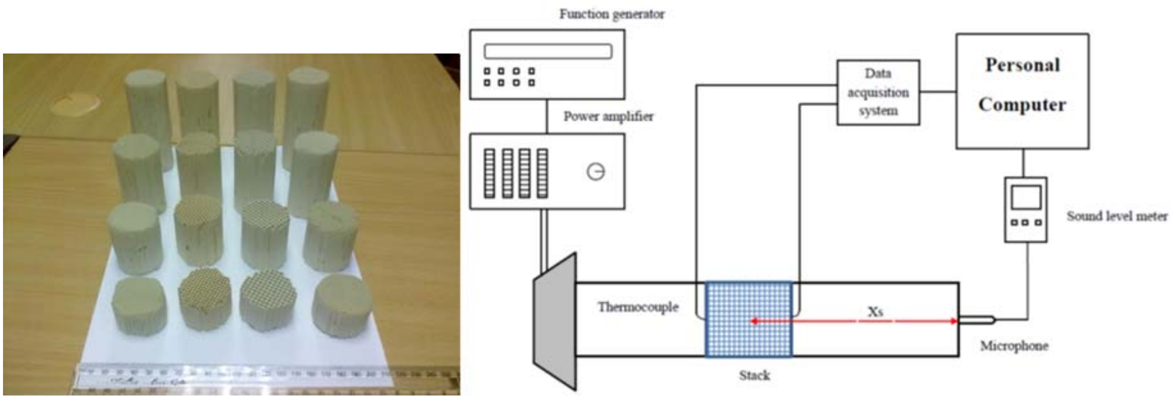

Figure 4: Stack samples used in the experiments and schematic diagram of experimental apparatus

The coefficient of performance (COP) of a thermo-acoustic refrigerator indicates how effective the device is in converting and producing cooling load by absorbing sound energy. The coefficient of performance is given by Equations 4 and 5. For all the stack lengths considered (Figure 4), a study conducted by Tartibu [11] shows that the values of COPR decrease as the distance from the pressure antinode (closed end) increases (Figure 5) (detailed description available in Tartibu's studies [11]). In particular, the shortest stack length shows the highest COPR. Interestingly, this behaviour has been observed by Herman and Travnicek [10] and Tartibu *et al.* [16] using mathematical modelling.

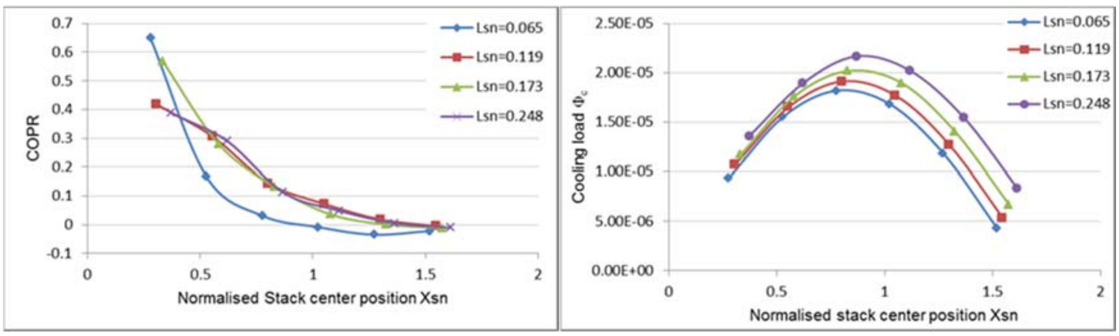

Figure 5: COPR and cooling load for four different honeycomb ceramic stack lengths (detailed description available in Tartibu's study [11])

The cooling load was calculated using Equation 3. Figure 5 shows the cooling load as a function of the normalised stack centre position obtained for four different stack lengths. The results obtained show a maximum cooling load when the stack is moved away from the pressure antinode (Figure 5). The results suggest the cooling load increases with the stack length. Contrary to the maximum COPR, increasing the stack length leads to an increase in cooling load for TAR (Figure 5), a result which concurs with previous studies by Herman and Travnicek [10] and Tartibu *et al.* [16] suggesting that there is a distinct optimum for maximum cooling and maximum coefficient of performance (detailed description available in Tartibu's studies [11]). Table 3 presents the parameters as estimated for the cordierite honeycomb ceramic stack used in this experiment. The normalised values are obtained from Tables 1 and 2.

Table 3: Estimated parameters of TAR

Lsn	Xsn						δkn	F [Hz]	Tm [K]	DR
0.065	0.280	0.528	0.777	1.024	1.272	1.520	0.135	135	250	0.025
0.119	0.307	0.555	0.803	1.051	1.299	1.547	0.168			
0.173	0.334	0.582	0.830	1.078	1.326	1.574	0.255			
0.248	0.372	0.620	0.868	1.116	1.364	1.612	0.291			

The experimental results have been compared with the mathematical programming results of the models. The ε-constraint method as formulated in Equation 10 was applied. For the problem formulation proposed in Equation 10, the following constraints (upper and lower bounds) have been enforced for the solver to carry out the search for the optimal solution in those ranges:

$X_{Sn}.lo = 0.280; \quad X_{Sn}.up = 1.612$

$\delta_{kn}.lo = 0.05; \quad \delta_{kn}.up = 0.1$

The thermal penetration depth range has been shortened to reduce the computational time (as that could take several days) since the result trends were more important. The normalised temperature difference (θ) is set to be equal to 0.030. The blockage ratio has been set to 0.8. More input parameters are shown in Table 3. Efficient solutions for the proposed model have been found using the AUGMENCON method and the LINDOGLOBAL solver. To save computational time, the early exit from the loops as proposed by Mavrotas [17] has been applied. The range of each three objective functions is divided into four intervals (five grid points). The normalised stack length has been arbitrarily given values of 0.065-0.119-0.173-0.248 successively. Table 4 reports the results obtained representing the optimal parameters, namely BR, δ_{kn} and X_{sn}, with the highest performance highlighted in bold. The maximum CPU time taken to complete the results was 324.981 seconds.

For a specific value of normalised temperature difference (θ), the trend of the COPR and the cooling load (Φ_c) is the same as those reported in Figure 5 (the trend lines are shown for visual guidance). These results suggest that the COPR is maximised closer to the closed end, while the cooling (Φ_c) reaches its maximum value away from the closed end.

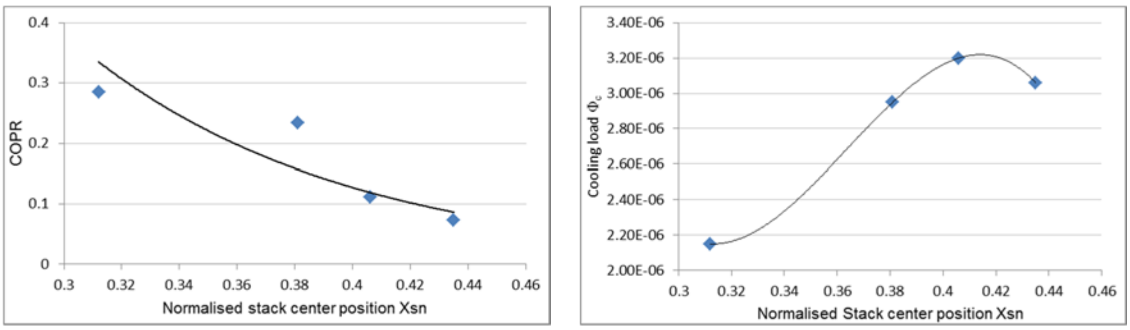

Figure 5: AUGMENCON results representing COPR and Φ_c for cordierite honeycomb ceramic stack

Table 4: Computation results obtained using AUGMENCON

	Results	1	2	3	4	5	6	7	8	9
L_{sn} = 0.065	BR	**0.800**	0.800	0.800	0.800	0.800	0.800	0.800	0.800	0.800
	δ_{kn}	**0.061**	0.075	0.081	0.100	0.100	0.100	0.100	0.075	0.075
	X_{Sn}	**0.312**	0.312	0.313	0.310	0.311	0.311	0.311	0.314	0.376
	Φ_C	**2.15E-06**	1.92E-06	1.69E-06	2.07E-06	2.88E-07	2.09E-06	3.04E-08	2.15E-07	1.55E-06
	COP	**9.372**	9.357	8.990	8.992	8.992	8.992	8.993	8.988	8.988
L_{sn} = 0.119	BR	**0.800**	**0.800**	0.800	0.800	0.800	0.800	0.800	0.800	0.800
	δ_{kn}	**0.050**	**0.100**	0.100	0.084	0.073	0.061	0.050	0.095	0.095
	X_{Sn}	**0.381**	**0.355**	0.340	0.377	0.379	0.381	0.382	0.376	0.376
	Φ_C	**1.35E-06**	**2.88E-06**	2.57E-06	2.27E-06	1.96E-06	1.66E-06	1.35E-06	2.88E-06	2.57E-06
	COP	**7.686**	**6.484**	6.677	5.361	5.362	5.363	5.364	5.360	5.360
L_{sn} = 0.173	BR	0.800	0.800	0.800	0.800	0.800	0.800	**0.800**	0.800	0.800
	δ_{kn}	0.050	0.089	0.089	0.085	0.073	0.062	**0.100**	0.097	0.097
	X_{Sn}	0.413	0.407	0.407	0.408	0.409	0.411	**0.406**	0.406	0.406
	Φ_C	1.44E-06	3.14E-06	2.80E-06	2.46E-06	2.12E-06	1.78E-06	**3.20E-06**	2.80E-06	3.14E-06
	COP	3.651	3.660	3.660	3.647	3.649	3.650	**3.661**	3.645	3.645
L_{sn} = 0.248	BR	**0.800**	**0.800**	0.800	0.800	0.800	0.800	0.800	0.800	0.800
	δ_{kn}	**0.050**	**0.089**	0.089	0.085	0.074	0.062	0.098	0.050	0.071
	X_{Sn}	**0.435**	**0.428**	0.428	0.430	0.432	0.433	0.427	0.435	0.400
	Φ_C	**1.43E-06**	**3.12E-06**	2.78E-06	2.44E-06	2.11E-06	1.77E-06	3.12E-06	1.43E-06	2.27E-06
	COP	**2.381**	**2.377**	2.377	2.377	2.379	2.380	2.375	2.381	2.886

CONCLUSION

This paper proposes the use of mathematical analysis and optimisation to model and optimise thermo-acoustic refrigerators (TARs). The General Algebraic Modelling System (GAMS) is used to implement a multi-objective optimisation approach, as this approach provides a fast estimate to initial design calculation of the geometrical configuration describing the stack of thermo-acoustic refrigerators. Four different parameters describing the geometry of the device – stack length, stack centre position, stack spacing and blockage ratio – have been studied. Nonlinear programming models with discontinuous derivatives (DNLPs) have been formulated and implemented in GAMS. An improved version of a multi-objective solution method, the ε-constraint method referred to as *augmented ε-constraint method* (AUGMENCON), has been applied. For different arbitrary values of stack length, this process generates optimal solutions describing geometry of the TAR, solutions which depend on the a priori design goal for maximum cooling or maximum coefficient of performance. This present research reveals the suitability of a multi-objective optimisation approach to model and optimise a TAR using the software GAMS. The test of the proposed models to evaluate their ability to predict the best parameters describing the geometry of the stack has revealed similar trend with experimental results, a finding which reinforces the application of the proposed approach in the design of thermo-acoustic refrigerators.

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