

# Phase Voltage Harmonic Imbalance in Asymmetrical Multiphase Machines with Single Neutral Point

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**Abstract**— Multiphase ( $n$ -phase) machines are often designed with  $l$  sub-windings on stator, each having  $k$  phases, and the machine is typically operated with  $l$  isolated neutral points. However, such a machine can also operate with a single neutral point, which improves the fault-tolerant properties. When a machine is inverter supplied, low order harmonics may be present due to the low switching frequency and nonlinear inverter properties. Moreover, low order zero-sequence harmonics can be deliberately injected to increase dc bus voltage utilisation. This paper investigates a phenomenon that has not been reported so far in relation to asymmetrical multiphase machines with a single neutral point, namely that the presence of balanced low order harmonics in leg voltages produces unbalanced phase voltage harmonics and consequently unbalanced phase current harmonics. By analysing the neutral point (common mode) voltage harmonics, imbalance in the phase voltage harmonics is explained. Analytical expressions for neutral point voltage harmonics and phase voltage harmonics are provided for asymmetrical machine configurations with a single neutral point having arbitrary numbers of sub-windings and sub-winding phases. Theoretical considerations are verified using simulations and experiments with asymmetrical twelve- and nine-phase loads with a single neutral point, respectively.

**Keywords**— Multiphase machines, phase voltage harmonics, harmonic imbalance, low-order harmonics

## I. INTRODUCTION

With variable speed drives reaching multi-MW power rating, multiphase machines offer numerous advantages over three-phase ones. Lower current and power per phase allow use of reduced rating switches, torque ripple is at the higher frequencies, and, above all, fault-tolerant operation is highly desirable in safety critical application [1, 2]. Some examples where advantages of the multiphase machines are of particular importance include fault-tolerant remote off-shore wind power generation systems [3, 4], increase of the dc link voltage by series connection of the voltage source inverters (VSIs) [5], multi-motor applications [6, 7], and electric ship propulsion [8] and generation [9].

Good fault tolerance is one of the most important benefits of the multiphase machines. In many cases use of a multiphase machine with a multiple three-phase winding topology has been considered as the most viable solution due to the readily available three-phase VSIs. Furthermore, the fault tolerant strategy can be very simple in the cases with multiple neutral points, since a whole three-phase winding set can be taken out of service in the case of a fault [10]. Of course, available power is reduced. On the other hand, research shows that

switching off an entire sub-winding is not an optimal solution and that configurations with the single neutral point provide superior performance in post-fault operation. By utilising flux/torque non-producing planes or by changing the transformation matrix, a smaller drop in available power under open-phase fault can be achieved while optimising operation for torque ripple or stator resistive loss minimisation [11].

If asymmetrical or split-phase machines are used, some additional benefits result as related to an easier manufacturing of the stator winding, since stator core can contain the same number of slots as for a three-phase machine [12], as long as the complete winding is formed as a set of three-phase sub-windings. Extensive research has been carried out on modelling and finding appropriate decoupling matrices for split-phase machines with both multiple and single neutral points [13-17]. Analysis of the torque ripple harmonics has been also carried out [12, 18]. Nevertheless, the properties of low order harmonics in this particular type of multiphase machines have not been explored in the past and this is the subject this paper deals with. As shown in the paper, harmonic imbalance in the phase voltages does not appear in asymmetrical six-phase machines, which have been by far the most frequent object of study in the multiphase drive area. Indeed, such an unbalance does not take place as long as the complete winding consists of only two sub-windings.

It is shown in this paper that balanced low-order leg voltage harmonics, usually present in high power multiphase drives with low PWM switching frequency, can produce unbalanced phase voltage harmonics in the case of the asymmetrical machines with a single neutral point. Unequal phase voltage harmonics may lead to unequal thermal losses among phases and uneven stress on the switching devices, which should be taken into consideration during the drive design stage. Starting from the analysis of the neutral point voltage harmonics, analytical expressions for phase voltage harmonics are provided and verified by simulations and experiments. The developed theory is general and it covers all cases of the asymmetrical machines with a single neutral point. The appearance of phase voltage harmonic imbalance requires existence of at least three sub-windings in the stator winding, as shown in the paper.

## II. ASYMMETRICAL MACHINE DESCRIPTION AND LEG VOLTAGE HARMONICS

A definition of an asymmetrical machine and phase displacement angles, in the case of an arbitrary number of phases, is provided first. Stator windings of an  $n$ -phase asym-



harmonics.

Expression (8) also shows that the amplitudes of the neutral point voltage normalised harmonics are not dependant on the number of phases in the sub-winding. This means that every normalised neutral point voltage harmonic  $h_n=h/k$  has the same amplitude for any asymmetrical machine with a single neutral point that has the same number of sub-windings ( $l$ ). Visualisation of the neutral point voltage normalised harmonics ( $h_n$ ) amplitudes in the case of the machines with 2, 3, 4, and 5 sub-windings is shown in Fig. 2 (all values are per-unit).

For the sake of clarity, an example can be made by comparing the nine-phase ( $k=3, l=3$ ) and the 15-phase ( $k=5, l=3$ ) machines, the second row in Fig. 2. For example, cases of the 1<sup>st</sup> and the 3<sup>rd</sup> normalised harmonics ( $h_n=h/k$ ) are the 3<sup>rd</sup> and the 9<sup>th</sup> harmonics of the 9-phase machine, while in the 15-phase case they are the 5<sup>th</sup> and the 15<sup>th</sup> harmonics. Since both machines have the same number of sub-windings, their normalised harmonics are the same. Therefore, the 3<sup>rd</sup> harmonic of the 9-phase machine has the same amplitude as the 5<sup>th</sup> harmonic of the 15-phase machine. The same applies for the 9<sup>th</sup> and the 15<sup>th</sup> harmonics of the 9-phase and 15-phase machines, respectively.

Even-order normalised harmonics are either zero or one, but since even harmonics cannot exist in symmetrical supply, they are not analysed here. It should be pointed out that (8) will become limit equation when  $h_n$  becomes integer multiple of  $2l$ . One way of solving this is to use L'Hôpital's rule, so instead of  $\sin(\pm\pi)$ ,  $\cos(\pm\pi)$  is obtained.

#### IV. PHASE VOLTAGE ANALYSIS

Phase voltage is calculated as a difference between leg voltage (3) and neutral point voltage (8) as follows:

$$v_{j,i,h}^{\text{phase}}(t) = A_h \cos\left(kh_n \left(\omega t + \frac{\pi}{n}(2l(i-1) + j - 1)\right)\right) - \frac{A_h}{l} \frac{\sin\left(h_n \frac{\pi}{2}\right)}{\sin\left(h_n \frac{\pi}{2l}\right)} \cos\left(kh_n \left(\omega t + (l-1)\frac{\pi}{2n}\right)\right) \quad (9)$$

When (9) is simplified and  $h_n$  is replaced by  $h/k$ , the expressions for amplitude and phase of the phase voltage harmonics become as follows:

$$A_{j,i,h}^{\text{phase}} = A_h \sqrt{1 + \frac{\left(\frac{\sin\left(h \frac{\pi}{2k}\right)}{\sin\left(h \frac{\pi}{2n}\right)}\right)^2}{l} - \frac{2}{l} \frac{\sin\left(h \frac{\pi}{2k}\right)}{\sin\left(h \frac{\pi}{2n}\right)} \cos\left(\frac{h}{n} j \pi + \frac{h}{k} i 2\pi - \frac{h(5l+1)}{2n} \pi\right)} \quad (10)$$

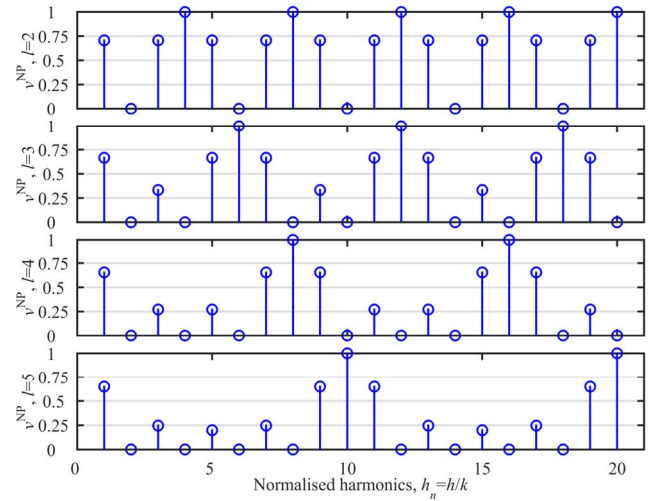


Fig. 2. Amplitudes of the neutral point voltage normalised harmonics for the machines with 2, 3, 4, and 5 winding sets.

$$\theta_{j,i,h}^{\text{phase}} = \text{atan2} \left\{ \frac{\frac{\sin\left(h \frac{\pi}{2k}\right)}{\sin\left(h \frac{\pi}{2n}\right)} \sin\left(h \frac{\pi}{2n}(l-1) - l \sin\left(h \frac{\pi}{n}(2l(i-1) + j - 1)\right)\right)}{\frac{\sin\left(h \frac{\pi}{2k}\right)}{\sin\left(h \frac{\pi}{2n}\right)} \cos\left(h \frac{\pi}{2n}(l-1) - l \cos\left(h \frac{\pi}{n}(2l(i-1) + j - 1)\right)\right)} \right\} + \pi \quad (11)$$

where, again,  $j$  and  $i$  designate the sub-winding and the phase within that sub-winding, respectively. When the expression for phase voltage amplitude (10) is analysed, it can be seen that all phases within one sub-winding have the same amplitude of the given voltage harmonic (because  $i$  is multiplied by  $2\pi$ , which is the period of the cosine function). On the other hand, voltage harmonic amplitudes in the phases in different sub-windings are different.

For the sake of clarity, this is demonstrated using the example of the nine-phase machine. Per-unit amplitudes of phase voltage 3<sup>rd</sup> harmonic, normalised with the amplitude of the leg voltage 3<sup>rd</sup> harmonic, are shown in Fig. 3. As expected, amplitudes of the 3<sup>rd</sup> harmonic of the phase voltages within each sub-winding are the same. On the other hand, there is a difference between phases of different sub-windings. Amplitudes of the 3<sup>rd</sup> harmonic of the phase voltages of the second sub-winding (2<sup>nd</sup>, 5<sup>th</sup>, and 8<sup>th</sup> phase) are different from the ones of the first sub-winding (1<sup>st</sup>, 4<sup>th</sup>, and 7<sup>th</sup> phase). As previously mentioned, this behaviour is easily explained when the phase angle of the neutral point voltage harmonics (8) is taken into consideration.

In the neutral point harmonic analysis, it has been shown that when the phase angle of the neutral point voltage is divided by the harmonic order, it is always equal to half of the phase shift angle of the first phase in the last sub-winding. Hence, the neutral point voltage is always positioned in the middle of the first group of the leg voltages, as shown schematically in Fig. 4. In this figure phase angles of the leg voltages are divided by the harmonic order; thus this analysis holds true for all harmonic orders.

The phase voltages are calculated as a difference between leg voltages and neutral point voltage. It can be seen that leg voltages are located at an equal angular distance from the neutral point voltage. Therefore, the corresponding phase voltages are equally influenced by the neutral point voltage and so have the same amplitude. In the case of the even number of sub-windings, Fig. 4 (left), each two pairs of the phase voltages have the same amplitude. On the other hand, in the case of the odd number of sub-windings, Fig. 4 (right), neutral point voltage coincides with one of the leg voltages, i.e. with  $(l-1)/2$ .

As stated before, phase voltage harmonics of the same order in one sub-winding have the same amplitude. Hence, the number of phases per sub-winding is irrelevant for the analysis. Consequently, only the first phases of all the sub-windings will be analysed, while the 2<sup>nd</sup>, 3<sup>rd</sup>, or any other phase in the sub-winding has the same amplitude of the considered phase voltage harmonic.

Amplitudes of the first five odd normalised phase voltage harmonics in the case of the machines with 2, 3, 4, and 5 sub-windings are given in Fig. 5. In the ideal case with a symmetrical supply, even harmonics do not exist. If even harmonics are present they are balanced and their amplitude is equal to zero or to the amplitude of the corresponding leg voltage harmonics. Hence, they are not of interest in the analysis and are omitted from Fig. 5.

Looking at Fig. 5, it can be seen that for the same number of sub-windings  $l$ , phase voltage harmonic unbalance is symmetrical around the imaginary line  $l/2$ , bold black line. If the line coincides with a phase number,  $l$  is odd and phase voltage harmonic amplitude of that phase has a unique value, while phase voltage harmonic amplitudes of the other phases are the same if they are at the same distance from  $l/2$ . Alternatively, if the line  $l/2$  is between two phases,  $l$  is even and then phases equidistant from the line  $l/2$  have the same value of the phase voltage harmonic amplitudes.

Examining the harmonic order in Fig. 5, it can be seen that pattern of unbalance in the phase voltage harmonics is repeated after each  $2l$  normalised harmonics. For example, if the machine has three sub-windings, the 1<sup>st</sup>, 7<sup>th</sup>, 13<sup>th</sup>, ... normalised harmonics of the phase voltages will have the same unbalance. If the given machine has three phases per sub-winding, for example a nine-phase machine, harmonics with the same unbalance pattern will be the 3<sup>rd</sup>, 21<sup>st</sup>, 39<sup>th</sup>, ..., effectively, the 3<sup>rd</sup>,  $(2n+3)$ <sup>th</sup>,  $(4n+3)$ <sup>th</sup> ... harmonic. It should be noted that in the case when there are two sub-windings, i.e.

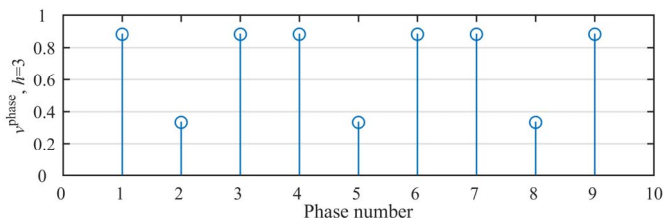


Fig. 3. The 3<sup>rd</sup> harmonic phase voltage amplitudes of the asymmetrical nine-phase machine with a single neutral point.

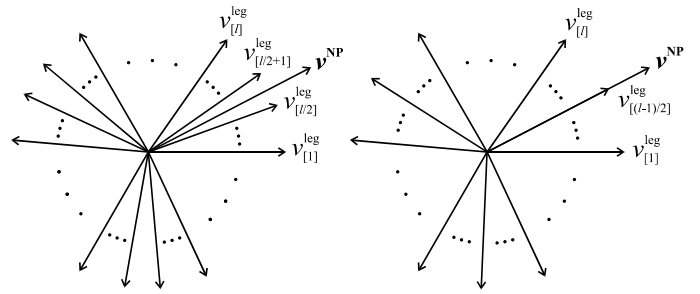


Fig. 4. Influence of the neutral point voltage on the phase voltages in the case of the even (left) and odd (right) number of winding sets.

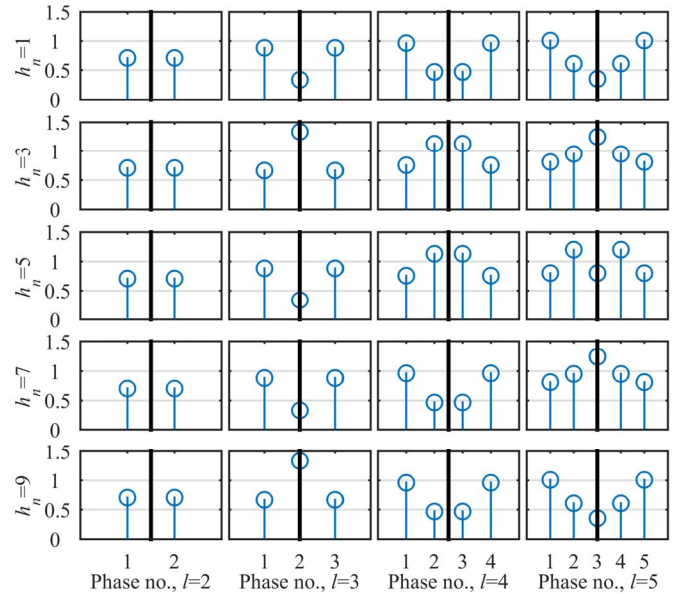


Fig. 5. Per-unit amplitudes of the 1<sup>st</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, 7<sup>th</sup> and 9<sup>th</sup> phase voltage normalised harmonics. The 1<sup>st</sup> phases of all sub-windings, for the machines with 2, 3, 4, and 5 winding sets, are shown.

a 6-, 10-, 14-phase machine, unbalance does not exist due to the fact that the phases of both sub-windings are affected in the same way by the neutral point voltage, as shown in Fig. 4 (left) and Fig. 5 (first column). However, the amplitudes of the phase voltage harmonics are different from the corresponding amplitudes of the leg voltage harmonics.

## V. SIMULATION AND EXPERIMENTAL RESULTS

To verify the analysis, simulations have been carried out for a 12-phase asymmetrical induction machine with a single isolated neutral point. The machine is considered ideal and balanced and is supplied by 12-phase two-level inverter in 180° conduction mode. As a consequence, the supply generates all odd-order harmonics. Dc link voltage is set to 600V and the fundamental frequency is 50Hz. Leg, phase and neutral point voltages, with the corresponding spectra, are shown in Fig. 6. Only the first phase is shown in time domain (first three plots), but all phases are included in the FFT analysis (bottom three plots). The first 21 harmonics are illustrated.

Even harmonics are omitted from the plot, since they are equal to zero, along with the dc component. It can be seen that

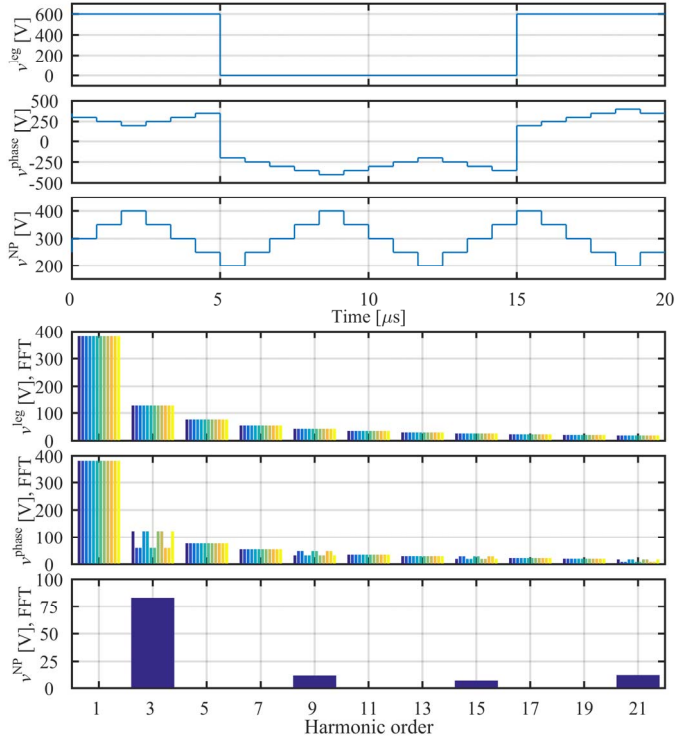


Fig. 6. Leg, phase and neutral point voltages with corresponding spectra for an asymmetrical 12-phase machine.

harmonics of the leg voltages are balanced, meaning that amplitudes of the leg voltages for the considered harmonic order have the same value. On the other hand, the imbalance exists in the case of some harmonics, such as the 3<sup>rd</sup>, 9<sup>th</sup>, 15<sup>th</sup>, and 21<sup>st</sup>, or, in terms of the normalised harmonics  $h_n=h/k$ , 1<sup>st</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, and 7<sup>th</sup>. Essentially, unbalance in the phase voltage harmonics is present for the harmonic orders that exist in the neutral point voltage.

The theory is verified by comparing the values of the phase voltage harmonics obtained by the simulation and equation (10). Leg voltage harmonic amplitudes obtained by FFT are used as input to the equation (10). The simulated and calculated amplitudes of phase voltage harmonics, for the harmonic orders where unbalance exists, are shown in Fig. 7. It can be seen that both simulation and calculation of the unbalanced phase voltage harmonics produce the same results, confirming the proposed theory.

Experimental verification has been performed using a nine-phase  $R-L$  load supplied from a bespoke nine-phase two-level voltage source inverter. The dc supply for the inverter is provided by a Sorensen SGI 600/25 dc voltage source. The inverter has hardware implemented dead time of 5 $\mu$ s. A simple carrier-based (CB) PWM is used as a modulation strategy. Inverter control is performed by dSPACE rapid prototyping platform. Measurements have been taken with Tektronix MSO2014 scope. Leg, phase and neutral point voltages are measured using a Tektronix P5205A active voltage differential probe, while phase currents have been measured using Tektronix TCP0030 active current probe. Resistance and inductance values of the used  $R-L$  load are 43 $\Omega$

and 250mH, respectively. The load is connected to form a single neutral point. Dc link voltage has been set to 300V.

To verify the proposed theory, references provided to the modulator consist of a component at fundamental frequency and the 3<sup>rd</sup>, 9<sup>th</sup>, 15<sup>th</sup>, and 21<sup>st</sup> harmonics. The amplitude of the fundamental is set to 60V, while all harmonics have the same amplitude of 20V. Fundamental frequency is 20Hz; hence harmonics are at 60Hz, 180Hz, 300Hz, and 420Hz, respectively. Switching frequency is 5kHz, which is in this case high enough so that the PWM switching process does not influence the harmonics of interest. Waveform and the spectrum of one of the references used to supply the  $R-L$  load are shown in Fig. 8. It should be noted that the reference settings are such as to ensure that the CB PWM stays in the linear modulation region.

Phase currents of the first and the second phase are shown in Fig. 9. It can be clearly seen that these two currents are different. Furthermore, in order to obtain more detailed insight into phase currents unbalance, spectra of all leg, phase, and neutral point voltages and phase currents for the 3<sup>rd</sup>, 9<sup>th</sup>, 15<sup>th</sup>, and 21<sup>st</sup> harmonics are shown in Fig. 10. Each vertical bar represents one phase. Dashed lines in the second row of Fig. 10 represent calculated harmonic amplitudes based on the measured amplitudes of the leg voltage harmonics as follows:

- blue dashed line – 9<sup>th</sup> harmonic of the 2<sup>nd</sup>, 5<sup>th</sup>, and 8<sup>th</sup> phases.
- black dashed line – 9<sup>th</sup> harmonic of 1<sup>st</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup>, and 9<sup>th</sup> phases.
- red dashed line – 3<sup>rd</sup>, 15<sup>th</sup>, and 21<sup>st</sup> harmonics of 2<sup>nd</sup>, 5<sup>th</sup>, and 8<sup>th</sup> phases.
- green dashed line – 3<sup>rd</sup>, 15<sup>th</sup>, and 21<sup>st</sup> harmonics of 1<sup>st</sup>, 3<sup>rd</sup>, 4<sup>th</sup>, 6<sup>th</sup>, 7<sup>th</sup>, and 9<sup>th</sup> phases.

Similarly, red and blue dashed lines in the bottom plot of Fig. 10 represent calculated harmonic amplitudes of the neutral point voltage. Red line represents 3<sup>rd</sup>, 15<sup>th</sup>, and 21<sup>st</sup> harmonics, while blue line represents 9<sup>th</sup> harmonic amplitude. It can be seen that unbalance in the phase voltage harmonics exists and measured harmonic amplitudes are in a good agreement with the predicted values. Small differences between the predicted and the measured values are predominantly due to the tolerances of the  $R-L$  load values.

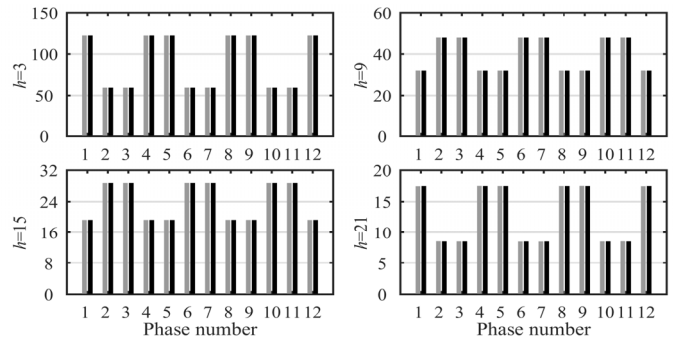


Fig. 7. Amplitudes of the 3<sup>rd</sup>, 9<sup>th</sup>, 15<sup>th</sup>, and 21<sup>st</sup> phase voltage harmonics (grey – simulation, black – calculation).

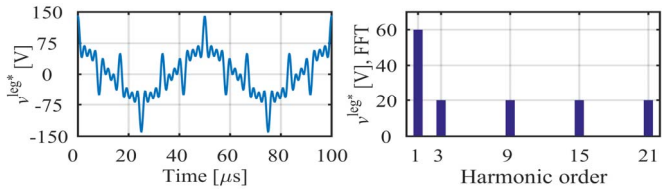


Fig. 8. Experimental results: Waveform and the spectrum of one of the references supplied to the inverter PWM unit.

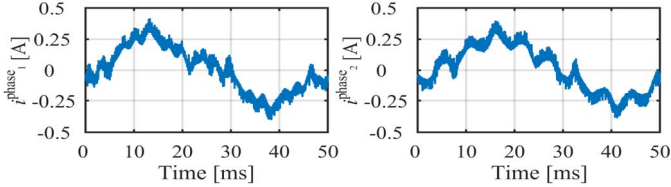


Fig. 9. Waveforms of the first (left) and second (right) phase currents.

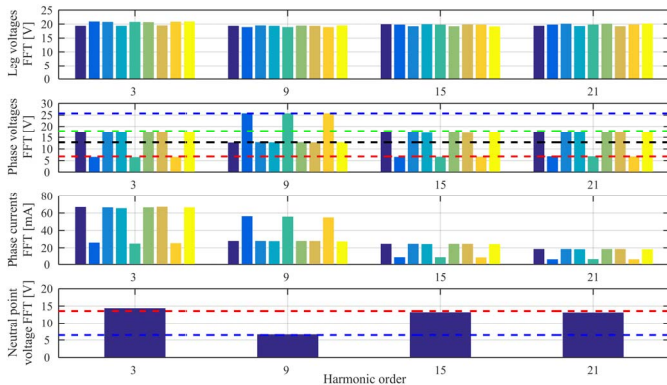


Fig. 10. Experimental results: Spectra of all leg, phase and neutral point voltages and phase currents for the selected harmonics in the case of the nine-phase  $R-L$  load.

## VI. CONCLUSION

The paper analyses harmonic properties of multiphase machines with asymmetrical winding design, where, although the winding is composed of a set of individual sub-windings, a single neutral point is created. It has been found that in such cases, even if the leg voltage harmonics have the same amplitudes, there is an unbalance in the phase voltage harmonics, and consequently phase current harmonics. Based on the definition of phase disposition angles of an asymmetrical multiphase machine, a general expression for neutral point voltage has been derived for an arbitrary number of phases and arbitrary harmonic order. Next, an expression for the amplitude and phase of the phase voltage harmonics for the general case has been arrived at. By analysing the neutral point voltage angle, a simple explanation has been presented for the unbalance that is seen in the phase voltage harmonics. Obtained equations are valid for any harmonic order in the case of any asymmetrical machine with a single neutral point.

Verification of the presented theory has been undertaken using simulations and experiments. Simulation has been carried out for the case of the twelve-phase inverter supplied machine in  $180^\circ$  conduction mode. Experiments have been performed with a nine-phase  $R-L$  load by deliberately injecting the harmonics of interest. The simulation and experimental

results are found to be in very good agreement with the proposed theory.

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