DOMAIN-SPECIFIC AND DOMAIN-GENERAL INFLUENCES ON EARLY MATHEMATICAL SKILLS: A LONGITUDINAL STUDY

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ABSTRACT

Two domain-specific quantity systems have been proposed; the "Precise Number System" for small precise numerical representations, and the "Approximate Number System" for imprecise numerical representations (Feigenson, Dehaene, & Spelke, 2004). The efficiency of these systems has been individually associated with numerical competence (Mazzocco, Feigenson, & Halberda, 2011; Schleifer & Landerl, 2011). Phonological awareness and VSSP functioning are domain-general cognitive skills which have been shown to contribute to distinct aspects of early numerical competence (Krajewski & Schneider, 2009; LeFevre et al., 2010). Krajewski and Schneider's model (2009) proposes three distinct developmental levels of early number skills; phonological awareness contributes to basic verbal number skills (Level I) while VSSP functioning and quantitative skills contribute to quantity to number-word linkage (Level II) and to early arithmetic skills (Level III). This thesis examines the longitudinal and independent contributions that domain-specific and domain-general cognitive skills make to early number skills and to two standardised mathematical attainment measures.

Verbal, visuo-spatial and quantitative skills were assessed in 129 children at the start of Reception Year. Precise quantity discrimination skills predicted performance and growth in children's ability to count objects (Level II), approximate quantity discrimination skills predicted performance and growth in reciting the number-word sequence (Level I) and the two domain-general cognitive skills predicted performance and growth in performing simple arithmetic skills (Level III) over an eighteen-month period. Also, approximate quantity discrimination skills, phonological awareness and VSSP functioning predicted performance in both mathematical attainment measures over a six-month period. However VSSP functioning predicted performance and growth in a specific mathematical attainment measure over an eighteen-month period.

Each cognitive skill seems to have a circumscribed role as a precursor of specific later number skills. This suggests that identifying deficits in these cognitive skills and designing targeted-intervention programmes for children in the very early stages of schooling could prevent later general mathematical deficits.

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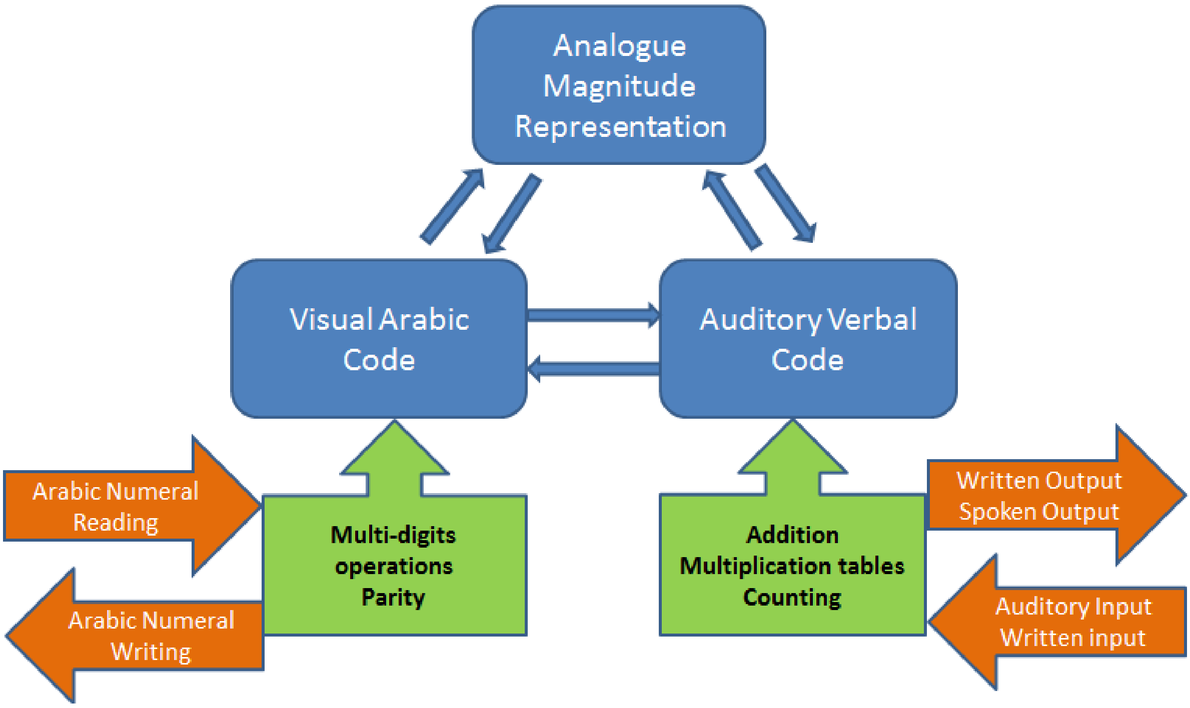
1. THEORETICAL CONTEXT, THESIS OVERVIEW AND AIMS

This chapter reviews three recent theoretical models of numerical processing that identify three key types of cognitive skills proposed to be critical for number processing; quantitative, verbal and visuo-spatial short-term memory skills. They suggest that the extent of the contribution of each of these three cognitive precursors to mathematical outcomes depends on the specific cognitive demands that the numerical tasks make. The first model is the triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene, Piazza, Pinel, & Cohen, 2003) and is based on behavioural, neuropsychological and neuroimaging data obtained with adults. Following this, two recent theoretical models of early arithmetic development, that have been partially or fully tested in children, are presented and discussed. Similarities and differences between the studies that empirically tested these models are highlighted. Lastly, this chapter discusses how the present study tests and expands these theoretical models.

1.1 THE TRIPLE-CODE MODEL OF NUMBER PROCESSING

The triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003) is based on converging evidence from behavioural, neuropsychological and neuroimaging studies conducted with adults. It proposes that humans are endowed with three distinct systems for numerical information processing. Two of these systems are considered domain-general; one system located in the left angular gyrus responsible for the verbal manipulation of numerical information and one system located in the posterior superior area of the intraparietal sulcus in charge of the visuo-spatial attention processes needed when a numerical task is being performed. The triple-code model proposes that the extent to which these two domain-general systems are implicated when performing numerical tasks would depend upon the specific demands of the numerical task. Thus, tasks that require manipulation of numerical information in verbal codes would make higher demands on the verbal-code system. For instance, reciting times tables requires number fact retrieval in the form of verbal codes (DeSmedt, Taylor, Archibald, & Ansari, 2010) and therefore verbal skills need to be recruited. Similarly, numerical tasks that require “keeping track” of sequential information would rely to a greater extent on the visuo-spatial attention-code system. For example, to perform a sequential counting task visuo-spatial attention skills are needed to keep active information about the counting process. This model also proposes a core-quantity system located in the bilateral horizontal segment of the intraparietal sulcus that is responsible for the representation and manipulation of analogue quantities. Unlike the two domain-general systems, this system is proposed to activate whenever numerical information is being processed, regardless of its presentation format. Thus, this core-quantity system is a potential domain-specific system for number processing (see figure 1.1).

Figure 1.1: Dehaene's (1992) diagram of the triple-code model of number processing

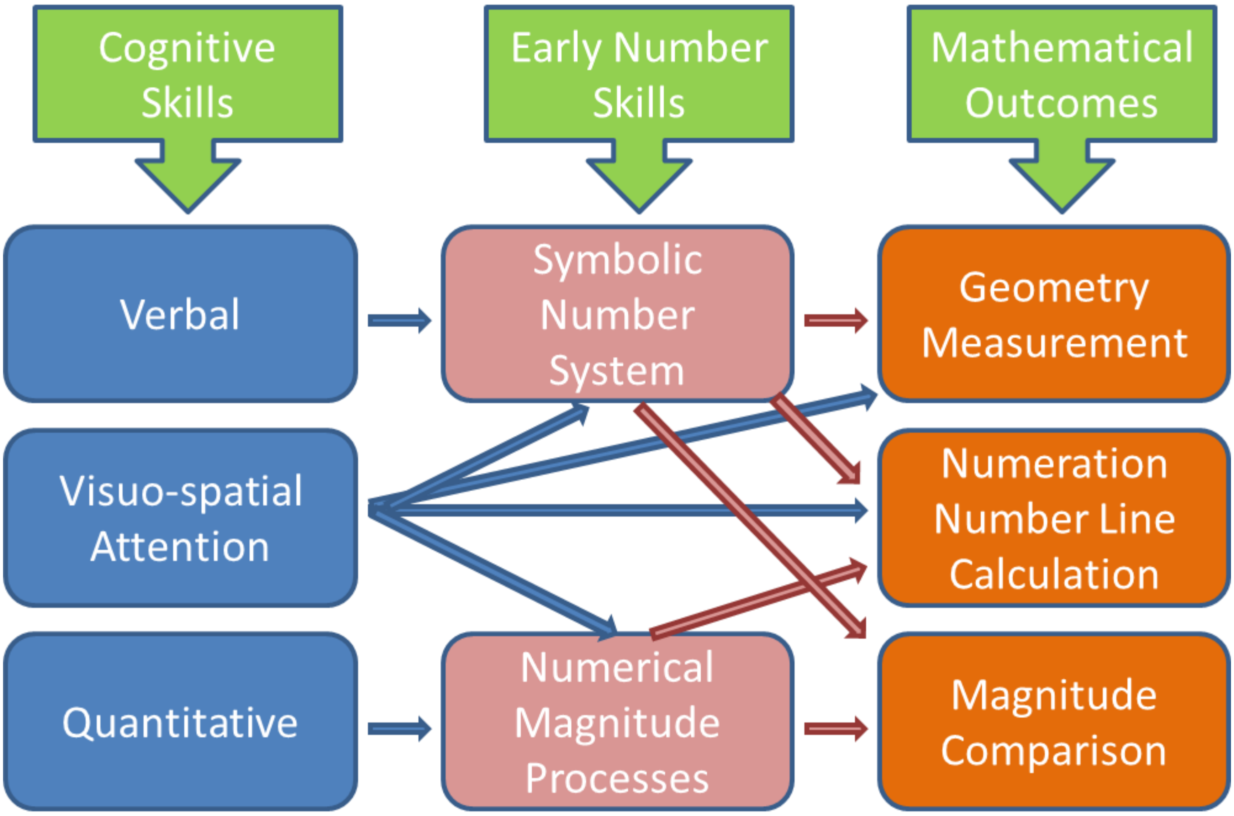


This theoretical model proposes a modular structure of the way numerical information is processed. It argues that both, domain-specific cognitive skills (quantity-processing system) and domain-general cognitive skills (verbal and visual-spatial attention systems) are implicated in processing numerical information. It also proposes that the extent of the contributions of verbal and visuo-spatial attention skills varies in relation to the specific numerical task demands, while quantitative skills contribute consistently, irrespective of the demands of specific mathematical tasks. This model predicts that, when a particular number system is damaged or malfunctioning, difficulties in performing specific number tasks that rely on the malfunctioning system will occur, but that performance on other number tasks that rely on non-affected number systems should not be substantially affected. It should be noted, however, that this theoretical model reveals little about the contributions that these three cognitive skills make to the early stages of mathematical development as it is based on behavioural, neuropsychological and neuroimaging data obtained with adults. This issue is addressed in the following models.

1.2 THE PATHWAYS TO EARLY MATHEMATICS MODEL

The pathways to early mathematics model (LeFevre et al., 2010) is based on the triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003) and explores its applicability to early mathematical development. In line with the triple-code model, LeFevre et al. (2010) propose that early quantitative, verbal and visuo-spatial attention skills make independent contributions to different early number skills and to performance on different mathematical attainment measures. The model also proposes that the extent to which these three cognitive precursors contribute to different aspects of numerical competence vary depending on each numerical task's demands. It argues that verbal skills contribute to children's performance on symbolic number tasks because language is a symbolic representational system and therefore similar rules apply when learning the conventional and culture-specific representations for number through mathematical instruction (hereafter formal number system).Verbal skills would also be expected to contribute to children's performance on all standardised mathematical attainment tests since the symbolic number system is always involved. The model also proposes that quantitative skills are involved in number tasks and standardised mathematical attainment measures that require accessing the analogue magnitude representations of numbers. Lastly, because visuo-spatial attention skills are needed to represent and/or retain the numerical information of the task regardless of its format, this model proposes that visuo-spatial attention skills make additional unique contributions to children's performance on numerical tasks where visuo-spatial orientation on an internal number line is needed. Therefore, visuo-spatial attention skills would be expected to contribute to a wide variety of standardised mathematical attainment measures (see figure 1.2).

Figure 1.2: LeFevre’s (2010) pathways to early mathematics’ model diagram



1.2.1 Empirical testing of the pathways to early Mathematics Model

LeFevre et al.’s (2010) model was tested in a two-year longitudinal study with pre-school and kindergarten children (median age five years for the preschool group and median age five years eleven months for the kindergarten group). The impact of quantitative, verbal and visuo-spatial short-term memory skills (hereafter visuo-spatial STM) on two early number skills tasks and performance on various standardised and research-based mathematical attainment measures was examined. Measures of the three cognitive precursors and participants’ early number skills were obtained at the beginning of the study. Participants’ early quantitative skills were assessed with an enumeration task in which children were presented with arrays ranging from one to six dots. Children’s response times (hereafter RTs) to articulate the number-word matching the exact number of dots presented was recorded. It has been argued that the ability to quickly and accurately enumerate small collections of up to three or four items (*subitising*) is different from counting and underpins later numerical skills and basic arithmetic skills (Butterworth, 1999, 2005, 2010). Note that subitising skills and their potential role in mathematical development are discussed in much greater detail in Chapter 2 of this thesis. Analysis of RTs revealed a clear discontinuity between children RTs for arrays containing one to three dots, for which the RTs slope showed no significant increase between one and two dots and a slight increase between two and three dots, and arrays containing four to six dots, where the RTs slope showed a significant increase with each additional dot presented. The median for the RTs of all trials presenting one, two or three dots was used as a measure of children's subitising latency. Children's verbal skills were assessed with two standardised tests assessing their receptive vocabulary and phonological awareness. Children's visuo-spatial STM skills were assessed with a child-friendly Corsi blocks task. At the same time as these cognitive assessments, both a verbal and a non-verbal early number task were also administered. In the verbal number task children were asked to correctly articulate the number-word matching a one-, two- or three-digit Arabic numeral presented on a screen. In the non-verbal number task children were asked to complete non-verbal arithmetic problems. They were presented with a set of objects, then this set was masked and another set of objects was added to or removed from the initial set. Children were then asked to form a set matching the number of objects in the hidden set using their own objects.

It was found that children's verbal skills made unique contributions to their performance on the verbal number task (number naming), but not to their performance on the non-verbal number task (non-verbal arithmetic). In contrast, children's subitising latency made unique contributions to their performance on the non-verbal number task (non-verbal arithmetic) but not to their performance on the verbal number task (number naming). Performance on the Corsi blocks task made unique contributions to variation in children's performance on the verbal and non-verbal number tasks. Thus, as hypothesised in their theoretical model, children's verbal skills contributed to their performance on a symbolic number task but not to their non-berbal arithmetic performance. Also, children's quantitative skills contributed to their performance on a number task that demanded the representation and manipulation of analogue quantities but not verbal skills. Visuo-spatial attention skills made independent unique contributions to both types of early number tasks.

Two years later participants' mathematical knowledge was assessed using standardised and research-based mathematical attainment measures. In line with their theoretical predictions, it was expected that the extent of the contribution of each cognitive precursor would vary depending on the specific cognitive demands that each mathematical attainment measure makes. Four standardised subtests of mathematical attainment were administered; the

Numeration, Measurement and Geometry subtests of mathematical knowledge from the KeyMaths Test-Revised (Connolly, 2000) and the Calculation subtest from the Woodcock-Johnson Test (hereafter WJ) of Achievement-Revised (Woodcock & Johnson, 1989). The Numeration subtest assesses children's knowledge of the numerical order, the Measurement subtest assesses children's ability to compare quantities and the Geometry subtest assesses children's processing and understanding of spatial arrays, sequencing and patterning. The Calculation subtest assesses computation skills. In addition, two computerised research-based measures of mathematical knowledge were administered. One was a computerised number line task where children were shown a target number and then asked to place the on-screen cursor where they estimated the target number would be on a line that presents a "1" on the extreme left and a "1,000" on the extreme right. Proximity of the children's selected location to the real target location was then analysed using linear regression analysis. The other research-based measure of mathematical knowledge was a symbolic comparison task where children were presented with pairs of distinct single-digit numbers varying in physical size for a maximum of three seconds. Then children were asked to indicate which digit represented a larger number.

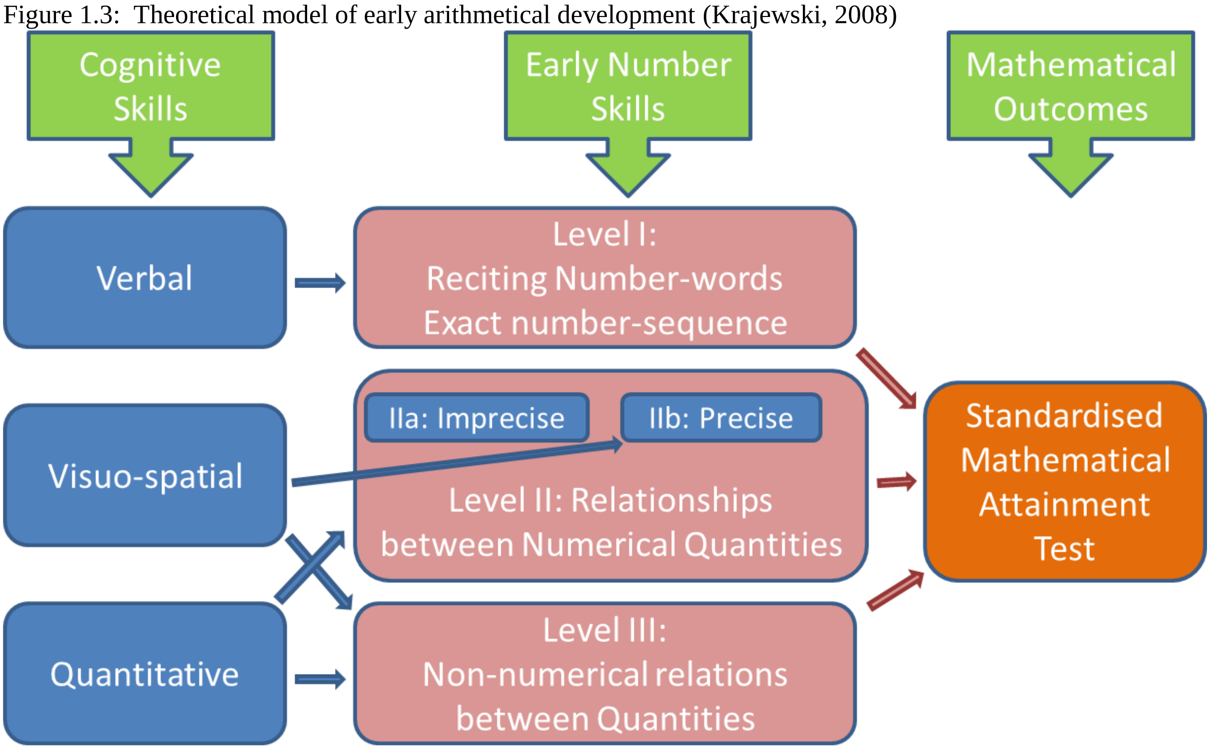
While the children's verbal skills, represented by a latent variable of children's receptive vocabulary, phonological awareness and number naming performance, made significant contributions to all standardised and research-based mathematical measures two years later, their quantitative skills and visuo-spatial STM skills only predicted specific outcome measures. In particular, the quantitative pathway, represented by a latent variable of children's subitising latency and non-verbal arithmetic performance, made significant contributions to both research-based measures but only predicted performance on the Numeration and Calculation subtests of the standardised mathematical attainment measures. No significant variance in the Geometry subtest or Measurement subtest was predicted by the quantitative pathway. The visuo-spatial STM skills' pathway, represented by children's performance on the Corsi blocks task, made significant contributions to all standardised mathematical attainment tests and to their number line performance but failed to predict children's performance on the symbolic comparison task. The separate and independent longitudinal contributions that the three mathematical cognitive precursors made to the different outcome measures was interpreted as evidence of their variable contribution depending on the characteristics and demands that the mathematical measures make.

LeFevre et al.’s (2010) study provides a comprehensive model of early mathematical development by including measures of domain-general and domain-specific cognitive skills. It proposes that different early number tasks make different cognitive demands and classify these as symbolic and non-verbal number skills. Their work also explores the relative contribution that children’s visuo-spatial STM skills, verbal skills and quantitative skills make to different mathematical attainment measures. LeFevre et al. (2010) acknowledge that two distinct quantity systems have been proposed; a quantity system for the processing of small and precise numerical magnitudes proposed by Butterworth and colleagues (Butterworth, 1999, 2005, 2010; Castelli, Glaser, & Butterworth, 2006; Gelman & Butterworth, 2005) and a quantity system for the processing of approximate quantities of numerical magnitudes proposed by Dehaene and colleagues (Dehaene, 1997; Dehaene, Molko, Cohen, & Wilson, 2004; Spelke & Dehaene, 1999). These two systems for number processing and their potential role in early mathematical development are discussed in much greater detail in Chapters 2 and 3 of this thesis. However, LeFevre et al. (2010) only included a measure of children’s precise enumeration speed to assess children’s quantitative skills. No measure representing children’s ability to make approximate numerical judgements was included. Moreover, in the subitising task children had to answer with a number-word and therefore their verbal skills and their pre-existing knowledge of the formal number system were to some extent involved. In addition, LeFevre et al. (2010) used combined measures of cognitive predictors and early number skills to examine how verbal skills and quantitative skills relate to different mathematical attainment measures. Finally, a single-task was used to assess children’s visuo-spatial STM skills. Latent variables are considered to be better representative measures than single-tasks because they suffer less contamination from the specific tasks demands than single-tasks measures (Bowey, 2005).

1.3 KRAJEWSKI AND SCHNEIDER’S MODEL OF EARLY ARITHMETICAL DEVELOPMENT

Krajewski and Schneider’s (2009) theoretical model of early arithmetical development proposes that children’s quantitative skills, verbal skills and visuo-spatial STM skills contribute to their early number skills and mathematical attainment performance. This model distinguishes three distinct developmental levels of early number skills (referred to as “quantity-number competences”, hereafter QNCs). QNCs Level I refer to children’s ability to articulate number-words and recite the number-word sequence correctly. QNCs Level II refers to the linkage of number-words to their quantity meaning. It is proposed that this linkage is completed in two phases; first, children link quantities to words with imprecision (Level IIa), then the precise number-word is linked to its exact quantity concept and children can distinguish which of two consecutive number-words refers to a larger quantity (Level IIb). QNCs Level III refers to children's ability to use exact number-words to describe the outcome of adding or subtracting one quantity and another. Krajewski and Schneider (2009) propose that children's verbal skills contribute to their performance on QNCs Level I because these are mainly a verbal process and do not require understanding of quantities. Children's quantitative skills and visuo-spatial STM skills would be expected to contribute to their performance on QNCs Level II because these require the linking of number-words with their quantity meaning and therefore quantity representations are needed. Children's quantitative skills and their visuo-spatial STM skills would also contribute to their performance on QNCs Level III because these require the representation and manipulation of quantities. All levels of QNCs are proposed to contribute to performance on standardised mathematical attainment tests (see figure 1.3).

Figure 1.3: Theoretical model of early arithmetical development (Krajewski, 2008)



1.3.1 Empirical testing of Krajewski and Schneider's (2009) theoretical model

Krajewski and Schneider (2009) conducted a three-year longitudinal study with German kindergarten children examining the influence of domain-general cognitive skills and early number skills on mathematical attainment. Data was collected at four different time points. At the first time point (children's mean age five years and seven months), two visuo-spatial sketch-pad (hereafter VSSP) measures and two phonological awareness measures were obtained. At the second time point (children's mean age five years and eleven months), three measures of children's basic verbal number skills (ability to recite the number word sequence forwards and backwards and to identify individual elements in the sequence) and a number naming measure (articulating the number-word matching various Arabic numerals) were obtained. These tasks were expected to provide valid measures of QNCs Level I. At the third time point (children's mean age six years and seven months), children were administered three groups of tasks assessing different aspects of their numerical knowledge (non-symbolic quantity comparison skills, correct use of number-words to non-symbolic quantities in a fixed sequence, symbolic quantity comparison skills and their skills at mapping Arabic numerals to their corresponding quantities). These tasks were expected to provide valid measures of QNCs Level II. At this time point, children were also asked to verbally report the numerical difference between pairs of arrays of dots and to represent with concrete objects the numerical outcome of a verbally presented arithmetical problem. These tasks were expected to provide valid measures of QNCs Level III. At the fourth time point (children's mean age eight years and eight months), children's arithmetic, applied mathematics and geometry knowledge was assessed with a modified version of the German Mathematics Test (hereafter DEMAT 2+) (Krajewski, Liehm, & Schneider, 2004).

In line with the predictions from their model, individual differences in phonological awareness significantly predicted children's performance on QNCs Level I tasks while children's individual differences in VSSP functioning failed to do so. In contrast, variation in VSSP functioning significantly predicted performance on the tasks representing QNCs Level II and Level III but not their performance on tasks representing QNCs Level I. QNCs Level I and QNCs Levels II and III made unique contributions to children's mathematical attainment in Grade 3. Thus, phonological awareness and VSSP functioning related differently to the three Levels of QNCs. Phonological awareness predicted the later acquisition of number-words and number sequence reciting skills and also contributed to verbal number tasks where representations or manipulations of quantities were not needed. Children's VSSP functioning predicted later ability to perform number tasks that demanded the representation and manipulation of abstract quantities.

Krajewski and Schneider's (2009) theoretical model extends previous research in various ways. First, it identifies three distinct developmental levels of early number skills, providing a more comprehensive, specific and coherent set of early number skills than LeFevre et al. (2010). The distinction between numerical sequence meaning (number-word reciting) and numerical cardinal meaning (quantity to number-word linkage) is supported by other developmental models of early mathematical competence (Fuson, 1988, 1992; Gelman & Gallistel, 1978; Wynn, 1992). These argue that numerical sequence meaning refers to being able to recite the number-word sequence and acknowledging the position of a number-word in the number sequence without necessarily accessing its cardinality meaning (Fuson, 1992; Gelman & Gallistel, 1978). In contrast, the numerical cardinal meaning refers to acknowledging the exact number of discrete items in a collection (Fuson, 1988, 1992). Therefore, these models consider rote citation of the number-words and understanding the absolute numerical value of a set of items distinct developmental number skills. This distinction is also supported by neuropsychological data in adults (Dehaene & Cohen, 1997; Delazer & Butterworth, 1997). Thus, the three distinct developmental levels of QNCs proposed by Krajewski and Schneider (2009) seem more appropriate than the distinction between symbolic number skills and non-verbal arithmetic proposed by LeFevre et al. (2010). Also, unlike LeFevre et al. (2010), Krajewski and Schneider's (2009) study examines the longitudinal contribution that children's cognitive precursors make to early number skills and uses latent variables to represent children's domain-general cognitive skills. However, Krajewski and Schneider (2009) used only one mathematical attainment measure and despite including children's early quantitative skills in their theoretical model as a cognitive predictor of later number skills and mathematical attainment, the predictive power of quantitative skills was not empirically tested.

1.4 STRENGTHS AND WEAKNESSESS OF THE PATHWAYS TO EARLY MATHEMATICS MODEL AND KRAJEWSKI AND SCHNEIDER'S MODEL OF EARLY ARITHMETICAL DEVELOPMENT

The triple-code model of number processing (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003) provides a solid comprehensive framework for exploring both the independent and relative contributions that quantitative, verbal and visuo-spatial STM skills make to different number skills and to mathematical attainment measures. This model suggests that domain-specific and domain-general cognitive skills make independent contributions to numerical performance and that the extent of their contributions varies in relation to the individual number task demands. The pathways to early mathematics model (LeFevre et al., 2010) and the theoretical model of early arithmetical development (Krajewski & Schneider, 2009) suggest that these two key aspects of the triple-code model of number processing are also applicable to early mathematics development. These two models were partially or fully tested in young children using longitudinal data in their respective studies. However, there are key methodological differences in the way these two models were tested.

Firstly, although both theoretical models consider quantitative skills to be key cognitive precursors of mathematics development, Krajewski and Schneider (2009) failed to include measures of these skills in their study and LeFevre et al. (2010) only included a measure of subitising skills, but not tasks tapping children's ability to make approximate numerical judgements. In addition, the subitising task employed by LeFevre et al. (2010) is not purely quantitative, as it demanded a number-word response from the children and therefore verbal skills and pre-existing knowledge of the formal number system were involved to some extent. Also, combined measures of cognitive skills and number tasks' performance were used to examine how verbal and quantitative skills relate to the different mathematical attainment measures. Therefore these variables could be contaminated by other domain-general cognitive skills such as memory or language. Secondly, while LeFevre et al. (2010) distinguish between symbolic number tasks and non-verbal arithmetic tasks, Krajewski and Schneider (2009) clearly provide a more coherent set of early number skills by proposing three distinct developmental levels of early number skills. This more detailed distinction aligns better with previous models of mathematical development (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1992) and is also supported by neuropsychological data (Dehaene & Cohen, 1997; Delazer & Butterworth, 1997). Thirdly, LeFevre et al. (2010) used a single-task measure to tap children's visuo-spatial STM skills which could potentially be contaminated by the specific task demands. In contrast, Krajewski and Schneider (2009) used latent variables to reflect children's domain-general cognitive skills, which may provide more accurate measures of these skills because they comprise the common variance from different tasks tapping the same construct and therefore reduce the measurement error from the specific single task demands (Bowey, 2005). Fourthly, Krajewski and Schneider (2009) include only one standardised mathematical attainment test as an outcome measure while LeFevre et al. (2010) include distinct standardised and research-based mathematical knowledge measures and therefore provide empirical evidence of the relative contribution that quantitative, verbal and visuo-spatial STM skills make to each of these outcome measures. Last, while LeFevre et al. (2010) only provide empirical evidence of the concurrent contribution of children's verbal, quantitative and visuo-spatial STM skills to their early number skills, Krajewski and Schneider (2009) provide empirical evidence of the longitudinal contribution that verbal and visuo-spatial STM skills make to children's distinct levels of early number skills between the ages of five and six years.

1.5 OVERALL AIMS AND THESIS OVERVIEW

The overall aim of the longitudinal study presented in this thesis was to examine the independent and unique contributions that young children's domain-specific and domain-general cognitive skills make to their early number skills and mathematical attainment using Krajewski and Schneider's (2009) theoretical model as a general framework. Krajewski and Schneider's (2009) model was selected over that of LeFevre et al. (2010) because it proposes three distinct developmental levels of early number skills and therefore is more in line with previous models of mathematical development (Fuson, 1988; Gelman & Gallistel, 1978; Wynn, 1992) which are also supported by neuropsychological data (Dehaene & Cohen, 1997; Delazer & Butterworth, 1997). Two distinct domain-specific systems for quantity processing have been proposed; one for small and precise magnitude representations and one for approximate magnitude representations (Feigenson et al., 2004). Current research addressing these two quantity processing systems and their relationships with early mathematical development is discussed in Chapters 2 and 3 of this thesis. The present study examined the unique and independent contributions that children's precise and approximate quantity discrimination skills make to their development of early number skills proposed by Krajewski and Schneider (2009) and to their mathematical attainment.

The relationships of verbal and visuo-spatial STM skills with early number skills and mathematical attainment were examined in both Krajewski and Schneider's (2009) and in LeFevre et al.'s (2010). Chapter 4 of this thesis discusses current research examining the impact of verbal and visuo-spatial STM skills on young children's early number skills and mathematical attainment. The present study aimed to examine the unique and independent contributions that children's visuo-spatial STM and verbal skills make to their early number skills and mathematical attainment.

Following these review chapters, Chapter 5 presents a pilot study that was conducted to test the criterion validity and reliability of the novel tasks designed to tap children's domain-specific systems for quantity processing and distinct early number skills. Refined aims are presented in Chapter 6 with an extensive rationale for the longitudinal study and its methodology. Chapters 7 and 8 present the longitudinal study results. A final extensive discussion of the findings and how the present study tested and expanded Krajewski and Schneider's (2009) work is presented in Chapter 9.

Throughout this thesis a number of abbreviations, acronyms and specific terms are used. Consequently, a glossary is provided at the end of this thesis to specify what these abbreviations, acronyms and specific terms refer to in the present work.

2. NUMBER SENSE: THE PRECISE NUMBER SYSTEM

This thesis aims to explore the separate and independent contributions that young children's verbal skills, visuo-spatial STM skills and quantitative skills make to their early number skills and mathematical attainment. Both Krajewski and Schneider's (2009) model and LeFevre et al.'s (2010) model stress that quantitative processing skills play a crucial role in early mathematical development. The concept of number sense stands for the early quantitative skills that enable us to intuitively grasp numerical information. This number sense is believed to be a domain-specific and biologically-determined skill for three main reasons. First, new-borns can already detect numerical differences in very small visual and aural arrays (Antell & Keating, 1983; Bijeljac-Babic, Bertoncini, & Mehler, 1993) and even actions (Wynn, 1996), suggesting that number sense emerges early in life before verbal abilities or formal mathematical instruction. Second, empirical evidence of the detection and discrimination of numerical information in non-human animals (Hanus & Call, 2007; Hauser, MacNeilage, & Ware, 1995; Pepperberg & Gordon, 2005) indicates the number sense is shared across species. Third, neuropsychological, electro-imagining and neuroimaging data point to the existence of specific brain networks specialized in processing numerical information (Dehaene, Dehaene-Lambertz, & Cohen, 1998) indicating that these skills have their own neural substrate.

This core knowledge perspective can be contrasted with a number of other perspectives of early cognitive development, including mathematical development. In the seminal theory of cognitive development Piaget proposed that children are born with only reflexive schemas which simply allow them to react in specific ways to specific stimuli. These schemas are later modified and developed through the processes of a number of functional invariants (e.g. assimilation and accommodation) creating sequence of clearly defined stages (sensory-motor, pre-operational, concrete operational and formal operational) in the development of knowledge and understanding. Carey (2004), as an alternative to Piaget's mechanism of functional invariants proposes a mechanism of development characterised as "bootstrapping"; which is heavily dependent on language. This view suggests that only through external symbols that need to be learnt through formal or informal instruction can our preverbal quantity representations represent natural number concepts. However, empirical findings from comparison studies between children whose culture lacks number words and English speakers suggest that number-words are not crucial in order to acquire the natural number concepts and that spatial strategies might be the ones supporting our number concepts (Butterworth & Reeve, 2008). A last alternative view comes from Geary (1995) who proposes two classes of cognitive abilities; primary cognitive abilities, which are supported by neurobiological systems that are specialised for the processing of domain-specific information, and secondary cognitive abilities that are a product of the co-operation of primary cognitive abilities that only develop through formal or informal instruction. Within the number domain, primary cognitive mathematical abilities would be our number sense, basic understanding of numerical cardinal meaning, counting and performing simple arithmetic. Secondary cognitive mathematical abilities would be the ability of mapping number-words to quantities and performing advanced arithmetic. Geary’s view strengthens the role of working memory as a key primary cognitive ability for mathematical development.

Regardless of what skills later support our preverbal quantity representations, solid theoretical views propose that our number sense is a key foundational skill upon which our basic arithmetic skills are built (Butterworth, 1999, 2005, 2010; Dehaene, 1997; Dehaene et al., 2004; Spelke & Dehaene, 1999). However, within the detection and discrimination of numerical information, precise and approximate numerical representations seem to be processed differently. This suggests the existence of two core quantification systems; the “Precise Number System” (hereafter PNS) and the “Approximate Number System” (hereafter ANS) (Feigenson et al., 2004). The PNS would represent small quantities precisely and rely on a number magnitude processing skill called “subitising” (Revkin, Piazza, Izard, Cohen, & Dehaene, 2008; Trick & Pylyshyn, 1994). The ANS would represent larger quantities imprecisely with a ratio signature in accord of Weber’s law and rely on an internal number line where numerical magnitudes are organised by size in a continuum (Dehaene, 1997; Huntley-Fenner & Cannon, 2000). Two plausible although nonexclusive theoretical proposals coexist nowadays; one proposes that our precise magnitude representations support later numerical and basic arithmetic skills (Butterworth, 1999, 2005, 2010), the other proposes that approximate magnitude representations underpin the later acquisition of early numerical and arithmetic skills (Dehaene, 1997; Dehaene et al., 2004; Spelke & Dehaene, 1999). Whilst these theoretical links have been postulated, empirical evidence examining the relationships between number sense and numerical competence remain controversial (Fuhs & McNeil, 2013; Gilmore et al., 2013; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Furthermore, so far no study has explored whether the efficiency of these two systems can explain separate unique variance in children's early number skills and mathematical attainment. Whether both of these systems explain independent and unique variance in young children's numerical competence that cannot be explained by domain-general mathematical related cognitive skills also remains to be explored.

This chapter presents a comprehensive although not exhaustive literature review of key behavioural, neuropsychological and neuroimaging studies examining adults' and children's precise enumeration skills for small quantities. It also reviews the main theories that have emerged; those proposing subitising is a domain-specific number skill, and those proposing subitising is a consequence of the limited capacity of a domain-general cognitive system. This chapter includes the limited evidence that has empirically linked children's precise numerical processing skills to their early number skills and mathematical attainment. The following chapter focuses on the ANS and its relationship with early number skills and mathematical attainment.

2.1 SUBITISING AND THE PRECISE NUMBER SYSTEM (PNS)

Subitising is defined as a cognitive process that enables the rapid, accurate and confident numerical judgment over small numerical sets without the need for *counting* (Kaufman, Lord, Reese, & Volkmann, 1949). When humans are asked to quickly and accurately enumerate visual sets of discrete items, RTs increase with the number of items presented (Akin & Chase, 1978; Chi & Klahr, 1975; Frick, 1987). If the set is only accessible for a brief period of time, accuracy drops as the set increases in number (Kaufman et al., 1949; Mandler & Shebo, 1982; Wolters, van Kempen, & Wijhuizen, 1987). Enumerating very small sets of up to three or four discrete items seems to be fast and accurate, with no significant increase in RTs and free from errors regardless of the number of items, while RTs and error rates increase gradually with every additional item for collections containing more than three or four discrete items (Akin & Chase, 1978; Chi & Klahr, 1975; Frick, 1987; Mandler & Shebo, 1982; Saltzman & Garner, 1948; Trick & Pylyshyn, 1994). These two trends reflect a drastic change in the quantification processes between numerical sets of up to three or four items and for larger numerical sets, and are seen by some as evidence of the existence of two distinct and qualitatively different quantification processes (Kaufman et al., 1949; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). Because enumerating a few discrete items is done very quickly with no significant increase in RTs per item, some researchers argue that processing the numerical information within a very small numerical set takes place in parallel, whilst enumerating larger sets is done serially, resulting in the observed increases in RTs and error rates (Schleifer & Landerl, 2011). However, this argument is not free from controversy (Balakrishnan & Ashby, 1991; Gallistel & Gelman, 1992) because RTs for the enumeration of small quantities also seem to increase with number, but this increase is very shallow (Balakrishnan & Ashby, 1991), and no discontinuity is found when subvocal articulation is prevented (Cordes, Gelman, Gallistel, & Whalen, 2001). Controversy hinges on whether the behavioural evidence is strong enough to support subitising as a domain-specific cognitive skill that underlies a unique numerical mechanism which is distinct from *counting*. Two-process theories argue that subitising is indeed a domain-specific cognitive skill distinct from counting. One-process theories argue that the discontinuities observed in RTs and accuracy rates in enumeration tasks are a consequence of the limited capacity of a domain-general cognitive skill and that both subitising and counting function under the same principles. Limitations in memory, perception and attention systems have been proposed as alternative explanations to the existence of two qualitatively different quantification processes.

2.2 BEHAVIOURAL EVIDENCE OF TWO QUANTIFICATION PROCESSES IN ADULTS

Early evidence suggesting that subitising is a domain-specific cognitive skill different from counting comes from Taves (1941), who conducted a visual enumeration task where arrays varying from two to 180 dots were presented for 200 msec. Participants had to report the number of dots presented and the degree of confidence in their responses. When arrays from one to about seven dots were presented participants were both accurate in their responses and highly confident, but these indices drastically decreased for larger quantities. Similarly, Kaufman et al. (1949) conducted a visual enumeration task where arrays ranging from one to 210 dots were presented for 500 msec. A clear discontinuity in participants’ accuracy and self-reported degree of confidence was found between arrays below and those above six dots. Interpretations in favour of the existence of two quantification processes were drawn; one process being fast, accurate and confident with a limited capacity of about six items identified as “subitising”, and the other process, *counting*, which appeared to be slower, less confident and more error-prone. Additional evidence towards a two-process theory was found by Chi and Klahr (1975), who administered a visual enumeration task to twelve adults and twelve children. For both groups, a discontinuity in the RTs slopes was found. However, the inflection point in the RTs curves for both groups occurred between three and five items. The authors supported the existence of at least two different quantification processes depending on the number of visual items presented but proposed that its limited capacity was smaller than what previously suggested (Kaufman et al., 1949). As Balakrishnan and Ashby (1991) would later note, if the criterion for accuracy is the number of items that can be enumerated correctly by the participant 50% of the time, then the subitising limit would be about seven items (Miller, 1956). However, if a stricter criterion is used, such as 99% of the time, then the subitising limit would be about three or four items. More recently, Revkin et al. (2008) administered two enumeration tasks in which two arrays were presented for 150 msec.; one presenting one to eight dots, and one presenting ten to 80 dots. Participants were faster in their responses for quantities ranging from one to eight dots. Also, while little variability was found in the RTs for quantities between one and four dots, RTs increased with any additional dot presented for arrays containing above four dots.

However, not all behavioural evidence with adults supports the existence of two distinct quantification processes for small and large collections. Saltzman and Garner (1948) found that RTs discontinuities disappear when certain features of the visual stimuli are controlled. Knowledge of the stimulus-range, practise, regularity in spacing and size of the stimuli were controlled for in their visual enumeration task presenting two to ten concentric circles for 500 msec. A consistent increase in participants' RTs was found as the number of dots presented increased, suggesting that discontinuities found in enumeration tasks reflect the methodology used. Balakrishnan and Ashby (1991) presented arrays of blocks on a computer screen and asked subjects to report the number of blocks presented. Participants' RTs and accuracy did not fit either a bilinear-two-process model that would support the existence of two distinct quantification processes or a log linear-single-process model that would indicate subitising and counting are not distinct quantification processes. Gallistel and Gelman (1992) found that RTs for small quantities also showed an increase with every additional visual item presented and interpreted the discontinuity in the RTs slopes as evidence of a shift between the usage of non-verbal tags to the usage of verbal tags in a serial counting process. Discontinuities were proposed to be due the different representations used to access the number name; subitising would only require mapping the preverbal representation (non-verbal tag) to its verbal one, while counting would require verbal retrievals, therefore taking longer time than subitising. Cordes et al. (2001) asked participants to quickly press a key as many times as was indicated by an Arabic numeral (from two to 35) presented on a screen. Subvocal articulation was prevented by participants saying “the” for each time they pressed the key. Results showed that the mean number of times participants pressed the key increased as the Arabic numeral increased. No significant differences were found between the regressions’ slopes for the number of presses for Arabic numerals within the subitising range and above, supporting the one-process theory.

2.3 ALTERNATIVE THEORIES OF THE EXISTENCE OF TWO DISTINCT QUANTIFICATION PROCESSES

Limitations in the capacity of different domain-general cognitive systems, such as attention, memory or perception have been proposed as plausible explanations for the discontinuities found in RTs and accuracy rates in visual enumeration tasks.

2.3.1 Subitising as a limited memory process

Researchers like Miller (1956) and Cowan (2001) interpret the discontinuities observed in enumeration tasks as evidence of the limited span of working memory. In accordance with this view, Klahr and Wallace (1973) postulated a three-quantification operator model in working memory, these being subitising, counting and estimating. Subitising was postulated to be used for small collections of items and to have a limited capacity of about five items. When asked to enumerate the items in a collection, short-term memory (hereafter STM) starts a serial self-terminated process to be matched with a stored subitising list in long-term memory (hereafter LTM). This list comprises a set of distinct quantitative symbols, each of which represents a cardinal numerosity. When no matches can be found between the result of the STM scan and the LTM labels, another quantification operator (counting or estimating) needs to commence in order to solve the enumeration task.

2.3.2 Subitising as a limited perceptual process

Atkinson, Campbell, and Francis (1976) found that subjects’ accuracy in enumerating collections within the subitising range fell from four to two items when the distance between the dots presented in the arrays was manipulated. Similarly, Akin and Chase (1978) administered an enumeration task controlling for compactness, symmetry, planarity and linearity in the visual stimuli (blocks). They concluded that a grouping process is responsible for the shallow RT slopes for arrays of items within the subitising range and that the visual structure of the stimuli has an impact on this grouping process. Mandler and Shebo (1982) suggested that the discontinuity found in RTs can be explained by acquired canonical patterns. This is, when a small array of dots is displayed, its items will be forming a familiar canonical shape (e.g. two dots a line, three dots a triangle, etc.) and discontinuities in RTs could be evidence of a perceptual process for the recognition of familiar patterns rather than a specific numerical skill. Simons and Langheinrich (1982) found RT discontinuities at four dots in a visual enumeration task regardless of the arrangement of the stimuli (linear, dice or random). Interpretations were that RT discontinuities are due to limitations in the perceptual process as a result of the usage of clustering, figural cues and scanning processes, instead of canonical patterns.

2.3.3 Subitising as a limited pre-attentive process

Trick and Pylyshyn (1994) proposed that subitising is the result of a limited pre-attentive mechanism based on Pylyshyn's (1989) FINSTs (Fingers of Instantiation) mechanism, "a primitive mechanism capable of individuating and dynamically indexing a small number of features (or features-clusters) in a visual field" (pg. 93). Trick and Pylyshyn (1994) suggest that subitising takes place after other pre-attention processes for external features have finished, such as colour or orientation of the stimuli, but before attention processes commence. When the visual stimuli exceeds the mechanism's limits, another process is needed (counting). Also, when attention is required to solve a visual task, the efficiency of the FINST mechanism would be compromised.

Thus, behavioural data on visual enumeration tasks with adults does not always show a clear discontinuity (Balakrishnan & Ashby, 1991; Cordes et al., 2001; Saltzman & Garner, 1948) and when it does the limited capacity of different cognitive systems has been proposed to account for the evidence often interpreted as a subitising process (Atkinson et al., 1976; Cowan, 2001; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). Nevertheless, the possible existence of two distinct quantification processes for small and larger numerical collections remains tenable (Chi & Klahr, 1975; Revkin et al., 2008).

2.4 BEHAVIOURAL EVIDENCE OF TWO QUANTIFICATION PROCESSES IN CHILDREN

Visual enumeration and discrimination paradigms have been employed to study young children's quantification processes; discontinuities in accuracy rates and RTs have been almost always reported.

2.4.1 Enumeration Paradigms

Chi and Klahr (1975) administered a visual enumeration task to a group of five- and six-year olds and a group of adults. A discontinuity in the RTs slopes between three and five items was found in both groups. The authors not only regarded this as evidence of the existence of two distinct quantification processes but also suggested their possible ontogeny. Comparable results and conclusions have been reported with children of seven-to-eight years (Svenson & Sjöberg, 1978) and with five-year-olds (Benoit, Lehalle, & Jouen, 2004) who were asked to report the number of dots presented on a screen. Fischer, Gebhardt, and Hartnegg (2008) compared 156 children with arithmetic skills' problems with 219 control children. Participants' ages in both groups ranged from seven to seventeen years. For both groups accuracy and speed were better for arrays displaying one to four dots and gradually deteriorated for larger quantities. Control children were faster and more accurate in the visual enumeration task than children presenting arithmetic difficulties. Interestingly, accuracy and speed improved with age in both groups. More recently, Schleifer and Landerl (2011) conducted a visual enumeration task with eight-, eleven- and fourteen-year-olds and adults in which RTs and eye-movement behaviour were recorded. For both variables they found a discontinuity between one to four dots and five to ten dots, providing evidence not only for the existence of two distinct enumeration processes but also suggesting that subitising is a parallel process and counting is a serial one.

However, Gelman and Tucker (1975) used an enumeration paradigm to examine whether three-, four- and five-years-olds engaged or did not in serial counting when asked to enumerate small collections ranging from two to five discrete items. A total of 144 children divided into three groups according to their age were asked to report the number of items presented on a card for either one sec., five sec. or 60 sec. Data analyses revealed an increase in accuracy with age and that the longer the stimuli were presented, the more accurate subjects were in their responses. A trend to count overtly regardless of the set size was observed in the younger group, even when time exposure was very limited. Older children (four- and five-year-olds) were more likely to engage in overt counting when larger sets were presented, but not with small sets. The authors concluded that under certain conditions children have a tendency to rely on counting rather than other quantification processes.

2.4.2 Discrimination paradigms

Discontinuities in accuracy and RTs have also been observed in young children using discrimination paradigms. Starkey and Cooper (1995) asked children aged between two and five years to identify the larger of two arrays of dots between an array presented throughout the trial and another array that would show for 200 msec. Accuracy analyses revealed that younger children in the sample seemed to be subitising arrays containing one to three dots, and older children arrays of one to four dots. Trick, Enns, and Brodeur (1996) used a discrimination paradigm with different age groups (six-, eight-, ten-, 22- and 72-year-olds) who were asked to determine if the array of dots presented had the same number of dots or one dot more than a sample array by pressing different keys. Stimuli pair displays in the discrimination condition were one versus two, three versus four, six versus seven and eight versus nine. Data analyses revealed an increase in RTs with every additional item presented, however, the slopes for stimuli displaying six to nine items were significantly steeper than the slopes for one to four items. Within the younger groups (six-, eight-, ten- and 22-year-olds), it was also found that RTs decreased with age for both small and large arrays. Interestingly, this decrease was more pronounced for arrays in the counting range.

Thus, children seem to be faster and are more accurate when enumerating small visual sets of items than when enumerating sets of four items or more (Starkey & Cooper, 1995; Svenson & Sjöberg, 1978) and also when having to identify the larger of two arrays containing up to three or four items than larger arrays (Starkey & Cooper, 1995; Trick et al., 1996). Behavioural evidence obtained with children also suggests that subitising is a developmental skill because speed and the number of discrete items they can subitise increase with age during childhood (Benoit et al., 2004; Fischer et al., 2008; Trick et al., 1996). Comparison studies contrasting children’s and adults’ performance in enumeration tasks show that adults are faster and more accurate than children but both groups show discontinuities in RTs and accuracy rates for the enumeration of small and large sets of visual items (Starkey & Cooper, 1995; Trick et al., 1996). Nevertheless, not all studies conducted with children support the interpretation of two distinct quantification processes (Gelman & Tucker, 1975) and it is noteworthy that a wide range of paradigms and methodologies have been used to assess subitising skills in children and adults.

2.5 BEHAVIOURAL, NEUROPSYCHOLOGICAL AND NEUROIMAGINING EVIDENCE OF TWO QUANTIFICATION PROCESSES IN CHILDREN AND ADULTS

Neuropsychology often studies disassociations, cases of patients with damaged or malfunctioning brain structures who are unable to perform certain task(s). If a patient with a damaged or malfunctioning brain structure is unable to perform a particular task, but can perform any other task, it is reasonable to believe that the damaged brain structure is responsible for the functions needed to perform that particular task. When a patient with certain brain damage or brain malfunction cannot perform a certain task, but is capable of performing other(s) and there is another patient with a different damaged or malfunctioning area who presents the opposite behavioural pattern, it is referred to as a double disassociation. Double disassociations are the strongest evidence that a brain structure might be responsible for a certain cognitive function because most certainly the impossibility of performing one task or the other is not due to the level of difficulty of the task but due to the area presenting damage or malfunction. Neuroimaging techniques measure brain activity to identify the structures that activate during certain cognitive processes. Two of the most commonly used techniques are Positron Emission Tomography (hereafter PET) and Functional Magnetic Resonance Imaging (hereafter fMRI). PET monitors the trace of glucose or fludeoxyglucose for a limited time. FMRI traces oxygenated and deoxygenated haemoglobin and offers a better resolution than PET.

Gerstmann syndrome is caused by left parietal lobe damage and characterised by the presence of difficulties in dealing with numbers (acalculia or dyscalculia), writing (agraphia or dysgraphia), naming or pointing the fingers (digital agnosia) and distinguishing between left and right. Therefore, the brain networks responsible for our understanding of numbers, letters, fingers and space seem to be hardwired in the left parietal lobe of the brain. However, neuropsychological and neuroimaging evidence suggests that only numbers above four are processed on the left hemisphere. For instance, Cipolotti, Butterworth, and Denes (1991) studied the case of C. G. who presented hypo-density in the left fronto-parietal region of her brain and was unable to perform any task that involved the manipulation of the numbers above four in any modality, despite her intelligence quotient (hereafter IQ) and other neuropsychological signs being normal. Although C. G. was unable to subitise, she preserved small numbers within the subitising range but not larger numbers. Pasini and Tessari (2001) asked subjects to enumerate different dot patterns ranging from one to sixteen dots. Then an Arabic numeral was presented in either the left or right side of the screen which either matched the number of dots or not. Faster responses were given when stimuli were presented on the left for quantities in the subitising range and on the right when stimuli exceeded the subitising range. On the assumption that faster responses are given when information is presented in the visual field controlled by the hemisphere processing that type of information, the authors suggested that processes of subitising occur in the right hemisphere and counting processes occur in the left hemisphere. Arp, Fagard, and Taranne (2006) compared the performance of four-to-eight cerebral-palsied and control children in a subitising task. Cerebral palsy children who presented a right-hemisphere lesion had a more limited subitising range than those who presented left-hemisphere or bilateral damage, again suggesting that certain brain networks in the right hemisphere could be responsible for subitising skills.

It has also been suggested that subitising does not rely on visual attention processes or at least not on the same processes as counting. Vuilleumier and Rafal (1999) asked subjects with right parietal lesions to perform two different tasks with the same stimuli; an enumeration task where a collection of shapes ranging from one to four was presented and a localization task where they had to report on which side the stimuli were presented. It was found that subjects performed better when asked to enumerate than when they were asked to simply indicate the location of the stimuli, even when stimuli in the enumeration task were presented in their neglected visual field. Bull, Blatto-Vallee, and Fabich (2006) compared performance on a subitising task consisting of arrays presenting one to six dots for 50 msec. in twenty deaf young adults and twenty controls. Because deaf adults seem to have better visual attention skills (Corina, Kritchevsky, & Bellugi, 1992; Rettenback, Diller, & Sireteaunu, 1999), it was hypothesized that if subitising relies on visual-attention processes deaf participants should perform better than controls. Results showed that accuracy was high for both groups within the subitising range and decreased when more than four dots were presented, but no significant advantage was found for the deaf group in the subitising task. Sathian et al. (1999) conducted a PET study where participants were asked to enumerate either zero, one, one to four or five to eight targets (vertical bars) presented among several horizontal bars in a display. RT data analysis confirmed that participants were subitising when one to four targets were presented. PET results revealed that when participants were engaged in counting, they activated networks from more brain regions compared to when they were subitising. Fink et al. (2001) found that subitising activates visual-attention brain networks but different from the ones that activate in other visual-attention tasks requiring decisions based on shapes. They compared the fMRI of nine participants on a quantity discrimination task and a shape visual yes-no decision task. In the quantity discrimination task, subjects were shown arrays of three, four or five dots for 300 msec. and were asked to report if four dots were presented. For the shape task, subjects were asked to indicate if the dots were forming a square shape. Behavioural data analyses showed that subjects' RTs increased as the number of dots presented increased and there was a discontinuity in RTs slopes, barely increasing for each additional item in the subitising range and showing a sharp increase for larger quantities. Their fMRI data showed that distinct brain networks activated depending on the tasks' demands; the quantity discrimination task activated the left inferior frontal cortex and visual processing areas while the shape task activated the temporo-patietal cortex, medial posterior cingulate cortex and left dorsolateral parietal cortex.

In addition, Nan, Knosche, and Luo (2006) asked participants to report the number of rectangles (targets) and ignore the circles (distracters) in displays of collections of discrete items. The number of targets in the displays varied from one to six and the number of distracters was either zero, equal to the number of targets or twice the number of targets presented. Behavioural results showed a consistent increase in the RTs with increasing number of targets and distracters and no clear discontinuity between small and large number of targets or distracters. However, when four to five targets were presented, participants found more difficulties to perform the task than when one to three targets were presented, suggesting that counting was more affected by the presence of distracters than was subitising. Their EEG results showed that counting and subitising seem to share many brain networks, but also that while counting seems to recruit spatial attention resources subitising does not.
Subitising and counting also seem to be qualitatively distinct quantification processes. For instance Dehaene and Cohen (1994) compared the performance of five patients with simultanagnosia with that of five matched controls in enumeration and visual search tasks. Simultanagnosia is a visual deficit that renders patients unable to perceive scenes as a whole although they can perceive individual objects in those scenes. Visual serial counting is

impaired in these patients because they cannot "keep track" of the items already counted. Simultanagnosia patients were able to subitise sets of one and two items and in some cases three discrete items whilst their performance was significantly worse than controls' in visual search tasks, suggesting that subitising might not be a serial process. Piazza, Giacomini, Le Bihan, and Dehaene (2003) asked subjects to perform an enumeration task and a colour naming task. Results showed that the same brain areas were active for the subitising and colour naming task but a sharp bilateral increase in fronto-parietal areas occurred in the enumeration task for quantities of four and above. These studies suggest that the process of subitising may rely on parallel mechanisms. However, not all neuroimaging evidence supports the subitising-counting dichotomy. Piazza, Mechelli, Butterworth, and Price (2002) administered a visual baseline task and a visual enumeration task consisting of arrays ranging from one to nine dots presented for 2,500 msec. Although RT slopes were sharper for the six-to-nine dot arrays, PET results revealed no differences in brain activation patterns for arrays presenting one to four dots or arrays presenting six to nine dots. Brain activation increased as the number of dots presented increased and brain networks in the extrastriate middle occipital and intraparietal areas were more active for counting than for subitising; however these were also more active for subitising than for the baseline condition. The authors raised the possibility that the same neural system could be involved in both counting and subitising.

Cognitive neuropsychology and neuroimaging studies have reached a common and more unitary agreement for the distinction of subitising and counting as qualitatively different quantification processes, although not absolutely free from controversy (Piazza et al., 2002). The vast majority of evidence suggests that the brain mechanisms responsible for subitising and counting might be embodied in different hemispheres; subitising processes seem to be hardwired in the right hemisphere while counting seems to be hardwired in the left hemisphere (Arp et al., 2006; Pasini & Tessari, 2001; Vuilleumier & Rafal, 1999). Research also finds that subitising does not (or at least not totally) depend on other brain networks responsible for visual-attention processes (Arp et al., 2006; Bull et al., 2006; Dehaene & Cohen, 1994), raising the possibility of the existence of very specialised brain networks for processing small numerical magnitudes. While subitising activates brain networks mainly in visual processing areas, counting seems to demand the activation of more brain networks (Piazza et al., 2002; Sathian et al., 1999), suggesting that enumerating small numerical magnitudes is a very automatic process that makes little additional cognitive demands. These differences speak in favour of the two distinct enumeration processes dichotomy and suggest that processing the numerical information of small quantities might be done in parallel while enumerating larger quantities might require engaging in a serial counting process.

2.6 SUBITISING AND EARLY MATHEMATICAL DEVELOPMENT

The defective number module hypothesis (Butterworth, 1999, 2005, 2010) argues that mathematics learning difficulties might occur due to a selective impairment of the biologically-determined domain-specific capacity to understand and represent small numerical magnitudes. Despite this theoretical link, only a limited body of research has associated small number processing with numerical competence in typically developing children. The defective number module hypothesis is mainly supported by comparison studies conducted with children presenting mathematical learning difficulties (hereafter MLD). For example, Landerl, Bevan, and Butterworth (2004) compared four groups of eight- and nine-year-olds diagnosed with either dyslexia, dyscalculia, or both disorders, with matched controls in diverse numerical tasks which included a visual enumeration task. Dyscalculic children were those who scored more than three standard deviations above the mean RTs of the control group on a computasired arithmetic task comprising single-digit additions, subtractions and multiplications. Dyscalculics were slower than dyslexics and controls at enumerating arrays ranging from one to three dots, and from four to ten dots and were also significantly slower at naming one- and two-digit numerals presented on a screen and at reading three-digit numerals from a sheet of paper after controlling for their colour naming speed. Results were interpreted in favour of the defective number module hypothesis (Butterworth, 1999, 2005, 2010). Fischer et al. (2008) conducted a comparison study with seven- to seventeen-year-olds presenting arithmetic skills problems and controls. Children were assigned to the low arithmetic skills group if they performed poorly on a standarised arithmetic skills test but not on reading and spelling tasks. Not only it was found that controls were faster and more accurate in a visual enumeration task than those presenting arithmetic difficulties but it was also found that the difference in performance between both groups increased with age.

Schleifer and Landerl (2011) administered a visual enumeration task to a group of second, third and fourth graders with dyscalculia and matched controls. Children were considrered dyscalculic if their performance was below 1.5 standard deviations from the age norms on a standardised test of arithmetic skills. Although in both groups and for every age group they found a discontinuty in RTs between the subitising range (one to three dots) and larger arrays, dyscalculics' RTs slopes for the subitising range were significantly steeper compared to controls'. In addition RT slopes decreased with age and were very similar for larger arrays in both goups, suggesting that MLD children present a specific and ongoing difficulty in processing only small quantities at high speed.

One study has examined the longitudinal contribution that children's enumeration speed for numerical collections of up to three discrete items makes to their later mathematical attainment (LeFevre et al., 2010). This study is described in detail in Chapter 1 of this thesis. Four- and five-year-olds' speed in enumerating one to three dots predicted their concurrent ability to manipulate quantities non-verbally and a combined masure of children's enumeration speed and their performance on a non-verbal arithmetic task predicted their performance on two standarised tests of mathematical attainment and on three research-based mathematical attainment measures two years later.

Taken together these studies provide evidence that faster enumeration of small quantities is related to children's numerical comptence and suggests that children's ability to process small quantities might be a foundational skill unpon which number and arithmetic skills are built. However, although all these studies used RTs as a common index of subitising skills, only Landerl et al. (2004) controlled for a naming speed measure. Some of these studies did not control for age (LeFevre et al., 2010) despite the evidence from previous research that age has an effect on subitising speed (Fischer et al., 2008). In addition, all tasks employed to assess subitising skills demanded a certain degree of verbal skills and also knowledge of the formal number system, thus not providing pure measures of children's quantitative skills.

2.7 DISCUSSION

A large body of research has consistently found discontinuities in accuracy rates and RTs slopes for the enumeration of small and large sets of discrete visual items in children and adults (Chi & Klahr, 1975). The vast majority of neuropsychological and neuroimaging studies also suggest that subitising and counting are qualitatively different processes and that they recruit different neural networks (Piazza et al., 2003). Subitising seems to activate fewer brain structures and to require less attention resources than counting, suggesting that it is an automatic and parallel process. In contrast, counting seems to be a serial process that makes higher attention demands (Fink et al., 2001; Sathian et al., 1999). Research conducted with children suggests that subitising and counting are developmental skills that improve with age (Fischer et al., 2008). In addition, subitising skills have been shown to predict children's concurrent performance on specific number skills and to contribute to later mathematical attainment performance (LeFevre et al., 2010).

It is worth noting that despite the vast literature on this small quantity processing skill, controversial arguments regarding subitising remain unresolved. First, the origin of subitising remains unclear; whether it is a domain-specific cognitive skill or it depends on a domain-general cognitive system. Second, its relationship with early number skills and mathematical attainment during early stages of schooling needs to be examined in a more rigorous way. This thesis explores the unique and independent contributions that young children's speed in making accurate two-choice numerical judgements about non-symbolic numerical collections of up to three discrete items make to their later number skills and mathematical attainment. Krajewski and Schneider's (2009) theoretical model proposes that children's foundational quantity discrimination skills contribute to specific early number skills that require quantity representations (quantity to number-word linkage, QNCs Level II) and quantity manipulations (understanding of relationships between numerical quantities, QNCs Level III). LeFevre et al. (2010) found that children's speed in enumerating small collections of up to three dots predicted children's concurrent non-verbal arithmetic performance and moreover contributed to specific mathematical attainment measures two years later. One of the aims of this thesis was to examine whether children's speed in making non-symbolic numerical judgements over small quantities contributes to their later performance on any of the three levels of early number skills proposed by Krajewski and Schneider (2009) and/or to distinct aspects of later mathematical attainment.

3. NUMBER SENSE: THE APPROXIMATE NUMBER SYSTEM

In the previous chapter precise quantity discrimination skills and studies examining their relationship with early mathematical development were discussed. However, it has been proposed that alongside precise quantity discrimination skills, humans are also endowed with approximate quantity discrimination skills (Xu, 2003). Precise and approximate quantity discrimination skills seem to rely on distinct core systems for quantity processing (Feigenson et al., 2004). This chapter focuses on our approximate quantity discrimination skills and their relationship with early number skills and mathematical attainment. It has been proposed that our approximate numerical skills rely on mental magnitudes that are represented in an organised mental continuum (Dehaene & Changeux, 1993; Moyer & Landauer, 1967) and that activate automatically in the presence of numerical information (Dehaene & Akhavein, 1995). These analogical numerical representations are believed to form a “number line” where small numerical magnitudes would be represented very precisely and vagueness would increase proportionally for larger numerical magnitudes (Dehaene & Cohen, 1991, 1995; Dehaene, Dupoux, & Mehler, 1990). Evidence of this mental number line comes from three main behavioural effects which had been observed. First, the longer the numerical distance between two numerical representations, the easier it is to discriminate them (numerical distance effect, hereafter NDE) (Moyer & Bayer, 1976). Second, as numerosity increases, the greater the difficulty in discriminating two numerical representations of equal numerical distance (numerical size effect, NSE) (Moyer & Landauer, 1967). Third, large numbers are associated with right side responses and smaller numbers are associated with left side responses in Western adults (Spatial-Numerical Association of Response Codes, SNARC) (Dehaene, Bossini, & Giraux, 1993). These three effects have been observed with both symbolic and non-symbolic number tasks (Dehaene et al., 1993), across stimuli formats (Barth, La Mont, Lipton, & Spelke, 2005; Hauser, Dehaene, Dehaene-Lambertz, & Patalano, 2002) and even when just a same-different judgment is required (Duncan & McFarland, 1980), suggesting that humans rely on this analogue mental number line for multiple number tasks.

However, unlike precise quantity discrimination skills (discussed in detail in Chapter 2) approximate numerical discriminations do not seem to depend on the absolute number of items presented but on the numerical ratio difference between the numerical magnitudes; the smaller the ratio difference between two numerical representations, the harder it is to discriminate them and as the numerical difference increases, discrimination improves proportionally in accord with Weber's Law (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Whalen, Gallistel, & Gelman, 1999). Also, unlike small non-symbolic quantity discrimination skills, non-symbolic approximate numerical discriminations do not seem to be affected by continuous features of the stimuli such as surface area or contour length (Xu et al., 2005, but see also Solstesz et al., 2010). These two distinctive signatures for quantity discrimination suggest that we possess distinct abstract representations for precise and approximate numerical magnitudes (Feigenson et al., 2004; Xu, 2003; Xu, Spelke, & Goddard, 2005). This distinction is also supported by neuropsychological and neuroimaging data (Dehaene, Spelke, Stanescu, Pinel, & Tsivkin, 1999; Lemer, Dehaene, Spelke, & Cohen, 2003). It has been proposed that our magnitude representations for approximate numerical discriminations underpin the later acquisition of early numerical and arithmetic skills (Dehaene, 1997; Dehaene et al., 2004; Spelke & Dehaene, 1999). A limited body of evidence has explored the relationship between approximate quantity discrimination skills and early number skills (Mussolin, Nys, Leybaert, & Content, 2012), although more work has examined the relationship between non-symbolic approximate quantity discrimination skills and children's performance on standardised mathematical attainment tests. Some studies find that approximate non-symbolic quantity discrimination skills are associated with mathematical attainment (Halberda, Mazzocco, & Feigenson, 2008) and even are causal predictors of later mathematical attainment in young children (Libertus, Feigenson, & Halberda, 2013). However these findings have not been consistently replicated (Fuhs & McNeil, 2013; Gilmore et al., 2013).

This chapter presents a comprehensive although not exhaustive literature review of behavioural, neuropsychological, electro-imagining and neuroimaging studies that support the distinction of abstract representations for precise and approximate numerical magnitudes. It also reviews the contrasting findings in favour and those against the association between non-symbolic approximate quantity discrimination skills and early number skills and mathematical attainment in children, including comparison studies with MLD children and correlational, retrospective and longitudinal studies conducted with typically developing children. Finally, it addresses the methodological issues that could account for the non-converging findings across studies.

3.1 BEHAVIORAL EVIDENCE OF THE APPROXIMATE NUMBER SYSTEM IN ADULTS

Moyer and Landauer (1967) asked young adults to identify the larger of two single-digit Arabic numerals presented simultaneously. RTs and error rates decreased as the numerical distance between the numerals increased suggesting that subjects were transcoding the numerical symbolic representations into analogue magnitude representations organised by size. The same behavioural pattern have been consistently found (Banks, Fujii, & Kayra-Stuart, 1976; Parkman, 1971) including when comparing two-digit Arabic numerals (Dehaene et al., 1990; Hinrichs, Yurko, & Hu, 1981), using priming paradigms (Marcel & Forrin, 1974), with cross-matching tasks between Arabic numerals and dot patterns (Buckley & Gillman, 1974), between Arabic numerals and number-words (Dehaene & Akhavein, 1995), between Arabic numerals and numbers retained in memory (Sekuler, Rubin, & Armstrong, 1971) and even when numerical information is irrelevant to the task (Dehaene & Akhavein, 1995; Henik & Tzelgov, 1982; Tzelgov, Meyer, & Henik, 1992).

Thus, distance and size effects seem to occur irrespective of the input and output modality of the stimuli and support the contention that analogue magnitude representations organised by size are employed for approximate numerical judgements. Distance and size effects are also reflected in adults' accuracy rates and RTs when performing approximate numerical judgements over non-symbolic quantities (Halberda & Feigenson, 2008). If the stimuli to be judged are visually presented for an unlimited time, RTs increase as the numerical ratio difference between the two numerical magnitudes decreases (e. g. Hinrichs et al., 1981; Buckeley & Gilman, 1974). When the stimuli are presented for a limited time which prevents verbal counting, accuracy decreases as the numerical ratio difference between the two numerical magnitudes decreases in accord with Weber's Law (van Oeffelen & Vos, 1982). This ratio-dependent feature when making approximate numerical discriminations has even been found in adults who do not have arithmetical training and who have a very restricted number lexicon (Pica, Lemer, Izard, & Dehaene, 2004) suggesting that the ability to make approximate numerical judgements does not depend on knowledge of the formal number system or other domain-general cognitive skills such as language. It has also been found that Western adults give faster left-side responses when presented with small magnitude representations and faster right-side responses when presented with large magnitude representations (see Fias & Fisher, 2005; Wood et al., 2008, for reviews) suggesting that these analogue magnitude representations organised in a continuum also have a spatial dimension (Dehaene et al., 1993). Because this effect is compromised in the presence of visuo-spatial working memory demands when adults perform symbolic magnitude comparisons, it has been suggested that visuo-spatial WM skills might play a role in numerical magnitude representations, at least in adults (van Dijck, Gevers, & Fias, 2009).

3.2 BEHAVIORAL EVIDENCE OF THE APPROXIMATE NUMBER SYSTEM IN CHILDREN

Six-month-old infants seem to possess different representations for precise and approximate numerical magnitudes (Xu, 2003; Xu et al., 2005) and they can already detect numerical changes in the stimuli even when other continuous variables have changed but not vice versa (Brannon, Abbot, & Lutz, 2004). However, their discrimination fails if the ratio difference between the numerical sets decreases from 2.0 to 1.5 (Lipton & Spelke, 2003; Xu et al., 2005; Xu & Spelke, 2000). In children RTs and error rates decrease as the numerical difference between the numerical representations increase with symbolic (Duncan & McFarland, 1980; Sekuler & Mierkiewicz, 1977) and with non-symbolic (Barth et al., 2005; Halberda & Feigenson, 2008) numerical representations. In addition, Halberda and Feigenson (2008) found that the numerical ratio difference that can be accurately discriminated increases with age; being 3:4 for pre-schoolers and gradually increasing up to 10:11 in adulthood.

3.3 NEUROPSYCHOLOGICAL, ELECTRO-IMAGINING AND NEURO-IMAGINING EVIDENCE OF THE APPROXIMATE NUMBER SYSTEM

Single and double-dissociations for exact and approximate numerical abilities have been reported. Warrington (1982) reported the case of D.R.C., a 61-year-old man who presented a haematoma in the left posterior parieto-occipital region. His intelligence, memory and ability to read, write and remember numbers were all normal. D.R.C could estimate the number of dots visually presented, approximate the solution to arithmetic problems and accurately judge the larger of two-digit Arabic numerals. However, he was significantly slowed and inaccurate at performing exact arithmetic, using laborious and inefficient strategies to arrive to the answer. A similar case was reported by Dehaene and Cohen (1991). N.A.U. was a 41-year-old man who presented hypo-density in the left temporo-parieto-occipital area and although he could not do simple one-digit arithmetic calculations, he was capable of judging whether the result of an arithmetic problem was "approximately" correct or not. Only a few years later, Dehaene and Cohen (1997) reported a double disassociation; B.O.O. was a 60-year-old right-handed woman with a left subcortical lesion and M.A.R. was a 61-year-old left-handed man with a lesion in his left inferior parietal lobe whose hemispheric specialization was inverse to the right-handed standard. While B.O.O.'s performance on quantity knowledge tasks was within normal limits, M.A.R. showed a clear impairment in number judgments, proximity tasks and bisection of numerical intervals. However, M.A.R.'s performance on verbal number tasks was significantly better than B.O.O.'s, who was unable to recite number sequences, the alphabet or arithmetic tables correctly. Lemer et al. (2003) reported a similar double disassociation; a patient with left intraparietal and right occipital brain damage who performed very poorly in approximation, symbolic and non-symbolic magnitude comparison tasks but who could perform exact arithmetical tasks and a patient with left temporal hypo-metabolism who had intact approximation abilities but whose abilities for exact calculations were severely affected.

Neuroimaging and electro-imagining studies also support such a distinction. For instance Dehaene et al. (1999) conducted a study in which bilingual subjects were trained on either exact or approximate two-digit additions in one language and who were then tested on trained and untrained additions in the other language. Behavioural results showed that while subjects' performance on exact arithmetic for the non-trained additions was affected, the performance on untrained approximate arithmetic was not, being very similar to performance on trained approximate additions. Thus exact arithmetic seemed to recruit verbal sources while approximate arithmetic seemed to be language-independent. Neuroimaging results obtained with fMRI and event-related potential (records electric potentials at high speed) also revealed different brain activation patterns when participants were processing exact and approximate arithmetic; while the latter showed greater activation of the bilateral parietal lobes, the former activated strictly left hemisphere regions. Stanescu-Cosson et al. (2000) found an increase in the activation of bilateral intra-parietal areas when subjects were presented with large numbers and asked to perform exact calculations. Piazza, Mechelli, Price, and Butterworth (2006) found that the right intra-parietal cortex was significantly more active during estimation tasks than in exact counting tasks regardless of the modality of presentation of the stimuli. Pinel, Dehaene, Rivière, and Lebihan (2001) asked subjects to decide whether the visually presented Arabic numeral or number-word was larger or smaller than 65 by pressing a key. PET results revealed a significantly increased activation in the right intra-parietal areas with decreased numerical distance between stimuli and target number, regardless of the stimuli-format presentation. Thus, neuroimaging and electro-imagining evidence also support the existence of two distinct routes for number processing; the left intra-parietal cortex that activates when performing exact numerical tasks and the right intraparietal cortex that activates particularly in the presence of approximate numerical tasks.

3.4 APPROXIMATE QUANTITY DISCRIMINATION SKILLS AND EARLY MATHEMATICS DEVELOPMENT

It has been proposed that early approximate numerical discriminations underpin the later acquisition of early numerical and arithmetic skills (Dehaene, 1997; Dehaene et al., 2004; Spelke & Dehaene, 1999) and that poor approximate numerical representations are a core aspect of MLD (Mazzocco et al., 2011). Approximate numerical discriminations are usually assessed with non-symbolic approximate quantity discrimination tasks in young children. In these tasks children are asked to select the larger of two non-symbolic collections of discrete items by either naming the colour of the items in the larger collection (e. g. Libertus et al., 2011) or by pressing a left or right key depending on the side on which the larger array is presented (e. g. Piazza et al., 2010). These tasks are accepted measures of ANS precision in young children. However, despite the extensive body of research examining the relationship between approximate quantity discrimination skills and children's early mathematics development, this link remains controversial. Some theoretical views propose that an underlying deficit in the representational system for approximate numerical magnitudes is a core aspect of MLD (the defective number module hypothesis, Butterworth, 1999, 2005, 2010). An alternative theoretical view proposes that MLD are due to a deficit in accessing the numerical information from symbolic numerical representations rather than a deficit in the representational system for approximate numerical magnitudes per se (the access deficit hypothesis, Rousselle & Noel, 2007). This last view is supported by studies that find a relationship between children's symbolic approximate quantity discrimination skills and performance on standardised mathematical attainment measures, but which find no relationship between children's non-symbolic approximate quantity discrimination skills and their performance on standardised mathematical attainment measures (e. g. Rousselle & Noel 2007).

Furthermore, some authors do not attribute the relationship between performance on ANS tasks and mathematical attainment to the efficiency of children's analogue representational system for numbers (Soltész et al., 2010). This last theoretical view suggests that non-symbolic approximate quantity discrimination tasks might be unintentionally tapping domain-general cognitive aspects in children, such as inhibition control skills (Fuhs & McNeil, 2013; Gilmore et al., 2013). First the evidence both in favour and against the association of non-symbolic approximate quantity discrimination skills with early number skills in children is reviewed. Then studies that have examined the relationship between non-symbolic approximate quantity discrimination skills and mathematical attainment in MLD children and correlational and longitudinal studies conducted with typically developing children are reviewed. Lastly, potential reasons for the non-converging findings across studies are discussed.

3.4.1 Approximate quantity discrimination skills and their relationship with early number skills in young children

Whilst there are numerous studies examining the relationship between general mathematical attainment and approximate non-symbolic quantity discrimination skills, there are very few studies which examine how approximate non-symbolic quantity discrimination skills relate to distinct early number skills. Mussolin et al. (2012) assessed non-symbolic approximate quantity discrimination skills, number-sequence reciting skills and performance on a battery comprising a wide range of early number skills in four groups of children (three-, four-, five- and six-year-olds). Children's accuracy on a non-symbolic quantity discrimination task consisting of identifying the larger of two arrays ranging from three to 24 discrete items was associated with children's performance on the number sequence reciting task and the numerical tasks battery even when individual differences in age, IQ, verbal skills and visuo-spatial and verbal STM skills were controlled for. However, Piazza et al. (2010) suggested that non-symbolic approximate quantity discrimination skills relate differently to different early number skills. They compared the performance of kindergarten (children aged between three and six years) and school-aged children (children aged between eight and twelve years) with and without dyscalculia on a non-symbolic approximate quantity discrimination task. Dyscalculics were diagnosed after a poor performance on a dyscalculia battery test (Biancardi & Nicoletti, 2004) despite other cognitive skills being normal. Children had to identify the larger of two non-symbolic large arrays simultaneously presented until response by pressing a key; one array contained either sixteen or 32 dots and the other array ranged from twelve to 40 dots. For all participants, accuracy rates increased as the numerical ratio difference between the sets decreased and performance improved with age. Participants' *w* parameter (this is the minimum change needed in the numerical ratio presented by the two numerical magnitudes to be correctly discriminated) was calculated. It was found that dyscalculic children performed significantly worse than their typically developing peers. Interestingly, when the tasks comprised in the dyscalculia battery test were grouped in relation to the cognitive demands they made (transcoding skills, quantity and relational-based skills, simple arithmetic facts retrieval skills and complex written and oral calculation skills), group differences were found only for quantity and relational-based symbolic skills. Thus, non-symbolic approximate quantity discrimination skills related to symbolic number comparison skills that demanded accessing the number semantics, but not to other number tasks. Soltész et al. (2010) raised the possibility that the numerical representations are supported by a domain-general cognitive skill rather than by a domain-specific core quantity system. They administered an approximate non-symbolic quantity discrimination task to a group of four-to-seven-year-olds in which they had to identify the larger of two arrays ranging from four to eighteen dots by pressing a key. Children's accuracy and RTs in this task did not relate to their symbolic or counting knowledge skills. Interestingly, accuracy was significantly predicted by participants' memory for numbers and for words. Thus, although limited research which has examined whether performance on non-symbolic approximate quantity discrimination tasks is related to early number skills, it does not reach a unitary agreement.

3.4.2 Approximate quantity discrimination skills and their relationship with performance on standardised mathematical attainment tasks in young children

A large body of research has examined the relationship between approximate quantity discrimination skills and children's performance on standardised mathematical attainment tests, however this relationship remains controversial. This section reviews evidence in favour and against the association of non-symbolic approximate quantity discrimination skills with mathematical attainment in typically developing children.

3.4.2.1 Comparison studies

Mazzocco et al. (2011) found that fourteen-year-olds with very low mathematics achievement scores (participants who obtained scores below the 10th percentile consistently for at least five years since kindergarten in the Test of Early Mathematics Abilities (hereafter TEMA) 2 (Ginsburg & Baroody, 1990) or in the WJ Revised Calculation subtest (Woodcock & Johnson, 1990)) performed significantly worse than their typical, high and low achievement peers (who obtained scores between the 10th and the 25th percentile consistently for at least five years since kindergarten) in a large approximate quantity discrimination task. Participants were asked to identify the larger of two spatially intermixed sets containing five to sixteen blue and yellow dots simultaneously presented for 200 msec. Group differences were found between very low achievers and the other three groups in the *w* parameter. These differences remained after controlling for concurrent individual differences in other cognitive skills (lexical access and STM) and even after controlling for individual differences in other cognitive skills measures (non-word reading decoding, executive function, visual memory and visual perception) obtained in previous years. The authors suggested that individual differences in the efficiency of children's approximate number sense could account for their mathematical attainment performance over their school-age years.

However, some comparison studies fail to find group differences in performance on approximate non-symbolic quantity discrimination tasks and yet find group differences in symbolic quantity comparison tasks. For instance Rousselle and Noel (2007) compared eight-year-olds MLD children and controls in different numerical tasks including a symbolic and a non-symbolic quantity discrimination task. Group assignment criteria depended on the composite score obtained on a battery of tests comprising six subtests: number writing, number comparison, transcoding, untimed addition and subtraction and a timed addition test. Those obtaining a composite score below the 15th percentile were classified as MLD. In the non-symbolic comparison task children had to identify the larger of two arrays containing from six to 28 discrete visual items and in the symbolic comparison task they had to identify the larger of two single-digit Arabic numerals presented simultaneously. For both tasks stimuli were present until response. Significant group differences between MLD children and controls in accuracy and RTs were found for the symbolic number task, while for the non-symbolic comparison task no significant differences between these groups were found. Similar results were found by Iuculano, Tang, Hall, and Butterworth (2008). They compared the performance of eight- and nine-year-olds dyscalculics and controls on symbolic and non-symbolic tasks. Participants' assignment to the dyscalculic group depended on their performance on the *Dyscalculia Screener* (Butterworth, 2003). Children were asked to perform a small symbolic quantity comparison task where they had to select the larger of two single-digit Arabic numerals (symbolic task) and a small non-symbolic quantity comparison task where they had to select the larger of two arrays ranging from one to nine squares (non-symbolic task). In addition, they were asked to perform a large approximate non-symbolic comparison task presenting arrays ranging from ten to 58 dots (Barth et al., 2005). Children's efficiency in the low numeracy group did not differ from their typically developing peers in the small and large approximate tasks, although they were significantly worse than their peers in the small symbolic comparison task. Similarly, Landerl and Kölle (2009) found no significant group differences in RTs between typically developing children in second, third and fourth grade and a small group of dyscalculic children in a single-digit comparison task nor in a non-symbolic comparison task. However, dyscalculic children were significantly slower and less accurate in selecting the larger of two two-digit numerals even after controlling for their verbal and non-verbal IQ. Children were identified as dyscalculics if they scored 1.5 standard deviations below the mean on the Heidelberger Rechentest standardised arithmetic test (Haffner, Baro, Parzer, & Resch, 2005). DeSmedt and Gilmore (2011) also found that first graders who scored below the 25^{th} percentile on the Math Up to 10 curriculum-based standardised general mathematics achievement test (Dudal, 2000) were significantly slower than their typically achieving peers on a symbolic comparison task, even after controlling for individual differences in baseline speed. However, no group differences were found on a non-symbolic quantity discrimination task consisting of identifying the larger of two simultaneously presented arrays ranging from one to nine dots using the same regression model. Thus, the vast majority of comparison studies fail to find the difference between low and high achievers reported by Mazzocco et al. (2011).

3.4.2.2 Correlational studies with typically developing children

Nordman, Bull, Davidson, and Church (2009, September) asked five-year-olds to identify by pressing a key the larger of two arrays simultaneously presented side to side on a screen for 2,000 msec. Each array contained five to 35 dots. Accuracy rates increased as the numerical ratio difference between the sets decreased and there were large individual differences in performance across subjects. Children's Weber Just Noticeable Difference (hereafter JND, the precision with which the child can respond with 75% accuracy) was obtained for every child. A higher Weber JND correlated with concurrent higher scores on the Number Operations and Mathematical Reasoning standardised subtests of the Wechsler Individual Achievement Test Second Edition (hereafter WIAT-II^{UK}) (Wechsler, 2002). The same association was found by Libertus et al. (2011) who asked 174 pre-schoolers to identify the largest of two spatially intermixed sets containing four to fifteen blue and yellow dots simultaneously presented for 2,000 msec. by naming the colour of the larger array. Regressions analyses revealed that children's accuracy and children's *w* parameter significantly predicted their performance on the TEMA 3 (Woodcock, McGrew, & Mather, 2001) even after controlling for their RTs, their age and a measure of their vocabulary.

However, Inglis, Attridge, Batchelor, and Gilmore (2011) suggested that the relationship between non-symbolic approximate quantity discrimination skills and mathematical achievement changes with age. A group of seven-to-nine-year-olds and a group of adults (mean age of 23 years) were asked to select the larger of two arrays ranging from seven to 22 dots presented simultaneously for 1,500 msec. Accuracy rates increased as the numerical ratio difference between the sets decreased for all participants. Children were administered the WJ III revised Calculation subtest and the Matrix Reasoning subtest of the WASI (Wechsler, 1999) while adults were administered different numerical subtests of the WJ III battery (Woodcock et al., 2001), the Matrix Reasoning subtest of the WASI and two numerical tasks consisting of written arithmetic problems (Evans & Handley, 1999; Usiskin, 1982). Only in children did the *w* parameter predict performance on the Calculation subtest after controlling for age and Matrix Reasoning performance. The *w* parameter failed to predict any of the outcome measures in adults using the same regression model. Similar findings have been recently reported by Bonny and Lourenco (2013) with younger participants. Non-symbolic approximate quantity discrimination skills were assessed in a large group of three-to-five-year-olds with a task consisting of the identification of the larger of two simultaneously presented arrays; one containing eight dots and another array ranging from four to twelve dots. Analyses including the whole sample revealed that children's accuracy predicted their performance on the TEMA 3 even when their receptive vocabulary was controlled. However, when the same regression model was conducted for each age group (three-, four- and five-year-olds), five-year-olds approximate non-symbolic acuity did not predict their mathematical attainment performance, suggesting that the relationship between children's ANS precision and mathematical attainment is non-linear and weakens over development.

Nevertheless and just as with comparison studies, null findings of the relationship between children's performance on approximate non-symbolic quantity discrimination tasks and their mathematical attainment have simultaneously been reported. Holloway and Ansari (2009) administered a symbolic and a non-symbolic comparison task to a large group of six-year-olds. In the symbolic task, children had to select the larger of two single-digit Arabic numerals ranging from one to nine and in the non-symbolic task participants had to select the larger of two arrays ranging from one to nine squares. Children's performance on the Mathematics Fluency and Calculation subtests of the WJ III tests (Woodcock et al., 2001) were used as outcome measures. The NDE in the symbolic comparison task (indexed as RTs) predicted children's performance on both mathematical attainment measures even after controlling for individual differences in age, processing speed and other cognitive skills such as reading. However, the NDE in the non-symbolic comparison task failed to predict significant variance in any of the outcome measures using the same regression model. Similarly, Sasanguie, De Smedt, Defever, and Reynvoet (2012) found that kindergartens' and first, second and sixth graders' efficiency (RTs adjusted to accuracy) in a symbolic comparison task where they had to identify the larger of two single-digit numerals predicted their mathematic achievement level. In contrast, efficiency in a non-symbolic comparison task where participants had to identify the larger of two arrays of dots ranging from one to nine failed to predict children's mathematical attainment. Very recently, Sasanguie et al. (2013) have found that scores on a curriculum-based standardised achievement test for mathematics from the Flemish Student Monitoring System (Dudal, 2000) in a large group of six-to-eight-year-olds were predicted by their RTs on a symbolic quantity comparison task, but neither their *w* parameter nor their mean accuracy on a non-symbolic comparison task similar to the one employed by Piazza et al. (2010) predicted their maths performance. These studies provide support for the defective access hypothesis (Rousselle & Noel, 2007), suggesting that young children's difficulties in maths could be due to a specific deficit in accessing the number semantics from symbolic numerical representations. Thus, results from studies conducted with typically developing young children examining the relationship between approximate non-symbolic quantity discrimination skills and mathematical attainment are also controversial.

3.4.2.3 Retrospective and longitudinal studies

Halberda et al. (2008) assessed fourteen-year-olds with the approximate non-symbolic quantity discrimination task described in section 3.4.2.1 (Mazzocco et al., 2011). Accuracy rates increased as the numerical ratio difference between the sets decreased and performance varied widely across the sample. Participants' *w* parameter predicted their performance on the TEMA 2 (Woodcock & Johnson, 1989) and the WJ revised calculation subtest (Woodcock & Johnson, 1989) since kindergarten, even after controlling for other cognitive skills measures such as naming speed or IQ. In addition, Mazzocco et al. (2011) conducted a longitudinal study with seventeen pre-schoolers who were asked to select the larger of two arrays of discrete objects that appeared simultaneously for either 1,200 msec. or for 2,500 msec. depending on participants' age group. Again, accuracy rates increased as the numerical ratio difference between the sets increased and performance varied widely across the sample. Two years later, participants were administered the TEMA 3, the WASI (Wechsler, 1999) and three tests of Rapid Automatized Naming (Denckla & Rudel, 1976). Children's percentage of correct responses on the non-symbolic approximate quantity discrimination task accounted for significant variance in children's performance on the TEMA 3 two years later after controlling for their age and display times. Furthermore, Libertus et al. (2013) have recently published a longitudinal study conducted with a large group of pre-schoolers where they used the non-symbolic approximate quantity discrimination task described in section 3.4.2.2 (Libertus et al., 2011). Children were administered this and other tasks (TEMA-3, Form A of the Peabody Picture Vocabulary Test, Dunn & Dunn, 2007) in two occasions six months apart. At the latter time point an attention measure and a memory span forward and backward task were also administered. Participants' percentage of correct responses increased across time points and their RTs decreased across time points. Also, accuracy decreased as the numerical ratio difference decreased. It was found that pre-schoolers' accuracy and RTs on the non-symbolic approximate quantity discrimination task at the start of the study were unique and causal predictors of their later performance on the TEMA 3 even when individual differences in age, initial math abilities and vocabulary were accounted for. Thus, retrospective and longitudinal studies suggest that non-symbolic approximate quantity skills contribute, and are causal predictors of, young children's mathematical attainment.

3.4.3 The role of inhibition control skills in ANS tasks

Inhibition skills are a well-established predictor of early mathematics development (Blair & Razza, 2007; Bull & Scerif, 2001; Clark, Pritchard, & Woodward, 2010; St Clair-Thompson & Gathercole, 2006; Welsh, Nix, Blair, Bierman, & Nelson, 2010). Very recently, two studies have suggested that the relationships identified between children's performance on non-symbolic approximate quantity discrimination tasks and their mathematical attainment could be an artefact of the relationship between their inhibition control skills and their mathematical attainment. They suggest that ANS tasks could unintentionally tap inhibition control skills because children have to inhibit responding to the larger physical set in incongruent trials that is, when physical aspects of the numerical set are inversely related to the number of items presented. Fuhs and McNeil (2013) examined the non-symbolic approximate quantity discrimination skills of a large group of pre-schoolers from low-income homes (participants' families met the federal poverty guidelines). Children had to identify the larger of two sets ranging from one to 30 stars that varied in size intra and inter set and remained visible until the child responded. Children's mathematical attainment, inhibitory control and receptive vocabulary were also assessed. Mathematical attainment was assessed with the TEMA 3. Inhibitory control was assessed with three tasks: the Head/Feet task (McCabe, Rebello-Britto, Hernandez, & Brooks-Gunn, 2004), the Day/Night task (Gerstadt, Hong, & Diamond, 1994) and the Knock/Tap task (Hughes, 1998). Receptive vocabulary was assessed with the Peabody Picture Vocabulary Test (Dunn & Dunn, 2007). Children's accuracy in the non-symbolic task did not predict significant variance over and above their vocabulary. However, children's inhibitory control predicted unique variance over and above both their vocabulary and their accuracy on any of the trials types of the non-symbolic quantity discrimination task.

In addition, Gilmore et al. (2013) examined mathematical attainment with the Numerical Operations subtest of the WIAT-II^{UK} (Wechsler, 2002), inhibition control skills with the NEPSY-II Inhibition subtest (Korkman, Kirk, & Kemp, 2007) and performance on a non-symbolic approximate quantity discrimination task in a group of seven-to-ten-year-olds. In their quantity discrimination task children had to identify the larger of two arrays ranging from five to 28 dots that were simultaneously presented for 1,500 msec. Children's inhibition control skills always predicted unique variance in their mathematical attainment regardless of the order in which the two independent variables were entered into the regression model.

However, the children's *w* parameter or accuracy in the quantity discrimination task failed to predict unique variance in children's mathematical attainment when either was entered after inhibition control skills in the regression model. Thus, it could be that non-symbolic approximate quantity discrimination tasks are unintentionally tapping inhibition control skills and their relationship with mathematical attainment is due to them tapping inhibition rather than the efficiency of children's approximate quantity representations.

3.5 APPROXIMATE QUANTITY DISCRIMINATION SKILLS AND THEIR INCONSISTENT RELATIONSHIP WITH EARLY MATHEMATICAL DEVELOPMENT ACROSS STUDIES

Previous sections in this chapter show that while some comparison, correlational and longitudinal studies have identified a relationship between non-symbolic approximate quantity discrimination skills and mathematical attainment (Libertus et al., 2013; Mazzocco et al., 2011) other comparison and correlational studies have not found this relationship (Holloway & Ansari, 2009; Landerl & Kölle, 2009) and some suggest that this relationship is not consistent over development (Bonny & Lourenco, 2013). Potential reasons for the conflicting findings across studies could lie in the different methodologies, metrics and analyses used to examine the relationship between children's ANS precision and their mathematical competence. Firstly, methodological aspects of the ANS tasks vary across studies. For example, although the majority of the non-symbolic quantity discrimination tasks present two arrays simultaneously and for a brief period of time to prevent the use of verbal counting strategies, there are exceptions (Piazza et al., 2010; Rousselle & Noel, 2007). If arrays are presented for a period of time that does not prevent children from using alternative quantification strategies then it is difficult to discriminate whether the non-symbolic quantity discrimination task is actually tapping ANS efficiency or simply providing a measure of the efficiency of other domain-general cognitive systems. For instance, children with good verbal counting skills could be opting for fast serial counting strategies or a combination of subitising and verbal counting instead of relying on their ANS to respond to untimed quantity discrimination tasks. Also, the number of discrete items and the numerical ratio difference displayed between arrays vary widely across studies. Only in some of the ANS tasks was the numerical ratio difference between arrays systematically varied (Soltész et al., 2010) and while some tasks depicted numerical ratios easy to discriminate (Holloway & Ansari, 2009; Sasanguie et al., 2013), others presented much harder numerical ratios (Halberda et al., 2008). Second is the issue of accuracy, RTs or *w* parameter being used as indices of children's ANS precision. Some ANS metrics seem to have a stronger relationship with mathematical attainment measures than others. For instance, Libertus et al. (2011) found that children's accuracy on a non-symbolic approximate quantity discrimination task was a better predictor of their mathematical attainment than RTs or *w* parameter. Third, and as suggested by Piazza et al. (2010), the ANS may have a stronger relationship with some number skills than others. Therefore, the strength of the relationship between ANS and mathematical attainment could vary in relation to the standardised mathematical attainment test used. Finally, there is no consistency in the regression models used across studies. Some studies control for domain-general cognitive skills but do not specifically control for working memory skills (Libertus et al., 2011). However, it has been suggested that the mental number line relied upon when making approximate numerical discriminations has a spatial dimension (Dehaene et al., 1993) and also that non-symbolic quantity discrimination tasks might unintentionally tap inhibition motor skills (Fuhs & McNeil, 2013; Gilmore et al., 2013). If ANS is a domain-specific number system, it should predict variance in children's number skills and mathematical attainment that cannot be explained by other domain-general cognitive systems.

Moreover, findings may differ because different age groups have been studied. Some cross-sectional studies examining children of different ages find that the relationship between ANS precision and mathematical attainment changes over development (Bonny & Lourenco, 2013; Inglis et al., 2011).

3.6 DISCUSSION

Adults with and without formal arithmetical training can discriminate the larger of two large numerical sets without counting (Pica et al., 2004). Similar approximate quantity discrimination skills have been found in preverbal infants (Lipton & Spelke, 2003) and animals (Gallistel, 1990). Approximate quantity discrimination skills seem to improve with age (Halberda & Feigenson, 2008) and to recruit a neural substrate in the brain distinct from that supporting exact numerical processing (Dehaene & Cohen, 1997; Dehaene et al., 1999). Thus, like small and precise numerical processing skills approximate numerical skills seem to be present very early in humans, to be shared with other animal species, to be supported by specific brain networks and to improve during childhood. Nevertheless, two distinctive signatures make the ANS distinct from the PNS; First, approximate non-symbolic quantity discrimination skills do not seem to be affected by physical features of the stimuli (Xu, Spelke et al. 2005, but see also Solstesz et al., 2010 for an opposite opinion). Second, the correct approximate discrimination depends on the numerical ratio difference between the numerical magnitudes presented (Piazza et al., 2004; Whalen et al., 1999). These differences between PNS and ANS suggest that we possess distinct abstract representations for small and large numerical magnitudes (Feigenson et al., 2004; Xu, 2003; Xu et al., 2005) and such a distinction is supported by neuropsychological, electro-imagining and neuroimaging data (Dehaene et al., 1999; Lemer et al., 2003). Approximate magnitude representations have been proposed to underpin our later symbolic numerical competence (Dehaene, 1997; Dehaene et al., 2004; Spelke & Dehaene, 1999). Nevertheless, the relationship between ANS precision and early number skills and mathematical attainment remains controversial. Clearly studies have employed diverse tasks and metrics to tap ANS precision, as well as different regression models that controlled for different cognitive skills. These different methodological approaches could well account for the non-converging findings.

If PNS and ANS are distinct domain-specific systems for numerical representations that underpin later numerical competence (Butterworth, 1999, 2005, 2010; Dehaene, 1997; Dehaene et al., 2004; Feigenson et al., 2004; Spelke & Dehaene, 1999) these two systems should make unique and independent longitudinal contributions to children's early number skills and mathematical attainment measures. However, this assumption has not been explored previously. The longitudinal study presented in this thesis will examine whether the precise and approximate quantity discrimination skills make differential contributions to early number skills and mathematical attainment.

4. DOMAIN-GENERAL COGNITIVE SKILLS: PHONOLOGICAL AND VISUO-SPATIAL SKILLS

Phonological processing and visuo-spatial STM skills were identified by Krajewski and Schneider (2009) and LeFevre et al. (2010) as key cognitive precursors of early number skills and mathematical attainment in young children. Krajewski and Schneider (2009) and LeFevre et al. (2010) propose that these two domain-general cognitive skills have different relationships with different early number skills depending on the cognitive demands the number tasks make. In particular, Krajewski and Schneider (2009) propose that phonological awareness contributes to basic verbal number skills such as reciting the number-word sequence (Level I QNCs), whilst the VSSP functioning contributes to the process of linking quantity representations to number-words and to understanding number relationships (levels II and III QNCs, respectively). Krajewski and Schneider (2009) and LeFevre et al. (2010) also suggest that phonological processing and visuo-spatial STM skills contribute to children's mathematical attainment. However, standardised mathematical attainment tests usually comprise a wide range of number skills and therefore do not allow to be determinate the extent to which different cognitive skills are involved. This chapter focuses on how phonological processing and visuo-spatial STM skills relate to early number skills and mathematical attainment in young children. It first presents studies examining number skills competence in dyslexic children. Then, studies examining the relationship between phonological awareness and early number skills and mathematical attainment in typically developing children are discussed. This chapter also presents a literature review of key studies examining the relationships of visuo-spatial STM skills in children presenting MLD. Then, studies examining the relationship between VSSP functioning with early number skills and mathematical attainment in typically developing children are also discussed.

4.1 PHONOLOGICAL PROCESSING ABILITIES AND EARLY MATHEMATICAL DEVELOPMENT

Phonological processing skills are involved in the manipulation (phonological awareness), retention (phonological memory) and retrieval (rate of access to phonological information) of phonological codes. These skills are well-established predictors of early reading attainment (de Jong, 2007; Hulme, Snowling, Caravolas, & Carroll, 2005; Melby-Lervåg, Lyster, & Hulme, 2012). Nevertheless, phonological processing skills also seem to be good early predictors of children’s mathematical attainment (DeSmedt et al., 2010; Hecht, Torgeson, Wagner, & Rashotte, 2001). This section reviews studies conducted with dyslexic children because their weak phonological processing has been associated with weaknesses in particular number tasks that demand the processing of verbal codes.

4.1.1 Comparison studies of children with dyslexia

Dyslexic children have poor phonological processing abilities (Bradley & Bryant, 1985; Bruck, 1990; Griffiths & Snowling, 2001; Manis, Custodio, & Szeszulski, 1993; Olson, Kliegel, Davidson, & Foltz, 1985; Snowling, 2000; Swan & Goswami, 1997; Vellutino, Scanlon, & Spearing, 1995). The weak phonological representations hypothesis (Simmons & Singleton, 2008) postulates that dyslexic children perform worse than their peers in number tasks that demand the manipulation of verbal codes due to their poor phonological processing abilities, but that their performance in number tasks which do not require phonological processing may not necessarily be affected. For instance, dyslexic children should struggle with multiplication tasks because they require the retrieval of phonological representations from LTM, but not with place value tasks because they do not make any verbal demands (DeSmedt et al., 2010; Simmons & Singleton, 2009). This hypothesis is supported by studies conducted with dyslexic children who performed worse than their peers in specific number skills where verbal skills are directly involved. For example, Turner Ellis, Miles, and Wheeler (1996) found that school-aged dyslexic boys were slower than their aged-matched controls at retrieving answers to multiplication problems. Similarly, Miles (1983) found that children with dyslexia experienced more difficulties in reciting the times tables than their typically developing peers from the age of seven onwards. Geary, Hamson, and Hoard (2000) found that first-graders presenting with reading difficulties were significantly slower at retrieving monosyllabic familiar words and also experienced greater difficulties than their peers in retrieving the answers to visually presented simple addition problems. More recently, Simmons and Singleton (2009) found that eleven-year-olds with dyslexia were slower and less accurate in arithmetic fact retrieval than their typically developing peers but not in a place value understanding test, where no verbal skills were involved. Boets and De Smedt (2010) found that third-graders with dyslexia performed significantly worse than their peers in single-digit multiplications and subtractions. Thus, much evidence suggests that the quality of children’s phonological representations impacts their performance on specific number tasks where phonological processing abilities are needed.

4.2 PHONOLOGICAL AWARENESS AND EARLY MATHEMATICAL DEVELOPMENT

Phonological awareness refers to the specific ability to represent and manipulate the phonological structure of language. Phonological awareness is usually assessed in young children with tasks where the child is asked to identify initial phonemes, rhyming words or to segment or blend specific syllables or phonemes within real words. While some consider phonological awareness a STM skill (Passolunghi & Siegel, 2001; Wagner, Torgesen, Laughon, Simmons, & Rashotte, 1993), others regard phonological awareness as a cognitive skill distinct from memory (Windfuhr & Snowling, 2001). The latter approach is adopted here because it is supported by empirical data obtained with children (Alloway, Gathercole, Willis, & Adams, 2004; Gathercole, Tiffany, Briscoe, & Thorn, 2005; Gathercole, Willis, & Baddeley, 1991; Muter & Snowling, 1998). Like other phonological processing skills, phonological awareness predicts reading abilities in young children (Gathercole et al., 2005; Hecht et al., 2001). It has been suggested that phonological awareness contributes to the linkage between phonemes and graphemes when learning to read (see Cain, 2010 for a discussion of this issue), so it is plausible that this skill also contributes to the linkage between phonemes and symbolic numerical representations. Hecht et al. (2001) found that phonological awareness predicted greater variance in both mathematical attainment performance and growth in school-aged children than phonological memory and rate of access of phonological representations. This section reviews studies which examine the relationships between phonological awareness and distinct early number skills in typically developing children. It then reviews studies examining the relationships between children's phonological awareness and their performance on standardised mathematical attainment tests. It concludes by discussing possible reasons why phonological awareness consistently predicts specific number skills in which phonological representations are needed but does not consistently predict performance on standardised mathematical attainment tests.

4.2.1 The relationship between phonological awareness and specific number skills

Studies conducted with young typically developing children have found that phonological awareness predicts performance on specific number tasks where verbal skills are involved. For example, in LeFevre's (2010) study (described in detail in Chapter 1 of this thesis) kindergarteners' phonological awareness predicted unique variance in children's concurrent ability to name Arabic numerals but not in their concurrent ability to perform non-verbal arithmetic. In Krajewski and Schneider's (2009) study (also described in detail in Chapter 1 of this thesis) phonological awareness predicted unique variance in children's scores on a composite measure comprising verbal number tasks such as reciting the number-word sequence forward and backward, identifying elements within the number-word sequence or naming Arabic numerals four months later. In contrast phonological awareness did not predict children's scores on a composite measure comprising number tasks that demanded abstract quantity manipulation such a non-symbolic comparison task or performing non-verbal arithmetic problems (Krajewski and Schneider, 2009). These studies support the weak phonological representations hypothesis (Simmons & Singleton, 2008) in that phonological awareness is involved in number tasks that demand verbal codes' processing but not in other number tasks where phonological representations are not needed.

4.2.2 The relationship between phonological awareness and mathematical attainment

There is also evidence that phonological awareness contributes to young children's performance on standardised tests of mathematical attainment. For example, Leather and Henry (1994) found that seven-year-olds performance on the arithmetic subtest of the WISC-R (Wechsler, 1974) was best predicted by a composite of their phonological awareness skills. Simmons, Singleton, and Horne (2008) found that kindergartener's phonological awareness and VSSP functioning were unique predictors of performance on the Number Skills Test of the British Ability Scales (hereafter BAS) (Elliott, Murray, & Pearson, 1983) a year later. In LeFevre et al.'s (2010) study (described in detail in Chapter 1 of this thesis) a combined measure of phonological awareness and a number naming task in five-year-olds predicted performance two years later on all standardised tests and research-based measures of mathematical attainment included in the study. However, the relationship between phonological awareness and performance on standardised mathematical attainment tests in young children has not always been replicated. For instance, Passolunghi, Mammarella, and Altoé (2008) found that phonological awareness assessed at the beginning of first grade did not predict mathematical performance on a standardised mathematical test by the end of first grade although measures of central executive and phonological loop functioning did. In Krajewski and Schneider's (2009) study (described in detail in Chapter 1 of this thesis) the impact of kindergarteners' phonological awareness on a standardised mathematical attainment test three years later was indirect and mediated by their performance on early verbal number skills assessed at midway through kindergarten.

Thus, phonological awareness makes both concurrent and longitudinal contributions to specific number tasks that require the manipulation of verbal codes during early stages of schooling. However, evidence of the longitudinal contributions that phonological awareness makes to young children's performance on standardised mathematical attainment tests is controversial. There are a number of reasons why phonological awareness may not be consistently related to general mathematical attainment. First, language factors such as participants' language structure (Italian) in Passolunghi et al. (2008) may account for the null findings. It has been suggested that Italian and Spanish children have higher central executive functioning demands when performing verbal number tasks than other children because number-words in these languages are larger in length. Variance in these measures might be more greatly influenced by the central executive functioning in Spanish and Italian children and the role for phonological awareness might be smaller (Raghubar, Barnes, & Hecht, 2010). Second, the lack of a direct relationship between phonological awareness and performance on a standardised mathematical attainment test in Krajewski and Schneider's (2009) study could be due to the length of time which elapsed between cognitive precursors and outcome measures. Studies reporting direct relationships between phonological awareness and mathematical attainment are either concurrent or the time which passed between the collection of the cognitive precursors and when the outcome measures were obtained are considerably shorter (Simmons et al., 2008). Because there is evidence that children recruit different cognitive skills to perform the same number task depending on their age (McKenzie, Bull, & Gray, 2003; Rasmussen & Bisanz, 2005), it could be that the longitudinal contributions that phonological awareness make to mathematical attainment in young children are less substantial over time and therefore do not remain significant over a three-year period. Finally, null findings could also be due to the nature of the standardised mathematical attainment test used. According to the weak phonological representations hypothesis (Simmons & Singleton, 2008), the quality of children's phonological representations impacts only on number tasks that demand the processing of verbal codes. Because mathematical attainment tests usually comprise a wide variety of number tasks the strength of the relationship between phonological awareness and a specific standardised mathematical attainment test may vary in relation to the nature of the items that the mathematical test comprises.

4.3 WORKING MEMORY AND EARLY MATHEMATICAL DEVELOPMENT

Working memory refers to the ability of simultaneously storing and processing information (Baddeley, 2000). Different theoretical models of working memory co-exist nowadays (see Miyake and Shah, 1999, for a review). In this chapter Baddeley's (2000) multi-component working memory model is used as a framework because each of its components has been related to academic success (Holmes & Adams, 2006) and empirical findings suggest that it can be applied to children (Alloway et al., 2005). The multi-component model (Baddeley, 2000) is based on behavioural and neuropsychological evidence and identifies three distinct systems to account for how information is manipulated and stored in memory; the central executive, the VSSP and the phonological loop. The central executive component is a supervisory system in charge of attention processes and responsible for inhibiting, shifting and updating information (Baddeley, 1996; Miyake et al., 2000). The central executive component is supported by the episodic buffer which merges incoming information that requires attention resources with automatic information to produce integrated episodes (Baddeley, 2000). The central executive component controls two limited-capacity storage systems: the VSSP component and the phonological loop component. The VSSP component is responsible for the visual and spatial memory processes and supports the generation, retention and manipulation of visuo-spatial information (Logie, 1995). The phonological loop component stores phonological information and is in charge of maintaining articulatory rehearsal processes active (Baddeley, 1986).

There is a large body of research associating working memory functioning with early mathematics development in children (Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Mabbott & Bisanz, 2008; McLean & Hitch, 1999; Passolunghi & Siegel, 2001; Raghubar et al., 2010; Wilson & Swanson, 2001). This section reviews studies that compared MLD children with their typically developing peers in visuo-spatial STM tasks. Within research of mathematical development visuo-spatial STM is usually viewed in the context of Baddeley's (2000) multi-component model of working memory. To assess VSSP functioning children are usually asked to recall in the same order or manner the locations where visually presented stimuli have previously been shown for a limited time (e.g. block recall, mazes memory, visual pattern test).

4.3.1 Comparison studies conducted with MLD children

Ozols and Rourke (1988) found that seven-to-eight-year-olds presenting with only math difficulties had worse visuo-perceptual and visuo-spatial skills than children presenting reading and spelling difficulties or children presenting difficulties in both academic domains. McLean and Hitch (1999) compared the performance of nine-year-olds with specific arithmetic difficulties to a group of age-matched controls. Children performing low (below 25% of the raw scores) on a standardised mathematical attainment test but performing at an average level on a standardised reading test were assigned to the arithmetic difficulties group. It was found that children in the arithmetic difficulties group performed significantly worse than their peers on a VSSP working memory task. Murphy, Mazzocco, Hanich, and Early (2007) determined retrospectively the presence of MLD depending on children's performance on a standardised mathematical attainment test in a group of kindergarten and first graders. Children were classified as typically developing children, children with MLD (who scored between the 11^{th} and the 22^{nd} percentile during at least two years) and children presenting severe mathematical disabilities (who scored below the 10^{th} percentile during at least two years). Analyses of performance in kindergarten and first grade revealed that children in the MLD and severe mathematical disabled groups performed significantly worse than their peers on visuo-spatial STM tasks. Thus, visuo-spatial STM skills seem to contribute to mathematics performance and deficits in visuo-spatial STM may account for the low mathematical performance in MLD children.

4.4 VSSP FUNTIONING AND EARLY MATHEMATICAL DEVELOPMENT

It is been suggested that the VSSP provides a mental workspace where information is encoded, retained and manipulated during calculation procedures and/or transcribed into qualitatively different representations or procedural rules (Noël, Fias, & Brysbaert, 1997; Simmons, Willis, & Adams, 2012; Trbovich & LeFevre, 2003). Krajewski and Schneider (2009) and LeFevre et al. (2010) found VSSP functioning to be an early precursor of children's early number skills. VSSP functioning has also been found to be a strong predictor of young children's mathematical performance (Holmes & Adams, 2006; Raghubar et al., 2010). This section reviews studies examining the relationships between VSSP functioning and early number skills in typically developing children. Studies where VSSP functioning has been associated with young typically developing children' performance on standardised mathematical attainment tests are then discussed, as well as possible reasons why VSSP functioning does not consistently predict performance on these measures.

4.4.1 The relationship between VSSP functioning and specific number skills

McKenzie et al. (2003) found that six-year-olds performance on a Corsi blocks task correlated with their accuracy on simple arithmetic problems presented aurally and that under VSSP functioning demands, children's arithmetic performance deteriorated significantly. Swanson and Beebe-Frankenberger (2004) found that a working memory composite score comprising performance on five different working memory tasks where two of them were visuo-spatial STM tasks (visual matrix and mapping and directions task) significantly predicted unique variance in children's calculation and word-problem solving over and above verbal STM skills and executive functioning. Rasmussen and Bisanz (2005) found that pre-schoolers' performance on a Corsi block task was a unique predictor of their performance on non-verbal arithmetic problems. Also, in Krajewski and Schneider's (2009) study (described in detail in Chapter 1), kindergarteners' VSSP functioning predicted their performance four months later on a composite measure comprising different number tasks that involved quantity representation or manipulation, such as identifying which of two number-words represents a larger quantity or reporting the numerical difference between pairs of arrays of dots. In contrast, kindergarteners' VSSP functioning failed to predict their performance on a composite measure comprising different verbal number tasks (Krajewski and Schneider, 2009). In LeFevre et al.'s (2010) study (described in detail in Chapter 1), five-year-olds' performance on a Corsi blocks task predicted their concurrent performance on a number task in which they were asked to match a number-word with its corresponding Arabic numeral and on non-verbal arithmetic problems. In addition, in Simmons et al.'s (2012) study, VSSP functioning was a unique predictor of performance on a number writing task and a symbolic magnitude comparison task in a group of first and third graders, whilst no other working memory component predicted unique variance on these tasks.

4.4.2 The relationship between VSSP functioning and mathematical attainment

Holmes and Adams (2006) and Holmes, Adams, and Hamilton (2008) found that VSSP functioning predicted performance on the National Curriculum-based mathematical skills test in seven-, eight- and nine-year-olds respectively. Simmons et al. (2008) found that five-year-olds performance on a Corsi span task was a unique predictor of children's performance a year later on the Number Skills Test of the BAS (Elliott et al., 1983). Bull, Espy, and Wiebe (2008) found that VSSP functioning in a large group of four-year-olds was the only unique predictor of their mathematical performance assessed at the end of the first year with the Performance Indicators in Primary School. However, in Krajewski and Schneider's (2009) study (described in detail in Chapter 1) the impact of kindergarteners' VSSP functioning on the DEMAT2+ three years later was indirect and mediated by their performance on early non-verbal number skills assessed in kindergarten. As discussed in section 4.2.2 of this chapter, null results in this case could be due to the length of time elapsed between the collection of the cognitive precursors and outcome measures or due to the extent to which items in the mathematical attainment test applied required VSSP functioning skills. Therefore, studies consistently find that children's VSSP functioning predicts their concurrent and longitudinal performance on number tasks that demand quantity representation and manipulation of analogue quantities and even their number writing skills. VSSP functioning almost consistently predicts children's concurrent and later performance on standardised mathematical attainment tests.

In addition to the relationships between VSSP functioning and number skills and mathematical attainment, studies have also linked central executive functioning (Bull, Johnson, & Roy, 1999; Bull & Scerif, 2001; McKenzie et al., 2003) and phonological loop functioning (Adams & Hitch, 1997; Jarvis & Gathercole, 2003; Noël, Seron, & Trovarelli, 2004; Rasmussen & Bisanz, 2005; Towse & Houston-Price, 2001) with early mathematical development. Nevertheless, studies have also demonstrated that the contributions of VSSP functioning to number skills and mathematical attainment in young children are independent of other working memory components (Bull et al., 2008; McKenzie et al., 2003; Rasmussen & Bisanz, 2005; Simmons et al., 2012)

4.5 DISCUSSION

Phonological awareness and VSSP functioning seem to both be key domain-general abilities that independently predict mathematical success in young children. Studies conducted with non-typically developing children suggest that visuo-spatial STM and phonological processing skills contribute to early number skills and mathematical attainment performance (Boets & De Smedt, 2010; Murphy et al., 2007; Simmons & Singleton, 2009). Studies conducted with young typically developing children find that phonological awareness predicts their concurrent and later verbal number skills (Krajewski & Schneider, 2009; LeFevre et al., 2010). Similarly, studies conducted with young typically developing children also find that VSSP functioning predicts their concurrent and later early number skills where analogue magnitude representations or transformation of numerical information into procedural rules is needed (Krajewski & Schneider, 2009; Rasmussen & Bisanz, 2005). In addition some studies find that phonological awareness and VSSP functioning also predict concurrent and later performance on mathematical attainment tests (Leather & Henry, 1994) and that they account together for a large substantial variance in young children's later mathematical attainment (Simmons et al., 2008). However, not all studies find direct relationships between VSSP functioning or phonological awareness and children's later performance on standardised mathematical attainment tests (Krajewski & Schneider, 2009; Passolunghi et al., 2008). It could be that the longitudinal contributions that VSSP functioning and phonological awareness make to mathematical attainment are not consistent over long developmental periods. Cross-sectional studies suggest that children recruit distinct cognitive skills to perform the same numerical tasks depending on their age (McKenzie et al., 2003; Rasmussen & Bisanz, 2005). However, it could also be that standardised tests are less informative of these specific relationships because they include many types of number skills.

5. THE PILOT STUDY: DESIGN AND VALIDITY OF THE NOVEL TASKS

This thesis aims to explore the impact of quantitative, verbal and visuo-spatial STM skills on young children's early number skills and mathematical attainment over the early stages of schooling using Krajewski and Schneider's (2009) theoretical model of early arithmetical development as a general framework. Novel tasks to assess children's quantitative and early number skills were designed. A pilot study was conducted to test the validity and reliability of these novel tasks. The specific aims of this pilot study were:

- To test the three novel Early Number Skills' tasks designed to tap the three distinct developmental levels of QNCs proposed in Krajewski and Schneider (2009), determining whether they provide distinct valid measures of these skills over early stages of schooling.

- To determine whether performance on each of the three early number tasks explain unique variance in children's scores on the two different standardised mathematical attainment measures; The Numerical Operations and Mathematical Reasoning subtests of the WIAT-II^{UK}. Thus, determining whether these tasks are valid measures of individual early number skills.

- To analyse whether the precise and approximate quantity discrimination skills tasks tap the two distinct domain-specific quantity systems; PNS and ANS, respectively (Feigenson et al., 2004).

- To determine whether children's performances on the precise quantity discrimination skills task and on the approximate quantity discrimination skills task make independent and unique contributions to the three Early Number Skills' tasks and to children's performance on the two standardised mathematical attainment measures of the WIAT-II^{UK}.

5.1 EARLY NUMBER SKILLS TASKS

Early number skills refer to numerical abilities that children acquire through formal instruction of the formal number system, such as being able to recite the number-word sequence, counting discrete objects or solving basic arithmetic problems. These early number competences have been found to be good early predictors of mathematical attainment in young children (Krajewski & Schneider, 2009; LeFevre et al., 2010; Östergren & Träff, 2013; Sarnecka & Carey, 2008)

5.1.1 Rationale for the design of the Early Number Skills' tasks

In order to design different tasks that would tap distinct early number skills, Krajewski and Schneider's (2009) theoretical model of early arithmetical development was adopted because it provides a comprehensive yet specific and precise classification of early number skills. This theoretical model purposes three different developmental levels of early QNCs:

Number-word isolated from quantities (QNC Level I): Refers to the ability to correctly articulate the number-words and recite the number-word sequence, but not necessarily connecting these words with the quantity concept they refer to. To assess this skill, children were asked to recite the number-word sequence forward from different start points.

Imprecise and precise quantity to number-word linkage (QNC Level IIa and IIb): Refers to the acquisition of the number-words' quantity meaning. First, children connect number-words to quantities imprecisely ("a bit", "much", "very much") (QNC Level IIa). Eventually, this connection becomes precise and children distinguish which number-word refers to a larger quantity of two consecutive number-words in the number-word sequence (QNC Level IIb). To assess children's understanding of the number-words' exact quantity meaning, they were asked to verbally report the exact number of discrete items presented on laminated sheets.

Number relationships (QNC Level III): Children understand that quantities can be composed and decomposed into other exact quantities and that these outcomes can be referred to with a number-word. To assess whether children understand the relations between quantities they were asked to perform simple abstract arithmetic (additions and subtractions).

The different Early Number Skills' tasks designed for the study were expected to provide appropriate distinct measures of the three different developmental levels of QNCs proposed by Krajewski and Schneider (2009). These authors suggested that verbal skills contribute to the knowledge of the number-words (QNC Level I) while quantity discrimination skills and visuo-spatial STM skills contribute to the linkage of number-words to their precise quantity concept (QNC Level II) and to children's early arithmetic skills (QNC Level III). Also, that performance on Level I, II and III of QNCs contribute to children's mathematical attainment.

5.2 QUANTITY DISCRIMINATION SKILLS TASKS

It has been proposed that our quantity discrimination skills are domain-specific and biologically-determined (Butterworth, 1999, 2005, 2010; Dehaene, 1997; Dehaene et al., 2004; Spelke & Dehaene, 1999). Two distinct systems for the representation of precise and approximate numerical magnitude representations have been proposed; the PNS and the ANS (Feigenson et al., 2004). The former supports the precise abstract representations of a few discrete items and the latter supports the approximate abstract representation of numerical magnitudes (see Chapters 2 and 3 for a more detailed review of these systems and their relationship with mathematical development). Enumeration tasks and small non-symbolic quantity discrimination tasks have been employed to tap young children's PNS (Svenson & Sjöberg, 1978; Trick et al., 1996). Better performance on these tasks has been associated with better numerical competence in children (Arp et al., 2006; Fischer et al., 2008; Landerl et al., 2004; LeFevre et al., 2010; Schleifer & Landerl, 2011). Non-symbolic approximate quantity discrimination tasks have been employed to tap children's ANS, and better performance on these tasks has also been associated with better numerical competence (Libertus et al., 2011; Mazzocco et al., 2011; Nordman et al., 2009, September) although this finding has not always been replicated (Holloway & Ansari, 2009; Sasanguie et al., 2013).

5.2.1 Rationale for the design of the Precise Quantity Discrimination skills task

Whilst enumerating small collections of discrete items is a fast and accurate process; enumerating large collections is slower and error-prone (Akin & Chase, 1978; Chi & Klahr, 1975; Frick, 1987; Mandler & Shebo, 1982; Trick & Pylyshyn, 1994). Analyses of children's accuracy and RTs in enumeration tasks show a clear discontinuity for sets of up to three or four items and above (Arp et al., 2006; Benoit et al., 2004; Chi & Klahr, 1975; Fischer et al., 2008; Landerl et al., 2004; LeFevre et al., 2010; Schleifer & Landerl, 2011; Starkey & Cooper, 1995; Svenson & Sjöberg, 1978). This suggests that the process of enumerating small sets of up to three or four discrete items is done in parallel, and for larger numerical sets a serial quantification process (vocal or subvocal counting) is needed (Arp et al., 2006; Bruandet, Molko, Cohen, & Dehaene, 2004; Piazza et al., 2003; Sathian et al., 1999). Children's higher accuracy rates and faster RTs in enumerating small quantities have been associated with better numerical competence (Arp et al., 2006; Fischer et al., 2008; Landerl et al., 2004; LeFevre et al., 2010; Schleifer & Landerl, 2011). However, tasks employed to assess children's subitising skills usually make additional cognitive demands or involve understanding or using the formal number system to a certain extent (LeFevre et al., 2010). Studies using discrimination paradigms with children have also found discontinuities in accuracy rates and RTs between sets containing up to three items and above (Starkey & Cooper, 1995; Trick et al., 1996). However, these studies used symbolic numerical representations (Durand, Hulme, Larkin, & Snowling, 2005) and/or presented the two small non-symbolic numerical collections sequentially (Starkey & Cooper, 1995; Trick et al., 1996) and/or stimuli in these tasks presented collections containing more than three discrete items (Iuculano et al., 2008). Therefore all these measures are potentially contaminated by children's knowledge of the formal number system, memory skills or verbal counting abilities. Nevertheless, studies consistently find that children are faster and/or more accurate with the enumeration of at least up to three discrete items (Arp et al., 2006; Benoit et al., 2004; Chi & Klahr, 1975; Fischer et al., 2008; Landerl et al., 2004; Schleifer & Landerl, 2011; Svenson & Sjöberg, 1978; Trick et al., 1996), suggesting that their subitising limit is at least up to three discrete items.

In the present study, the design of the Precise Quantity Discrimination task intends to provide a measure of children enumeration speed for quantities they can subitise. Sets were presented for unlimited time. Consequently, children's speed and not accuracy could discriminate whether they were relying on their subitising skills to respond or whether they were engaging in serial counting or any other not-as-efficient strategies. As a general rule, children's accuracy is used when stimuli is presented for a limited time (Benoit et al., 2004; Gelman & Tucker, 1975; Starkey & Cooper, 1995) and children's RTs are used when stimuli is presented for a long time (Iuculano et al., 2008) or until the child responds (Chi & Klahr, 1975; Durand et al., 2005; LeFevre et al., 2010; Schleifer & Landerl, 2011; Svenson & Sjöberg, 1978; Trick et al., 1996). Sets were presented simultaneously and children responded by pressing a key so that there were few cognitive demands from other cognitive systems. Furthermore no symbolic number representations were employed or required. The aim of this task was to examine whether children's speed completing accurate numerical judgements on non-symbolic sets containing no more than three discrete items could predict their performance on the Early Number Skills' tasks and the two standardised mathematical attainment measures. In addition a measure of children's RTs over two-choice non-numerical judgements was obtained in order to statistically control for their baseline RTs.

5.2.2 Rationale for the design of the Approximate Quantity Discrimination skills task

Pre-schoolers' performance on non-symbolic approximate quantity discrimination tasks has been associated with their numerical competence (Libertus et al., 2011; Mazzocco et al., 2011; Murphy et al., 2007; Nordman et al., 2009, September), although not all studies find this association (Rousselle & Noel, 2007; Sasanguie et al., 2012; Sasanguie et al., 2013), or at least not a linear association throughout development (Bonny & Lourenco, 2013; Inglis et al., 2011). Different approximate non-symbolic quantity discrimination tasks have been used to tap young children's ANS precision, these differ in stimuli presentation time, number of discrete items and level of difficulty in the numerical ratios displayed (see section 3.5 of this thesis for a discussion of the different methodologies employed to tap ANS precision in children). For the present study, Nordman et al.'s (2009, September) ANS task was adapted to tap children's non-symbolic approximate quantity discrimination skills. Two large non-symbolic numerical sets above the subitising range (five to 35 discrete items) were simultaneously presented for 2,000 msec. during which children had to make their choice by pressing a key. Because the average time for five- and six-year-olds to correctly enumerate an additional item outside the subitising range is approximately 1,000 msec. (Chi & Klahr, 1975; LeFevre et al., 2010; Svenson & Sjöberg, 1978; Trick et al., 1996) a time window of 2,000 msec. makes the use of serial counting unlikely, forcing participants to rely on their ANS to solve the task. Yet, this time seems to be sufficient for young children to make approximate quantity judgements (see Inglis et al., 2011). In addition, a measure of participants' ability to count discrete visual items with no time restriction was obtained (Counting Objects task of the Early Number Skills) to determine whether children are even capable of verbally counting the items should they opt for this strategy.

Stimuli in both computerized quantity discrimination tasks were non-symbolic and a two-key choice response method was used. Children were not presented with, nor required to employ, symbolic numerical representations (Arabic numerals or number-words). In addition, discrimination paradigms with simultaneous presentation of the two numerical sets were chosen because it has been suggested that sequential presentation could demand working memory load to retain and compare the numerical information of the sets (Gilmore, Attridge, & Inglis, 2011). Thus performance on both tasks is unlikely to be contaminated by children's knowledge of the formal number system or other domain-general cognitive skills such as language or memory.

5.3 METHODOLOGY

5.3.1 Participants

63 children (29 males) from Reception Year and Year 1 classes of a primary school in North West of England successfully completed all the assessments (see table 5.1 for details of age and gender distribution). 64 children were initially recruited but one female from Reception Year did not complete all the assessments due to continuing non-attendance on the scheduled assessment days and her data has not been included in the final analysis. Based on the school Ofsted report at the time (http://www.ofsted.gov.uk) the proportion of pupils eligible for Free School Meals was in line with the national average. Three participants (two in Reception Year and one in Year 1) were identified by the school as having Special Education Needs. Data from these participants have been included in the analysis because it made the sample representative of the school population as a whole.

Table 5.1: Descriptive statistics of participants' age and gender for Reception Year, Year 1 and overall sample

	Age in months			**Gender**		**Sample**
Year Group	**Mean**	**SD (months)**	**Range**	**Males**	**Females**	**Total**
Reception Year	5 years, 3 months	3.73 months	54.3-69.1	13	17	30
Year 1	6 years, 2 months	3.86 months	66.2-81.3	16	17	33
Total	5 years, 9 months	6.92 months	54.3-81.3	29	34	63

Note1. Months' information is given as units

5.3.2 Procedure

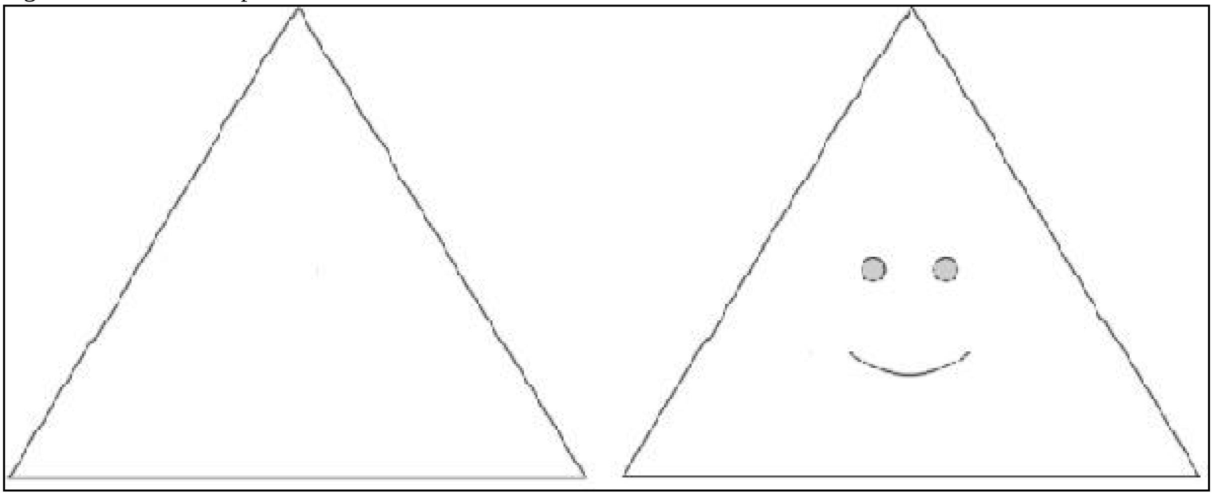
Written consent was gained from the school´s head-teacher and from the parents or guardians of the children. Prior to any assessment, the children were given a brief explanation of the study and the tasks individually and their verbal assent was needed to commence. The participants had their Precise and Approximate Quantity Discrimination Skills, Early Number Skills, mathematical and reading attainment assessed in three individual sessions of a maximum of fifteen minutes during school hours. Computer tasks were combined with oral and/or pencil-and-paper tasks in each session. The order of the presentation of the sessions was randomised across children to counterbalance effects of order.

5.3.3 Computerised Tasks

5.3.3.1 Baseline task

A baseline measure of children´s RTs on a two-choice non-numerical computerised task was obtained. The task was designed with E-Prime in a game-format. Two triangles were presented simultaneously side-by-side on a white screen until participants responded. On each pair, one triangle had eyes and a smile ("happy triangle") and the other triangle was empty (see figure 5.1 for an example). Children were told that they would see two triangles, one on each side of the screen at the same time and that they needed to press as fast and accurately as they could one of the two keys of a response box matching the side (left or right) on which the "happy triangle" appeared. This task consisted of three practise trials and twelve experimental trials in which the right answer appeared six times on each side in a random order. Feedback for each trial was given immediately after each response with a white screen where either a "well done" message or a "oh no, the happy triangle was on the other side" message appeared for 2,000 msecs. Then the following trial automatically started. Children's accuracy and RTs were recorded.

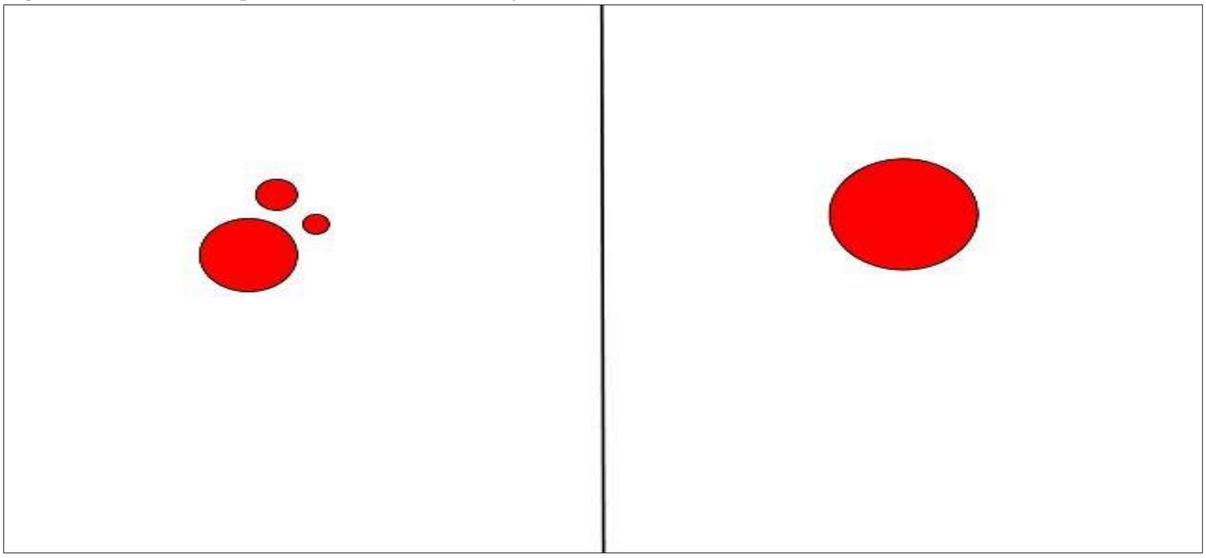
Figure 5.1: Trial example of the Baseline task



5.3.3.2 Precise Quantity Discrimination Skills' task

Two arrays ranging from one to three red circles were simultaneously presented side-by-side on a white screen and separated by a black horizontal line (see figure 5.2 for an example). For each trial, the number of circles in the sets was always different. Children were told that they would see two groups of circles one on each side of the screen at the same time and that they needed to press as fast and accurately as they could one of the two keys of a response box matching the side (left or right) where more circles appeared. This task consisted of four practise trials and 36 experimental trials; twelve trials presenting a comparison of one against two circles, twelve trials presenting a comparison of one against three circles and twelve trials presenting a comparison of two against three circles. Contour length and surface area were controlled for so that these continuous variables could not be associated with the correct response. Six trials were presented in which there was the same surface area in both sets, six trials in which the left set contained the larger surface area, six trials in which the right set contained the larger surface area, six trials presented the same contour length in both sets, six trials on which the left set presented the larger contour length and six trials on which the right set presented the larger contour length. The correct answer appeared the same number of times on each side of the screen in random order. Feedback was given immediately after each response for each trial with a white screen where either a "well done" message or a "oh no, there were more circles on the other side" message automatically appeared for 2,000 msecs. Then the following trial automatically started. Children's accuracy and RTs were recorded.

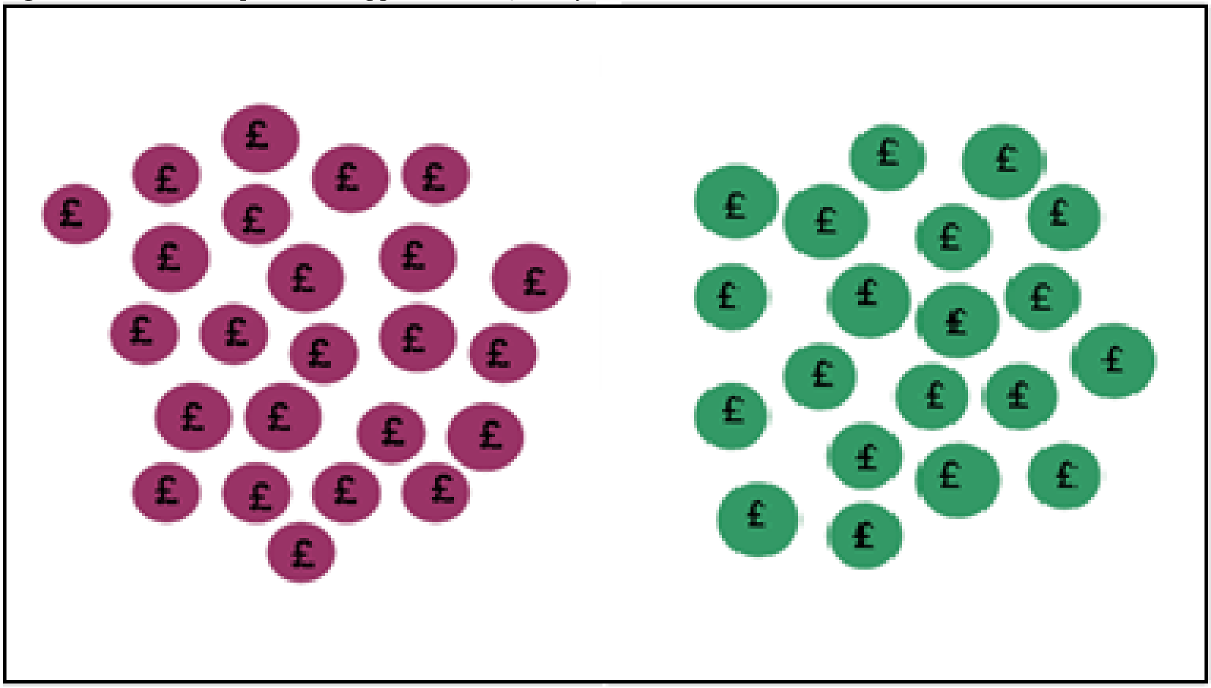
Figure 5.2: Trial example of the Precise Quantity Discrimination skills' task



<u>5.3.3.3 Approximate Quantity Discrimination Skills' task</u>

Nordman et al.'s (2009) non-symbolic approximate quantity discrimination task was adapted using E-Prime. Children were told that they needed to help a pirate to buy a new boat by collecting as many coins as possible. Two sets of circles were simultaneously presented for 2,000 msec. side-by-side on a white screen. During this time, participants had to press the left or right key of a response box matching the side where the larger set in number was presented. Children were told that two groups of coins would appear on each side of the screen at the same time, and that they needed to press as fast and accurately as they could one of the two keys of a response box matching the side (left or right) on which more coins appeared. Circles within each set were either green or purple and contained the sterling pound symbol (£). Circles varied in size inter and intra set (see figure 5.3 for an example).

Figure 5.3: Trial example of the Approximate Quantity Discrimination skills' task



This task consisted of four practise trials and 110 experimental trials. The number of circles in each set varied from five to 35, with each pair depicting a numerical ratio difference ranging from 1.0 (Weber =0) to 1.5 (Weber = 0.5) (see table 5.2). The experimental trials were divided into four blocks, each block presenting 27 trials (two blocks) or 28 trials (two blocks) in random order. The correct answer appeared the same number of times on each side of the screen in random order. Feedback was given immediately after response for each trial. Feedback consisted on the presentation of either a chest full of coins (if a correct response was given) or an empty chest (if an incorrect response was given) on a white screen for 2,000 msecs. If a response was not given within 2,000 msecs. an incorrect answer was automatically recorded and no feedback was given. The following trial automatically started after feedback or when there was no response within 2,000 msecs. Encouraging feedback was given after each block by presenting a picture of a boat that got bigger in size on a white screen. Children were allowed to take breaks between blocks so the picture of the boat remained on the screen until the child indicated that he or she was ready to play again. Then the experimenter pressed a key to start the new block and the first trial started immediately. Correct responses given within the time limit were recorded.

Table 5.2: Approximate Quantity Discrimination skills task design: Stimulus pairing for each numerical ratio difference, Weber fraction value and number of trials

Weber	Ratio	Stimulus pairings (left answer required)					Stimulus pairings (right answer required)					Total times
0.5	**1.5**	9:6	-	18:12	27:18	-	-	10:15	-	-	20:30	20
0.4	**1.4**	-	14:10	21:15	-	-	-	10:14	-	-	-	12
0.3	**1.3**	-	13:10	-	-	30:23	7:9	-	14:18	20:26	-	20
0.2	**1.2**	6:5	-	-	24:20	30:35	5:6	-	15:18	20:24	25:30	28
0.1	**1.1**	11:10	-	22:20	32:29	-	-	11:12	-	-	30:33	20
0.0	**1.0**	5:5	9:9	15:15	24:24	30:30	7:7	10:10	22:22	25:25	31:31	10

Note1. Numeric ratio = n2/n1, n2 being the larger set
Note2. Weber Fraction value = (n2-n1)/n1, n2 being the larger set

5.3.4 Non-computerised tasks

5.3.4.1 Early Number Skills' tasks

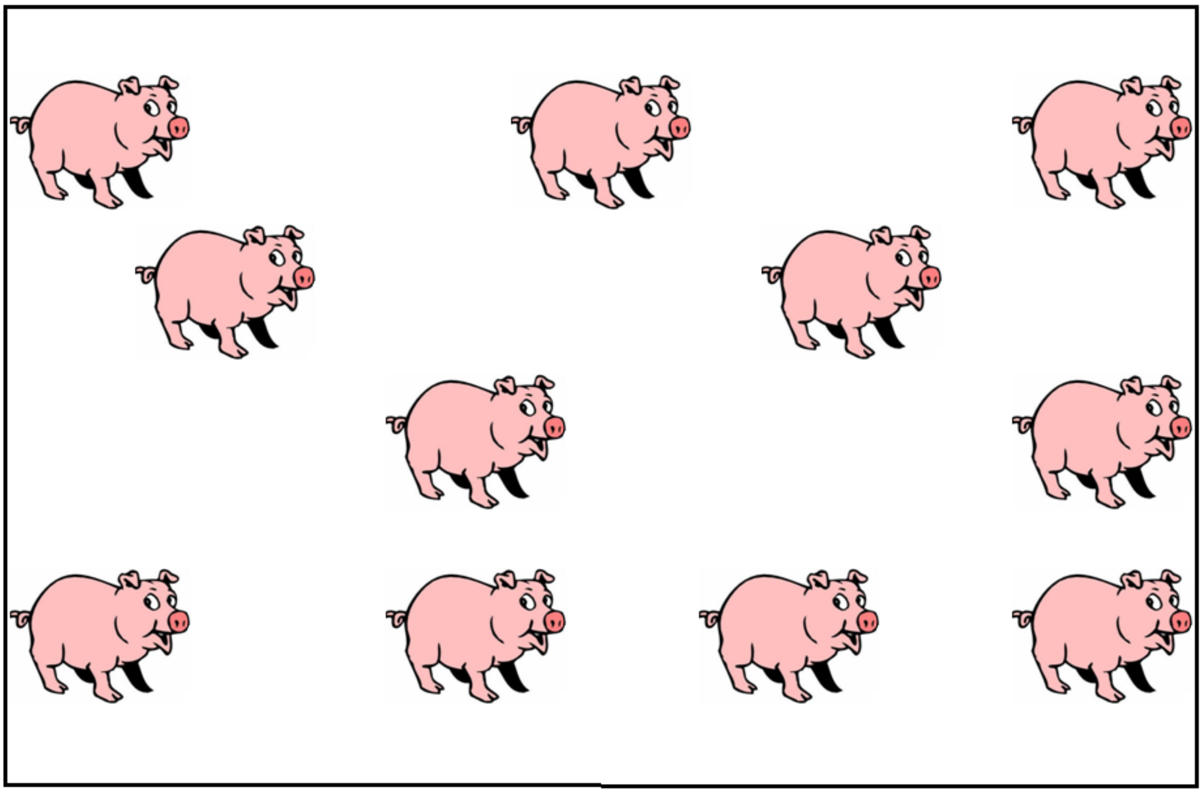
- Knowledge of the number sequence (KNS)

Children were asked to recite the number-word sequence starting from one by being asked by the researcher "Can you count? Can you show me how high you can count up to?" and were stopped at twenty. If participants missed or made less than four consecutive errors they were then asked to recite the number sequence starting from numbers 25, 65 and 75 and scored on their ability to recite the six following number-words. They were stopped after four incorrect or missing number-words. One point was given for each number-word given in the correct order. Consequently, a maximum of 50 points could be obtained. No points were given for misplaced or missing number-words.

- Counting objects (CO)

Children were asked to count pictures of animals presented randomly spread on a laminated sheet. The picture of the animals varied between sheets. The researcher asked the child *"How many (name of the animal presented) are there?"* (see figure 5.4 for an example).

Figure 5.4: Counting Objects task example



The task consisted of twenty trials presented in increasing difficulty showing from three to 97 pictures of animals of approximately 2.5 cm^2 (see table 5.3). The first fifteen trials were presented on A4 laminated sheets. The last five trials were presented on A3 laminated sheets. Children were told they could touch the pictures to count them if needed. One point was given for each set counted correctly. Children were stopped after four consecutive incorrect answers.

Table 5.3: Number of discrete items (pictures of animals) presented for each item on the Counting Objects task

Item no.	**Items displayed**	**Item no.**	**Items displayed**	**Item no.**	**Items displayed**	**Item no.**	**Items displayed**
Item 1	3	Item 6	11	Item 11	20	Item 16	35
Item 2	5	Item 7	13	Item 12	23	Item 17	42
Item 3	7	Item 8	15	Item 13	25	Item 18	51
Item 4	8	Item 9	16	Item 14	27	Item 19	66
Item 5	10	Item 10	18	Item 15	30	Item 20	97

- Story Problems (SP)

The researcher read an arithmetical problem in the format of a brief story to the child while showing a picture that related to the story context but showed no concrete support for the answer. A request for the result of a simple arithmetic calculation was made at the end of each story with a direct question (e.g. *“Four people live in a house. Three people move in with them. How many people live in the house now?”* While a picture of a house was presented to the child). The researcher re-read the story to the child only if he/she asked for it. This task consisted of twenty story problems. The first part consisted of ten addition problems presented in increasing difficulty. The first five story problems in this part requested adding two single-digit numbers, resulting in a single-digit outcome and the last five story problems requested adding a two-digit number to a single-digit number resulting in a two-digit outcome not larger than twenty. The second part consisted of ten subtraction story problems presented in increasing difficulty. The first five story problems in this part requested subtracting a single-digit number from a single-digit number, resulting in a single-digit outcome and the last five story problems requested the subtraction of a single-digit number from a two-digit number, resulting in a two digit outcome no lower than eleven. Children were stopped after four consecutive incorrect answers on each part. One point was given for each correct answer.

5.3.4.2. Mathematical Attainment Measures

Two subtests of the WIAT-II^{UK} were administered in two different sessions.

- The Numerical Operations subtest

This subtest assesses children’s arithmetic competence and includes tasks such as numeral identification, numeral position, numeral writing and solving written arithmetic problems on paper.

- The Mathematical Reasoning subtest

This subtest assesses children’s ability to manipulate and apply numerical information in different contexts. The tasks are presented orally and supported with illustrations. They include counting, identifying shapes and solving numerical problems.

5.3.4.3 Reading Attainment Measure

Children were administered the Word Reading subtest of the WIAT-II^{UK}. This test assesses children's printed letters identification, knowledge of letter names and single word reading.

5.4 RESULTS

5.4.1 Descriptive statistics

5.4.1.1 Computerised tasks

- Precise Quantity Discrimination Skills' task and Baseline task

A measure of children's RTs was obtained calculating the mean RT for all correct responses taking less than two times the interquartile range from the median (hereafter Precise QDS Trimmed RT) for each child. This was done in order to eliminate influence of outliers due to children's distractibility rather than representing time needed to process numerical information. Similar data trimming procedures were suggested by Tukey (1977) and have been performed in visual-attention studies exploring whether subjects are engaging in parallel or serial processes (Pylyshyn & Storm, 1988; Simon, Peterson, Patel, & Sathian, 1998; Wei, Lü, Müller, & Zhou, 2008) and with adults responding to numerical information (Carreiras, Carr, Barber, & Hernandez, 2010). Similar although more idiosyncratic data trimming has also been performed with young and older children's latencies in response to non-symbolic numerical stimuli in order to eliminate very fast responses that are highly unlikely for subjects' age or responses taking too long due to distractibility (Butterworth, 2003; Gebuis, Herfs, Kenemans, De Haan, & Van der Smagt, 2009; Libertus et al., 2011).

In addition, a computerised baseline measure of children's speed in making accurate two-choice non-numerical discriminations was obtained calculating the mean RT for all correct responses taking less than two times the interquartile range from the median (hereafter Baseline Trimmed RT) for each child. This measure was obtained with the aim of controlling for individual differences in non-numerical two-choice discrimination tasks so that a no contaminated measure of the time needed to make numerical discriminations could be obtained. A similar baseline speed measure has been used with children to adjust their RTs when responding to numerical stimuli to their individual latencies in computerised tasks (see Butterworth, 2003).

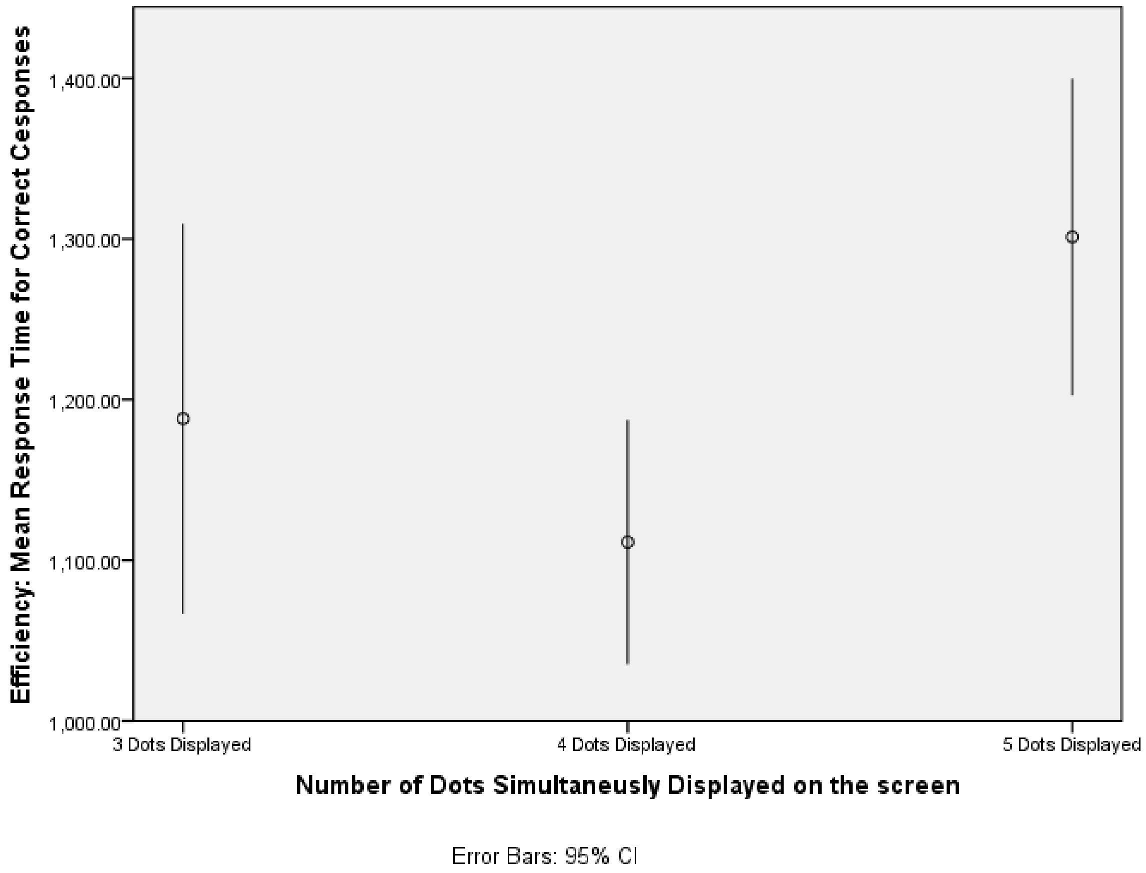
Descriptive statistics for children's Baseline and Precise Quantity Discrimination Skills' Accuracy and Mean Trimmed RT were obtained (see table 5.4).

Table 5.4: Descriptive statistics for the Precise QDS' task and the Baseline task (n=63)

Test	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min.**	**Max.**
Precise QDS						
Precise QDS Accuracy (max.36)	33.71	3.08	-9.48	18.32	18	36
Precise QDS Trimmed RT (msec.)	1176.15	328.95	6.33	8.40	750.53	2521.68
Baseline Task						
Baseline Accuracy (max.12)	11.30	0.91	-4.71	3.57	8	12
Baseline Trimmed RT (msec.)	746.50	240.90	9.57	19.09	374.71	1866.08

Accuracy rates in both tasks are very high with all participants scoring above chance, however, participants' speed varied widely across the sample. In order to explore whether children employed a parallel or a serial enumeration strategy to identify which set contained more items, a repeated measures analysis of variance was conducted. Three Trimmed RT measures for when the total number of dots presented on the screen equalled three (one against two), four (one against three) and five (two against three) were entered. Children's Trimmed RT for each type of trial were not normally distributed for three dots (D (63) = .22, <.001) or four dots (D (63) = .14, <.01) but they were for five dots (D (63) = .11, *p*=.05). Data failed to meet sphericity assumptions (χ^2 (2) =22.69, *p*<.001) so values given are following Greenhouse Geisser corrections (ε=.76). There was a significant effect of the number of dots presented in the trials on children's Trimmed RT F(1.53, 94.61) = 13.94, *p*<.001). Children were significantly more efficient on trials presenting three dots (M=1166.73, SD=452.29) and four dots (M=1103.49, SD=274.37) than on trials presenting five dots (M=1269.99, SD=337.44). There was evidence for linear (F(1,62)=7.80, *p*<.01) and non-linear (F(1,62)=26.64, *p*<.001) trends in the data. If children had solely employed a serial counting strategy one would expect there to be an increase in the RTs with every additional dot presented on the screen and only the linear trend would be significant. However, the data shows a quadratic trend as the number of dots presented in the task increases (see figure 5.5).

Figure 5.5: Trimmed RT for correct responses when three dots (one against two), four dots (one against three) and five dots (two against three) where displayed irrespective of the side where the correct answer was presented



- Approximate Quantity Discrimination Skills' task

Accuracy analyses for the non-symbolic Approximate Quantity Discrimination task (hereafter Approximate QDS) show a normal distribution (see table 5.5).

Table 5.5: Descriptive statistics for the Approximate QDS' task.

Test	Mean	SD	ZSkewness	ZKurtosis	Min.	Max.
Approximate QDS Accuracy (max.100)	70.63	10.23	-0.51	0.64	50	94

To test whether difficulty increases as the numerical ratio difference associated with each pair of non-symbolic quantities decreases, the number of left responses given by participants in each trial was analysed for each numerical ratio bin. Results were plotted (see figure 5.6).

Figure 5.6: Proportion of left responses (Z-scores) at each Weber Fraction (ratio bin)

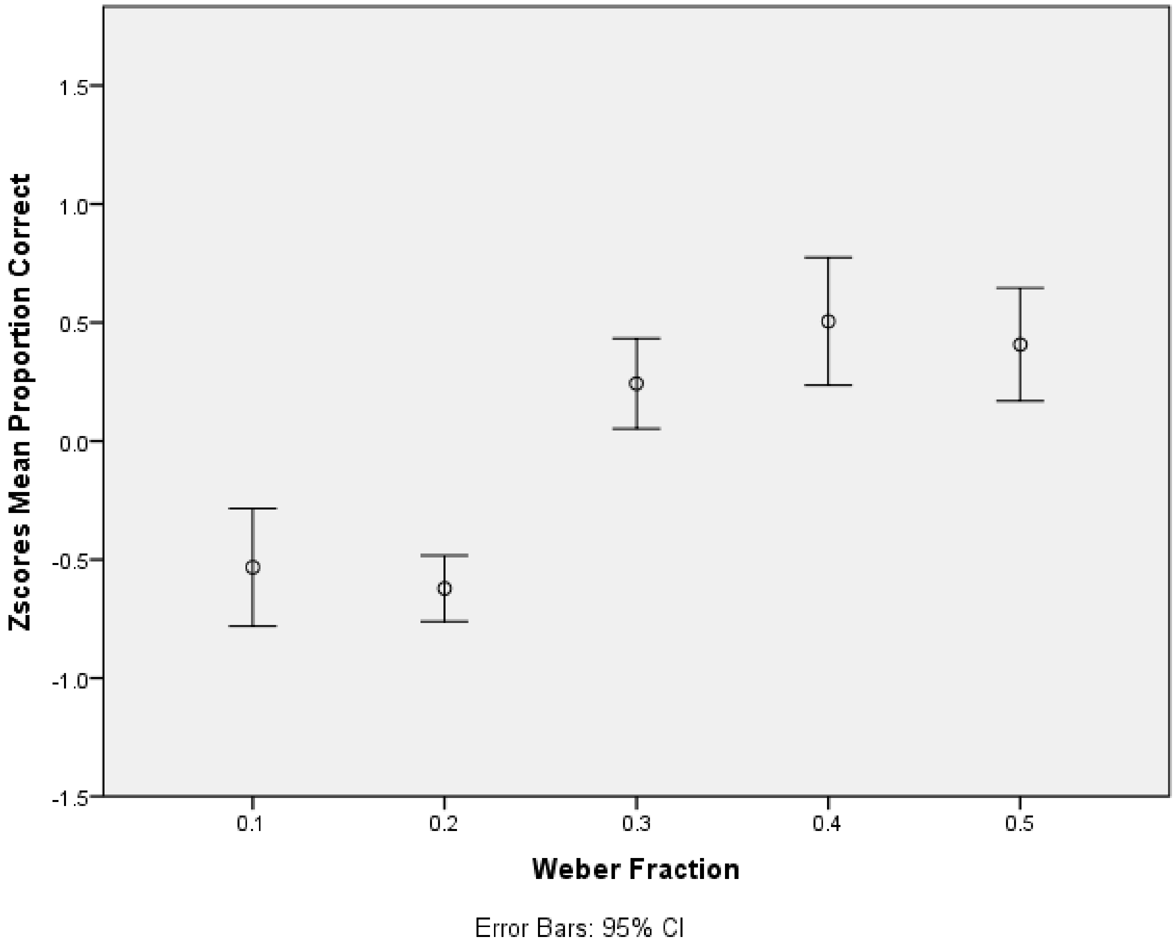


Figure 5.6 shows that the number of correct responses given decreased as the numerical ratio difference associated with each pair of non-symbolic quantities decreased.

5.4.1.2 Non-computerised tasks

- Early Number Skills' tasks

Descriptive statistics for children's scores on the Knowledge of the Number Sequence task, the Counting Objects task and the Story Problems tasks assessing their Early Number Skills were obtained (see table 5.6).

Table 5.6: Descriptive statistics for the three novel Early Number Skills' tasks

Early Number Skills Tasks	Mean	SD	ZSkewness	ZKurtosis	Min	Max
Knowledge of Number Sequence (max. 50)	32.56	6.98	-4.46	1.34	12	38
Counting Objects (max. 20)	12.05	4.14	-2.20	0.17	1	19
Story Problems (max. 20)	8.05	4.78	-0.40	-1.73	0	17

Note 1. For the Counting Objects task, a mean of 12.05 stands for 23 discrete items counted correctly (see table 5.3 in section 5.3.4.1 for further clarification)

Results show that only the Story Problems task of the three Early Number Skills tasks is normally distributed. The spread of scores in the Knowledge of the Number Sequence task shows a negatively skewed distribution and strong ceiling effects. The spread of scores in the Counting Objects task also shows a negatively skewed distribution although less severe and with no ceiling effects.

- Standardised Attainment tasks

Descriptive statistics for children's raw and standard scores on the Numerical Operations, Mathematical Reasoning and Word Reading subtests of the WIAT-II^{UK} were obtained (see table 5.7)

Table 5.7: Descriptive statistics for the Mathematical attainment (n=58) and Reading attainment (n=63) raw and standard scores

Standardised Attainment Measures	Mean	SD	ZSkewness	ZKurtosis	Min	Max
Numerical Operations WIAT-II^{UK} raw scores	8.22	2.43	-1.86	0.39	2	12
Numerical Operations WIAT-II^{UK} standard	100	11.09	1.19	1.61	68	125
Mathematical Reasoning WIAT-II^{UK} raw scores	17.79	5.45	1.23	1.17	6	34
Mathematical Reasoning WIAT-II^{UK} standard	103.15	13.31	-1.06	-0.68	73	128
Word Reading WIAT-II^{UK} raw scores	29.89	29.17	2.73	-1.00	2	103
Word Reading WIAT-II^{UK} standard	89.51	24.88	0.23	-1.61	45	144

Children's mathematical attainment scores in the Numerical Operations and the Mathematical Reasoning subtests of the WIAT-II^{UK} are broadly similar to the UK average. Means for both subtests are based on a sub-sample of 58 children as five participants in Reception Year were too young for the test norms to be applied. Children's raw scores were used for further analyses. Participants' reading attainment was assessed as below the UK average using the WIAT-II^{UK}. Reasons for these anomalous results are discussed later.

5.4.2 Correlations and partial correlations for all the predictor variables and outcome measures

Correlations and partial correlations controlling for participants' age in months for all predictor variables and outcome measures were obtained (see table 5.8).

Table 5.8: Correlations (above the diagonal) and partial correlations controlling for age in months (below the diagonal) for the Quantity Discrimination Skills' tasks with the three Early Number Skills' tasks and the two mathematical attainment measures (n=63, df=60)

Variable	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1.Age (months)	.059	-.330**	.258*	.345**	.424***	.449***	.504***	.440***
2.Precise Quantity Discrimination Skills' Accuracy	-	-.296*	.431***	.207	.183	.197	.262*	.182
3.Precise Quantity Discrimination Skills' Trimmed RT	-.293*	-	-.431***	-.562***	-.453***	-.525***	-.631***	-.604***
4.Approximate Quantity Discrimination Skills' Accuracy	.432***	-.379**	-	.345**	.385**	.384**	.560***	.353**
5.Knowledge of the Number Sequence	.199	-.506***	.282*	-	.541***	.646***	.724***	.649***
6.Counting Objects	.174	-.366**	.315*	.464***	-	.534***	.690***	.586***
7.Story Problems	.191	-.447***	.310*	.586***	.425***	-	.715***	.693***
8.Numerical Operations WIAT-II^{UK}	.270*	-.570***	.515***	.679***	.609***	.634***	-	.711***
9.Mathematical Reasoning WIAT-II^{UK}	.173	-.541***	.276*	.590***	491***	.618***	.631***	-

***p<=.05; **p<=.01; ***p<=.001**

All significant correlations are positive with the exception of the variables measuring children's RTs, where correlations are negative, indicating that the older the children are, the faster they can respond to computerised discrimination tasks and the better they performed on the outcome measures. After controlling for children's age in months, the predictor variables remain significantly correlated to each other. Children's performance on each of the three Early Number Skills tasks remains significantly and modestly correlated to each other and performance on each of the two mathematical attainment measures also remains significantly and modestly correlated to each other. Regarding children's performance on the quantity discrimination skills' tasks, children's accuracy on the Precise Quantity Discrimination skills task does not share significant variance with any of the Early Number Skills or with performance on the Mathematical Reasoning subtest of the mathematical attainment measures. In contrast, after controlling for age in months, Precise Quantity Discrimination Skills' Trimmed RT shares significant variance with all outcome measures, the strength of these correlations being either modest or weak. Children's Approximate Quantity Discrimination skills' Accuracy correlates with all predictor variables and outcome measures before and after controlling for age in months, the strength of these correlations being either modest or weak. Children's performances on each of the three Early Number Skills tasks relate modestly to each other after controlling for their age in months. Children's performance on the mathematical attainment measures are highly correlated to each other after controlling for their age in months.

5.4.3 Correlations and partial correlations for the Precise Quantity Discrimination Skills' task with all the predictor variables and outcome measures

5.4.3.1 Partial correlations for Baseline Trimmed RT with all the predictor variables and all the outcome measures

Partial correlations controlling for participants' age in months for Baseline Trimmed RT with all the predictor variables and all outcome measures were obtained (see table 5.9).

Table 5.9: Partial correlations for Baseline Trimmed RT with all predictor variables and outcome measures controlling for age (n=63, df=60)

	Quantity Discrimination Skills			**Early Number Skills**			**Maths attainment**	
	PQDS Acc.	**PQDS Trimmed RT**	**AQDS Acc.**	**KNS**	**CO**	**SP**	**NO**	**MR**
Baseline Trimmed RT	.024	.694***	-.338**	-.478*	-.433***	-.321*	-.562***	-.410***

***p<=.05; **p<=.01; ***p<=.001**

After controlling for age in months, Baseline Trimmed RT shares significant variance with all predictor variables except for Precise Quantity Discrimination Skills' Accuracy. Baseline Trimmed RT also shares significant variance with all outcome measures. The strength of all these correlations is modest or weak except for Precise Quantity Discrimination Skills' Trimmed RT that shares a large amount of variance with Baseline Trimmed RT. Therefore, additional partial correlations controlling for participants' age in months and Baseline Trimmed RT were conducted for Precise Quantity Discrimination Skills' Trimmed RT and Accuracy with all the outcome measures (see table 5.10).

Table 5.10: Partial correlations for Precise Quantity Discrimination Skills' Accuracy and Trimmed RT with Approximate Quantity Discrimination Skills' Accuracy and all outcome measures controlling for age and Baseline Trimmed RT (n=63, df=59)

	Approximate QDS Acc.	**KNS**	**CO**	**SP**	**NO**	**MR**
Precise QDS Accuracy	.467***	.240	.205	.210	.342**	.201
Precise QDS Trimmed RT	-.213	-.275*	-.101	-.329**	-.303*	-.391**

***p<=.05; **p<=.01; ***p<=.001**

After controlling for age in months and Baseline Trimmed RT, Precise Quantity Discrimination skills' Accuracy shares significant variance with Approximate Quantity Discrimination skills' Accuracy and with performance on the Mathematical Reasoning subtest. After controlling for age in months and Baseline Trimmed RT, Precise Quantity Discrimination skills' Trimmed RT still shares significant variance with children's performance on the Knowledge of the Number Sequence task, the Story Problems task and both mathematical attainment measures. Children's Precise Quantity Discrimination Trimmed RT after controlling for their Baseline Trimmed RT was used as a measure of their speed of precise non-symbolic numerical judgements for further analyses.

5.4.6 Regressions

5.4.6.1 The relationships between the three Early Number Skills' tasks and the two mathematical attainment measures

Two different linear regression analyses for each mathematical attainment measure were conducted to explore whether children's scores in the three Early Number Skills' tasks could explain unique variance in their scores on the two different standardised mathematical attainment measures. The two regression models consisted of two blocks where age in months was always introduced in the first block and the three Early Number Skills' tasks (Knowledge of Number Sequence, Counting Objects and Story Problems) were introduced together in the second block.

- The Numerical Operations subtest

In the first regression analysis, the raw scores of the Numerical Operations subtest were introduced as the criterion variable (see table 5.11).

Table 5.11: Forced entry regression analysis examining the prediction of the raw score of the Numerical Operations subtest of the WIAT-II^{UK} from the scores in the Early Number Skills' tasks over and above participants' age in months.

		B	**SE**	**Beta**	***t***	**Sig.**
Block 1	F(1, 61)=20.720, p<.001					
	Age in months	.177	.039	.504	4.552	.000
Block 2	F(4, 58)=36.912, p<.001					
	Knowledge of Number Sequence	.117	.033	.335	3.497	.001
	Counting Objects	.179	.052	.304	3.420	.001
	Story Problems	.140	.050	.275	2.792	.007

Note. R^2 = .25 for block 1, ΔR^2= .46

Over and above children's age in months, children's performance on the Early Number Skills' tasks together explained an additional 46.4% of children's variance in the Numerical Operations subtest. All three Early Number Skills were unique predictors.

- The Mathematical Reasoning subtest

In the second regression analysis, the raw scores of the Mathematical Reasoning subtest were introduced as the criterion variable (see table 5.12).

Table 5.12: Forced entry regression analysis examining the prediction of the raw score of the Mathematical Reasoning subtest of the WIAT-II[UK] from the scores in the Early Number Skills' tasks over and above participants' age in months.

		B	SE	Beta	*t*	Sig.
Block 1	F(1, 61)=14.673, p<.001					
	Age	.347	.091	.440	3.831	.000
Block 2	F(4, 58)=.20.919, p<.001					
	Knowledge of Number Sequence	.209	.090	.267	2.311	.024
	Counting Objects	.269	.141	.204	1.906	.062
	Story Problems	.420	.136	.368	3.099	.003

Note. R^2 = .19 for block 1, ΔR^2= .40

Over and above children's age in months, performance on the Early Number Skills' tasks together explained an additional 39.7% of children's variance on the Mathematical Reasoning subtest. Knowledge of the Number Sequence and Story Problems were unique predictors.

5.4.6.2 The relationships between the two Quantity Discrimination Skills' tasks and the three Early Number Skills' tasks

Three different linear regression analyses for each Early Number Skills' task were conducted to explore whether children's Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy could explain unique variance in their performance on each of the standardised mathematical attainment measures. The two regression models consisted of two blocks where age in months and Baseline Trimmed RT were always introduced in the first block and Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy were introduced together in the second block. Baseline Trimmed RT was introduced in the first step of the model so that individual differences in responding to non-numerical two-choice discrimination tasks was controlled for given that Precise Quantity Discrimination Skills' Trimmed RT (and not accuracy) was used as a predictor variable.

- Knowledge of the Number Sequence task

A linear regression analysis was conducted to explore whether children's Quantity Discrimination Skills could explain unique variance in their performance on the Knowledge of the Number Sequence task (see table 5.13).

Table 5.13: Forced entry regression analysis examining the prediction of Knowledge of Number Sequence from Precise QDS Trimmed RT and Approximate QDS Accuracy over and above age in months and Baseline Trimmed RT

		B	**SE**	**Beta**	***t***	**Sig.**
Block 1	F(2, 60)=14.128, p<.001					
	Age in months	.274	.110	.246	2.253	.028
	Baseline Trimmed RT	-.013	.003	-.460	-4.216	.000
Block 2	F(4, 58)=8.771, p=.080					
	Precise QDS Trimmed RT	-.007	.003	-.310	-1.984	.052
	Approximate QDS Accuracy	.057	.080	.083	.711	.480

Note. R^2 = .32 for block 1, ΔR^2= .06

Over and above children's age in months and Baseline Trimmed RT, performance on the Quantity Discrimination Skills' tasks together explained an additional 5.7% of children's variance on the Knowledge of the Number Sequence task. However the additional variance explained was not statistically significant.

- Counting Objects task

A linear regression analysis was conducted to explore whether children's Quantity Discrimination Skills could explain unique significant variance in their performance on the Counting Objects task (see table 5.14).

Table 5.14: Forced entry regression analysis examining the prediction of Counting Objects from Precise QDS Trimmed RT and Approximate QDS Accuracy over and above age in months and Baseline Trimmed RT

		B	**SE**	**Beta**	***t***	**Sig.**
Block 1	F(2, 60)=15.010, p<.001					
	Age in months	.202	.065	.337	3.124	.003
	Baseline Trimmed RT	-.007	.002	-.402	-3.721	.000
Block 2	F(4, 58)=8.230, p=.280					
	Precise QDS Trimmed RT	-.001	.002	-.074	-.467	.643
	Approximate QDS Accuracy	.067	.048	.166	1.409	.164

Note. R^2= .33 for block 1, ΔR^2= .03

Over and above children's age in months and Baseline Trimmed RT, children's performance on the Quantity Discrimination Skills' tasks together explained an additional 2.9% of children's variance on the Counting Objects task. However the additional variance explained was not statistically significant.

- Story Problems task

A linear regression analysis was conducted to explore whether children's Quantity Discrimination Skills could explain unique significant variance in their performance on the Story Problems task (see table 5.15).

Table 5.15: Forced entry regression analysis examining the prediction of Counting Objects from Precise QDS Trimmed RT and Approximate QDS Accuracy over and above age in months and Baseline Trimmed RT

		B	**SE**	**Beta**	***t***	**Sig.**
Block 1	$F(2, 60)=11.881$, $p<.001$					
	Age in months	.266	.077	.385	3.442	.001
	Baseline Trimmed RT	-.006	.002	-.294	-2.629	.011
Block 2	$F(4,58)=.8.869$, $p=.016$					
	Precise QDS Trimmed RT	-.005	.002	-.366	-2.350	.022
	Approximate QDS Accuracy	.071	.054	.153	1.310	.195

Note. R^2 = .28 for block 1, ΔR^2= .10

Over and above children's age in months and Baseline Trimmed RT, performance on the Quantity Discrimination Skills' tasks together explained an additional 9.6% of children's variance on the Story Problems task. The proportion of variance predicted was statistically significant and Precise Quantity Discrimination skills' Trimmed RT was a unique predictor.

5.4.6.3 The relationships between the two Quantity Discrimination Skills' tasks and the two mathematical attainment measures

Two different linear regression analyses for each mathematical attainment measure were conducted to explore whether children's scores on the Quantity Discrimination Skills' tasks could explain unique independent variance in their performance on each of the mathematical attainment measures. The two regression models consisted of two blocks where age in months and Baseline Trimmed RT were always introduced in the first block and the two Quantity Discrimination Skills' tasks were introduced together in the second block.

- Numerical Operation subtest

A linear regression analysis was conducted to explore whether children's Quantity Discrimination Skills could explain unique significant variance in their performance on the Numerical Operations subtest (see table 5.16).

Table 5.16: Forced entry regression analysis examining the prediction of the Numerical Operations subtest of the WIAT-II^{UK} from Precise QDS Trimmed RT and Approximate QDS Accuracy over and above age in months and Baseline Trimmed RT

		B	**SE**	**Beta**	***t***	**Sig.**
Block 1	F(2, 60)=28.698, p<.001					
	Age in months	.139	.039	.367	3.607	.001
	Baseline Trimmed RT	-.005	.001	-.451	-4.437	.000
Block 2	F(4, 58)=22.005, p=.001					
	Precise QDS Trimmed RT	-.002	.001	-.236	-1.895	.063
	Approximate QDS Accuracy	.069	.022	.291	3.124	.003

Note. R^2 = .49 for block 1, ΔR^2= .11

Over and above children's age in months and Baseline Trimmed RT, performance on the Quantity Discrimination Skills' tasks together explained an additional 11.4% of children's variance on the Numerical Operations subtest. The proportion of variance predicted was statistically significant and Approximate Quantity Discrimination skills' Accuracy was a unique predictor.

- Mathematical Reasoning subtest

A linear regression analysis was conducted to explore whether children's Quantity Discrimination Skills could explain unique significant variance in their performance on the Mathematical Reasoning subtest (see table 5.17).

Table 5.17: Forced entry regression analysis examining the prediction of the Mathematical Reasoning subtest of the WIAT-II[UK] from Precise QDS Trimmed RT and Approximate QDS Accuracy over and above age in months and Baseline Trimmed RT

		B	**SE**	**Beta**	***t***	**Sig.**
Block 1	F(2, 60)=14.742, p<.000					
	Age in months	.283	.085	.359	3.316	.002
	Baseline RT	-.009	.002	-.377	-3.483	.001
Block 2	F(4, 58)=11.212, p=.007					
	Precise QDS RT	-.007	.002	-.450	-3.033	.004
	Approximate QDS Accuracy	.038	.059	.072	.649	.519

Note. R^2 = .33 for block 1, ΔR^2= .11

Over and above children's age in months and Baseline Trimmed RT, performance on the Quantity Discrimination Skills' tasks together explained an additional 10.7% of children's variance on the Mathematical Reasoning subtest. The proportion of variance predicted was statistically significant and Precise Quantity Discrimination skills' Trimmed RT was a unique predictor.

5.6 DISCUSSION

This pilot study examined the validity of the novel tasks designed to be used in the longitudinal study, where the independent and unique longitudinal contributions of domain-general and domain-specific cognitive skills on young children's Early Number Skills and mathematical attainment was examined. This pilot study explored whether the three novel early number tasks provide valid and appropriate measures of the three developmental levels of early QNCs proposed by Krajewski and Schneider (2009). Also, it explored whether these three novel number tasks make unique and independent contributions to the two standardised mathematical attainment measures included in the study. In addition, this pilot study explored whether the two non-symbolic Quantity Discrimination Skills' tasks designed to tap the two different core quantity systems proposed by Feigenson et al. (2004) provide accurate measures of these systems. It also explored whether they make unique and independent contributions to the three Early Number Skills tasks and to the two standardised mathematical attainment measures once children's age and individual differences in responding to non-numerical two-choice discrimination tasks was controlled for.

5.6.1 Criterion validity of the standardised attainment measures

Participants' scores on the Numerical Operations and Mathematical Reasoning subtests of the WIAT-II^{UK} are broadly similar to the UK average and provide a good spread of scores. However, participants' reading attainment assessed with the Word Reading subtest of the WIAT-II^{UK} was below the UK average. This test, included to test the specificity of the relationships found in the present study, does not provide a valid accurate measure of children's reading attainment because it is not indicative of their teacher-assessed performance or in line with their standardised mathematical attainment scores. A plausible explanation for these results is that in 26 of the first 30 items in this subtest (where a discontinue rule of seven consecutive incorrect or missing answers applies), participants are asked to report the letter name from a printed letter symbol. The vast majority of the children could identify the letter sound, but were unable to report the letter name because they were learning to read with the phonics method and being taught to produce letter sounds from a printed letter symbol not the letter names. Thus the Early Word Recognition subtest of the York Assessment of Reading for Comprehension (hereafter YARC) (Hulme et al., 2009) was used in the longitudinal study to test the specificity of the relations between the domain-specific and domain-general cognitive predictors and the variables of interest.

5.6.2 Criterion validity of the three novel Early Number Skills' tasks

The three early number tasks designed to assess the three levels of early QNCs proposed in Krajewski and Schneider´s (2009) theoretical model of early arithmetical development are highly correlated with each other but yet explain independent and unique variance in both of the standardised mathematical attainment measures (with the exception of the Counting Objects task that misses levels of significance when predicting performance on the Mathematical Reasoning subtest, p=0.062). Thus, they provide appropriate measures of the three distinct levels of QNCs at this age and their criterion validity is satisfactory. Nevertheless, the Knowledge of the Number Sequence task shows a negatively skewed distribution of scores and strong ceiling effects. Consequently, more difficult items were added to this task for the longitudinal study in order to obtain a better spread of scores. The scores' distribution in the Counting Objects task is also negatively skewed although not severely and shows no ceiling effects. For these reasons and because at a practical level, making children count more than a hundred objects would consume valuable time during the data collection sessions, this task has not been modified for the longitudinal study. Although the Story Problems task provided a good spread of scores, for the longitudinal study the researcher did not re-read the story to the child, not even if requested. Instead, participants were asked to focus on remembering information given and as a last resource to have a guess. This is because some children might not ask for repetition due to shyness while others would. Thus, in an attempt to provide more consistency with the administration procedures, participants listened to each story problem only once.

5.6.3 Construct validity of the Quantity Discrimination Skills' tasks

The non-symbolic Precise Quantity Discrimination Skills' task was designed to tap children's Precise Number System functioning. It has been argued that the process of subitising depends on this particular system supporting abstract and precise numerical representations (Feigenson et al., 2004). Because children's subitising range is about three or four discrete items (Arp et al., 2006; Benoit et al., 2004; Chi & Klahr, 1975; Fischer et al., 2008; Landerl et al., 2004; Schleifer & Landerl, 2011; Svenson & Sjöberg, 1978; Trick et al., 1996), the Precise Quantity Discrimination Skills' task only presented sets containing up to three items. It was found that the total number of items presented had a significant effect on children's numerical discriminations' latancies in this task. However, if children were employing a serial counting strategy in order to respond a continuous linear increase with every additional dot presented on the screen would be expected. In contrast, it took children overall less time to discriminate one dot against three dots than one dot against two dots or two dots against three dots, suggesting that children were indeed employing an alternative quantification strategy different from vocal or sub-vocal counting and were relying on a parallel enumeration process. This explanation could account for the discontinuity in their latencies as the number of items increased. The fact that children's individual latencies on a computerised two-choice non-numerical discrimination task were controlled for and that both sets to be compared were presented simultaneously makes it unlikely that these results are due to the efficacy of children's domain-general cognitive systems. Also, because stimuli in the task were non-symbolic and a key respond method was used children's performance is unlikely to be influenced by individual differences in knowledge of the formal number system.

The non-symbolic Approximate Quantity Discrimination Skills' task seems to efficiently tap children's ANS precision for several reasons. First, pairs of non-symbolic sets in this task were presented simultaneously for 2,000 msec. and the smallest set presented in this task contained five items. The average time for five- and six-year-olds to correctly enumerate an additional item outside the subitising range is approximately 1,000 msec. (Chi & Klahr, 1975; LeFevre et al., 2010; Svenson & Sjöberg, 1978; Trick et al., 1996). Therefore, if children had been relying on serial enumeration strategies (serial counting or a combination of subitising and serial counting) they would not have had enough time to make accurate numerical judgements. Second, a measure of participants' ability to count discrete visual items with no time restriction was obtained (Counting Objects task). On average children could correctly count up to fifteen discrete items and in more than 55% of experimental trials on this task at least one of the sets contained more than fifteen dots (see table 5.2 in section 5.3.3.3). Third, children's performance on this task shows the typical ratio-signature (decrease of correct responses as the numerical ratio difference between the numerical sets decreases) observed in previous studies conducted with young children (Halberda & Feigenson, 2008). Finally, children's accuracy rates are fairly similar to those reported by Libertus et al. (2013) in which pre-schooler's accuracy was 65.10%, with a standard deviation of 15.15. In the present study, the mean accuracy rate across the sample is 70%, with a standard deviation of 10.23 and children in the present sample were slightly older. For these reasons, the non-symbolic Approximate Quantity Discrimination skills' task employed in the present study is considered to be tapping children's Approximate Number System precision. Participants' employment of other alternative quantification strategies such as serial counting could not have been sufficiently efficient to respond accurately.

5.6.4 Relationships between Quantity Discrimination Skills' tasks and the outcome measures

Children's Precise Quantity Discriminations skills' Trimmed RT makes unique independent contributions to the Story Problems task of the Early Number Skills and to their performance on the Numerical Operations subtest. These results support Krajewski and Schneider's (2009) theoretical proposal that early quantity discrimination skills contribute to children's Level III QNCs. However, although included in their theoretical model, Krajewski and Schneider (2009) did not assess quantity discrimination skills to explore how they relate to the three levels of QNCs; this aspect makes the present study pioneering. Results also support previous studies where a relationship between children's ability to precisely and accurately enumerate or discriminate small numerical representations and their mathematical attainment was found (Arp et al., 2006; Durand et al., 2005; Fischer et al., 2008; Landerl et al., 2004; LeFevre et al., 2010; Schleifer & Landerl, 2011). Nevertheless, no relationship was found between children's Trimmed RT in making precise numerical discriminations over small quantities and their performance on the Mathematical Reasoning subtest, suggesting that the impact of these early quantitative skills might be relative to certain aspects of mathematical attainment but not all.

In contrast, Approximate Quantity Discrimination Skills' Accuracy failed to predict performance on any of the Early Number Skills' tasks at a significant level but predicted their scores in the Numerical Operation subtest of mathematical attainment. These results are in line with previous findings from Nordman et al. (2009, September) where five-year-olds performance on a very similar non-symbolic approximate quantity discrimination task predicted their scores in the Numerical Operation mathematical attainment subtest. The relationship between approximate non-symbolic numerical discrimination skills and performance on standardised mathematical attainment measures is also in line with other recent studies that used different standardised attainment measures to the ones used in the present study (Gilmore, McCarthy, & Spelke, 2010; Libertus et al., 2011; Mazzocco et al., 2011). It is worth noting that after controlling for age in months and Baseline Trimmed RT, Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy did not share significant variance, suggesting they are independent measures. The fact that the two Quantity Discrimination Skills' tasks relate differently to the two mathematical attainment measures further supports the idea that these two early quantity discrimination skills could work as distinct precursors of different mathematical abilities.

5.6.5 Causality of the relationships identified

The relationships found in this study are concurrent and therefore their causality cannot addressed; although it seems reasonable that early quantity discrimination skills are causal predictors of children's learning of the formal number system and their mathematical attainment because number sense emerges early in life and prior to verbal skills or formal mathematical instruction (Antell & Keating, 1983; Bijeljac-Babic et al., 1993; Wynn, 1996; Xu, 2003; Xu et al., 2005), this concurrent study cannot fully support this view. It could be that better early number competence improves ANS acuity and that children's learning of the formal number system helps developing early precise quantity discrimination skills. Only using longitudinal data can the causality of these relationships be examined to determine whether children's early quantitative skills are causal predictors of their later numerical competence and mathematical attainment.

6. LONGITUDINAL STUDY: RATIONALE, AIMS AND METHODOLOGY

Krajewski and Schneider’s (2009) theoretical model of early arithmetical development was adopted for this longitudinal study as a general framework to explore how domain-general and domain-specific cognitive skills relate to children’s early number skills and to their mathematical attainment during the early stages of schooling. Krajewski and Schneider’s (2009) theoretical model proposes that children’s early quantity discrimination skills, phonological awareness and VSSP functioning are each independent early precursors of distinct QNCs. These researchers differentiate three distinct developmental levels of early QNCs; basic numerical skills (Level I), quantity to number-word linkage (Level II) and linking quantity relations with number-words (Level III). QNCs Level I refer to children’s ability to articulate the number-words and to their ability to recite the number-word sequence. This developmental level would be predicted by children’s phonological awareness because it facilitates their acquisition of language (Cain, 2010), and consequently facilitates the acquisition of number-words. QNCs Level II refer to children’s ability to use the acquired basic verbal numerical skills to refer to quantities, this developmental levels has two phases; imprecise quantity to number-word linkage (QNCs Level IIa), when children first refer to quantities imprecisely (“a bit”, “much”, “very much”) and precise quantity to number-word linkage (QNCs Level IIb), when children can refer to quantities using the exact number-words that match that specific quantity (“one”, “twenty”, “a hundred”). QNCs Level II would be predicted by children’s quantity discrimination skills but also by their visuo-spatial STM skills, as these have been found to play a fundamental role in children’s ability to grasp the cardinality concept (Ansari et al., 2003). Last, QNCs Level III refer to children’s early calculation skills, when they understand that quantities can be composed and decomposed into other quantities and they can use number-words to express the outcome of simple arithmetic problems. This developmental level would also be predicted by children’s VSSP functioning and their quantity discrimination skills.

As discussed in Chapter 1 of this thesis, Krajewski and Schneider (2009) partially tested their theoretical model. They explored how VSSP functioning and phonological awareness assessed at the start of kindergarten (five years and seven months) relate to their QNCs Level I at midway through kindergarten and to their QNCs Levels II and III at the end of kindergarten. They also explore how children’s three levels of QNCs relate to their mathematical attainment three years later. Despite the fact that their theoretical model integrates quantity discrimination skills and makes predictions about how these skills relate to the distinct QNCs, no quantity discrimination skills tasks were included in their study. It was found that children's phonological awareness significantly predicted children's performance on QNCs Level I tasks. QNCs Level II and Level III tasks were best predicted by children's VSSP functioning. However, because no quantity discrimination skills tasks were included in their study, no evidence of how children's quantity discrimination skills relate to their later QNCs and mathematical achievement was provided.

6.1 RATIONALE AND AIMS OF THE THESIS

This longitudinal study uses Krajewski and Schneider's (2009) theoretical model as a framework to simultaneously address the contribution that domain-general and domain-specific cognitive skills make to young children's early number skills and mathematical attainment. The current study has several aims.

6.1.1 To explore the relationships between domain-general cognitive skills and the Early Number Skills

First, it aims to explore whether phonological awareness and VSSP functioning differentially predict unique and independent variance in children's later Early Number Skills. VSSP functioning facilitates the abstract representation and manipulation of quantities, linking quantities to number-words and is a good predictor of children's early arithmetic skills (Ansari et al., 2003; Huttenlocher, Jordan, & Levine, 1994; Krajewski & Schneider, 2009; Rasmussen & Bisanz, 2005). Phonological awareness is a well-established precursor of reading (Badian, 2000; Bowey, 2005; Mann & Liberman, 1984) and also plays an important role in early mathematics development because some numerical tasks require accessing, retaining and manipulating phonological representations (Hecht et al., 2001; Simmons et al., 2008). These two domain-general cognitive skills were selected for the study because Krajewski and Schneider (2009) found they make a significant contribution to children's later performance on the distinct developmental levels of QNCs. In addition, phonological awareness and VSSP functioning together captured substantial variation in young children's arithmetic performance a year later, over and above their concurrent vocabulary and nonverbal reasoning (Simmons et al., 2008). Therefore, these two predictors seem to be key abilities that underpin mathematical success in young children.

Multiple early number skills have been used in previous studies exploring the impact of children's cognitive skills on their early numerical competence, however, only some of these studies categorise the tasks, usually into either verbal or non-verbal/quantitative tasks (LeFevre et al., 2010; Rasmussen & Bisanz, 2005). No coherent and consistent set of early number skills has yet been proposed to examine whether different cognitive skills relate to different types of early number skills. Krajewski and Schneider's (2009) distinction of three developmental levels of QNCs provides a more specific classification for the study of children's early number skills that is also supported by different developmental models of early mathematical competence (Fuson, 1988, 1992; Gelman & Gallistel, 1978; Wynn, 1992) and neuropsychological data (Dehaene & Cohen, 1997; Delazer & Butterworth, 1997). Three Early Number Skills' tasks were designed to represent each of the three theoretical developmental levels of QNCs proposed by Krajewski and Schneider (2009). In the present study, in order to assess children's QNCs Level I they were asked to recite the number-word sequence forward from different start points (Knowledge of the Number Sequence task). This task was purely verbal and no visual support was provided. To assess children's QNCs Level II a counting task was administered where they were asked to report the exact number of discrete items visually presented to them (Counting Objects task). To assess QNCs Level III children were read an arithmetic problem in a story format where contextual but no concrete visual support for the answer was provided. In order to respond children were expected to make simple abstract calculations (additions and subtractions). Based on Krajewski and Schneider's (2009) model, it was expected that children's phonological awareness would make independent and unique longitudinal contributions to their performance on the Knowledge of the Number Sequence task over a six-month period and over an eighteen-month period. It was also expected that children's VSSP functioning would make independent and unique longitudinal contributions to their performance on the Counting Objects and Story Problems task over a six-month period and over an eighteen-month period.

6.1.2 To explore the relationships between the domain-general cognitive skills and the two mathematical attainment measures

Standardised mathematical attainment tests usually include tasks which draw on a combination of different cognitive skills. Therefore, no distinction of the extent of the contribution that each cognitive skill makes to different aspects of children's mathematical attainment can be made. Young children's VSSP functioning has been found to be a good predictor of their performance on mathematical attainment tasks (Bull et al., 2008; DeSmedt et al., 2009; Simmons et al., 2008), and so has phonological awareness (Hecht et al., 2001; Simmons et al., 2008). However, the extent of their specific contributions to specific mathematical attainment tasks remains unclear due to the use of mixed tests to assess children's mathematical attainment. Because reasoning about numerical information and solving written arithmetic problems have been found to tap distinct key aspects of mathematical achievement in young children (Nunes, Bryant, Barros, & Sylva, 2012), the present study uses the Mathematical Reasoning subtest and the Numerical Operations subtest of the WIAT-II^{UK} (Wechsler, 2002) to assess children's performance on standardised mathematical attainment measures. These two measures were particularly selected because the majority of the items in the Mathematical Reasoning subtest focus particularly on the nature of the numerical relations and are verbally presented and/or require a verbal response from the child; therefore making heavy demands on verbal and quantitative skills. In contrast, the majority of the items in Numerical Operation subtest present printed arithmetic problems and little or no verbal instructions are given, therefore making heavy demands on visual and writing skills. The fact that these two mathematical attainment measures differ in presentation format, stimuli and response method allow determining the extent to which VSSP functioning and phonological awareness play differential roles in predicting children's mathematical attainment.

It was expected that both domain-general cognitive skills would predict children's performance on the Mathematical Reasoning subtest because items in this test demand numerical and verbal processing and verbal responses from the child. It was expected that children's VSSP functioning would predict their performance on the Numerical Operation subtest of the WIAT-II^{UK} because Camos (2008) and Simmons et al. (2012) found the VSSP functioning is involved in writing procedural rules. Although LeFevre et al. (2010) proposes that phonological processing skills contribute to number tasks because language is a symbolic representational system and therefore similar rules apply when learning the formal number system, in the present study it was only expected that VSSP functioning would predict children's performance on the Numerical Operation subtest of the WIAT-II^{UK} because in this test they would only need to access the semantic quantity representation but not the phonological representation from the printed symbolic number representations.

6.1.3 To explore the relationships between domain-specific cognitive skills and the Early Number Skills

The current study also includes two quantity discrimination skills' tasks because as discussed in Chapters 2 and 3 of this thesis two distinct domain-specific quantity systems have been proposed; one for precise numerical representations and one for approximate numerical representations (Feigenson et al., 2004). Therefore, two computerised Quantity Discrimination Skills' tasks were designed to provide appropriate distinct measures of children's precise and approximate quantity discrimination skills. In both tasks, children viewed two non-symbolic quantities presented simultaneously on each side of the screen. To respond they had to press as fast as they could a left or right key matching the side on which the larger quantity appeared. Thus children's performance on these two Quantity Discrimination Skills' tasks is unlikely to be contaminated by their verbal skills, knowledge of the formal number system or working memory skills, this last could act as an extraneous variable when stimuli is presented serially (Gilmore et al., 2011).

To obtain a measure of children's precise quantity discrimination skills only arrays of up to three dots were presented until the child responded and their accuracy and RTs were recorded. Both children's accuracy rates and RTs have previously been used as an index of their precise numerical representation skills. Because in this task stimuli were presented until the child responded, accuracy rates would not make a distinction between the children that were accurate and fast from the ones who were accurate but employed a not-as-efficient strategy. Thus, the mean RTs for all correct responses after eliminating the influence of outliers (responses taking longer than two times the interquartile range from the median for each child) was obtained (Precise QDS Trimmed RT). This data trimming was conducted in order to eliminate the influence of outliers due to children's distractibility, rather than representing time needed to process numerical information (Butterworth, 2003; Gebuis et al., 2009; Libertus et al., 2011). In addition, children's accuracy and RTs on a computerised two-choice non-numerical discrimination task (Baseline) were also recorded and their mean RTs for all correct responses after eliminating the influence of outliers was also obtained (Baseline Trimmed RT). Children's speed in discriminating small quantities was calculated by using their Precise Quantity Discrimination Skills' Trimmed RT after controlling for their Baseline Trimmed RT. Controlling for children's baseline speed helps to adjust their RTs when responding to numerical stimuli to their individual latencies in computerised tasks (see Butterworth, 2003).

To obtain a measure of children's Approximate Quantity Discrimination skills, children had to identify the larger of two non-symbolic large numerical sets within the limited duration of the stimuli presentation (2,000 msec.). The limited presentation time intended to prevent the use of verbal counting strategies so that children had to rely on their approximate quantity discrimination skills to respond. However, this time seems to be sufficient for young children to make approximate numerical judgements over non-symbolic quantities (Inglis et al., 2011). Because pre-schooler's accuracy in a large non-symbolic quantity discrimination task has been found to be the most highly related estimate (above Weber fraction and RTs) with their maths ability measured with a standardised test (see Libertus et al., 2011) accuracy scores on this task were used for further analysis.

Butterworth (1999, 2005, 2010) suggests that precise numerical discrimination skills serve as a foundation to our later number skills and basic arithmetic skills. Other views propose that large approximate numerical discrimination skills are the ones that support the later acquisition of early symbolic number skills (Barth et al., 2005; Condry & Spelke, 2008; Dehaene, 1997; Dehaene et al., 1998). However, there is limited evidence associating precise quantity discrimination skills with children's early number skills. So far children's speed in accurately quantifying non-symbolic arrays of up to three discrete items has been found to predict their ability to perform abstract arithmetic (LeFevre et al., 2010). However, the task used to assess children's subitising skills demanded responding with a number-word and therefore verbal skills were involved to a certain extent. Also, no naming speed basline measure was controlled for. Thus this measure could potentially be contaminated by extraneous factors such as individual differences in verbal skills or individual latencies. Children's ability to make approximate numerical judgements over non-symbolic quantities has been found to predict their performance on number tasks that demand accessing the number semantics (Piazza et al., 2010), their number sequence reciting task and other numerical tasks (Mussolin et al., 2012). It was expected that children's performance on both Quantity Discrimination Skills' tasks would relate to their performance on the three Early Number Skills' tasks, but due to the limited evidence examining these relationships and the fact that the two Quantity Discrimination Skills' tasks are novel tasks specifically designed to eliminate contamination from extraneous variables, no further specific predictions could be made. The independent longitudinal contributions that each Quantity Discrimination Skill task makes to the different Early Number Skills over a six-month period and over an eighteen-month period were explored.

6.1.4 To explore the relationships between the domain-specific cognitive skills and the two mathematical attainment measures

The current study also aimed to explore whether the two distinct Quantity Discrimination Skills' tasks predict unique variance of children's performance on the two standardised mathematical attainment tasks selected for the study. Better (faster and/or more accurate) precise quantity discrimination skills over small quantities in young children have been associated with better mathematical attainment in studies comparing MLD and typically achieving children (Fischer et al., 2008; Landerl et al., 2004). Children's accuracy on large non-symbolic approximate quantity discrimination tasks has also been associated with better mathematical performance (Halberda et al., 2008; Libertus et al., 2011; Mazzocco et al., 2011), although not without controversy (Gilmore et al., 2013; Holloway & Ansari, 2009; Sasanguie et al., 2012; Sasanguie et al., 2013; Soltész et al., 2010). However, no study up until now has attempted to explore the independent and unique longitudinal contributions that each of these two Quantity Discrimination Skills' tasks make to children's mathematical attainment over a six-month period and over an eighteen-month period. The absence of studies exploring how these two core systems relate to children's mathematical achievement has already been highlighted in recent literature (Libertus et al., 2011). Presumably, if these two quantity discrimination skills rely on different domain-specific number systems (Feigenson et al., 2004), they could support different aspects of children's later mathematical attainment, but no evidence of this presumption has been explored up to this date.

6.1.5 To explore the domain-general and the domain-specific cognitive skills' unique contributions to the three Early Number Skills and to the two mathematical attainment measures

The absence of studies exploring how domain-specific and domain-general cognitive skills relate to children mathematical achievement has been recently stated (Raghubar et al., 2010). The present study therefore explores the independent and unique contributions that the two types of precursors included in the study (domain-general and domain-specific cognitive skills) make to children's Early Number Skills and to the different mathematical attainment measures included in the study over a six-month period and over an eighteen-month period. Both types of precursors were expected to make independent and unique contributions to children's Early Number Skills and mathematical attainment. However, due to the lack of studies exploring the impact of these two groups of precursors simultaneously, no specific or further predictions could be made nor whether their contributions would be consistent over time. It could be that in the presence of domain-general cognitive skills, quantity discrimination skills do no longer predict unique variance in children's performance on the variables of interest, or indeed vice-versa.

6.1.6 To explore the specificity of the relationships between cognitive precursors and mathematical outcomes

The Early Word Recognition subtest of the YARC Early Reading Test (Hulme et al., 2009) was included in the study to determine the specificity of the relationship between the precursors and the variables of interest. Children's phonological awareness is a domain-general cognitive skill that has been identified as a solid predictor of their latter reading attainment (Gathercole et al., 2005; Hecht et al., 2001). It was therefore expected that children's reading performance would be predicted by their phonological awareness. However, because the two core systems for numerical representations have been claimed to be domain-specific and independent from language (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003; Gelman & Butterworth, 2005), children's performance on the two Quantity Discrimination Skills' tasks are expected to predict their performance on the Early Number Skills' tasks and mathematical attainment measures, but not their reading attainment. If results challenge these expectations, and Quantity Discrimination Skills can also predict later reading attainment in young children, it could be that quantity discrimination skills are in fact broader precursors of later scholastic attainment.

6.1.7 To explore the domain-general and the domain-specific cognitive skills' unique contributions to the three Early Number Skills and the two mathematical attainment measures over and above General Conceptual Abilities

Two additional measures tapping children's verbal conceptual ability (Naming Vocabulary) and non-verbal conceptual ability (Picture Similarities) from the BAS-II II^{UK} (Elliott, Smith, & McCulloch, 1996) were also included in the study with the aim of determining whether the predictor variables are specific precursors of early mathematical development or whether they do not predict later mathematical performance over and above General Conceptual Abilities. Although the contamination of extraneous variables was minimised by using novel quantity discrimination tasks that improve previous methodologies used to tap the two distinct domain-specific systems, tasks' demands could unintentionally tap additional cognitive abilities as well as the variables of interest. Therefore, measures of children's General Conceptual Abilities are controlled for to eliminate possible ambiguity in the findings due to uncontrolled shared variance between the variables of interest and general cognitive abilities that typically predict scholastic success. Results from these analyses would determine whether the domain-specific and the domain-general cognitive skills are specific precursors of early mathematical development or whether they do not predict later mathematical performance over and above General Conceptual Abilities.

6.1.8 To explore the causality of the relationships between cognitive predictors and outcome measures

Longitudinal studies exploring the relationships between domain-general cognitive skills and mathematical performance suggest that phonological awareness and VSSP functioning are causal predictors of children's later mathematical attainment (Simmons et al., 2008). However, only one longitudinal study explored the causality of the relationships between phonological awareness and later mathematical attainment in young children, confirming that phonological awareness is a causal predictor of school-aged children's mathematical attainment (Hecht et al., 2001). No study has explored whether VSSP functioning is a causal predictor of children's early number skills and mathematical attainment. Regarding quantity discrimination skills, only Libertus et al. (2013) have confirmed the role of non-symbolic approximate quantity discrimination skills as causal predictors of mathematical attainment over a six-month period in pre-schoolers. However, no study has explored whether precise quantity discrimination skills are a causal predictor of children's early number skills and mathematical attainment.

The present study explores whether children's domain-specific and domain-general cognitive skills assessed when children have undergone very limited formal mathematical instruction can predict children's growth in the Early Number Skills and mathematical attainment measures. Analyses of growth are highly restrictive because they control for auto-regressor effects (tendency of a variable to later predict itself). Thus growth analyses examine whether the predictor variable can explain significant variance in the criterion variable over and above predicting itself. It is worth noting that if results from growth analyses are significant, emphasis can be placed in the contention that the predictor variable can predict growth in the criterion variable and consequently, is acting as a *causal predictor* of the criterion variable. Because children's performance on Early Number Skills' tasks and on mathematical attainment measures may not necessarily depend on the same cognitive processes at different points over development it is important to explore the contributions of such cognitive processes while controlling for the autoregressive effects so that their contributions are not inflated (see Bowey, 2005 for further clarification on analyses of growth).

It was expected that performance on the Quantity Discrimination Skills' tasks would predict children's growth in the Early Number Skills and mathematical attainment measures over an eighteen-month period because these skills are present in new-borns (Antell & Keating, 1983; Bijeljac-Babic et al., 1993; Wynn, 1996), in adults with a very restricted lexicon for number words (Pica et al., 2004) and even other animals species (Hanus & Call, 2007; Hauser et al., 1995; Pepperberg & Gordon, 2005). Because quantity discrimination skills seem to emerge early in life prior to verbal abilities or formal instruction they could influence later mathematical development, but this does not mean they should. It could be that learning to use the formal number system contributes to make more precise and accurate quantity discriminations. Because research shows that quantity discrimination skills develop over time (Fischer et al., 2008; Halberda & Feigenson, 2008), it could also be that these skills and learnt number skills contribute to one another throughout the early years and that their relationship is to a certain extent reciprocal. Results of the growth analyses shed light upon whether early quantity discrimination skills are in fact *causal predictors* of children's early number skills and mathematical attainment. Phonological awareness has been found to be a causal precursor of school-aged children's mathematical attainment (Hecht et al., 2001) and approximate non-symbolic quantity discrimination skills' accuracy has been found to be a causal precursor of mathematical attainment over a six-month period in preschool years (Libertus et al., 2013). Thus, the domain-specific and the domain-general cognitive precursors were expected to be *causal predictors* of children's Early Number Skills and mathematical attainment.

6.2 METHODOLOGY

6.2.1 Participants

A total of 131 children from Reception Year classes of five primary schools in the North West of England were recruited for the study. Data was collected in three time points; children completed two time points while in Reception Year (at the start and at the end of the academic year) and one time point while in Year 1 (at midway through the academic year). Time 1 (T_1) assessments took place between the months of November and January and 129 children (69 males) successfully completed all tasks. Two females were unable to complete some of the tasks; one due to school absence on the scheduled assessment days and one who did not desire to continue with the assessments. Time 2 (T_2) assessments took place between the months of May and July and 128 children (68 males) successfully completed all the tasks. One male was absent during the scheduled assessment days. Time 3 (T_3) assessments took place between the months of January and March and 126 children (68 males) successfully completed all the tasks. Two females were absence because their families moved abroad (see table 6.1 for details of age and gender distribution). Data from children who did not complete all tasks at a certain time point has not been included from the incomplete time point onwards.

Table 6.1: Descriptive statistics of participants' age and gender in T_1, T_2 and T_3 and overall sample

		Age in months			Gender		
Time	**Year**	**Mean (in months)**	**SD**	**Range**	**Males**	**Females**	**Total**
T_1	Reception	4 years, 8 months (56.9m)	4.05	50 – 64	69	60	129
T_2	Reception	5 years, 2 months (62.5m)	3.93	56 – 70	68	60	128
T_3	Year 1	5 years, 10 months (70.9m)	3.97	64 - 78	68	58	126

Note. Months' information is given as units

The socio-economic level of children varied across the sample (see the proportion of pupils eligible for Free School Meals at each participating school based on the last schools Ofsted reports prior commencing the study, http://www.ofsted.gov.uk) (see table 6.2).

Table 6.2: Proportion of pupils eligible for Free School Meals and number of participants attending the schools at each time point

Proportion entitled to FSM	**Schools**	**Participants T_1**	**Participants T_2**	**Participants T_3**
Below UK Average	2	72	72	70
In line with UK Average	2	39	39	39
Above UK Average	1	18	17	17

6.2.2 Procedure

Written consent was gained from the schools´ head-teachers and from the parents or guardians of the children. Prior to any assessment, a brief explanation of the study and the tasks was given to each child individually and his or her verbal assent was needed to commence. Children were assessed individually in sessions of approximately twenty minutes in a quiet area of the school during school hours. At the first time point, participants had their domain-general cognitive skills (VSSP functioning and phonological awareness), domain-specific cognitive skills (non-symbolic Precise and Approximate Quantity Discrimination Skills), Early Number Skills (Knowledge of the Number Sequence, Counting Objects and Story Problems) and mathematical attainment (The Numerical Operations and Mathematical Reasoning subtests of the Wechsler Individual Achievement Test, Wechsler, 2002) assessed in three different sessions. At the second time point, participants' Early Number Skills and mathematical attainment were assessed in two different sessions. At the third time point, participants' non-symbolic Precise and Approximate Quantity discrimination Skills, Early Number Skills, mathematical attainment, verbal and non-verbal general conceptual ability and reading attainment were assessed in three different sessions (see table 6.3). At the first and third time points computer tasks were combined with verbal and/or pencil-and-paper tasks in each session. The order of the presentation of the sessions for all time points was randomised across children to counterbalance effects of order.

Table 6.3: Tasks administered at each time point

	SKILL	TASK	T_1	T_2	T_3
Predictor Variables	**Domain-general Cognitive Skills**	Block recall and Mazes memory (AWMA)	√		
		Phoneme segmentation and Rhyme awareness (PIPA)	√		
	Domain-specific Cognitive Skills	Baseline Reaction Time (BL)	√		√
		Precise Quantity Discrimination (PQD)	√		√
		Approximate Quantity Discrimination (AQD)	√		√
Outcome Variables	**Early Number skills**	Knowledge of the number sequence (KNS)	√	√	√
		Counting Objects (CO)	√	√	√
		Story Problems (SP)	√	√	√
	Mathematical Attainment	Mathematical Reasoning subtest (WIAT-II^{UK})	√	√	√
		Numerical Operations subtest (WIAT-II^{UK})	√	√	√
Generic Variables	**General Conceptual Ability**	Picture similarities (BAS II^{UK})			√
		Naming vocabulary (BAS II^{UK})			√
Specificity Variable	**Reading Attainment**	Early word recognition (YARC)			√

6.2.3 Tasks

6.2.3.1 Predictor Variables

- Domain-general cognitive skills tasks

Four tasks were administered to assess children's domain-general cognitive skills. To assess children's VSSP functioning two working memory tasks from the Automatic Working Memory Assessment (Alloway, 2007) (hereafter AWMA) were administered:

1. *Block recall subtest*: In this test the child sees an image presenting eight blocks and a finger tapping a certain number of blocks sequentially. The child is then encouraged to use his or her finger to tap the same blocks in the same order.

2. *Mazes memory subtest*: In this test the child is presented with a schematic figure of a man inside a black maze and a red line shows the path to get out. Three seconds later, the path line disappears and the child is encouraged to use his or her finger to trace the route that the line indicated.

Both tasks consist of three practice trials and seven levels of increasing difficulty presenting up to six different items. The test automatically terminates after three incorrect answers on items of the same level of difficulty are given.

To assess children's phonological awareness two oral subtests from the Preschool and Primary Inventory of Phonological Awareness (hereafter PIPA) (Dodd, Crosbie, MacIntosh, Teitzel, & Ozanne, 2000) were administered:

1. *Syllable segmentation*: This test assesses the ability to process real words sub-lexically. The experimenter says a word while showing a picture of five drums, the child is then asked to reproduce the word by touching a drum for each syllable. This test consists of four practice items and twelve experimental items.

2. *Rhyme awareness*: This test assesses the ability to identify words that are phonologically similar. The experimenter presents four pictures of different objects and names them to the child who is asked to identify the object's name that does not rhyme with the others. This test consists of two practice items and twelve experimental items.

For both tasks all items are administered and one point is given for each correct response.

- Domain-specific cognitive skills tasks

The Baseline task, the Precise Quantity Discrimination Skills' task and the Approximate Quantity Discrimination Skills' task were administered in different sessions at the first and third time points of data collection. The Baseline task and the Precise Quantity Discrimination Skills' task administered in the pilot study were administered in this longitudinal study with no modifications. For the Approximate Quantity Discrimination Skills' task, ten pairs (five mirrored pairs) were presented for each numerical ratio difference depicted in the task. This modification was done to obtain a balanced measure of children's accuracy on each numerical ratio presented (see table 6.4) because in the pilot an unequal numbers of trials were presented from each ratio bin.

Table 6.4: Approximate Quantity Discrimination Skills' task design: Stimulus pairing for each Weber fraction (numerical ratio difference) depicted and number of trials

Weber Fraction	**RO**	**Stimulus pairings (left answer required)**					**Stimulus pairings (right answer required)**				
0.5	1.5	9:6	15:10	18:12	27:18	30:20	6:9	10:15	12:18	18:27	20:30
0.4	1.4	7:5	14:10	21:15	28:20	35:25	5:7	10:14	15:21	20:28	25:35
0.3	1.3	9:7	13:10	18:14	26:20	30:23	7:9	10:13	14:18	20:26	23:30
0.2	1.2	6:5	12:10	18:15	24:20	30:35	5:6	10:12	15:18	20:24	25:30
0.1	1.1	11:10	12:11	22:20	32:29	33:30	10:11	11:12	20:22	29:32	30:33
0.0	1.0	5:5	9:9	15:15	24:24	30:30	7:7	10:10	22:22	25:25	31:31

Note$_1$. Weber Fraction value = $(n_2-n_1)/n_1$, n_2 being the larger set
Note$_2$. Numeric ratio (RO) = n_2/n_1, n_2 being the larger set

6.2.3.2 Outcome Variables

- Early Number Skills' tasks

The Knowledge of the Number Sequence of the Early Number Skills' tasks was modified for this longitudinal study from the initial design in the pilot study by adding more difficult items to prevent ceiling effects; children were asked to recite the number sequence from one and were stopped at number twenty, then asked to recite the number sequence starting from numbers 25, 65, 95 and 155. The same scoring and stopping rules from the pilot study were applied. In addition, for the Story Problems task the researcher only read the story to the child once. This was done in an attempt to provide more consistency in the administration procedures because some children may not verbally express their need to listen the story again while others might.

- Mathematical attainment tasks

The Numerical Operations and The Mathematical Reasoning subtests from the Wechsler Individual Achievement Test Second Edition (Wechsler, 2002) were administered in two different sessions. The standardised norms for administration and scoring of the WIAT-II^{UK} were applied.

- Generic variables

To assess children's verbal and non-verbal general conceptual ability two tasks from the Early Years Core Scales of the BAS-II II^{UK} (Elliott et al., 1996) were administered in different sessions:

1. *Picture similarities*: Measures non-verbal reasoning ability. The child is shown a row of four different pictures and given a card with an extra picture. The child is encouraged to place the card under the picture that better relates to it.

2. *Naming vocabulary*: Assesses spoken vocabulary. The child is shown pictures of different objects in sequential order and encouraged to name them.

Specific stopping rules are applied for each of the subtests depending on the child's age and the number of correct and incorrect responses given.

- Reading

To assess children's reading attainment, the Early Word Recognition subtest of the YARC Early Reading Test (Hulme et al., 2009) was administered. This test assesses single-word reading, it contains fifteen words that are phonemically regular and fifteen words that are phonemically irregular. One point is given for each word read correctly and the test is terminated after ten consecutive reading errors. This test was selected with a view to obtain an accurate measure of children's reading attainment.

7. LONGITUDINAL STUDY:

EXPLORATION OF THE DATA AND PRELIMINARY DATA ANALYSES

This chapter presents the results from the preliminary data exploration analyses conducted. First, descriptive statistics for all variables included in the longitudinal study are presented and discussed. Then, concurrent correlations and partial correlations controlling for age in months for all variables administered at each time point are presented and discussed. Last, correlations and partial correlations controlling for age in months for reading attainment with all predictor variables are presented and discussed.

7.1 DESCRIPTIVE STATISTICS

Descriptive statistics of children's raw scores, and standardised, percentile, ability and T-scores where applicable, were obtained for each task administered.

7.1.1 Predictor variables: Domain-general cognitive skills (administered at Time 1):

Descriptive statistics of children's raw scores, standardised scores and percentiles scores for the Syllable Segmentation and Rhyme Awareness subtests of PIPA (Dodd et al., 2000) were obtained (see table 7.1)

Table 7.1: Descriptive statistics for the Syllable Segmentation and Rhyme Awareness subtests of the PIPA at T_1 (n=129)

Phonological awareness (PIPA)	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min**	**Max**
Syllable Segmentation (raw scores)	4.56	2.36	1.81	0.33	0	11
Syllable Segmentation (standard scores)	8.45	2.21	1.05	2.81	2	15
Syllable Segmentation (percentile)	33.20	21.80	0.21	1	0	95
Rhyme awareness (raw scores)	5.00	2.53	1.71	-1.62	0	11
Rhyme awareness (standard scores)	9.31	2.13	2.09	-0.33	5	15
Rhyme awareness (percentile)	41.98	23.46	2.09	-1.71	5	95

Children's raw scores on the Syllable Segmentation and Rhyme Awareness subtests are normally distributed. The standard scores on both tasks denote normal phonological awareness skills in the sample overall, however, the percentile scores' means for both tasks are below average, especially for the Syllable Segmentation subtest. This could be due to the fact that assessments took place while participants' were being taught to identify and isolate words' phonemes and found it particularly difficult to change to identify and isolate words' syllables.

Descriptive statistics of children's raw scores, standardised and percentiles scores for the Block Recall and Mazes Memory subtests of the AWMA (Alloway, 2007) were obtained (see table 7.2).

Table 7.2: Descriptive statistics for the Block Recall and Mazes Memory subtests of the Automatic AWMA at T_1 (n=129)

AWMA subtests	Mean	SD	ZSkewness	ZKurtosis	Min	Max
Block Recall (raw scores)	10.78	3.16	0.76	3.95	0	22
Block Recall (standard scores)	95.74	16.47	-12.38	33.59	0	138
Block Recall (percentile)	41.18	24.56	2.81	-0.95	0	99
Mazes Memory (raw scores)	6.05	4.53	2.48	-0.45	0	21
Mazes Memory (standard scores)	85.97	32.43	-8.95	6.62	0	137
Mazes Memory (percentile)	39.19	26.64	1.48	-1.98	0	99

Children's raw scores on the Block Recall and Mazes Memory subtests are not normally distributed. Block Recall raw scores show a leptokurtic distribution, however, standard and percentile scores are within the average limits. Mazes Memory raw scores are positively skewed however, standard and percentile scores are within the average limits.

Principal Component Analysis (hereafter PCA) was conducted with the four domain-general cognitive skills to reduce the number of predictors. The raw scores for Syllable Segmentation, Rhyme Awareness, Block Recall and Mazes Memory subtests were entered into a two fixed-factor principal component analysis (see table 7.3 for correlations between variables).

Table 7.3: Correlations for the raw scores of the Syllable Segmentation and Rhyme Awareness subtests of the PIPA and Mazes Memory and Block Recall subtests of the AWMA (n=129)

	PIPA Rhyme Awareness	**AWMA Mazes Memory**	**AWMA Block Recall**
PIPA Syllable Segmentation	.220**	.118	.155*
PIPA Rhyme Awareness	-	.137	.216**
AWMA Mazes Memory		-	.279***

***p<=.05; **p<=.01; ***p<=.001 (Sig. 1-tailed)**

Kaiser-Meyer-Olkin value was acceptable (KMO=.62) according to Kaiser (1974). Bartlett's test of sphericity was significant ($\chi^2(6)=25.44$, $p<.001$), indicating that it is appropriate to conduct factor analysis on these variables. Results from the PCA analysis were a first factor that included the AWMA measures; with factor loadings of .74 ($R^2=.54$) for Block Recall and .84 ($R^2=.70$) for Mazes Memory and a second factor that included the PIPA measures; with factor loadings of .81 ($R^2=.66$) for Syllable Segmentation and .72 ($R^2=.52$) for Rhyme Awareness. All factor loadings are greater than .36, and the squared factor loading values are greater than 0.4 and therefore represent substantive significant values (Stevens, 1992). The resulting scores from the PCA analysis were used as VSSP functioning and phonological awareness variables for further analyses. Descriptive statistics of children's VSSP functioning and phonological awareness scores from the PCA analysis were obtained (see table 7.4).

Table 7.4: Descriptive statistics for the visuo-spatial sketch-pad functioning (VSSP) and phonological awareness (PA) scores from the PCA analysis (n=129)

Variable	**ZSkewness**	**ZKurtosis**	**Min**	**Max**
VSSP	2.86	1.69	-1.91	3.34
PA	1.86	-0.45	-2.24	2.58

Note. For all PCA variables the mean is 0 and the SD is 1

7.1.2 Predictor variables: Domain-specific cognitive skills tasks (administered at Time 1 and Time 3):

Tests of difference were conducted when the same task was administered at two different time points to explore whether scores were significantly different at each time point. Because the study sample is sufficiently large and therefore the assumption of normal distribution is not required (see Lumley et al., 2002), related samples t-tests were conducted to compare the scores between time points.

7.1.2.1 Baseline task and Precise Quantity Discrimination Skills' tasks

As in the pilot study, the mean RTs for the Baseline (BL) task and for the Precise Quantity Discrimination Skills' (Precise QDS) task were obtained calculating the mean RTs for all correct responses for each child. Baseline Trimmed RT and the Precise Quantity Discrimination Skills' Trimmed RT values were obtained calculating the mean RTs for all correct responses taking less than two times the interquartile range from the median for each child. This was done in order to eliminate the influence of outliers due to children's distractibility, rather than representing the time needed to process numerical information (see appendix 1 for differences in Baseline data distribution prior and post data trimming at Time 1 and at Time 3, and appendix 2 for differences in Precise Quantity Discrimination Skills' data distribution prior and post data trimming at Time 1 and Time 3).

Descriptive statistics of children's accuracy and Trimmed RT in the Baseline task at Time 1 and at Time 3 were obtained (see table 7.1)

Table 7.1: Descriptive statistics of children's accuracy scores and Trimmed RT in the Baseline task (BL) task at T_1 (n=129) and at T_3 (n=126)

Baseline Task	**N**	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min.**	**Max.**
BL Accuracy (max.12) T_1	129	10.55	1.90	-11.05	19.40	0	12
BL Accuracy (max.12) T_3	126	11.45	0.91	-11.77	20.26	7	12
BL Trimmed RT (msec.) T_1	128	888.37	289.50	7.48	8.31	309.66	2111.25
BL Trimmed RT (msec.) T_3	126	679.30	152.42	8.23	1.32	453.55	1465.10

Children's accuracy scores in the Baseline task were not normally distributed neither at Time 1 (D (126) = .233, p<.001), or at Time 3 (D (126) =.338, p<.001). Children's were significantly more accurate on this task as they got older (t=-5.138, df=125, p<.001). Children's Baseline Trimmed RT was also not normally distributed neither at Time 1 (D (125) = .122, p<.001), or at Time 3 (D (125) =.099, p<.005). Children's were significantly faster on this task as they got older (t=8.709, df= 124, p<.001). Only baseline Trimmed RT was used as a control variable because it aimed to control for the individual differences in responding to non-numerical two-choice discrimination tasks when Precise Quantity Discrimination Skills' Trimmed RT (and not accuracy) was used as a predictor variable.

Descriptive statistics of children's accuracy scores and trimmed RTs in the Precise Quantity Discrimination Skills' task at Time 1 and at Time 3 were also obtained (see table 7.2).

Table 7.2: Descriptive statistics for accuracy and Trimmed RT in the Precise Quantity Discrimination Skills' (PQDS) task at T_1 and at T_3

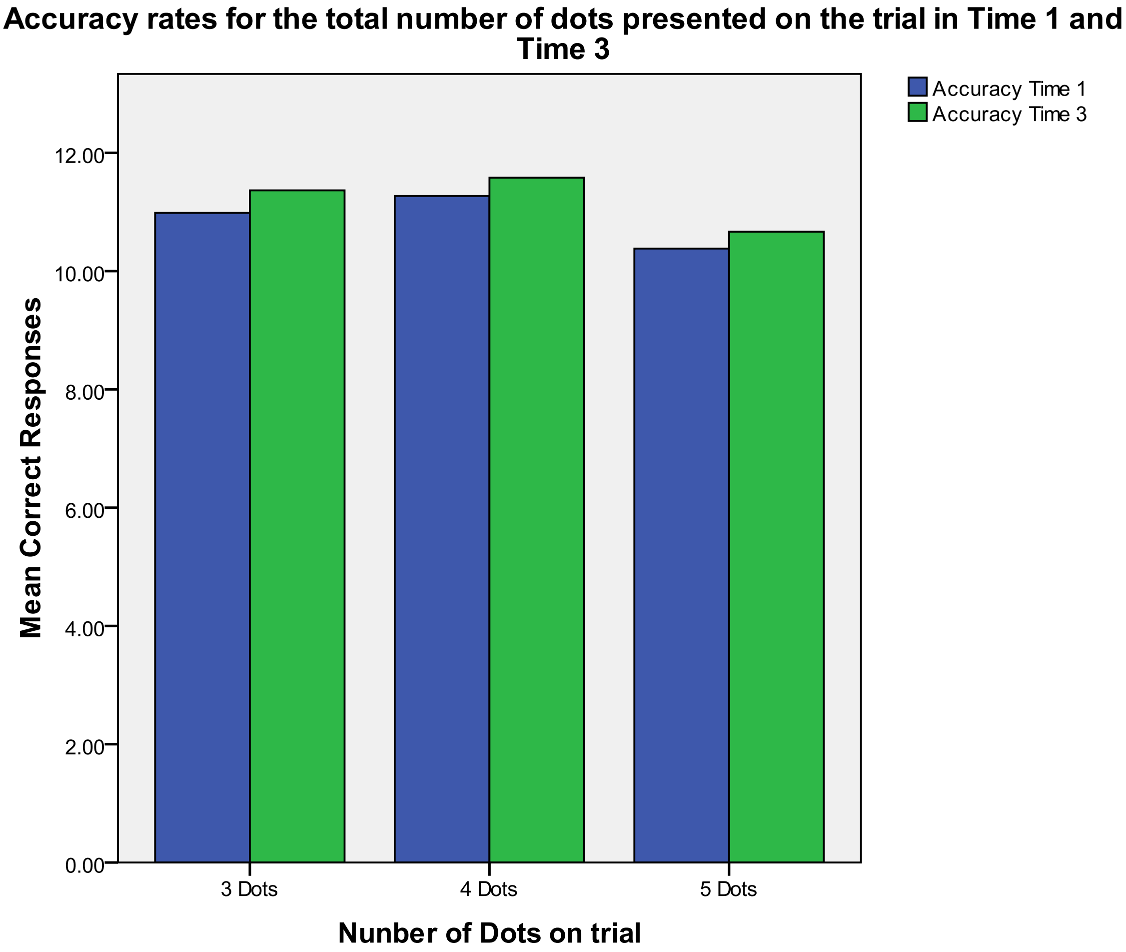
Precise QDS	**N**	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min.**	**Max.**
PQDS Accuracy (max.36) T_1	129	32.59	4.52	-8.67	6.31	18	36
PQDS Accuracy (max.36) T_3	126	33.61	2.82	-8.59	9.79	22	36
PQDS Trimmed RT (msec.) T_1	129	1555.25	796.82	12.38	18.88	521.06	4953.54
PQDS Trimmed RT (msec.) T_3	126	932.72	172.12	3.77	3.07	577.59	1587.16

Children's accuracy scores in the Precise Quantity Discrimination Skills' task were not normally distributed either at Time 1 (D (126) = .247, p<.001), or at Time 3 (D (126) = .214, p<.001). Children were significantly more accurate on this task as they got older (t=-2.303, df=125, p<.05). Children's Trimmed RT in the Precise Quantity Discrimination Skills' task were not normally distributed at Time 1 (D (126) = .170, p<.001), but they were normally distributed at Time 3 (D (126) =.075, p=.077). Children were significantly faster on this task as they got older (t=8.830, df= 125, p<.001).

Further analyses on children's accuracy rates and Precise Quantity Discrimination Skills were conducted. First, children's accuracy scores were analysed depending on the total number of dots presented in the trial; three dots (one against two dots), four dots (one against three dots) and five dots (two against three dots) to explore whether levels of difficulty differed significantly across the different types of trials. Children's accuracy scores for each trial type at Time 1 were not normally distributed neither for three dots (D (129) =.289<.001), four dots (D (129)=.399<.001) or five dots (D(129)=.230,<.001). Data failed to meet sphericity assumptions (χ^2 (2) =26.39, p<.001) so values given are following Greenhouse Geisser corrections (ε=.84). Significant effects are followed by paired comparisons under Bonferroni correction. There was a significant effect of the number of dots presented in the trial on children's accuracy rates F(1.68, 215.56)=32.59, p<.001). Children were significantly more accurate on trials presenting four dots (M=11.25, SD=1.41) than on trials presenting three dots (M=10.98, SD=1.56), and on these they were significantly more accurate than on trials presenting five dots (M=10.35, SD=2.01). There was evidence for linear (F(1,128)=26.20, p<.001) and non-linear (F(1,128)=40.872, p<.001) trends in the data.

Children's accuracy scores for each type of trial at Time 3 were not normally distributed neither for three dots (D (126) = .355<.001), four dots (D (126) =.436<.001) or five dots (D (126) =.225,<.001). Data failed to meet sphericity assumptions (χ^2 (2)=.12.91, p<.01) so values given are following Greenhouse Geisser corrections (ε=.910). Significant effects are followed by paired comparisons under Bonferroni correction. There was a significant effect of the number of dots presented in the trial on children's accuracy rates F(1.820, 227.504) =38.730, p<.001). Children were significantly more accurate on trials presenting three dots (M=11.36, SD=1.00) and four dots (M=11.58, SD=.98) than on trials presenting presenting five dots (M=10.67, SD=1.46). Accuracy rates on trials presenting three and four dots were not significantly different from each other. There was evidence for linear (F(1,125)=35.154, p<.001) and non-linear (F(1,125)=43.871, p<.001) trends in the data.

Figure 7.1: Accuracy rates for the PQDS' task when the total number of dots presented on the screen were 3 (1 against 2 and 2 against 1), 4 (1 against 3 and 3 against 1) and 5 (2 against 3 and 3 against 2), regardless the side where the correct answer was presented at T_1 and T_3

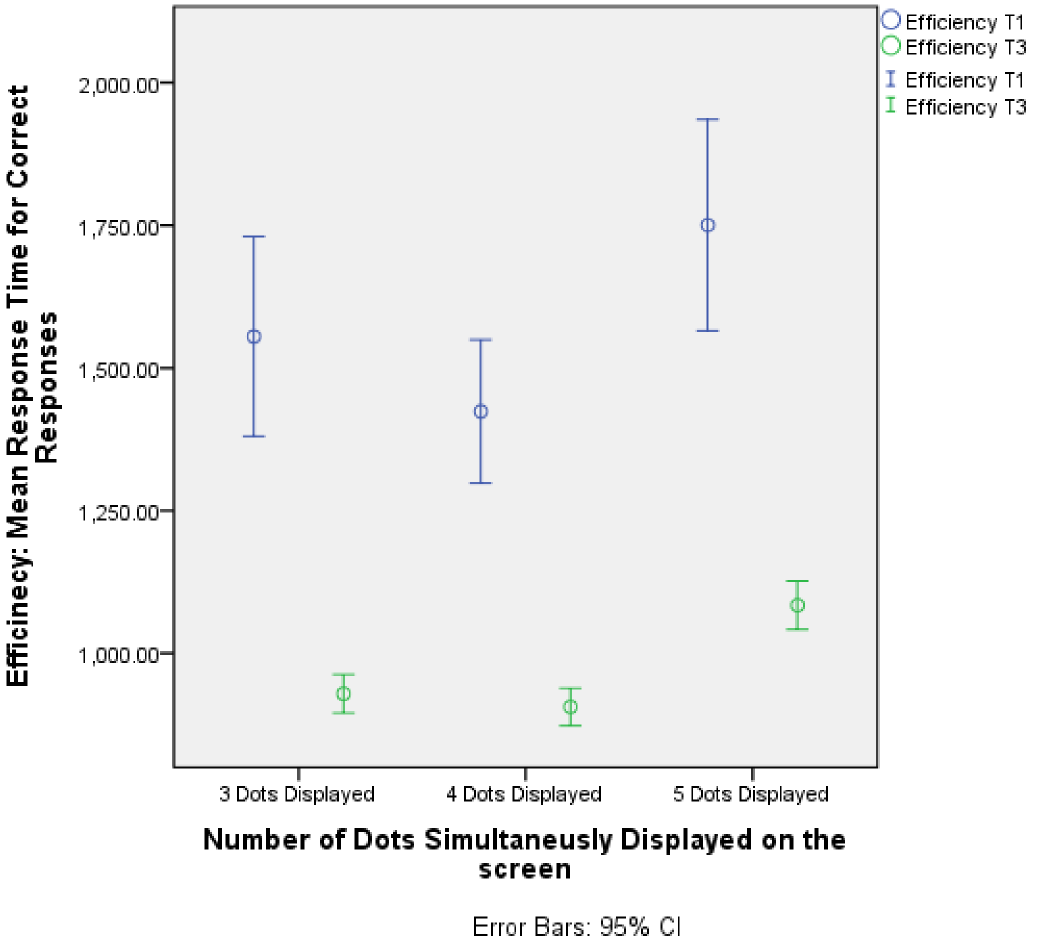


Second, children's Precise Quantity Discrimination Skills' Trimmed RT was analysed depending on the total number of dots presented in the trial; three dots (one against two dots), four dots (one against three dots) and five dots (two against three dots) to explore whether they employed different enumeration strategies to respond to these trials. Children's Precise Quantity Discrimination Skills' Trimmed RT for each type of trial at Time 1 was not normally distributed neither for three dots (D (129) = .212, *p*<.001), four dots (D (129) = .168, *p*<.001) or five dots (D (129) = .225, *p*<.001). Data failed to meet sphericity assumptions (χ^2 (2)=16.24, *p*<.001) so values given are following Greenhouse Geisser corrections (ε=.89). Significant effects are followed by paired comparisons under Bonferroni correction. There was a significant effect of the number of dots presented in the trial on children's Precise Quantity Discrimination Skills' Trimmed RT F(1.79, 228.56) = 13.27, *p*<.001). Children were significantly faster on trials presenting four dots (M=1425.87, SD=707.91), than on trials presenting three dots (M=1566.26, SD=998.69), and on these they were significantly faster than on trials presenting five dots (M=1742.34, SD=1038.36). There was evidence for linear (F(1,128)=6.202, p<.05) and non-linear (F(1,128)=27.009, p<.001) trends in the data.

Children's Precise Quantity Discrimination Skills' Trimmed RT at Time 3 were normally distributed for trials presenting three dots (D (126) = .070, p>.05) and four dots (D (126) = .072, p>.05) and not normally distributed for trials presenting five dots (D (126) = .084, p<.05). Data failed to meet sphericity assumptions (χ^2 (2)=16.29, p<.001) so values given are following Greenhouse Geisser corrections (ε=.89). There was a significant effect of the number of dots presented in the trial on children's Precise Quantity Discrimination Skills' Trimmed RT F(1.78, 222.60) = 170.97, p<.001). Children were significantly faster in trials presenting four dots (M=874.86, SD=163.33), than on trials presenting three dots (M=899.01, SD=169.13), and on these they were significantly faster than on trials presenting five dots (M=1051.93, SD=220.99). There was evidence for linear (F(1,125)=188.851, p<.001) and non-linear (F(1,125)=146.881, p<.001) trends in the data.

If children employed a serial counting strategy to solve this task, a significant increase in their Precise Quantity Discrimination Skills' Trimmed RT would be expected with every new dot presented, as it would be an additional item to be counted. Children employed the same or even less time to make numerical judgements between one against three dots than one against two dots. This suggests that children were probably employing a parallel strategy rather than a serial verbal counting strategy when making numerical discriminations over arrays presenting one against two dots and arrays presenting one against three dots. The time children employed in making numerical judgements between two against three dots was at both time points significantly longer than the time they employed when they were asked to discriminate one against two dots, suggesting that at least some children could be employing serial counting to accurately respond to these trials. It is also worth noting that not only RTs decreased from Time 1 to Time 3 in this task but also that the difference in time needed to make numerical judgements over arrays presenting one against two dots and arrays presenting one against three dots significantly decreases from Time 1 to Time 3, to the point that at time three, the difference in time between these two types of trials is very subtle (see figures 7.6). Children's Precise Quantity Discrimination Skills' Trimmed RT at Time 1 will be used for further analyses so that influence of outliers due to distractibility is minimised. In addition, their Baseline Trimmed RT will also be controlled for so that their individual latencies in making accurate non-numerical discriminations cannot contaminate the results.

Figure 7.2: Trimmed RT in the Precise Quantity Discrimination task when the total number of dots presented on the screen were 3 (1 against 2 and 2 against 1), 4 (1 against 3 and 3 against 1) and 5 (2 against 3 and 3 against 2), irrespective of the side where the correct answer was presented at T_1 and T_3



7.1.2.2 *Approximate Quantity Discrimination Skills' task*

The total number of correct responses given with exception of the pairs depicting the same number of dots on both arrays (Weber 0, ratio 1) at Time 1 and Time 3 was obtained for each child (see table 7.3).

Table 7.3: Descriptive statistics for the Approximate Quantity Discrimination Skills' (AQDS) task at T_1 and T_3 (max.100)

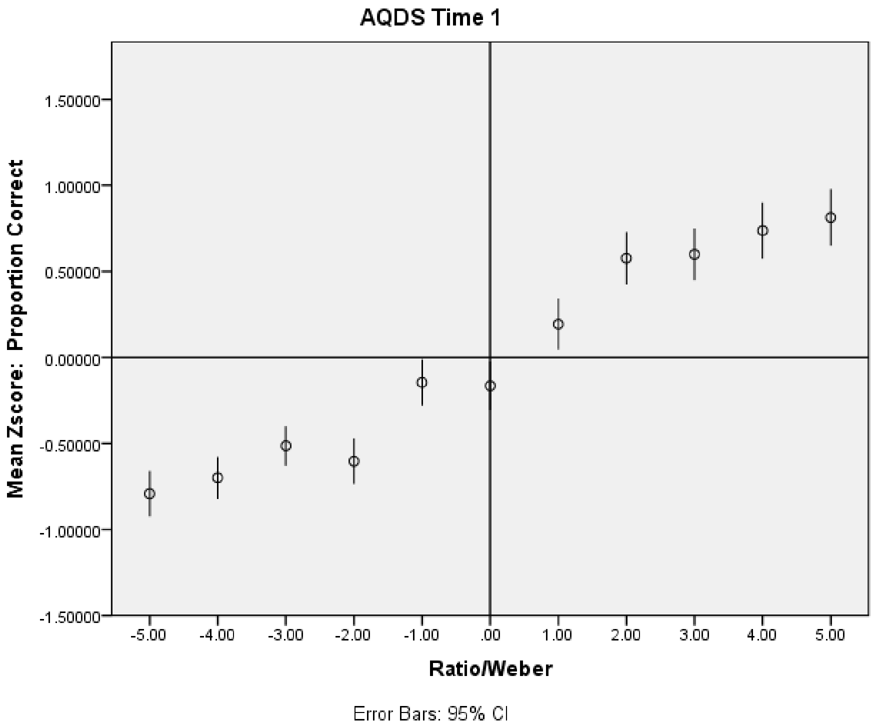
Approximate QDS	N	Mean	SD	ZSkewness	ZKurtosis	Min.	Max.
AQDS Accuracy - T_1	129	59.29	10.84	-1.19	-1.50	29	79
AQDS Accuracy - T_3	126	70.65	9.92	-3.27	1.58	40	88

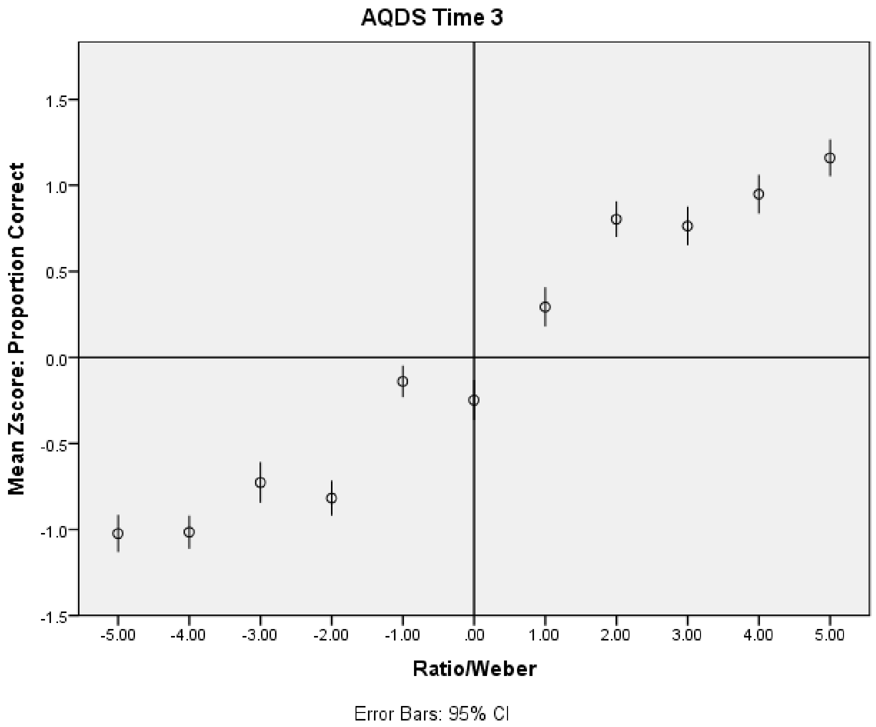
Children's accuracy scores in the Approximate Quantity Discrimination Skills' task were normally distributed at Time 1 (D (126) = .072, p=.18), and negatively skewed at Time 3 (D

(126) = .086, p=.182). Children's were significantly more accurate on this task when they were older (t=10.906, df= 125, p<.001). There are no floor or ceiling effects at any time point.

The number of leftward responses for each ratio difference (Weber function) depicted in the task was analysed taking into account the side where the correct response was presented. As expected, children's accuracy rates fall into a Weber's Law pattern; the proportion of accurate responses (represented in Z-scores) decreases as the ratio difference decreases from zero to five (Weber function increases) and increases as the ratio difference increases from zero to five (Weber function decreases) at both time points (see Figures 7.3 and 7.4).

Figures 7.3 and 7.4: Children's proportion of leftward responses for the five pairs presented depicting each Weber value on the AQDS task at T_1 (on the left) and T_3 (on the right)





At both time points children performed better when the numerical ratio difference between the two arrays was large (Weber 5) than when the numerical ratio difference between the two arrays was small (Weber 1), regardless the side where the correct response was presented. Graphs also reveal a general tendency to response with the right hand. The variance in children's responses on the five different pairs depicting different numerical ratio values on each side decreases from Time 1 to Time 3, suggesting that children's approximate quantity discrimination skills improve over time.

A partial correlation controlling for age in months and Baseline Trimmed RT at Time 1 was conducted to explore whether children's Precise Quantity Discrimination Skills' Trimmed RT was related to their Approximate Quantity Discrimination Skills' Accuracy. There was no significant relationship between these two measures ($r = -.079$, p (two-tailed) $=.379$) (see tables 7.15 and 7.16 in section 7.2.1). Consequently it is not appropriate to create a single composite variable with these two predictor variables.

7.1.3 Outcome variables

7.1.3.1 Early Number Skills' tasks (Administered at Times 1, 2 and 3):

The three Early Number Skills' tasks are novel tasks and are not standardised. They were designed to tap the three distinct developmental levels of QNCs proposed by Krajewski and Schneider (2009) at three different time points. It was expected that they would capture children's improvement on these skills over time by showing an increase in their accuracy while avoiding flooring and ceiling effects. Analyses of variance were conducted when the same task was administered at three different time points to explore whether scores were significantly different at each time point. Because the study sample is sufficiently large and therefore assumption of Normal distribution is not required (see Lumley, Diehr et al. 2002), repeated measures ANOVA were conducted to compare scores between time points.

- Knowledge of the Number Sequence (KNS)

Descriptive statistics for children's total scores on the Knowledge of the Number Sequence task (KNS) at each time point were obtained (see table 7.8).

Table 7.8: Descriptive statistics for the Knowledge of the Number Sequence task (QNC Level I) at T_1, T_2 and T_3.

Knowledge of the Number Sequence	**N**	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min**	**Max**
KNS - T_1 (min. 0, max.50)	129	24.10	9.36	4.24	0.57	9	48
KNS - T_2 (min. 0, max.50)	128	31.49	10.34	0.28	-2.69	8	50
KNS - T_3 (min. 0, max.50)	126	39.56	10.23	-4.45	-1.16	15	50

Children's accuracy rates in the Knowledge of Number Sequence task were not normally distributed neither at Time 1 (D (129) = .18, p<.001), Time 2 (D (128) = .13, 001) or Time 3 (D (126) = .24, p<.001). Data failed to meet sphericity assumptions (χ^2 (2)=9.688, p<.01) so values given are following Greenhouse Geisser corrections (ε=.93). Children scored significantly higher in this task as they got older, F(1.86, 232.525) = 141.759, p<.001). Although additional items were added to the initial task administered in the pilot, at Time 3 there are subtle ceiling effects.

- Counting Objects (CO)

Descriptive statistics for children's total scores on the Counting Objects (CO) task at each time point task were obtained (see table 7.9)

Table 7.9: Descriptive statistics for the Counting Objects task (QNC Level II) at T_1, T_2 and T_3

Counting Objects (CO)	**N**	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min**	**Max**
CO - T_1 (min. 0, max.20)	129	8.16	3.56	0.38	-2.09	1	17
CO - T_2 (min. 0, max.20)	128	10.80	3.50	-2.28	-0.40	1	19
CO - T_3 (min. 0, max.20)	126	13.14	3.59	-3.86	.95	2	19

Note 1. Means stand for 15 (8.16), 18 (10.80) and 25 (13.14) discrete items counted correctly (see table 5.3 in section 5.3.4.1 for further clarification).

Children's accuracy rates in the Counting Objects task were not normally distributed neither at Time 1 (D (129) = .10, p<.01), Time 2 (D (128) = .14, p<.001) or Time 3 (D (126) = .13, p<.001). Data met sphericity assumptions (χ^2 (2)=2.432, p=.296). Children scored higher in this task as they got older, F(1.96, 245.237) = 150.327, p<.001). There are no floor or ceiling effects at any time point.

- Story Problems (SP)

Descriptive statistics for children's total scores on the Story Problems (SP) task at each time point task were obtained (see table 7.10)

Table 7.10: Descriptive statistics for the Story Problems task (QNCs Level III) at T_1, T_2 and T_3 (min. 0, max. 20)

Story Problems (SP)	**N**	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min**	**Max**
SP - T_1 (min. 0, max. 20)	129	3.19	1.89	6.23	8.67	0	12
SP - T_2 (min. 0, max. 20)	128	4.88	2.94	4.90	2.95	0	16
SP – T_3 (min. 0, max. 20)	126	8.40	4.10	0.27	-2.18	0	18

Children's accuracy rates in the Story Problems task were not normally distributed (D (126) neither at Time 1 (D (129) = .18, p<.01), Time 2 (D (128) = .17, p<.001) or Time 3 (D (126) = .12, p<.001). Data failed to meet sphericity assumptions (χ^2 (2)=19.99, p<.001) so values given are following Greenhouse Geissser corrections (ε=.87). Children scored higher in this task as they got older, F(1.741, 217.590) = 146.401, p<.001). There are no floor or ceiling effects at any time point.

7.1.3.2 Standardised mathematical attainment measures (Administered at Times 1, 2 and 3):

For both mathematical attainment subtests of the WIAT-II (Wechsler, 2002), descriptive statistics for the standard scores are based on a sub-sample of 51 and 78 children at Time 1 and Time 2 respectively, as the rest of the children in the sample (78 and 41 children, respectively) were too young for the test norms to be applied. The norms could be applied to all children at Time 3.

- Mathematical Reasoning subtest WIAT-II^{UK} (MR WIAT-II^{UK})

Descriptive statistics for the raw scores and standard scores for the Mathematical Reasoning subtest of the WIAT-II^{UK} at Time 1, 2 and 3 were obtained (see table 7.11).

Table 7.11: Descriptive statistics for the Mathematical Reasoning subtest (MR WIAT-II^{UK}) at T_1, T_2 and T_3

Mathematical Reasoning	**N**	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min**	**Max**
MR WIAT-II^{UK} raw scores T_1	129	10.85	3.33	1.19	0.05	4	21
MR WIAT-II^{UK} standard T_1	51	100.90	8.74	-1.94	2.06	79	123
MR WIAT-II^{UK} raw scores T_2	128	14.63	4.12	-0.57	-0.95	4	25
MR WIAT-II^{UK} standard T_2	87	108.13	10.78	-2.00	0.94	76	132
MR WIAT-II^{UK} raw scores T_3	126	18.59	4.42	0.54	1.02	6	32
MR WIAT-II^{UK} standard T_3	126	102.87	12.70	-1.09	-0.30	70	132

Children's accuracy raw scores on the Mathematical Reasoning subtest WIAT-II^{UK} were normally distributed at Time 2 ($D(126)=.061$, $p=.200$) but not normally distributed at Time 1 ($D(126)=.097$, $p<.01$) or at Time 3 ($D(126)=.082$, $p<.05$). Data met sphericity assumptions ($\chi^2(2)=3.417$, $p=.181$). Children scored significantly higher as they got older, $F(2,250) = 301.276$, $p<.001$). Children's performance is broadly similar to the UK average at every time point.

- Numerical Operations subtest WIAT-II^{UK} (NO WIAT-II^{UK})

Descriptive statistics for the raw scores and standard scores of the Numerical Operation subtest of the WIAT-II^{UK} were obtained (see table 7.12).

Table 7.12: Descriptive statistics for the Numerical Operation subtest (NO WIAT-II^{UK}) at T_1, T_2 and T_3

Numerical Operations	**N**	**Mean**	**SD**	**ZSkewness**	**ZKurtosis**	**Min**	**Max**
NO WIAT-II^{UK} raw scores T_1	129	4.84	2.11	-2.95	-1.31	0	8
NO WIAT-II^{UK} standard T_1	51	96.90	7.99	-2.18	0.35	78	109
NO WIAT-II^{UK} raw scores T_2	128	6.92	1.74	-1.86	3.09	0	11
NO WIAT-II^{UK} standard T_2	87	101.15	8.63	1.11	-1.00	83	120
NO WIAT-II^{UK} raw scores T_3	126	8.32	1.94	-1.41	-1.32	4	13
NO WIAT-II^{UK} standard T_3	126	97.59	10.10	-1.90	0.53	68	120

Children's accuracy raw scores on the Mathematical Reasoning subtest WIAT-II^{UK} were not normally distributed neither at Time 1 ($D(129) = =.17$, $p<.001$), Time 2 ($D(128) = .17$, $p<.001$) or Time 3 ($D(126) = .13$, $p<.001$). Data met sphericity assumptions ($\chi^2(2)=3.506$,

p=.173). Children scored higher as they got older, $F(2, 250) = 194.190$, $p<.001$). There are no floor or ceiling effects at any time point. Overall, children's performance is broadly similar to the UK average at every time point.

7.1.4 Generic and Specificity variables (Administered at Time 3)

7.1.4.1 Generic variables: Verbal and non-verbal general conceptual ability

Descriptive statistics for the ability scores and T-scores of the Picture Similarities subtest and the Naming Vocabulary subtest of the BAS-II^{UK} (Elliott et al., 1996) at Time 3 were obtained (see table 7.13). Ability scores are standardised scores with an average of 100 and a standard deviation of fifteen. T-scores are scaled scores between one and a hundred, the average range being between 43 and 56 (see BAS-II^{UK} scoring manual for further information).

Table 7.13: Descriptive statistics for the Picture Similarities subtest and the Naming Vocabulary subtest of the BAS-II^{UK} at T_3 (n= 126)

British Ability Scales II	Mean	SD	ZSkewness	ZKurtosis	Min	Max
Picture Similarities BAS-II^{UK} ability scores	87.31	8.56	-4.09	14.60	41	111
Picture Similarities BAS-II^{UK} T-scores	55.65	8.45	2.14	1.72	28	80
Naming Vocabulary BAS-II^{UK} ability scores	116.64	14.07	.32	1.88	78	161
Naming Vocabulary BAS-II^{UK} T-scores	56.09	10.40	-0.23	0.69	26	80

Children's ability scores on the Picture Similarities subtest are negatively skewed but their T-scores on this test are normally distributed. Children's ability scores and T-scores on the Naming Vocabulary subtest are normally distributed. Overall, children's performance on these two subtests is broadly similar to the national average. Children's ability scores on both tests were used for further analysis.

7.1.4.2 Specificity variable: Reading Attainment

Descriptive statistics for the raw scores and standard scores of the Single-word Reading subtest of the YARC Early Word Recognition Test (Hulme et al., 2009) at Time 3 were obtained (see table 7.14).

Table 7.14: Descriptive statistics for the single-word reading test of the YARC Early Reading Test at T_3 (n= 126)

YARC Early Reading Test	Mean	SD	ZSkewness	ZKurtosis	Min	Max
Single-word reading raw scores	19.63	7.25	-1.36	-2.21	4	30
Single-word reading standard scores	104.66	11.77	-0.68	-0.91	74	127

Children's raw scores show a leptokurtic distribution; however standard scores are normally distributed. Children's performance on this task is broadly similar to the UK average. Children's raw scores were used for further analyses.

7.2 CONCURRENT CORRELATIONS AND PARTIAL CORRELATIONS

Correlations, and partial correlations controlling for age in months for all variables administered at each time point were obtained. Because the study sample is sufficiently large and therefore assumption of normal distribution is not required (see Lumley et al., 2002), Pearson's Product Moment was conducted for the concurrent correlations. Age in months was controlled for because it was significantly correlated with all four predictor variables (Precise Quantity Discrimination Trimmed RT, Approximate Quantity Discrimination Accuracy, VSSP and PA) at Time 1. Additional partial correlations at Time 1 and at Time 3 were conducted for children's Precise Quantity Discrimination Trimmed RT controlling for their age in months and Baseline Trimmed RT to explore whether children's Precise Quantity Discrimination Trimmed RT was related to their performance on the outcome measures once their Baseline Trimmed RT was accounted for.

7.2.1 Concurrent correlations and partial correlations controlling for age at Time 1

Concurrent correlations and partial correlations controlling for participants' age in months at Time 1 (and also controlling for Baseline Trimmed RT at Time 1 for PQDS Trimmed RT) for all tasks administered at Time 1 were conducted (see tables 7.15 and 7.16).

Table 7.15: Correlations (above the diagonal, n=129) and partial correlations controlling for age in months at T_1 (below the diagonal, df=125) for the baseline (BL) accuracy and Trimmed RT, the Precise Quantity Discrimination Skills' (PQDS) accuracy and Trimmed RT, the Approximate Quantity Discrimination Skills' (AQDS) accuracy, VSSP functioning (VSSP), phonological awareness (PA), the Early Number Skills (KNS, CO, SP) Mathematical Reasoning subtest of the WIAT II raw scores (MR WIAT-II^{UK}) and Numerical Operations subtest of the WIAT-II^{UK} raw scores (NO WIAT-II^{UK}) at T_1.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Age (months)	-.012	-.285***	.103	-.338***	.298***	.335***	.236**	.404***	.212*	.184*	.293***	.364***
1.BL Accuracy	-	.222*	.056	-.036	.190*	.115	.085	.179*	.111	.071	.076	.154
2.BL Trimmed RT	.224*	-	.055	.392***	-.152	-.174*	-.209*	-.116	-.182*	-.009	-.100	-.271**
3.PQDS Accuracy	.094	.089	-	-.009	.299***	.191*	.083	.156	.168	.106	.351***	.256**
4.PQDS Trimmed RT	-.044	.328***	.027	-	-.190*	-.257**	-.192*	-.259**	-.319***	-.143	-.186*	-.331***
5.AQDS Accuracy	.173	-.073	.283***	-.098	-	.238**	.194*	.320***	.263**	.138	.277***	.376***
6.VSSP	.101	-.088	.167	-.161	.153	-	.000	.161	.221*	.098	.298***	.401***
7.PA	.046	-.153	.061	-.123	.133	-.086	-	.395***	.270**	.263**	.302***	.339***
8.Knowledge of the Number Sequence	.155	.000	.125	-.142	.229**	.029	.337***	-	.470***	.340***	.429***	.469***
9.Counting objects	.076	-.130	.151	-.269**	.214*	.163	.231**	.430***	-	.177*	.453***	.455***
10.Story Problems	.083	-.046	.089	-.088	.089	.039	.230**	.296***	.143	-	.312***	.283***
11.MR WIAT-II^{UK}	.086	-.018	.338***	-.096	.208*	.222*	.250**	.355***	.418***	.275**	-	.604***
12.NO WIAT-II^{UK}	.078	-.186*	.236**	-.241**	.301***	.318***	.279***	.378***	.415***	.236**	.558***	-

***p<=.05; **p<=.01; ***p<=.001**

Table 7.16: Partial correlations controlling for age in months at T_1 and for the Baseline Trimmed RT at $T_1$for the Precise Quantity Discrimination Skills (PQDS) Trimmed RT with baseline (BL) accuracy, the Precise Quantity Discrimination Skills' (PQDS) accuracy, the Approximate Quantity Discrimination Skills' (AQDS) accuracy, VSSP functioning (VSSP), phonological awareness (PA), Early Number Skills (KNS, CO, SP), Mathematical Reasoning of the WIAT-II^{UK} raw scores (MR WIAT-II^{UK}) and Numerical Operations of the WIAT-II^{UK} raw scores (NO WIAT II) at T_1 (n= 129, df = 124)

	BL Accuracy	**PQDS Accuracy**	**AQDS Accuracy**	**VSSP**	**PA**	**KNS**	**CO**	**SP**	**MR WIAT-II^{UK}**	**NO WIAT-II^{UK}**
PQDS Trimmed RT	-.128	-.002	-.079	-.141	-.078	-.150	-.242**	-.109	-.096	-.194*

*p<=.05; **p<=.01; ***p<=.001

All significant correlations are positive with exception of the variables measuring children's RTs, in this case correlations are negative, indicating that the older the children are, the faster they can respond to computerised discrimination tasks and the better they perform on the outcome measures. Age in months significantly correlates with all variables expect with Precise Quantity Discrimination Skills' Accuracy. After controlling for children's age in months, none of the predictor variables remain significantly correlated to each other. Children's performances on the three Early Number Skills' tasks remain significantly correlated to each other with exception of the Counting Objects task with the Story Problems task. The strength of these correlations is modest for the Knowledge of the Number Sequence task with the Counting Object task and weak for the Knowledge of the Number Sequence with Story Problems task. The mathematical attainment measures remain significantly and modestly correlated to each other after controlling for participants' age. Regarding children's performance on the Quantity Discrimination Skills' tasks, children's Precise Quantity Discrimination Skills' Trimmed RT remains significantly correlated to their performance on the Counting Objects task but not with their performance on the Knowledge of the Number Sequence or Story Problems tasks neither after controlling for their age, or after controlling for their age and Baseline Trimmed RT. However, children's Approximate Quantity Discrimination Skills' Accuracy remains significantly correlated to their performance on the Knowledge of the Number Sequence and Counting Objects task. The strength of all these correlations is weak.

Children's performances on each of the three Early Number Skills' tasks still share significant variance with performance on the two mathematical attainment measures after controlling for age. The strength of these correlations varies from weak to modest. Regarding children's domain-general cognitive skills, while VSSP functioning does not share significant variance with any of the three Early Number Skills' tasks after controlling for age, their phonological awareness remains significantly correlated with their performance on the three Early Number Skills' tasks. However, the strength of these correlations is weak. Also, children's domain-general and domain-specific cognitive skills remain significantly correlated to their performance on both mathematical attainment measures after controlling for their age with exception of performance on the Precise Quantity Discrimination Trimmed RT that does no longer share any significant variance with their performance on the Mathematical Reasoning subtest. The strength of all partial correlations for the domain-general and the domain-specific cognitive skills are weak.

7.2.2 Concurrent correlations and partial correlations controlling for age at Time 2

Concurrent correlations and partial correlations controlling for participants' age in months at Time 2 for all tasks administered at Time 2 were conducted (see table 7.17).

Table 7.17: Correlations (above the diagonal) and partial correlations controlling for age in months at T_2 (below the diagonal) for the Early Number Skills' tasks and mathematical attainment measures administered at T_2 (n=128, df=125)

Task	**KNS**	**CO**	**SP**	**MR WIAT-II^{UK}**	**NO WIAT-II^{UK}**
Age In Months	.434***	.207*	.280***	.357***	.307***
Knowledge of the Number Sequence	-	.415***	.457***	.471***	.440***
Counting objects	.369***	-	.305***	.436***	.476***
Story Problems	.387***	.263**	-	.558***	.480***
Mathematical Reasoning WIAT-II^{UK}	.376***	.396***	.511***	-	.547***
Numerical Operations WIAT-II^{UK}	.357***	443***	.431***	.492***	-

***p<=.05; **p<=.01; ***p<=.001**

Age in months significantly correlates with all variables administered at Time 2. After controlling for children's age in months performances on all three Early Number Skills' tasks remain weakly correlated to each other. The mathematical attainment measures remain modestly correlated to each other. Children's performances on each of the Early Number Skills' tasks also remain correlated to their performance on both mathematical attainment measures. The strength of these partial correlations varies from modest to weak.

7.2.3 Concurrent correlations and partial correlations controlling for age at Time 3

Concurrent correlations and partial correlations controlling for participants' age in months at Time 3 (and also controlling for Baseline Trimmed RT at Time 3 for PQDS Trimmed RT) were conducted (see table 7.18).

Table 7.18: Correlations (above the diagonal, n=126) and partial correlations controlling for age in months at T_3 (below the diagonal, df=123) for the baseline (BL) accuracy and Trimmed RT, the Precise Quantity Discrimination Skills' (PQDS) Accuracy and Trimmed RT, the Approximate Quantity Discrimination Skills' (AQDS) Accuracy, the Early Number Skills' tasks (KNS, CO, SP), Mathematical Reasoning (MR WIAT-II^{UK}) and Numerical Operations (NO WIAT-II^{UK}) raw scores, single-word reading scores (YARC), Picture Similarities (PS BAS-II^{UK}) and Naming Vocabulary (NV BAS-II II^{UK}) at T_3.

	1	2	3	4	5	6	7	8	9	10	11	12	13
Age In Months	-.006	-.343***	.095	-.358***	.105	.168	.120	.342***	.306***	.388***	.272**	.069	.108
1.BL Accuracy	-	.052	.166	.129	.077	.196*	.218*	.016	.063	.075	.094	-.029	.063
2.BL Trimmed RT	.053	-	-.060	.435***	-.129	-.231**	-.187*	-.380***	-.395***	-.366***	-.301***	-.060	-.090
3.PQDS Accuracy	.168	-.029	-	.369***	.267**	.227*	.227*	.270**	.252**	.232**	.182*	.107	.218*
4.PQDS Trimmed RT	.136	.356***	.434***	-	-.160	-.137	-.016	-.190*	-.227*	-.178*	-.203*	-.078	-.146
5.AQDS Accuracy	.078	-.100	.259**	-.132	-	.187*	.217*	.333***	.430***	.233**	.140	.118	.259**
6.Knowledge of the Number Sequence	.200*	-.187	.215*	-.084	.172	-	.412***	.425***	.310***	.344***	.498***	.030	.140
7.Counting objects	.220*	-.157	.218*	.030	.207*	.400***	-	.226*	.307***	.335***	.326***	-.001	.199*
8.Story Problems	.019	-.297***	.254**	-.077	.318**	.397***	.199*	-	.608***	.613***	.483***	.140	.233**
9.MR WIAT-II^{UK}	.068	-.324***	.235**	-.132	.420***	.276**	.286***	.563***	-	.396***	.367***	.159	.368***
10.NO WIAT-II^{UK}	.083	-.269**	.213*	-.046	.210*	.307***	.315***	.555***	.316***	-	.525***	.029	.183*
11.YARC	.100	-.230**	.163	-.117	.116	.477***	.307***	.431***	.310***	.473***	-	-.087	.225*
12.PS BAS-II II^{UK}	-.029	-.038	.102	-.057	.112	.018	-.009	.124	.145	.002	-.110	-	.086
13.NV BAS-II^{UK}	.064	-.057	.210*	-.116	.251**	.125	.188*	.210*	.354***	.154	.204*	.079	-

***p<=.05; **p<=.01; ***p<=.001**

Table 7.19: Partial correlations controlling for age in months and Baseline Trimmed RT at T_3 and for the Precise Quantity Discrimination Skills' (PQDS) Trimmed RT with Baseline (BL) Accuracy, the Precise Quantity Discrimination Skills' (PQDS) Accuracy, the Approximate Quantity Discrimination Skills' (AQDS) Accuracy, the Early Number Skills (KNS, CO, SP), Mathematical Reasoning of the WIAT-II^{UK} raw scores (MR WIAT-II^{UK}) and Numerical Operations of the WIAT-II^{UK} raw scores (NO WIAT-II^{UK}), single-word reading scores (YARC), Picture Similarities (NV BAS-II^{UK}) and Naming Vocabulary (NV BAS-II^{UK}) at T_3 (n= 126, df = 122)

	BL Acc.	**PQDS Acc.**	**AQDS Acc.**	**KNS**	**CO**	**SP**	**MR WIAT-II^{UK}**	**NO WIAT-II^{UK}**	**YARC**	**PS BAS II**	**NV BAS II**
PQDS Trimmed RT	.126	.476***	-.103	-.019	-.093	-.033	-.019	-.056	-.038	-.046	-.102

***p<=.05; **p<=.01; ***p<=.001**

All significant correlations are positive with exception of the variables measuring children's RTs. In this case correlations are negative indicating that the older the children are the faster they can respond to computerised discrimination tasks and the better they perform on the outcome measures. Age in months significantly correlates with children's Precise Quantity Discrimination Skills' Trimmed RT, with both mathematical attainment measures and with reading performance. Children's performances on the Quantity Discrimination Skills' tasks do not correlate with each other either before or after controlling for their age. Children's performances on each of the Early Number Skills' tasks remain significantly correlated to each other after controlling for their age. Children's performances on both mathematical attainment measures also remain significantly correlated to each other after controlling for their age. From the Quantity Discrimination Skills' tasks, only children's Approximate Quantity Discrimination Skills' Accuracy remains significantly correlated to their performances on the Counting Objects task, Story Problems task and both mathematical attainment measures. Children's performances on each of the Early Number Skills' tasks remain significantly correlated to their performance on both mathematical attainment measures after controlling for their age. In addition children's reading performance remains significantly correlated to their performance on the three Early Number Skills tasks, both mathematical attainment measures and their performance on the Naming Vocabulary subtest. Only Naming Vocabulary subtest, but not Picture Similarities, shares significant variance with children's Approximate Quantity Discrimination Skills' Accuracy, the Counting Objects task, the Story Problems task, the Mathematical Reasoning subtest and their reading attainment.

7.2.4 Correlations and partial correlations for reading attainment with all predictor variables assessed at Time 1

Partial correlations controlling for participants' age in months at Time 1 (and also Baseline Trimmed RT for PQDS Trimmed RT) were also conducted to explore the relationships between the four cognitive precursors (Precise Quantity Discrimination Skills' Trimmed RT, Approximate Quantity Discrimination Skills' Accuracy, VSSP functioning and phonological awareness) and children's reading attainment assessed at Time 3 (see table 7.20).

Table 7.20: Partial correlations for children's scores on the Word Recognition subtest of the YARC Early Reading Test with the two Quantity Discrimination Skills' tasks (PQDS Trimmed RT and AQDS Accuracy) and the two domain-general skills (VSSP and PA) controlling for participants' age in months at T_1 (n=128, df = 125)

Tasks	Age T_1	Age T_3	PQDS Trimmed RT	AQDS Acc.	VSSP	PA
Single-word reading YARC	$.259^{**}$	$.272^{**}$	$-.214^{*}$	$.227^{*}$	.149	$.225^{*}$

***p<=.05; **p<=.01**
Note. Baseline Trimmed RT at T_1 was also partialled out for PQDS Trimmed RT

All significant correlations are positive with exception of children's Precise Quantity Discrimination Skills' Trimmed RT. In this case correlations are negative, indicating that the better children are at reading the faster they can respond to computerised discrimination tasks. Age in months at Time 1 and age in month at Time 3 significantly correlate with children's performance on the Single-word Reading subtest of the YARC Early Word Recognition Test, the strength of these correlations being very similar. Children's performance on the Early Word Recognition Test correlates significantly with their Approximate Quantity Discrimination Skills' Accuracy and their phonological awareness at Time 1 after controlling for their age in months at Time 1.

7.3 DISCUSSION

Children's performance on all standardised measures is broadly similar to the UK average. It can therefore be assumed that the study sample is representative of the population under investigation.

Children's Precise Quantity Discrimination Skills' Accuracy and Trimmed RT reveal that overall they can make precise judgments over two small non-symbolic quantities at the start of Reception Year and that on average they employ 1,555 msec. to respond. Given that the average time for five- and six-year-olds to correctly enumerate an additional item outside the subitising range is approximately 1,000 msec. (Chi & Klahr, 1975; LeFevre et al., 2010; Svenson & Sjöberg, 1978; Trick et al., 1996) and that the minimum number of dots presented in a trial on this specific task was three (one against two dots), it is highly unlikely that they were employing a serial exhaustive enumeration strategy to respond. In addition, the analyses conducted on the time they employed to discriminate the larger set, depending on the number of dots presented, also indicates that it took them more time to correctly discriminate one against two dots than one against three dots, thus not needing additional time for each new item to be counted and showing no continuous linear increase as the number of dots presented in a single trial increases. Consequently this task is tapping some “intuitive” strategy to make precise discriminations over small quantities, rather than tapping children’s serial counting speed. Children’s Approximate Quantity Discrimination Skills’ Accuracy aligns with previous ANS studies conducted with children of a similar age (see Libertus et al., 2013), suggesting that Approximate Quantity Discrimination Skills’ Accuracy in the present study is tapping children’s ANS functioning. Children’s performance at the start of Reception Year on both Quantity Discrimination Skills’ tasks correlates with at least one of the standardised mathematical attainment measures, which indicates their criterion validity is satisfactory. In addition, the Quantity Discrimination Skills’ tasks do not share significant variance with each other including when age in months is controlled for nor when age in months and Baseline Trimmed RT are controlled for. This suggests that each Quantity Discrimination Skills’ task is in fact tapping distinct skills thus supporting Feigenson et al.’s (2004) proposal that each of these skills actually rely on two distinct systems of quantity representation. Neither of these tasks correlate with either children’s VSSP functioning or phonological awareness, supporting the idea that these two skills are domain-specific and dissociable from language (Cohen, Dehaene, Chochon, Lehericy, & Naccache, 2000) and from STM (Butterworth, 1996).

For both tasks an improvement in performance from Time 1 to Time 3 is evident; children took significantly less time to make precise quantity discriminations over small non-symbolic numerical sets at midway through Year 1 than at the start of Reception Year and were significantly more accurate at identifying the larger of two large non-symbolic sets at midway through Year 1 than at the start of Reception Year. Given that the minimum time between the occasions children were administered these tasks was fourteen months, it is highly unlikely that improvement is due to practise. Alternatively, this data provides longitudinal evidence of the development of these specific skills throughout the early years and supports previous cross-sectional results where children were faster and more accurate the older they were (Benoit et al., 2004; Fischer et al., 2008; Gelman & Tucker, 1975; Holloway & Ansari, 2009; Svenson & Sjöberg, 1978; Trick et al., 1996) and results from a longitudinal study where pre-schoolers’ non-symbolic approximate quantity discrimination skills’ precision improved over a six-month period (Libertus et al., 2013). This evidence highlights the importance of conducting analyses to determine whether the relationships between quantity discrimination skills and children’s performance on early number skills and mathematical attainment measures are causal as well as the need to control for age in regression models predicting children's mathematical outcomes from their quantitative skills.

The PCA analysis was satisfactory and resulted in two latent variables representing the phonological awareness and the VSSP functioning measures respectively. The use of latent variables instead of single-task measures provides two main advantages. First, the number of predictors in the study is reduced and so the participant-test ratio improves. Second, measures are less contaminated with specific task demands than those obtained from single tests (see Bowey, 2005). Both domain-general cognitive skills measures obtained from the PCA analysis correlate with children's performance on both mathematical attainment measures, suggesting they contribute to mathematical attainment.

Results show that the three Early Number Skills' tasks designed to represent the distinct developmental levels of QNCs proposed by Krajewski and Schneider (2009) provide a good spread of scores while avoiding flooring or ceiling effects at any time point. Nevertheless, children's scores capture their improvement on these tasks over a six-month period and over an eighteen-month period. At each time point all three of these measures correlate with children's performance on at least one of the standardised mathematical attainment measures, highlighting their criterion validity.

The specificity measure administered at midway through Year 1 shows different patterns of relationships with the predictor variables; after controlling for children's age in months, reading performance shares significant variance with performance on both Quantity Discrimination Skills' tasks and phonological awareness. The fact that phonological awareness is related to children's later reading performance corroborates previous findings which propose that this skill is an early precursor of literacy development (Gathercole et al., 2005; Hecht et al., 2001). It was not expected that children's reading attainment would share significant variance with their performance on both Quantity Discrimination Skills' tasks, mainly because it has been proposed that these are domain-specific skills and independent from language (Cohen et al., 2000; Lemer et al., 2003). However, similar results were found by Holloway and Ansari (2009) with a group of six-year-olds. The authors suggested that these findings could be due to the common demands that reading and quantity discrimination tasks make, such as visual attention processes (Facoetti, Paganoni, Turatto, Marzola, & Mascetti, 2000).

Regarding the general conceptual ability measures administered at midway through Year 1, children's verbal General Conceptual Abilities share significant variance with their performance on the Counting Objects and Story Problems tasks and with their performance on the Mathematical Reasoning subtest. In contrast, children's non-verbal General Conceptual Abilities do not share significant variance with any of the variables of interest.

8. LONGITUDINAL STUDY RESULTS:

DOMAIN-GENERAL AND DOMAIN-SPECIFIC COGNITIVE SKILLS PREDICTING EARLY NUMBER SKILLS AND MATHEMATICAL ATTAINMENT

After preliminary analysis of the data partial correlations between predictor variables measured at the start of Reception Year (T_1) and outcome measures assessed at the end of Reception Year (T_2) and at midway through Year 1 (T_3) were obtained to explore how the domain-general and domain-specific cognitive skills relate to the outcome measures over different time periods. Following this, multiple regression analyses were conducted to explore the independent and unique contributions that the domain-general and domain-specific cognitive skills make to the three Early Number Skills' tasks and the two mathematical attainment measures over a six-month period and over an eighteen-month period. Analyses for each type of cognitive predictors (either domain-general or domain-specific) were first conducted independently for ease of comparison to previous findings, which have included solely domain-general predictors (e. g. Simmons et al., 2008) or have included solely domain-specific predictors (e. g. Libertus et al., 2011). Finally the unique and independent contribution of all four predictors to each of the outcome measures was explored simultaneously. The specific analyses conducted are explored in detail below.

8.1 STATISTICAL ANALYSIS STRATEGY

Different groups of regression analyses were conducted. First, regression analyses were conducted with only the domain-general cognitive skills as predictors. Then, regression analyses were conducted only with the domain-specific cognitive skills as predictors. Last, regression analyses with the domain-general and the domain-specific cognitive skills together as predictors were conducted.

8.1.1 Regression analyses with only the domain-general cognitive skills as predictors

Multiple regression analyses were conducted to determine whether the domain-general cognitive skills at the start of Reception Year (T_1) *independently* predict children's performance on the Early Number Skills and mathematical attainment measures at the end of Reception Year (T_2) and at midway through Year 1 (T_3). These analyses aimed to determine whether children's VSSP functioning and phonological awareness relate differently to each of the Early Number Skills and mathematical attainment measures over a six-month period and over an eighteen-month period. They will also determine whether the results obtained with the novel Early Number Skills' tasks align with previous findings where phonological awareness and VSSP functioning predict specific early number skills (e.g. Krajewski & Schneider, 2009; LeFevre et al., 2010).

8.1.2 Regression analyses with only the domain-specific cognitive skills as predictors

Multiple separate regression analyses were conducted to explore whether the domain-specific cognitive skills at the start of Reception Year (T_1) *independently* predict children's performance on the Early Number Skills and mathematical attainment measures at the end of Reception Year (T_2) and at midway through Year 1 (T_3). These analyses aim to determine whether the present results align with previous studies that have examined the relationships between children's subitising skills or children's ANS precision and their numerical competence (e.g. LeFevre et al., 2010; Libertus et al., 2013).

8.1.3 Regression analyses with the domain-general and the domain-specific cognitive skills as predictors

Regression analyses were conducted including all four precursors in the regression model simultaneously to determine the specific contribution that each type of cognitive skill makes to each of the outcome measures over a six-month period and over an eighteen-month period. The aim was to explore whether the two domain-specific and the two domain-general cognitive skills measured at the start of Reception Year (T_1) *independently* predict children's performance on the Early Number Skills and mathematical attainment measures at the end of Reception Year (T_2) and at midway through Year 1 (T_3) or whether any of the four cognitive precursors no longer predicts unique variance once all four predictors are included in the regression model. An additional regression analysis was also conducted to determine whether the domain-specific and domain-general cognitive skills at the start of Reception Year (T_1) predict children's reading attainment at midway through Year 1 (T_3). This additional regression was conducted with the aim of exploring whether children's Precise and Approximate Quantity Discrimination Skills also relate to later reading attainment or they are specific precursors of mathematical outcomes.

All regression analyses simultaneously exploring the longitudinal contributions of the four precursors to the outcome measures over an eighteen-month period described above were re-conducted controlling for children's General Conceptual Abilities to explore whether the domain-general and domain-specific cognitive skills still explain independent and unique variance in the outcome measures when children's General Conceptual Abilities are controlled for.

Lastly, the causality of the relationships found between domain-specific and domain-general cognitive skills and outcome measures was investigated by exploring whether the cognitive skills that predicted significant variance in the outcome measures could also predict *growth* on these variables. First, whether children's cognitive skills that predicted significant variance in the Early Number Skills over an eighteen-month period could also predict growth on these measures over this time period was explored. Second, whether the domain-specific and domain-general cognitive skills that predicted performance over a six-month period on the mathematical attainment measures could also explain growth in these measures over this period was explored. These additional analyses were conducted for ease of comparison with a recent study that has conducted growth analyses for approximate quantity discrimination skills on pre-schoolers' mathematical attainment over a six-month period (Libertus et al., 2013). Finally, whether children's cognitive skills that predicted significant variance in the outcome measures over an eighteen-month period could also predict growth over this time period was examined.

Age in months at the start of Reception Year (T_1) was controlled for in all regression models because this variable shared significant variance with all predictor variables at time 1 (see table 7.15 in section 7.2.1). In addition, when the predictor variable was children's Precise Quantity Discrimination Skills' Trimmed RT, their Baseline Trimmed RT was also partialled out.

8.2 THE RELATIONSHIPS BETWEEN COGNITIVE PRECURSORS AND OUTCOME MEASURES

8.2.1 The relationships between cognitive precursors and Early Number Skills

8.2.1.1 The relationships between domain-general cognitive skills at the start of Reception Year with the Early Number Skills at the end of Reception Year and at midway through Year 1

Partial correlations controlling for participants' age in months at Time 1 were conducted to explore the relationships between children's VSSP functioning and phonological awareness with their performance on the three Early Number Skills' tasks at the end of Reception Year and at midway through Year 1 (see table 8.1).

Table 8.1: Partial correlations for children's VSSP functioning (VSSP) and Phonological Awareness (PA) with the three Early Number Skills' tasks at T_2 and T_3 controlling for participants' age in months at T_1 (n=128, df = 125 for T_2; n=126, df = 121 for T_3)

	Early Number Skills T_2			Early Number Skills T_3		
Variable	**KNS T_2**	**CO T_2**	**SP T_2**	**KNS T_3**	**CO T_3**	**SP T_3**
VSSP	.113	.143	$.206^{*}$	.138	.146	$.269^{**}$
PA	$.310^{***}$	$.211^{*}$	$.224^{*}$	$.259^{**}$	.104	$.253^{**}$

*p<=.05; **p<=.01; ***p<=.001

Children's phonological awareness shares significant variance with all three Early Number Skills' tasks at Time 2 and with the Knowledge of the Number Sequence task and Story Problems task at Time 3. Children's VSSP functioning only shares significant variance with children's scores in the Story Problems task at Time 2 and at Time 3. The strength of all these correlations is weak or very weak.

Two different linear regression analyses for each Early Number Skill were conducted to explore whether children's VSSP functioning and phonological awareness could explain unique variance in their performance on each of the Early Number Skills tasks at the end of Reception Year and at midway through Year 1 (see table 8.2).

Table 8.2: Forced entry regression analyses examining the prediction of the Early Number Skills performance at T_2 and at T_3 from VSSP functioning and PA over and above age in months

	Knowledge of the Number Sequence (KNS)				Counting Objects (CO)				Story Problems (SP)			
	KNS T_2		KNS T_3		CO T_2		CO T_3		SP T_2		SP T_3	
Variable	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**
Step 1. Control variable												
Age T_1 (months)	.430***		.174		.181*		.111		.289***		.335***	
R^2 by control variable		*.185****		*.030*		*.033**		*.012*		*.084****		*.112****
Step 2. D-G Cog. Skills												
VSSP functioning	.138		.172		.172		.167		.234**		.298***	
PA	.301***		.278**		.229**		.122		.242**		.272***	
Unique R^2 by D-G Cog. Skills		*.095****		*.091***		*.069***		*.035*		*.094****		*.133****

***p<=.05; **p<=.01; ***p<=.001**

The ANOVA conducted on the final model predicting KNS T_2 is $F(3,124)=16.043$, $p<.001$
The ANOVA conducted on the final model predicting KNS T_3 is $F(3,122)=5.587$, $p<.001$
The ANOVA conducted on the final model predicting CO T_2 is $F(3,124)=4.667$, $p<.01$
The ANOVA conducted on the final model predicting CO T_3 is $F(3,122)=2.016$, $p=.115$
The ANOVA conducted on the final model predicting SP T_2 is $F(3,124)=8.909$, $p<.001$
The ANOVA conducted on the final model predicting SP T_3 is $F(3,122)=13.226$, $p<.001$

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 values represent the combined predictive impact of the two Domain-general Cognitive Skills while accounting for the control variables.

Over and above children's age in months, the two domain-general cognitive skills together explained 9.5% and 9.1% of variance in the Knowledge of the Number Sequence task at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with phonological awareness being a unique predictor at both time points. Using the same regression model both domain-general cognitive skills together explained 6.9% and 3.5% of variance in the Counting Objects task at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression model at the end of Reception Year was statistically significant with phonological awareness being the unique predictor. Lastly, the two domain-general cognitive skills together explained 9.4% and 13.3% of variance on the Story Problems task at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with both domain-general cognitive skills being unique predictors at both time points. This suggests that phonological awareness and VSSP functioning are distinct predictors of children's performance on the three Early Number Skills' tasks and that each domain-general cognitive skill relates differently to the different Early Number Skills included in the study. The variance predicted by the two domain-general cognitive skills remains fairly similar over a six-month period and over an eighteen-month period for the Knowledge of the Number Sequence task, however, it drops substantially when predicting performance on the Counting Objects task at midway through Year 1 and it increases substantially when predicting performance on the Story Problems task at midway through Year 1. Therefore, the longitudinal contributions that these two cognitive precursors make to the different Early Number Skills vary over time.

8.2.1.2 The relationships between domain-specific cognitive skills at the start of Reception Year with the Early Number Skills at the end of Reception Year and at midway through Year 1

Partial correlations controlling for participants' age in months at Time 1 were conducted to explore the relationships between children's Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy with their performance on the three Early Number Skills' tasks at end of Reception Year and at midway through Year 1 (see table 8.3).

Table 8.3: Partial correlations for Precise Quantity Discrimination Skills (PQDS) Trimmed RT and Approximate Quantity Discrimination Skills (AQDS) Accuracy with the three Early Number Skills' task at T_2 and T_3 controlling for participants' age in months at T_1 (n=128, df = 123 for T_2 and n=126, df=121 for T_3)

Variable	**KNS T_2**	**CO T_2**	**SP T_2**	**KNS T_3**	**CO T_3**	**SP T_3**
PQDS Trimmed RT T_1	-.113	-.216*	-.091	-.133	-.282**	-.150
AQDS Accuracy T_1	.316***	.288***	.269**	.253**	.209*	.241**

*p<=.05; **p<=.01; ***p<=.001
Note. Baseline Trimmed RT at T_1 was also partialled out for PQDS Trimmed RT

At Time 2 and Time 3, Precise Quantity Discrimination Skills' Trimmed RT shares significant variance with children's performance on the Counting Objects task after controlling for their age in months and Baseline Trimmed RT at Time 1. Approximate Quantity Discrimination Skills' Accuracy shares significant variance with children's scores on the three Early Number Skills' tasks at both time points. The strength of all these correlations is weak or very weak.

Two different linear regression analyses for each Early Number Skill were conducted to explore whether children's Quantity Discrimination Skills could explain unique significant variance in their performance on each of the Early Number Skills' tasks at the end of Reception Year and at midway through Year 1 (see table 8.4).

Table 8.4: Forced entry regression analyses examining the prediction of the Early Number Skills' tasks at T_2 and at T_3 from PQDS and AQDS over and above age in months and Baseline Trimmed RT

	Knowledge of the Number Sequence (KNS)				Counting Objects (CO)				Story Problems (SP)			
	KNS T_2		KNS T_3		CO T_2		CO T_3		SP T_2		SP T_3	
Variable	**B**	**R²**	**B**	**R²**	**B**	**R²**	**B**	**R²**	**B**	**R²**	**B**	**R²**
Step 1. Control variables												
Age T_1 (months)	.418***		.165		.168		.106		.269**		.303***	
Baseline Trimmed RT	-.052		-.027		-.044		-.018		-.070		-.109	
R² by control variables		*.190****		*.031*		*.034*		*.013*		*.088***		*.123****
Step 2. QDS												
PQDS Trimmed RT	-.091		-.125		-.216*		-.298**		-.076		-.138	
AQDS Accuracy	.276***		.237*		.276**		.195*		.257**		.227**	
Unique R² by QDS		*.079***		*.067**		*.114****		*.113****		*.067***		*.066***

***p<=.05; **p<=.01; ***p<=.001**

The ANOVA conducted on the final model predicting KNS T_2 is $F(4,122)=11.258$, $p<.001$
The ANOVA conducted on the final model predicting KNS T_3 is $F(4,120)=3.257$, $p<.05$
The ANOVA conducted on the final model predicting CO T_2 is $F(4,122)=5.308$, $p=.001$
The ANOVA conducted on the final model predicting CO T_3 is $F(4,120)=4.300$, $p<.01$
The ANOVA conducted on the final model predicting SP T_2 is $F(4,122)=5.586$, $p<.001$
The ANOVA conducted on the final model predicting SP T_3 is $F(4,120)=7.010$, $p<.001$
Note. B Stands for the standardised regression coefficient; R² stands for the proportion of variance explained. The unique R² values represent the combined predictive impact of the two Quantity Discrimination Skills' tasks while accounting for the control variables.

After controlling for age in months and Baseline Trimmed RT, the two Quantity Discrimination Skills together explained 7.9% and 6.7% of variance in the Knowledge of the Number Sequence task at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with Approximate Quantity Discrimination Skills' Accuracy being a unique predictor at both time points. Using the same regression model both Quantity Discrimination Skills together also explained 11.4% and 11.3% of variance in the Counting Objects task at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with both Quantity Discrimination Skills being unique predictors at both time points. Lastly, the two Quantity Discrimination Skills together explained 6.7% and 6.6% of variance in the Story Problems task after controlling for age in months and Baseline Trimmed RT at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with Approximate Quantity Discrimination Skills' Accuracy being a unique predictor at both time points. This suggests that Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy are distinct predictors of children's performance on Early Number Skills' tasks and that they relate differently to the different Early Number Skills. The variance predicted by the two Quantity Discrimination Skills' tasks remains fairly similar over a six-month period and over an eighteen-month period for the Counting Objects and Story Problems tasks, however, it decreases substantially when predicting performance on the Knowledge of the Number Sequence task, indicating that the longitudinal contributions made by these two skills to the different Early Number Skills vary over time.

When comparing how domain-general cognitive skills and Quantity Discrimination Skills relate to children's later Early Number Skills, results indicate that greater variance in performance on the Knowledge of the Number Sequence and Story Problems tasks is predicted by their domain-general cognitive skills than by their domain-specific cognitive skills at the end of Reception Year and at midway through Year 1. However, the Quantity Discrimination Skills predict greater variance than the domain-general cognitive skills in the Counting Objects task at the end of Reception Year and at midway through Year 1. This suggests that the three Early Number Skills draw on somewhat different cognitive skills.

8.2.1.3 The relationships between domain-general and domain-specific cognitive skills at the start of Reception Year and the Early Number Skills at the end of Reception Year and at midway through Year 1

Three additional groups of linear regression analyses were conducted to explore whether the cognitive precursors remained as unique predictors of performance once all predictor variables; domain-general cognitive skills (VSSP functioning and phonological awareness) and domain-specific cognitive skills (Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy) were entered together in the regression model (see table 8.5).

Table 8.5: Forced entry regression analyses examining the prediction of the three Early Number Skills' tasks at T_2 and at T_3 from the PQDS, AQDS, VSSP functioning and PA over and above age in months and Baseline Trimmed RT

	Knowledge of the Number Sequence (KNS)				Counting Objects (CO)				Story Problems (SP)			
	KNS T_2		KNS T_3		CO T_2		CO T_3		SP T_2		SP T_3	
Variable	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**
Step 1. Control Variables												
Age T_1 (months)	.418***		.165		.168		.106		.269**		.303***	
Baseline Trimmed RT	-.052		-.027		-.044		-.018		-.070		-.109	
R^2 by control variables		*.190****		*.031*		*.034*		*.013*		*.088***		*.123****
Step 2. QDS + DG Cog. Skills												
PQDS Trimmed RT	-.062		-.092		-.188*		-.278**		-.035		-.084	
AQDS Accuracy	.239**		.196*		.243**		.173		.209*		.167*	
VSSP functioning	.080		.120		.106		.103		.193*		.263**	
PA	.252**		.236**		.180*		.079		.207*		.245**	
Unique R^2 by QDS + DG Cog. Skills		*.137****		*.123***		*.148****		*.125***		*.127****		*.162****

***p<=.05; **p<=.01; ***p<=.001**

The ANOVA conducted on the final model predicting KNS T_2 is $F(6,120)=9.739$, $p<.001$
The ANOVA conducted on the final model predicting KNS T_3 is $F(6,118)=3.561$, $p<.01$
The ANOVA conducted on the final model predicting CO T_2 is $F(6,120)=4.451$, $p<.001$
The ANOVA conducted on the final model predicting CO T_3 is $F(6,118)=3.145$, $p<.01$
The ANOVA conducted on the final model predicting SP T_2 is $F(6,120)=5.470$, $p<.001$
The ANOVA conducted on the final model predicting SP T_3 is $F(6,118)=7.858$, $p<.001$

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 values represent the combined predictive impact of the two Quantity Discrimination Skills Tasks and the two domain-general Skills tasks while accounting for the control variables.

After controlling for age in months and Baseline Trimmed RT, the four cognitive precursors together explained 13.7% and 12.3% of variance in the Knowledge of the Number Sequence task at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with Approximate Quantity Discrimination Skills' Accuracy and phonological awareness being unique predictors at both time points. Using the same regression model the four predictors together explained 14.8% and 12.5% of variance in the Counting Objects task at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with both Quantity Discrimination Skills and phonological awareness being unique predictors at the end of Reception Year but only Precise Quantity Discrimination Skills' Trimmed RT being a unique predictor at midway through Year 1. Lastly, the two Quantity Discrimination Skills and the two domain-general cognitive skills together explained 12.7% and 16.2% of children's variance in the Story Problems task at the end of Reception Year and at midway through Year 1 respectively after controlling for age in months and Baseline Trimmed RT. The proportion of variance predicted by the regression models was statistically significant with both domain-general cognitive skills and Approximate Quantity Discrimination Skills' Accuracy being unique predictors at both time points.

Thus, all the variables that predicted unique and independent variance in children's performance on the Early Number Skills at the end of Reception Year and at midway through Year 1 in the separate analyses, remained as unique predictors of the same Early Number Skills' performance at the end of Reception Year and at midway through Year 1 when all predictors were entered together with the exception of children's Approximate Quantity Discrimination Skills' Accuracy which no longer predicts unique variance in children's performance on the Counting Objects task at midway through Year 1.

8.2.1.4 The relationships between domain-general and domain-specific cognitive skills at the start of Reception Year and the Early Number Skills at midway through Year 1 over and above General Conceptual Abilities

Correlations for age in months at Time 1 and partial correlations controlling for participants' age in months at Time 1 were conducted to explore the relationships between the four cognitive precursors (Precise Quantity Discrimination skills' Trimmed RT, Approximate

Quantity Discrimination Skills' Accuracy, VSSP functioning and phonological awareness) and children's performance on the Picture Similarities subtest and the Naming Vocabulary subtest of the BAS-II^{UK} (see table 8.6).

Table 8.6: Partial correlations for children's ability scores on the Picture Similarities (PS) subtest and on the Naming Vocabulary (NV) subtests of the BAS-II^{UK} with the two Quantity Discrimination Skills' tasks (PQDS Trimmed RT and AQDS Accuracy) and the two domain-general cognitive skills (VSSP and PA) controlling for participants' age in months at T_1 (n=128, df = 125)

Variable	***Age T_1***	**PQDS Trimmed RT**	**AQDS Accuracy**	**VSSP**	**PA**
PS BAS-II^{UK}	.078	.068	.055	.272**	.057
NV BAS-II^{UK}	-.553***	.137	.279**	.081	.193*

*p<=.05; **p<=.01; ***p<=.001

Note. Baseline Trimmed RT at time 1 was also partialled out for PQDS Trimmed RT

Age in months at Time 1 shares a significant proportion of variance with children's performance on the Naming Vocabulary subtest. After controlling for age in months at Time 1, Naming Vocabulary shares significant variance with phonological awareness. Children's performance on the Picture Similarities subtest shares significant variance with VSSP functioning. Children's performance on the Naming Vocabulary subtest also correlates significantly with Approximate Quantity Discrimination Skills' Accuracy. Thus, each of the children's General Conceptual Abilities shares significant variance with a distinct domain-general cognitive skill. However, the strength of all these correlations is weak.

A linear regression analysis for each Early Number Skill was conducted to explore whether the Quantity Discrimination Skills and the domain-general cognitive skills that were unique predictors of children's performance on the three Early Number Skills' tasks could still explain unique significant variation in performance on these tasks, over and above their General Conceptual Abilities at midway through Year 1 (see table 8.7).

Table 8.7: Forced entry regression analyses examining the prediction of the three Early Number Skills' tasks at T_3 from the PQDS, AQDS, VSSP functioning and PA over and above age in months, Baseline Trimmed RT and General Conceptual Abilities

	Knowledge of the Number Sequence T_3		**Counting Objects T_3**		**Story Problems T_3**	
Variable	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**
Step 1. Control Variables						
Age T_1 (months)	.165		.106		$.303^{***}$	
Baseline Trimmed RT	-.027		-.018		-.109	
R^2 by control variables		*.031*		*.013*		*$.123^{***}$*
Step 2. General Conceptual Abilities						
Picture Similarities Ability Scores	.002		-.026		.093	
Naming Vocabulary Ability Scores	.116		$.192^{*}$		$.196^{*}$	
R^2 by control General Conceptual Abilities		*.013*		*.036*		*$.049^{*}$*
Step 3. QDS + DG Cog. Skills						
PQDS Trimmed RT	-.090		$-.290^{**}$		-.099	
AQDS Accuracy	$.195^{*}$		.132		.139	
VSSP functioning	.126		.094		$.241^{**}$	
PA	$.237^{*}$		.052		$.223^{**}$	
Unique R^2 by QDS + DG Cog. Skills		*$.110^{**}$*		*$.108^{**}$*		*$.122^{***}$*

*p<=.05; **p<=.01; ***p<=.001

The ANOVA conducted on the final model predicting KNS T_3 is $F(8,116)=2.639$, $p=.01$

The ANOVA conducted on the final model predicting CO T_3 is $F(8,116)=2.690$, $p<.01$

The ANOVA conducted on the final model predicting SP T_3 is $F(8,116)=6.055$, $p<.001$

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 value by General Conceptual Abilities represents the combined predictive impact of the two General Conceptual Abilities tasks while accounting for the control variables. The unique R^2 value by Quantity Discrimination Skills and Domain-General Cognitive skills represents the combined predictive impact of the Quantity Discrimination Skills' tasks and the Domain-general Cognitive Skills tasks while accounting for the control variables and the General Conceptual Abilities.

The four cognitive precursors together explained 11% of the variance in the Knowledge of the Number Sequence task at midway through Year 1 over and above their age in months, Baseline Trimmed RT and General Conceptual Abilities. The proportion of variance predicted by the regression model was statistically significant with Approximate Quantity Discrimination Skills' Accuracy and phonological awareness being unique predictors. Using the same regression model both types of predictors together explained 10.8% of variance in the Counting Objects task at midway through Year 1. The proportion of variance predicted by the regression model was statistically significant with Precise Quantity Discrimination Skills' Trimmed RT being a unique predictor. Lastly, the four cognitive precursors together explained 12.2% of variance in the Story Problems task at midway through Year 1. The proportion of variance predicted by the regression model was statistically significant with phonological awareness and VSSP functioning being unique predictors after controlling for children's age in months, Baseline Trimmed RT and General Conceptual Abilities.

Thus the cognitive skills that were unique predictors of children's performance on the three Early Number Skills' tasks in the independent analyses where age or age and Baseline Trimmed RT were controlled for, remained as unique predictors of their performance on the three Early Number Skills' tasks over and above the children's age in months, their Baseline Trimmed RT and their General Conceptual Abilities over an eighteen-month period. The additional variance that domain-general and domain-specific cognitive skills together explained on the three Early Number Skills' tasks at midway through Year 1 is substantially smaller when General Conceptual Abilities are controlled for.

8.2.2 The relationships between cognitive precursors and the standardised attainment measures

8.2.2.1 The relationships between domain-general cognitive skills at the start of Reception Year with the two mathematical attainment measures at the end of Reception Year and at midway through Year 1

Partial correlations controlling for participants' age in months at Time 1 were conducted to explore the relationships between children's VSSP functioning and phonological awareness with their performance on the two mathematical attainment measures at the end of Reception Year and at midway through Year 1 (see table 8.8).

Table 8.8: Partial correlations for the Precise Quantity Discrimination Skills (PQDS) Trimmed RT and Approximate Quantity Discrimination Skills (AQDS) Accuracy at T_1 with the three Early Number Skills' tasks and the two standardised mathematical attainment measures at T_2 and T_3 controlling for participants' age in months at T_1 (n=128, df = 125 for T_2; n=126, df = 121 for T_3)

	Mathematical Attainment T_2		**Mathematical Attainment T_3**	
Variable	**MRWIAT-II^{UK} T_2**	**NOWIAT-II^{UK} T_2**	**MR WIAT-II^{UK} T_3**	**NO WIAT-II^{UK} T_3**
VSSP	.253**	.214*	.271**	.202*
PA	.232**	.225*	.183*	.006

*p<=.05; **p<=.01

At Time 2, children's domain-general cognitive skills share significant variance with their performance on both mathematical attainment measures. At Time 3, children's VSSP functioning shares significant variance with their performance on both mathematical attainment measures, but children's phonological awareness only shares significant variance with their performance on the Mathematical Reasoning subtest of the WIAT-II^{UK}. The strength of all these correlations is weak or very weak.

Five different linear regression analyses were conducted to explore whether children's domain-general cognitive skills could explain significant unique variance in their performance on each of the mathematical attainment measures at the start of Reception Year and midway through Year 1 (see table 8.9).

Table 8.9: Forced entry regression analyses examining the prediction of MR WIAT-II^{UK} and NO WIAT-II^{UK} at T_2 and at T_3 from VSSP functioning and PA over and above age in months

	Mathematical Reasoning (MR) WIAT-II^{UK}				Numerical Operations (NO) WIAT-II^{UK}			
	MR WIAT-II^{UK} T_2		**MR WIAT-II^{UK} T_3**		**NO WIAT-II^{UK} T_2**		**NO WIAT-II^{UK} T_3**	
Variable	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**
Step 1.Control Variable								
Age T_1 (months)	.354***		.296***		.280***		.372***	
R^2 by control variable		*.126****		*.088****		*.078****		*.138****
Step 2. D-G Cog. Skills								
VSSP functioning	.277***		.297***		.243**		.203*	
PA	.248**		.206*		.244**		.024	
Unique R^2 by D-G Cog. Skills		*.114***		*.107****		*.098****		*.036*

***p<=.05; **p<=.01; ***p<=.001**

The ANOVA conducted on the final model predicting MR WIAT-II^{UK} T_2 is $F(3,124)=12.998$, $p<.001$
The ANOVA conducted on the final model predicting MR WIAT-II^{UK} T_3 is $F(3,122)=9.829$, $p<.001$
The ANOVA conducted on the final model predicting NO WIAT-II^{UK} T_2 is $F(3,124)=8.844$, $p<.001$
The ANOVA conducted on the final model predicting NO WIAT-II^{UK} T_3 is $F(3,122)=8.562$, $p<.001$
Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 values represent the combined predictive impact of the two domain-general Cognitive Skills while accounting for the control variable.

Over and above children's age in months, the two domain-general cognitive skills together explained 11.4% and 10.7 % of variance in the Mathematical Reasoning subtest at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with both domain-general cognitive skills being unique predictors at both time points. Using the same regression model both domain-general cognitive skills together explained 9.8% and 3.6% of variance in the Numerical Operations subtest at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression model at the end of Reception Year was statistically significant with both domain-general cognitive skills being unique predictors at the end of Reception Year but only VSSP functioning being a unique predictor at midway through Year 1. This pattern of findings suggests that the longitudinal contributions which phonological awareness and VSSP functioning make to children's performance on mathematical attainment measures differ in relation to the characteristics of the particular mathematical attainment measure. Whilst the impact of the domain-general cognitive skills on children's performance on the Mathematical Reasoning subtest remains broadly similar over a six-month period and over an eighteen-month period, the impact of the domain-general cognitive skills on children's performance on the Numerical Operations subtest decreases substantially over the same time period, suggesting that the unique and independent longitudinal contributions that the domain-general cognitive skills make to children's mathematical attainment vary over time.

8.2.2.2 The relationships between domain-specific cognitive skills at the start of Reception Year and the two mathematical attainment measures at the end of Reception Year and at midway through Year 1

Partial correlations controlling for participants' age in months at Time 1 were conducted to explore the relationships between children's Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy with the two mathematical attainment measures at the end of Reception Year and at midway through Year 1 (see table 8.10).

Table 8.10: Partial correlations for Precise Quantity Discrimination Skills' (PQDS) Trimmed RT and Approximate Quantity Discrimination Skills' (AQDS) Accuracy with the two standardised mathematical attainment measures at T_2 and at T_3 controlling for participants' age in months at T_1 (n=128, df = 123 for T_2; n=126, df=121 for T_3)

	Maths Attainment T_2		**Maths Attainment T_3**	
Variable	**MR WIAT-II^{UK} T_2**	**NO WIAT-II^{UK} T_2**	**MR WIAT-II^{UK} T_3**	**NO WIAT-II^{UK} T_3**
PQDS Trimmed RT T_1	-.186*	-.199*	-.092	-.193
AQDS Accuracy T_1	.263**	.286***	.220*	.193*

*p<=.05; **p<=.01; ***p<=.001
Note. Baseline Trimmed RT at T_1 was also partialled out for PQDS Trimmed RT

At Time 2, children's Quantity Discrimination Skills share significant variance with their performance on both of the mathematical attainment measures after controlling for age in months and Baseline Trimmed RT at Time 1. However, Approximate Quantity Discrimination Skills' Accuracy only shares significant variance with children's scores on both mathematical attainment measures at Time 3 under the same conditions. The strength of these correlations is weak or very weak.

Five different linear regression analyses were conducted to explore whether children's Quantity Discrimination Skills could explain unique variance in their performance on the mathematical attainment measures at the end of Reception Year and at midway through Year 1 (see table 8.11).

Table 8.11: Forced entry regression analyses examining the prediction of MR WIAT-II^{UK} and NO WIAT-II^{UK} at T_2 and at T_3 from PQDS and AQDS over and above age in months and Baseline Trimmed RT

	Mathematical Reasoning (MR) WIAT-II^{UK}				**Numerical Operations (NO) WIAT-II^{UK}**			
	MR WIAT-II^{UK} T_2		**MR WIAT-II^{UK} T_3**		**NO WIAT-II^{UK} T_2**		**NO WIAT-II^{UK} T_3**	
Variable	**B**	**R²**	**B**	**R²**	**B**	**R²**	**B**	**R²**
Step 1.Control Variables								
Age T_1 (months)	291***		.259**		.230*		.382***	
Baseline Trimmed RT	-.218*		-.124		-.172		.036	
R² by control variables		*.169****		*.102****		*.105****		*.139****
Step 2. QDS								
PQDS Trimmed RT	-.171		-.080		-.190*		-.186*	
AQDS Accuracy	.227**		.197*		.259**		.171*	
Unique R² by QDS		*.075***		*.043*		*.096****		*.058**

*p<=.05; **p<=.01; ***p<=.001

The ANOVA conducted on the final model predicting MR WIAT-II^{UK} T_2 is $F(4,122)=9.837$, $p<.001$

The ANOVA conducted on the final model predicting MR WIAT-II^{UK} T_3 is $F(4,120)=5.055$, $p=.001$

The ANOVA conducted on the final model predicting NO WIAT-II^{UK} T_2 is $F(4,122)=7.687$, $p<.001$

The ANOVA conducted on the final model predicting NO WIAT-II^{UK} T_3 is $F(4,120)=7.380$, $p<.001$

Note. B Stands for the standardised regression coefficient; R² stands for the proportion of variance explained. The unique R² values represent the combined predictive impact of the two Quantity Discrimination Skills while accounting for the control variables.

After controlling for age in months and Baseline Trimmed RT, the two Quantity Discrimination Skills together explained 7.5% and 4.3 % of variance in the Mathematical Reasoning subtest at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression model at the end of Reception Year was statistically significant with Approximate Quantity Discrimination Skills' Accuracy being a unique predictor. Using the same regression model both Quantity Discrimination Skills together also explained 9.6% and 5.8% of variance in the Numerical Operations subtest at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with both Quantity Discrimination Skills being unique predictors at both time points. This pattern of findings suggests that Precise Quantity Discrimination Skills' Trimmed RT and Approximate Quantity Discrimination Skills' Accuracy relate differently to different aspects of mathematical attainment in young children. The longitudinal contributions that the Quantity Discrimination Skills make to children's performance on both standardised mathematical attainment measures decreases substantially from a six-month period to an eighteen-month period.

When comparing the separate analyses exploring how domain-general cognitive skills and Quantity Discrimination Skills relate to children's performance on standardised attainment measures, domain-general cognitive skills predict greater variance in performance in the Mathematical Reasoning subtest over a six-month period and over an eighteen-month period than domain-specific cognitive skills. Quantity Discrimination Skills together predict almost the same amount of variance in performance on the Numerical Operation subtest as domain-general cognitive skills at the end of Reception Year, however, at midway through Year 1 Quantity Discrimination Skills predict greater variance than the domain-general cognitive skills on this test. These results seem to be consistent with the idea that the implication of different cognitive skills might vary for different standardised mathematical attainment measures that differ in the tasks' characteristics (Dehaene, 1992; Dehaene & Cohen, 1995; Dehaene et al., 2003; LeFevre et al., 2010).

8.2.2.3 The relationships between domain-general and domain-specific cognitive skills at the start of Reception Year and the two mathematical attainment measures at the end of Reception Year and at midway through Year 1 and with the reading attainment measure at midway through Year 1

Five additional linear regression analyses were conducted to explore whether the four cognitive precursors could explain children's mathematical attainment at the end of Reception Year and at midway through Year 1 and reading attainment at midway through Year 1 (see table 8.12).

Table 8.12: Forced entry regression analyses examining the prediction of the MR WIAT-II^{UK} and the NO WIAT-II^{UK} at T_2 and at T_3 and of the YARC reading at T_3 from the PQDS, AQDS, VSSP functioning and PA over and above age in months and Baseline Trimmed RT

	Mathematical Reasoning (MR) WIAT-II^{UK}				**Numerical Operations (NO) WIAT-II^{UK}**				**Reading**	
	MR WIAT-II^{UK} T_2		**MR WIAT-II^{UK} T_3**		**NO WIAT-II^{UK} T_2**		**NO WIAT-II^{UK} T_3**		**YARC T_3**	
Variable	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**
Step 1.Control Variables										
Age T_1 (months)	.291***		.259**		.230*		.382***		.245**	
Baseline Trimmed RT	-.218*		-.124		-.172		.036		-.046	
R^2 by control variable		*.169****		*.102****		*.105****		*.139****		*.069**
Step 2. QDS + D-G Cog. Skills										
PQDS Trimmed RT	-.129		-.034		-.153		-.165		-.182	
AQDS Accuracy	.180*		.147		.217*		.152		.172	
VSSP functioning	.213*		.256**		.174*		.157		.126	
PA	.186*		.165		.183*		-.006		.203*	
Unique R^2 by QDS + DG Cog. Skills		*.134****		*.111***		*.144****		*.079**		*.127***

***p<=.05; **p<=.01; ***p<=.001**

The ANOVA conducted on the final model predicting MR WIAT-II^{UK} T_2 is $F(6,120)=8.697$, $p<.001$
The ANOVA conducted on the final model predicting MR WIAT-II^{UK} T_3 is $F(6,118)=5.325$, $p<.001$
The ANOVA conducted on the final model predicting NO WIAT-II^{UK} T_2 is $F(6,120)=6.625$, $p<.001$
The ANOVA conducted on the final model predicting NO WIAT-II^{UK} T_3 is $F(6,118)=5.491$, $p<.001$
The ANOVA conducted on the final model predicting YARC T_3 is $F(4,120)=5.346$, $p=.001$
Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 values represent the combined predictive impact of the two Quantity Discrimination Skills Tasks and the two Domain-general Skills tasks while accounting for the control variables

After controlling for age in months and Baseline Trimmed RT the four cognitive precursors together explained 13.4% and 11.1% of variance in the Mathematical Reasoning subtest at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with Approximate Quantity Discrimination Skills' Accuracy and the two domain-general cognitive skills being unique predictors at the end of Reception Year and only VSSP functioning being a unique predictor at midway through Year 1. Using the same regression model the four cognitive precursors together explained 14.4% and 7.9% of variance in the Numerical Operations subtest at the end of Reception Year and at midway through Year 1 respectively. The proportion of variance predicted by the regression models was statistically significant with both domain-general cognitive skills and Precise Quantity Discrimination Skills' Trimmed RT being unique predictors at the end of Reception Year and no cognitive precursors are unique predictors at midway through Year 1. Lastly, the two Quantity Discrimination Skills and the two domain-general cognitive skills together explained 12.7% of reading attainment at midway through Year 1 after controlling for age in months and Baseline Trimmed RT. The proportion of variance predicted by this regression model was statistically significant with phonological awareness being a unique predictor.

Thus the variance predicted by many of the individual cognitive precursors in the separate analyses does not remain statistically unique when all variables are entered together in the regression models. Phonological Awareness is no longer a unique predictor of Mathematical Reasoning performance at midway through Year 1 and VSSP functioning is no longer a unique predictor of Numerical Operations performance at midway through Year 1. Children's Approximate Quantity Discrimination Skills' Accuracy is no longer a unique predictor of performance on any standardised mathematical attainment measures at midway through Year 1, and Precise Quantity Discrimination Skills' Trimmed RT is no longer a unique predictor of performance on the Numerical Operations subtest. Regarding children's reading attainment, only phonological awareness remains as a unique predictor of performance. Thus domain-specific and domain-general cognitive skills seem to be distinct and unique predictors of children's performance on mathematical attainment measures that differ in tasks' characteristics, however their independent and unique longitudinal contributions vary from a six-month period to an eighteen-month period. Phonological awareness predicts unique significant variance in children's reading attainment at midway through Year 1 when all variables are entered together in the regression models. Results from these three groups of regression analyses suggest that while children's domain-general cognitive skills predict their mathematical and reading attainment, their Quantity Discrimination Skills only predict their mathematical attainment.

8.2.2.4 The relationships between domain-general and domain-specific cognitive skills at the start of Reception Year with the two mathematical attainment measures at the end of Reception Year and at midway through Year 1 and with the reading attainment measure at midway through Year 1 over and above General Conceptual Abilities

A linear regression analysis for the Mathematical Reasoning subtest, the Numerical Operations subtest and the Single-word Reading subtest was conducted to explore whether the Quantity Discrimination Skills and the domain-general cognitive skills that were unique predictors of children's performance on these three attainment measures could still explain unique significant variation in performance over and above their General Conceptual Abilities at midway through Year 1 (see table 8.13).

Table 8.13: Forced entry regression analyses examining the prediction of the three attainment measures at T_3 from the PQDS, AQDS, VSSP functioning and PA over and above age in months and Baseline Trimmed RT and General Conceptual Abilities

	Mathematical Reasoning WIAT-II^{UK} T_3		**Numerical Operations WIAT-II^{UK} T_3**		**Reading YARC T_3**	
Variable	**B**	**R^2**	**B**	**R^2**	**B**	**R^2**
Step 1. Control Variables						
Age T_1 (months)	.259**		.382***		.245**	
Baseline Trimmed RT	-.124		.036		-.046	
R^2 by control variables		*.102****		*.139****		*.069**
Step2. General Conceptual Abilities						
Picture Similarities Ability Scores	.100		-.005		-.128	
Naming Vocabulary Ability Scores	.330***		.146		.210*	
R^2 by control General Conceptual Abilities		*.122****		*.021*		*.056**
Step 3. QDS + D-G Cog. Skills						
PQDS Trimmed RT	-.070		-.174		-.178	
AQDS Accuracy	.068		.121		.138	
VSSP functioning	.208*		.151		.152	
PA	.105		-.026		.189*	
Unique R^2 by QDS + D-G Cog. Skills		*.056*		*.069**		*.106***

***p<=.05; **p<=.01; ***p<=.001**

The ANOVA conducted on the final model predicting MR WIAT-II^{UK} T_3 is $F(8,116)=5.638$, $p<.001$

The ANOVA conducted on the final model predicting NO WIAT-II^{UK} T_3 is $F(8,116)=4.305$, $p<.001$

The ANOVA conducted on the final model predicting YARC T_3 is $F(8,116)=4.340$, $p<.001$

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 value by General Conceptual Abilities represents the combined predictive impact of the two General Conceptual Abilities Tasks while accounting for the control variables. The unique R^2 value by Quantity Discrimination Skills and Domain-General Cognitive skills represents the combined predictive impact of the Quantity Discrimination Skills tasks and the Domain-general Cognitive Skills tasks while accounting for the control variables and the General Conceptual Abilities.

The four cognitive precursors together explained 5.6% of variance in the Mathematical Reasoning subtest over and above age in months, Baseline Trimmed RT and General Conceptual Abilities. Only VSSP functioning remains as a unique predictor but the proportion of variance predicted by the regression model is not statistically significant. Using the same regression model both types of precursors together explained 6.9% of variance in the Numerical Operations subtest. The proportion of variance predicted by the regression model is statistically significant although none of cognitive precursors remains as a unique predictor. Lastly, the four cognitive precursors together explained 10.6% of variance in the Word-reading subtest over and above their age in months, Baseline Trimmed RT and General Conceptual Abilities. The proportion of variance predicted by the regression model is statistically significant with only phonological awareness being a unique predictor.

The cognitive precursors that predicted unique variance in children's performance on the attainment measures at midway through Year 1 remained as unique predictors of attainment when their impact was explored over and above children's General Conceptual Abilities. However, the additional variation predicted by the four cognitive precursors in the mathematical attainment measures models after controlling for children's age in months, Baseline Trimmed RT and after the inclusion of children's General Conceptual Abilities is only statistically significant when predicting performance on the Numerical Operations subtest. The additional proportion of variance predicted by the four cognitive precursors in the reading attainment model after controlling for children's age in months, Baseline Trimmed RT and after the inclusion of children's General Conceptual Abilities remains statistically significant, although the additional variation predicted is smaller than when General Conceptual Abilities were not controlled for.

8.3 Exploring the causality of the relationships found between the cognitive precursors and the outcome measures

8.3.1 Early Number Skills growth from the start of Reception Year to midway through Year 1 (eighteen-month period)

Regression analyses were conducted to examine whether the cognitive precursors that explained performance on the Early Number Skills at midway through Year 1 could also predict *growth* on these variables over this eighteen-month period. A regression analysis for each Early Number Skill was conducted.

The first analysis aimed to determine whether children's Approximate Quantity Discrimination Skills' Accuracy and phonological awareness can predict growth on the Knowledge of the Number Sequence from the start of Reception Year to midway through Year 1. Age in months was controlled for in the first step of the regression and children's performance on the Knowledge of the Number Sequence task at the start of Reception Year was controlled for in the second step of the regression. Children's Approximate Quantity Discrimination skills' Accuracy and phonological awareness at the start of Reception Year were entered in the third step of the regression to explore whether these cognitive predictors could explain Knowledge of the Number Sequence growth (see table 8.14).

Table 8.14: Forced entry regression analyses examining the prediction of Knowledge of the Number Sequence growth from T_1 to T_3 from Approximate Quantity Discrimination skills (AQDS) Accuracy and phonological awareness (PA) T_1 over and above age in months

	Knowledge of the Number Sequence T_3	
Variable	**B**	**R^2**
Step 1. 1 $F(1,124)=3.858$, $p=.052$		
Age in months T_1	.174	
R^2 by control variable		*.030*
Step 2. $F(2,123)=10.947$, $p<.001$		
Knowledge of the Number Sequence T_1	$.383^{***}$	
R^2 by Early Number Skill T_1		*$.121^{***}$*
Step 3. $F(4,121)=7.532$, $p<.001$		
AQDS Accuracy	$.177^{*}$	
PA	.150	
Unique R^2 by Cognitive Predictors T_1		*$.048^{*}$*

$^{*}p<=.05$; $^{**}p<=.01$; $^{***}p<=.001$

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 value by the early number skill represents the predictive impact of the Early Number Skill at T_1 while accounting for the control variable. The unique R^2 value by the cognitive predictors represents the predictive impact of the cognitive predictors at T_1 while accounting for the control variable and performance on the Early Number Skill at T_1.

Children's performance on the Knowledge of the Number Sequence task at the start of Reception Year predicts 12.1% of their performance on the same task at midway through Year 1, over and above their age in months at the start of Reception Year. Over and above age in months and performance on the Knowledge of the Number Sequence task at the start of Reception Year, Approximate Quantity Discrimination Skills' Accuracy and phonological awareness together predict an additional 4.8% of the variance in the Knowledge of the Number Sequence task. The proportion of variance predicted by the regression model was statistically significant with Approximate Quantity Discrimination Skills' Accuracy being a unique predictor.

The second analysis aimed to determine whether children's Precise Quantity Discrimination Skills' Trimmed RT can predict growth on the Counting Objects task from the start of Reception Year to midway through Year 1. Age in months and Baseline Trimmed RT at the start of Reception Year were controlled for in the first step of the regression and children's performance on the Counting Objects task at the start of Reception Year was controlled for in the second step of the regression. Children's Precise Quantity Discrimination Skills' Trimmed RT at the start of Reception Year was entered in the third step of the regression to explore whether this cognitive predictor could explain Counting Objects growth (see table 8.15).

Table 8.15: Forced entry regression analyses examining the prediction of Counting Objects growth from T_1 to T_3 from Precise Quantity Discrimination skills (PQDS) Trimmed RT at T_1 over and above age in months and Baseline Trimmed RT at T_1

	Counting Objects T_3	
Variable	**B**	**R^2**
Step 1. $F(2,122)=.779$, $p=.461$		
Age in months T_1	.106	
Baseline Trimmed RT T_1	-.018	
R^2 by control variables		*.013*
Step 2. $F(3,124)=16.861$, $p<.001$		
Counting Objects T_1	$.552^{***}$	
R^2 by Early Number Skill T_1		*$.282^{***}$*
Step 3. $F(4,120)=14.083$, $p<.001$		
PQDS Trimmed RT T_1	$-.182^{*}$	
Unique R^2 by Cognitive Predictor T_1		*$.025^{*}$*

$^{*}p<=.05$; $^{**}p<=.01$; $^{***}p<=.001$

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 value by the early number skill represents the predictive impact of the Early Number Skill at T_1 while accounting for the control variables. The unique R^2 value by the cognitive predictor represents the predictive impact of the cognitive predictors at T_1 while accounting for the control variables and performance on the Early Number Skill at T_1.

Children's performance on the Counting Objects task at the start Reception Year over and above their age in months and Baseline Trimmed RT at the start of Reception Year predicts 28.2% of their performance on the same task at midway through Year 1. Precise Quantity Discrimination Skills' Trimmed RT is a unique predictor of performance on this task after the inclusion of the Counting Objects task at the start of Reception Year in the model. Precise Quantity Discrimination Skills' Trimmed RT predicts 2.5% of the variance in the Counting Objects task *growth* from the start of Reception Year to midway through Year 1. The proportion of variance predicted was statistically significant.

The third analysis aimed to determine whether children's phonological awareness and VSSP functioning can predict growth on the Story Problems task from the start of Reception Year to midway through Year 1. Age in months was controlled for in the first step of the regression and children's performance on the Story Problems task at the start of Reception Year was controlled for in the second step of the regression. Children's VSSP functioning and phonological awareness at the start of Reception Year were entered in the third step of the regression to explore whether these cognitive predictors could explain Story Problems growth (see table 8.16).

Table 8.16: Forced entry regression analyses examining the prediction of Story Problems growth from T_1 to T_3 from VSSP functioning and PA at T_1 over and above age in months

	Story Problems T_3	
Variable	**B**	**R²**
Step 1. F(1,124)=15.629, *p*<.001		
Age in months T_1	.335***	
R² by control variables		*.112****
Step 2. F(2,123)=10.812, *p*<.001		
Story Problems T_1	.197*	
R² by Early Number Skill T_1		*.038**
Step 3. F(4,121)=10.683, *p*<.001		
VSSP functioning	.289***	
PA	.241**	
Unique R² by Cognitive Predictors T_1		*.111****

***p<=.05; **p<=.01; ***p<=.001**

Note. B Stands for the standardised regression coefficient; R² stands for the proportion of variance explained. The unique R² value by the early number skill represents the predictive impact of the Early Number Skill at T_1 while accounting for the control variable. The unique R² value by the cognitive predictors represents the predictive impact of the cognitive predictors at T_1 while accounting for the control variable and performance on the Early Number Skill at T_1

Children's performance on the Story Problems task at the start of Reception Year predicts 3.8% of their performance on the same task at midway through Year 1 over and above their age in months at the start of Reception Year. Both domain-general cognitive skills are unique predictors of performance on this task after the inclusion of the Story Problems task at the start of Reception Year in the model. Together they predict 11.1% of the Story Problems task *growth* from the start of Reception Year to midway through Year 1. The proportion of variance predicted by the regression model was statistically significant.

8.3.2 Mathematical attainment growth from the start of Reception Year to the end of Reception Year (six-month period)

Regression analyses were conducted to determine whether the domain-specific cognitive skills and the domain-general cognitive skills that could explain performance on the mathematical attainment measures at the end of Reception Year could also predict *growth* on these outcome measures. These additional analyses were conducted for ease of comparison with a recent study where pre-schooler's approximate quantity discrimination skills were found to predict mathematical attainment growth over a six-month period (Libertus et al., 2013). All cognitive precursors that explained unique variance in performance on the mathematical attainment measures at the end of Reception Year were entered together in the model to see whether they could also predict unique *growth* in each of the mathematical attainment measures that could not be explained by the other cognitive predictors (see table 8.17)

Table 8.17: Forced entry regression analyses examining the prediction of Mathematical Reasoning growth from T_1 to T_2 from Approximate Quantity Discrimination Skills' task (AQDS), VSSP functioning (VSSP) and Phonological Awareness (PA) at T_1 over and above age in months

	MR WIAT-II^{UK} T_2		**NO WIAT-II^{UK} T_2**	
Variable	**B**	**R^2**	**B**	**R^2**
Step 1. F(1,124)=11.924, *p*=.001				
Age in months T_1	.354***		.280***	
R^2 by control variable		.126***		.078***
Step 2. F(2,123)=28.824, *p*<.001				
MR WIAT-II^{UK}/NO WIAT-II^{UK} T_1	.565***		.545***	
R^2 by Mathematical attainment T_1		.288***		.257***
Step 3. F(3,122)=21.516, *p*<.001				
AQDS Accuracy	.122		.132	
VSSP functioning	.144		.065	
PA	.103		.089	
Unique R^2 by QDS + D-G Cog. Skills		.041*		.025

***p<=.05; **p<=.01; ***p<=.001**

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 value by the mathematical attainment measure represents the predictive impact of the mathematical attainment measure at T_1 while accounting for the control variable. The unique R^2 value by the cognitive predictor represents the predictive impact of the cognitive predictor at T_1 while accounting for the control variable and performance on the mathematical attainment measure at T_1.

Approximate Quantity Discrimination skills' Accuracy, VSSP functioning and phonological awareness together predict 4.1% of the Mathematical Reasoning *growth* from the start to the end of Reception Year. The proportion of variance predicted was statistically significant although none of the cognitive precursors are unique predictors. Approximate Quantity Discrimination skills' Accuracy and both domain-general cognitive skills together predict 2.5% of the Numerical Operations *growth* from the start to the end of Reception Year. The proportion of variance predicted was not statistically significant and none of the cognitive precursors are unique predictors.

8.3.3 Mathematical attainment growth from the start of Reception Year to midway through Year 1 (eighteen-month period)

Finally, a regression analysis was conducted to determine whether the domain-general cognitive skill (VSSP functioning) that explained performance on the Mathematical Reasoning subtest at midway through Year 1 could also predict *growth* on this variable over this eighteen-month period. Age in months was controlled for in the first step of the regression and children's performance on the Mathematical Reasoning subtest at the start of Reception Year was controlled for in the second step of the regression. Children's VSSP functioning at the start of Reception Year was entered in the third step of the regression to explore whether this cognitive precursor could explain Mathematical Reasoning growth (see table 8.18).

Table 8.18: Forced entry regression analyses examining the prediction of Mathematical Reasoning growth from T_1 to T_3 from VSSP functioning at T_1 over and above age in months

	Mathematical Reasoning WIAT-II^{UK} T_3	
Variable	**B**	**R^2**
Step 1. Control Variable		
Age in months T_1	.335***	
R^2 by control variables		*.088****
Step 2. Mathematical Attainment T_1		
Mathematical Reasoning WIAT-II^{UK} T_1	.197*	
R^2 by Mathematical Attainment T_1		*.231****
Step 3. Cognitive predictor T_1		
VSSP functioning	.289***	
Unique R^2 by Cognitive Predictor T_1		*.027**

$^{*}p<=.05$; $^{**}p<=.01$; $^{***}p<=.001$

Note. B Stands for the standardised regression coefficient; R^2 stands for the proportion of variance explained. The unique R^2 value by the Mathematical Attainment represents the predictive impact of the Mathematical Attainment measure at T_1 while accounting for the control variable. The unique R^2 value by the cognitive predictor represents the predictive impact of the cognitive predictor at T_1 while accounting for the control variable and performance on the mathematical attainment measure at T_1.

Children's performance on Mathematical Reasoning at the start of Reception Year predicts 23.1% of their performance on the same measure at midway through Year 1 over and above their age in months at the start of Reception Year. VSSP functioning is a unique predictor of performance on this measure after the inclusion of Mathematical Reasoning performance at the start of Reception Year in the model. VSSP functioning predicts 2.7% of Mathematical Reasoning *growth* from the start of Reception Year to midway through Year 1. The proportion of variance predicted was statistically significant with VSSP functioning being a unique predictor. Thus VSSP functioning seem to act as causal predictor of children's mathematical reasoning over this period.

8.4 DISCUSSION

Results show that both domain-specific cognitive skills and both domain-general cognitive skills included in the study make independent and unique contributions to children's performance on the Early Number Skills' tasks and to their performance on the different mathematical attainment measures. Results also indicate that the longitudinal contributions that these four cognitive precursors make to the different Early Number Skills are consistent from a six-month period to an eighteen-month period. All four cognitive precursors are causal predictors of Early Number Skills. The longitudinal contributions that the four cognitive precursors make to the different mathematical attainment measures vary from a six-month period to an eighteen-month period and only VSSP functioning predicts growth in Mathematical Reasoning over an eighteen-month period. In addition, the Quantity Discrimination Skills predict the variables of interest but not of reading attainment, supporting the domain-specificity of these quantitative skills.

8.4.1 The relationships between cognitive precursors and Early Number Skills

Regarding the domain-general cognitive skills, this study provides additional evidence that children's phonological awareness and VSSP functioning relate differently to different early number skills that make distinct cognitive demands (Krajewski & Schneider, 2009; LeFevre et al., 2010). Children's phonological awareness makes independent and unique longitudinal contributions to children's performance on the Knowledge of the Number Sequence task over a six-month period and over an eighteen-month period. Phonological awareness and VSSP functioning make independent and unique longitudinal contributions to children's performance on the Story Problems task over a six-month period and over an eighteen-month period. However, only children's phonological awareness was a unique predictor of their performance on the Counting Objects task and only over a six-month period. These results remained comparable when all the cognitive precursors were entered together in the regression model and even when General Conceptual Abilities were controlled for, highlighting the solidity of these findings. Thus the present results align only partially with Krajewski and Schneider's (2009) theoretical model and study results. The implication of phonological skills in numerical tasks that demand processing verbal codes has been theoretically proposed (Dehaene et al., 2003; Simmons & Singleton, 2008) and empirically demonstrated in young children (Hecht et al., 2001; Krajewski & Schneider, 2009; Leather & Henry, 1994; LeFevre et al., 2010). The Knowledge of the Number Sequence task assesses children's ability to recite the number-words in the correct order and does not require any kind of quantity representation or manipulation, therefore it is not surprising that only children's phonological awareness and not VSSP functioning was a unique predictor of performance for this task. VSSP functioning predicted performance on the Story Problems task, providing additional evidence of its implication in numerical tasks that demand quantity manipulations (Krajewski & Schneider, 2009; Rasmussen & Bisanz, 2005).

However, and in contrast to Krajewski and Schneider's (2009) theoretical model predictions and study results, in the current study phonological awareness also makes independent and unique contributions to children's performance on Early Number Skills' tasks representing QNCs Level III. This disparity could be due to the fact that verbal skills are directly involved in the Story Problems tasks. The Story problems task is verbally presented and demand a verbal response from the child. The two tasks representing QNCs Level III in Krajewski and Schneider's (2009) study were also verbally presented, one presumably required a verbal response (determine the numerical difference between three pairs of dot arrangements) but the other did not (solve word-problems with concrete materials). These differences in tasks' response method could be the reason why in the present study phonological awareness makes independent and unique contributions to children's performance on the Story Problems task while in Krajewski and Schneider's (2009) it only predicts performance on QNCs Level I tasks, where all task demanded verbal responses. Therefore, although the Story Problems tasks of the Early Number Skills assessment might also make additional cognitive demands, it seems reasonable that phonological awareness makes independent and unique contributions to children's performance on this task. Also in contrast to Krajewski and Schneider's (2009) theoretical model and study results, VSSP functioning did not significantly predict children's performance on the Counting Objects task designed to represent Krajewski and Schneider's QNCs Level IIb, which they propose should be predicted by children's VSSP functioning and quantitative skills. A plausible explanation could be that, although in this task children were presented with non-symbolic quantities, they did not necessarily represent them nor retain them visually. Instead, children could have been assigning a verbal label (number-word) to each item counted and reporting the final number-word assigned to the last item counted. This could mean that they were making greater demands on their verbal skills and requiring little or no VSSP functioning skills. Nevertheless, performance on the Counting Objects task is not significantly predicted by either of the domain-general cognitive skills at midway through Year 1.

Results of the current study provide preliminary evidence that the two Quantity Discrimination Skills' tasks included in the study relate differently to each of the Early Number Skills. On the face of these findings both theoretical views regarding the foundational role of quantity discrimination skills are supported; the one proposing that children's precise quantity discrimination skills support their later verbal numerical skills and basic arithmetic skills (Butterworth, 1999, 2005, 2010) and the one proposing that children's approximate discrimination skills support their later acquisition of early number skills (Condry & Spelke, 2008; Dehaene, 1997; Dehaene et al., 1998; Dehaene et al., 2004). However, whilst Precise Quantity Discrimination Skills' Trimmed RT predicted unique and independent variance in children's performance on the Counting Objects task, Approximate Quantity Discrimination Skills' Accuracy predicted unique and independent variance in Knowledge of the Number Sequence, even when the independent contribution of the four cognitive precursors was explored simultaneously, over a six-month period and over an eighteen-month period, and even and over and above General Conceptual Abilities. This pattern suggests that precise and approximate non-symbolic quantity discrimination skills play fundamental and distinct roles on early number skills' development. These findings are contradictory to Krajewski and Schneider's (2009) model predictions because they suggest that rote recitation of the number-word sequence evolves in isolation of quantity discrimination skills. The present findings challenge this theoretical view because children's ability to recite the number-word sequence was predicted by their Approximate Quantity Discrimination skills' Accuracy. Early recitation of the number-word sequence seems to not only be supported by phonological awareness abilities but also by quantity discrimination skills. These results align with those from Mussolin et al. (2012) where young children's accuracy in a non-symbolic approximate quantity discrimination task predicted their performance on a number sequence reciting task. Nevertheless, children's Precise Quantity Discrimination skills' Trimmed RT predicted independent and unique variance in performance on the Counting Objects task which is in line with Krajewski and Schneider model's (2009) predictions that quantity discrimination skills contribute to QNCs Level II.

8.4.2 The relationships between cognitive precursors and mathematical attainment

The four cognitive precursors relate differently to the two mathematical attainment measures included in the study supporting the idea that the relative contribution of these cognitive skills varies in relation to each number task's format, demands and response method (Dehaene et al., 2003; Krajewski & Schneider, 2009; LeFevre et al., 2010; Simmons & Singleton, 2008; Simmons et al., 2012). Thus the mathematical aspects that these two mathematical attainment measures tap should be treated as different mathematical attainment constructs in young children (Nunes et al., 2012).

The present study hypothesised that children's phonological awareness would contribute to children's performance on the Mathematical Reasoning subtest because verbal skills are particularly involved. When the independent contribution that each of the four cognitive precursors make to the mathematical attainment measures was explored simultaneously, phonological awareness was no longer a unique predictor of children's performance on any of the mathematical attainment measures over an eighteen-month period. It has been suggested that learning and assigning verbal labels to visual symbols such as the Arabic numerals is similar to developing lexical or sub-lexical mappings when children are learning to read (LeFevre et al., 2010). This could explain why phonological awareness makes unique contributions to this test over a short period. Children at the end of Reception Year could be still undergoing this mapping learning process, whilst by midway through Year 1 this mapping may be already mastered. It was also expected that children's VSSP functioning would contribute to their performance on the Numerical Operations subtest, because in this test calculation and writing skills are particularly involved and these abilities seem to rely heavily on VSSP functioning (Huttenlocher et al., 1994; Krajewski & Schneider, 2009; Rasmussen & Bisanz, 2005). It was also found that VSSP functioning was no longer a unique predictor of children's performance on the Numerical Operation subtest over an eighteen-month period. It could be that a shift in cognitive strategies occurs in the transition from Reception Year to Year 1 and other cognitive skills not included in the present study are better predictors of children's performance on standardised mathematical attainment measures by midway through Year 1. For instance McKenzie et al. (2003) and Rasmussen and Bisanz (2005) found that central executive functioning together with VSSP functioning contribute to children's arithmetical performance at this age. Because the present study uses Krajewski and Schneider's (2009) theoretical model of early arithmetical development as a general framework, it did not include a measure of children's central executive functioning. Nevertheless, VSSP functioning predicted unique variance of children's performance on the Mathematical Reasoning subtest over an eighteen-month period and the proportion of variance predicted by the four regression models predicting children's performance on the Mathematical Reasoning and the Numerical Operations subtests at the end of Reception Year and at midway through Year 1 is statistically significant.

8.4.3 Proportion of variance predicted by the domain-specific cognitive skills

In the present study, the proportion of variance that both Quantity Discrimination Skills' tasks predict in children's Mathematical Reasoning over and above age and Baseline Trimmed RT is 7.5% at the end of Reception Year but drops down to 4.3% at midway through Year 1. The proportion of variance that both Quantity Discrimination Skills' tasks predict in the Numerical Operations subtest is 9.6% at the end of Reception Year but drops down to 5.8% at midway through Year 1. The magnitudes of the proportion of variance predicted by non-symbolic approximate quantity discrimination skills in mathematical attainment in young children seem to vary considerably across studies (see Libertus et al., 2011). Halberda et al. (2008) found that the Weber function of fourteen-year-olds on an approximate non-symbolic quantity discrimination task predicted 16% of the variance in their performance on a standardised mathematical attainment test (TEMA 2, Woodcock & Johnson, 1989) and 20% on a different standardised mathematical attainment test (WJ revised calculation test, Woodcock & Johnson, 1990). Libertus et al. (2011) found that three-to-five year olds accuracy on an approximate non-symbolic quantity discrimination task predicted a unique 13% in performance on a standardised mathematical attainment test (TEMA 3) (Ginsburg & Baroody, 2003) that could not be explained by their speed-accuracy trade-offs, after controlling their age and vocabulary size. Thus, the results of the present study are more in line with those from Libertus et al. (2013) possibly due to the similarities in participants' age, the use of accuracy as an index of approximate quantity discrimination skills and the similarities in the regression models (Libertus et al., 2013 included children's Trimmed RT in a computerised task and their vocabulary and in the present study children's Baseline Trimmed RT and their phonological awareness was included in the regression model).

However, it is noteworthy that the variance predicted by children's Quantity Discrimination Skills in the present study is smaller than that reported by Libertus et al. (2011). The smaller percentages predicted in the present study could be due to the fact that in the present study age is also controlled for. It could also be that quantity discrimination skills are more strongly related to specific standardised mathematical attainment tests. While the present study used the Mathematical Reasoning and the Numerical Operations subtests of the WIAT-II^{UK} (Wechsler, 2002), Libertus et al. (2013) used the TEMA 3 (Ginsburg & Baroody, 2003). Also there are differences in elapse of time between the times predictors and outcome measures were obtained. The present study reports longitudinal contributions over a six-month and over an eighteen-month period while Libertus et al. (2011) report concurrent contributions. Nevertheless, the present findings corroborate the contention that non-symbolic approximate quantity discrimination skills' accuracy predicts unique and independent variance in children's mathematical attainment.

8.4.4 Causality and growth analyses

Regarding the causality of the relationships found, the domain-specific and domain-general cognitive skills included in the study have causal relationships with children's Early Number Skills, as they do not only predict performance but also growth on these outcome measures. It is worth noting that phonological awareness was a unique predictor of Knowledge of the Number Sequence performance over an eighteen-month period but failed to predict unique variance of Knowledge of the Number Sequence growth over the same time period. It could be that number sequence reciting makes higher verbal demands at early stages of schooling. Because growth analyses eliminate early variation of the Knowledge of the Number Sequence at the start of Reception Year, later growth on this number skill is only predicted by children's approximate quantity discrimination skills (see Bowey, 2005). When all three significant cognitive precursors of mathematical attainment performance over a six-month period (Approximate Quantity Discrimination skills' Accuracy, VSSP functioning and phonological awareness) were included in the regression model, none of them stood as a unique predictor of Mathematical Reasoning growth, although the proportion of variance predicted by the model is statistically significant and VSSP functioning just misses traditional levels of significance (.055). In addition to these findings, only VSSP functioning predicted unique variance in children's Mathematical Reasoning subtest performance and growth over an eighteen-month period. Thus, the present results suggest that children's VSSP functioning is also a causal predictor of mathematical attainment in young children while the role of the other three cognitive precursors included in the study as causal predictors of mathematical attainment is more equivocal.

8.4.5 The relationships between cognitive precursors and reading attainment

Only phonological awareness predicted independent and unique variance in children's reading attainment at midway through Year 1 when the independent contribution of all cognitive precursors was explored simultaneously and remained as such even after controlling for General Conceptual Abilities. These results do not only align with previous findings where phonological abilities have been found to play a crucial role in reading development (Gathercole et al., 2005; Hecht et al., 2001) but also suggest that the relationships identified between Quantity Discrimination Skills and the variables of interest are specific to the number domain.

9. DISCUSSION

This thesis is the first large-scale study to simultaneously address the separate and independent contributions that young children's domain-specific and domain-general cognitive skills make to their early number skills and mathematical attainment through their very early stages of schooling. It presents longitudinal data from 129 children assessed at the start and end of Reception Year and at midway through Year 1. Using Krajewski and Schneider's (2009) theoretical model of early arithmetical development as a general framework, this thesis first explored the independent and unique contributions that domain-specific and domain-general cognitive skills make to three early number tasks designed to tap the three distinct developmental levels of QNCs proposed by Krajewski and Schneider (2009) over a six-month period and over an eighteen-month period. The present study extended Krajewski and Schneider's (2009) work because it empirically tested the relationships between the two domain-specific number systems that have been proposed (Feigenson et al., 2004) with the three distinct developmental levels of QNCs (Krajewski & Schneider, 2009). Two novel non-symbolic Quantity Discrimination Skills' tasks were designed; a Precise Quantity Discrimination Skills' task to tap children's PNS and an Approximate Quantity Discrimination Skills' task to tap children's ANS. These two Quantity Discrimination Skills' tasks improve some of the previous methodology employed to tap the two core quantity systems (LeFevre et al., 2010; Libertus et al., 2011) because they used non-symbolic numerical sets presented simultaneously and children responded by pressing a key, so no verbal or memory demands were made, nor knowledge of the formal number system was required. Three novel Early Number Skills' tasks were designed to represent the three distinct developmental levels of QNCs (Krajewski & Schneider, 2009). It was found that the domain-specific cognitive skills and the domain-general cognitive skills relate differently to the different Early Number Skills. The independent and unique longitudinal contributions that the four cognitive precursors made to the Early Number Skills over a six-month period remained over an eighteen-month period and over and above General Conceptual Abilities.

The relationships found between domain-general cognitive skills and Early Number Skills were broadly similar to those predicted by Krajewski and Schneider (2009). However, the two Quantity Discrimination Skills clearly supported different Early Number Skills and partially contradicted Krajewski and Schneider's (2009) theoretical predictions. Precise Quantity Discrimination skills' Trimmed RT predicted children's ability to link quantities to number-words (QNCs Level II) and Approximate Quantity Discrimination skills' Accuracy predicted children's ability to recite the number-word sequence (QNCs Level I). Thus the present results provide preliminary evidence that precise and approximate non-symbolic quantity discrimination skills play differential roles in early number skills' development.

In addition to examining individual number skills, the current study explored the independent and unique longitudinal contributions that the domain-specific and domain-general cognitive skills make to children's performance on two standardised mathematical attainment measures. Children's speed or accuracy in enumerating three or four items (subitising) has been associated with their mathematical attainment (Landerl et al., 2004) and a longitudinal study has found that a combined measure of kindergarteners' enumeration speed over collections of up to three items and a non-verbal arithmetic task contributed to their mathematical attainment two years later (LeFevre et al., 2010). On the other hand, children's ability to make approximate numerical judgements over non-symbolic quantities has been associated with their concurrent and later mathematical attainment (Libertus et al., 2013; Mazzocco et al., 2011), although some studies suggest this relationship is not consistent over early stages of schooling (Bonny & Lourenco, 2013) and some studies fail to find a relationship at all (Holloway & Ansari, 2009). This thesis contributes and extends previous work that has explored the relationships between cognitive skills and mathematical attainment in several ways. First, it simultaneously explored the independent and unique longitudinal contributions that precise and approximate non-symbolic Quantity Discrimination Skills make to children's later mathematical attainment over a six-month period and over an eighteen-month period. Second, it explored the independent and unique longitudinal contributions that these two non-symbolic Quantity Discrimination Skills make to two different mathematical attainment measures that seem to tap distinct mathematical attainment constructs in young children (Nunes et al., 2012). Third, it simultaneously explored the independent and unique longitudinal contributions that these domain-specific cognitive skills and two domain-general mathematical related cognitive skills make to young children's mathematical attainment. Results show that both of the domain-specific cognitive skills and both of the domain-general cognitive skills relate differently to the different mathematical attainment measures, supporting the justification for the distinction of these two measures as distinct constructs of mathematical attainment in young children (Nunes et al., 2012).

When the independent contributions that the four cognitive precursors make to the distinct mathematical attainment measures were explored simultaneously, only Approximate Quantity Discrimination Skills' Accuracy of the two Quantity Discrimination Skills' tasks made unique longitudinal contributions to children's mathematical attainment and only over a six-month period, suggesting that non-symbolic quantity discrimination skills are not good long-term predictors of young children's mathematical attainment. In contrast, both domain-general cognitive skills made unique longitudinal contributions to children's mathematical attainment over a six-month period and VSSP functioning was a unique predictor of performance on the Mathematical Reasoning subtest over an eighteen-month period. Thus suggesting that VSSP functioning is a strong long-term predictor of young children's mathematical attainment.

This study also used growth analysis to examine whether the relationships identified are causal. Only one study has examined whether school-aged children's phonological awareness could predict *growth* on their mathematical attainment performance (Hecht et al., 2001) and only one study has examined whether pre-schooler's approximate quantity discrimination skills could predict *growth* on their mathematical attainment performance over a six-month period (Libertus et al., 2013). Both studies reported significant results. Growth analyses were conducted in the present study and revealed that cognitive precursors that made unique longitudinal contributions to the Early Number Skills over an eighteen-month period, also predicted their growth over this period. These results strengthen the argument that these cognitive precursors are causal predictors of early number skills. Regarding mathematical attainment, the model predicting Mathematical Reasoning growth over a six-month period from the four cognitive precursors explained a significant proportion of variance overall. However, no individual cognitive precursor stood as a unique predictor of Mathematical Reasoning growth. Therefore the role of these cognitive precursors as causal predictors of mathematical attainment over a six-month period is more equivocal. Nevertheless, VSSP functioning was a unique predictor of Mathematical Reasoning performance and *growth* over an eighteen-month period, strengthening the argument that it is causal predictor of mathematical attainment.

Children's phonological awareness measured at the start of the study predicted their reading performance eighteen months later even when the contributions of all four cognitive precursors were examined together and over and above children's General Conceptual

Abilities. In contrast, none of the Quantity Discrimination Skills predicted unique variance in children's reading attainment using the same regression model, suggesting that quantity discrimination skills are specific precursors of early mathematical development and not broader precursors of scholastic attainment. These results also corroborate the key role that phonological awareness plays in literacy development (Gathercole et al., 2005; Hecht et al., 2001).

9.1 THE RELATIONSHIPS BETWEEN DOMAIN-GENERAL COGNITIVE SKILLS AND EARLY NUMBER SKILLS

The present findings align with Krajewski and Schneider's (2009) model predictions and study results in that phonological awareness predicts children's number-word sequence reciting and VSSP functioning predicts children's simple arithmetic performance. However, results of the relationships between domain-general cognitive skills and Early Number Skills varied slightly to the results Krajewski and Schneider (2009) reported. When all four cognitive precursors were included in the regression model phonological awareness also predicted unique variance in children's performance on the Story Problems task over a six-month period, over an eighteen-month period and its contribution remained over and above children's General Conceptual Abilities. It has been proposed that phonological awareness plays a crucial role in number tasks where processing and manipulation of verbal codes is needed (Simmons & Singleton, 2008). Comparisons between Krajewski and Schneider's (2009) tasks and the task representing QNCs Level III (relations between numerical quantities) in the current study suggest that the present task (Story Problems) made higher verbal demands than those of Krajewski and Schneider's (2009). This is because in the Story Problems task items were verbally presented and demanded a verbal response. Therefore, it is not surprising that phonological awareness made independent and unique contributions to the Story Problems at the end of Reception Year and at midway through Year 1. Thus, the present results corroborate the fundamental role that phonological awareness plays in numerical tasks that demand accessing and processing phonological representations (Hecht et al., 2001; LeFevre et al., 2010; Simmons & Singleton, 2008) and the fundamental role that VSSP functioning plays in young children's early arithmetic skills (Krajewski & Schneider, 2009; McKenzie et al., 2003; Rasmussen & Bisanz, 2005; Simmons et al., 2008; Swanson & Beebe-Frankenberger, 2004).

Nevertheless, the current results are at odds with Krajewski and Schneider's (2009) theoretical proposal in that early number-word sequence reciting is solely supported by phonological awareness and that quantity discrimination skills only contribute to higher levels of QNCs where quantity representations or manipulations are needed. In the current study not only phonological awareness, but also Approximate Quantity Discrimination Skills' Accuracy, made unique independent contributions to rote citation of the number-word sequence over a six-month period and over an eighteen-month period, even when all cognitive precursors were included in the model and over and above their General Conceptual Abilities.

9.2 THE RELATIONSHIPS BETWEEN DOMAIN-SPECIFIC COGNITIVE SKILLS AND THE EARLY NUMBER SKILLS

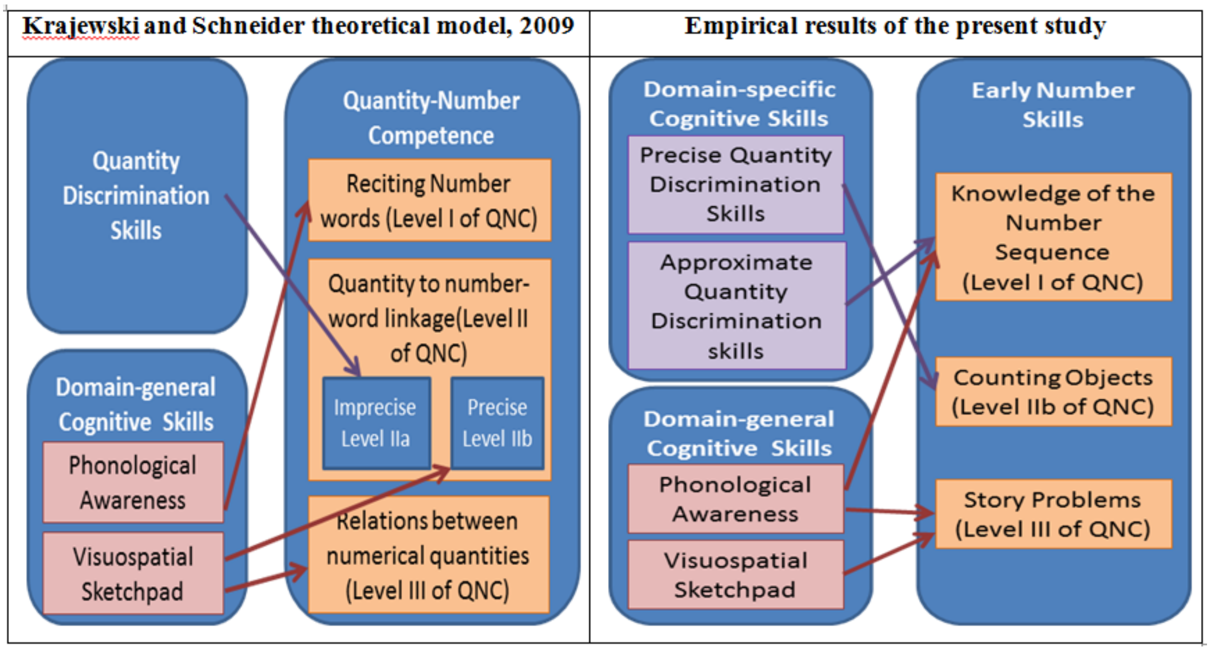
The present longitudinal study provides the first empirical evidence that precise and approximate quantity discrimination skills play differential roles in the development of different early number skills. Approximate Quantity Discrimination skills' Accuracy made unique longitudinal contributions to children's ability to recite the number-word sequence over a six-month period, over an eighteen-month period and over and above General Conceptual Abilities. This evidence directly challenges Krajewski and Schneider's (2009) theoretical model because they propose that the acquisition of the knowledge of the number sequence is asemantic. Distance effects have been repeatedly reported in behavioural experiments with children and adults (Halberda & Feigenson, 2008) and these are taken as evidence of the existence of a "mental number line", which is an analogue, internal presentation for number magnitude. This number line is a sequential structure where the numbers semantics are organised in a continuum and that supports our approximate numerical discrimination skills (Feigenson et al., 2004). It has been proposed that numerical representations on this analogue sequential structure are ordered by size but are noisy and overlap with each other (Cohen Kadosh, Tzelgov, & Henik, 2008; Dehaene & Changeux, 1993). An internal syntactic frame is needed for the production of number-words so that they can be ordered by number lexical class; this structure allows each term to have its own position within the class it belongs to (McCloskey, Sokol, & Goodman, 1986). Thus results from the present study suggest that the mental number line that supports approximate numerical judgments might also serve as an internal structure for acquiring numerical sequential meanings, including the lexical structure of the number-words.

Precise Quantity Discrimination skills' Trimmed RT predicted children's later ability to count objects over a six-month period, over an eighteen-month period and over and above children's General Conceptual Abilities. Counting objects is a complex numerical skill that requires mapping the discrete items in a set onto number-words in a one-to-one correspondence (Gelman & Gallistel, 1978). Subitising enables the rapid and precise enumeration of discrete items in small sets and supports our precise quantity discrimination skills (Feigenson et al., 2004). Presumably this skill requires a well-ordered and not noisy representation of magnitudes in order to facilitate quick and precise numerical discriminations over very small quantities. Consequently, precise quantity discrimination skills might underpin the early acquisition of the numerical cardinality meaning.

The distinction between sequence and cardinal numerical meaning is considered in different developmental models of mathematical development (Fuson, 1988; Krajewski & Schneider, 2009; Wynn, 1992) and supported by neuropsychological data with adults (Dehaene & Cohen, 1997; Delazer & Butterworth, 1997). However, never before has this distinction been associated with the two distinct systems for quantity representations that have been proposed (Feigenson et al., 2004). Two plausible although nonexclusive theoretical proposals coexist; one proposes that precise quantity discrimination skills are the ones supporting later numerical skills and basic arithmetic skills (Butterworth, 1999, 2005, 2010), the other suggests that approximate quantity discrimination skills underpin the later acquisition of early numerical skills and arithmetic (Dehaene, 1997; Dehaene et al., 2004; Spelke & Dehaene, 1999).

The present results suggest that approximate quantitative discrimination skills might act as a specific precursor for the acquisition of the numerical sequential meaning but not for the acquisition of numerical cardinal meaning and arithmetic skills. Butterworth (1999, 2005, 2010) argues that subitising underlies the development of children's formal number learning. However, findings of the current study indicate that subitising has a circumscribed role as a precursor of counting skills but no other early number skills such as early arithmetic skills. It is worth noting that the present findings do not rule out the possibility that children's subitising underpins their early and very basic arithmetic skills at very early stages of mathematical development and in the "absence" of any formal arithmetic instruction. Findings simply point in the direction that by the age of five, children rely on their domain-general cognitive skills and not on their "intuitive" quantitative skills to perform simple arithmetic. Thus, findings of the current study indicate that both domain-specific and domain-general cognitive skills play a fundamental role in early number skills' development. See figure 9.1 for a summary of the similarities and discrepancies of the present results with Krajewski and Schneider's (2009) theoretical model.

Figure 9.1: Similarities and discrepancies of the present results with Krajewski and Schneider's (2009) theoretical model

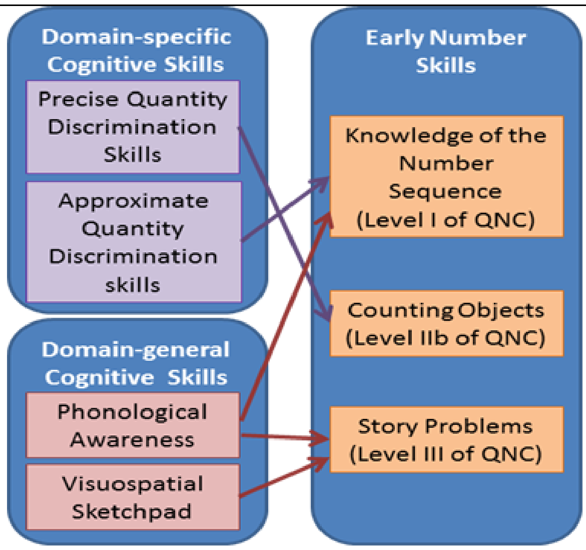
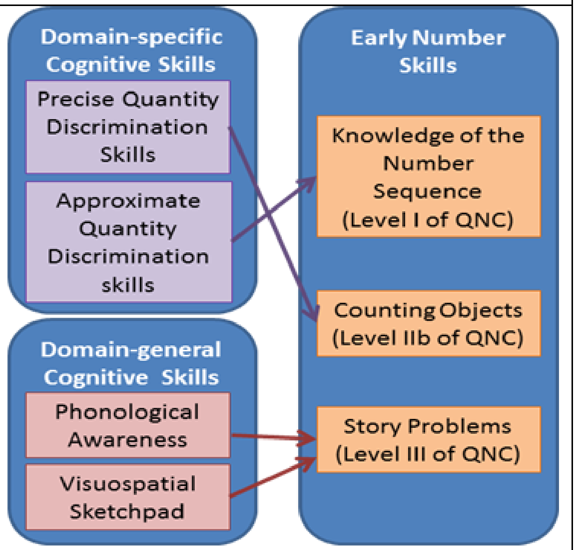


9.3 GROWTH ANALYSES AND CAUSALITY: EARLY NUMBER SKILLS

The four cognitive precursors included in the present study not only predicted children's Early Number Skills' performance but also predicted Early Number Skills' growth from the start of Reception Year to midway through Year 1. Studies have found that measures of children's phonological awareness and VSSP functioning at very early stages of formal education predict later number skills (Krajewski & Schneider, 2009). Similar longitudinal relationships have been reported between children's quantity discrimination skills and their early number skills (LeFevre et al., 2010; Piazza et al., 2010). Such longitudinal relationships intuitively suggest that the cognitive precursors are causal predictors of the outcome measures because at the time cognitive precursors were assessed, children had undergone formal mathematical instruction only for a very limited time. However, predictive studies in mathematical development do not tend to examine the predictors of growth, although there are a few exceptions (Hecht et al., 2001; Libertus et al., 2013). Thus, the possibility of predictor variables having reciprocal relationships with the outcome measures or even a reverse directionality to the one intuitively suggested remains tenable. It could be that as a consequence of the process of learning and using the formal number system, children “train” and improve their phonological, visuo-spatial and quantitative skills. A key feature of the present study is that it examined whether the cognitive precursors could also predict *growth* in the outcome measures. Growth analyses are a very strict methodology to confirm the role of cognitive precursors as causal predictors because predictor variables need to predict outcome measures over and above themselves. For this reason growth analyses are strong evidence of the role of precursors as causal predictors. However, because growth analyses are very conservative, it would not be wise to completely dismiss the role of cognitive precursors as causal predictors if they do not predict unique growth (see Bowey, 2005 for a full discussion on this issue).

In the present study, results from the growth analyses for the Early Number Skills indicate that Approximate Quantity Discrimination skills’ Accuracy is a causal predictor of children’s Knowledge of the Number Sequence, Precise Quantity Discrimination skills’ Trimmed RT is a causal predictor of children’s ability to count discrete objects and both domain-general cognitive skills are causal predictors of children’s performance on verbally presented simple arithmetic problems. Both quantity discrimination skills have been proposed to be early precursors of children’s later numerical competence because they are present very early in life (Xu & Spelke, 2000), in adult humans with a very restricted number lexicon (Pica et al., 2004) and in other animal species (Hanus & Call, 2007; Hauser et al., 1995; Pepperberg & Gordon, 2005). However, no study to date has empirically examined the causality of these relationships with early number skills. Because the cognitive precursors predicted unique variance in the Early Number Skills’ growth over an eighteen-month period it is possible to be very confident about their causal relationships with the Early Number Skills. However, phonological awareness did not predict unique variance in Knowledge of the Number Sequence growth (see figure 9.2 for a summary of the variables predicting unique variance in Early Number Skills’ growth over an eighteen-month period) despite the fact that it predicted performance over and above General Conceptual Abilities. It could be that knowledge of the number sequence reciting makes higher verbal demands at early stages of schooling but that later growth on number sequence reciting depends to a greater extent on children’s understanding of place value. Because growth analyses eliminate early variation of the Knowledge of the Number Sequence at the start of Reception Year, this could explain why only Approximate Quantity Discrimination skills’ Accuracy remains as a unique predictor of growth. However, the role of phonological awareness as a causal predictor of number-sequence reciting cannot completely be dismissed on the grounds that it does not predict unique variance in the Number Sequence growth and therefore its role as a causal predictor of number-sequence reciting is more equivocal (see Bowey, 2005).

Figure 9.2: Cognitive precursors predicting unique variance in Early Number Skills' performance (on the left) and unique variance in Early Number Skills' growth (on the right) from the Start of Reception Year to midway through Year 1.

Cognitive precursors predicting early number skills performance over an eighteen-month period	Cognitive precursors predicting early number skills growth over an eighteen-month period
	

9.4 THE RELATIONSHIPS BETWEEN DOMAIN-GENERAL COGNITIVE SKILLS AND MATHEMATICAL ATTAINMENT

Phonological awareness and VSSP functioning made independent and unique contributions to children's mathematical attainment, but they related differently to the two mathematical attainment measures, supporting the contention that these measures tap distinct constructs of mathematical attainment in young children (Nunes et al., 2012). When the independent and unique longitudinal contributions that the four cognitive precursors make to the two mathematical attainment measures were explored simultaneously, VSSP functioning and phonological awareness predicted unique variance in performance on the two standardised mathematical attainment measures over a six-month period. These results corroborate the proposal that mathematical attainment in young children is influenced by the quality of their phonological representations (Simmons & Singleton, 2008) and align with previous findings where phonological awareness predicted young children's performance on standardised mathematical attainment measures months or a year later (Leather & Henry, 1994; Simmons et al., 2008). They also corroborate the role of VSSP functioning as a predictor of young children's later performance on standardised mathematical attainment tests (Noël et al., 2004; Rasmussen & Bisanz, 2005; Simmons et al., 2008). However, only VSSP functioning predicted unique variance in Mathematical Reasoning over an eighteen-month period. Items in the Numerical Operations subtest are not all verbally presented and they do not require a verbal response (except for one item where children are asked to verbally report the number of pennies presented to them). It has been suggested that at early stages of formal mathematical learning, the process of mapping the numerical lexicon to the corresponding Arabic numerals is still under development and presumably requires phonological skills because it is a similar process to learning to read (LeFevre et al., 2010). Once this mapping has been made, children would need to rely little if at all on verbal skills to solve written arithmetic problems that do not require phonological processing skills. This could explain why phonological awareness no longer contributes to performance on the Numerical Operation subtest at midway through Year 1.

However, the finding that phonological awareness does not make unique longitudinal contributions to Mathematical Reasoning over an eighteen-month period was unexpected because items in the Mathematical Reasoning subtest are verbally presented and require verbal responses. It could be that a shift in cognitive strategies occurs in the transition from Reception Year to Year 1 and other cognitive skills not included in the present study might be better predictors of children's performance on the Mathematical Reasoning subtest at this developmental stage. For instance, previous studies indicate that central executive functioning together with VSSP functioning contribute to children's arithmetical performance at this stage (McKenzie et al., 2003; Rasmussen & Bisanz, 2005). Because the present study uses Krajewski and Schneider's (2009) theoretical model of early arithmetical development as a general framework, it did not include a measure of children's central executive functioning.

9.5 THE RELATIONSHIPS BETWEEN DOMAIN-SPECIFIC COGNITIVE SKILLS AND MATHEMATICAL ATTAINMENT

When all four cognitive precursors were included in the model, Precise Quantity Discrimination skills' Trimmed RT did not contribute to children's mathematical attainment at any time point, suggesting that this cognitive skill is not a good longitudinal predictor of children's later performance on standardised mathematical attainment tasks and challenging

LeFevre et al.'s (2010) findings where a combined measure of subitising skills and a non-verbal arithmetic task in five-year-olds made significant contributions to their performance on standardised and research-based mathematical attainment measures two years later. A plausible explanation for the contrasting findings could be that the task employed by LeFevre et al. (2010) to assess children's subitising did not control for children's age or naming speed. Especially when using RTs as an index of children's quantity discrimination skills it is crucial to control for their baseline speed on a task that demands a rapid response to stimuli presented in the same format. Otherwise speed measures of the variable of interest are likely to be largely contaminated by children's individual latencies. In addition, LeFevre et al.'s (2010) subitising task required the use of number-words and therefore was contaminated by children verbal skills. In addition, it was a combined measure of subitising skills and non-verbal arithmetic what predicted later mathematical attainment and not just subitising skills. Therefore this relationship is likely to be largely contaminated by the efficiency of other cognitive skills such as VSSP functioning.

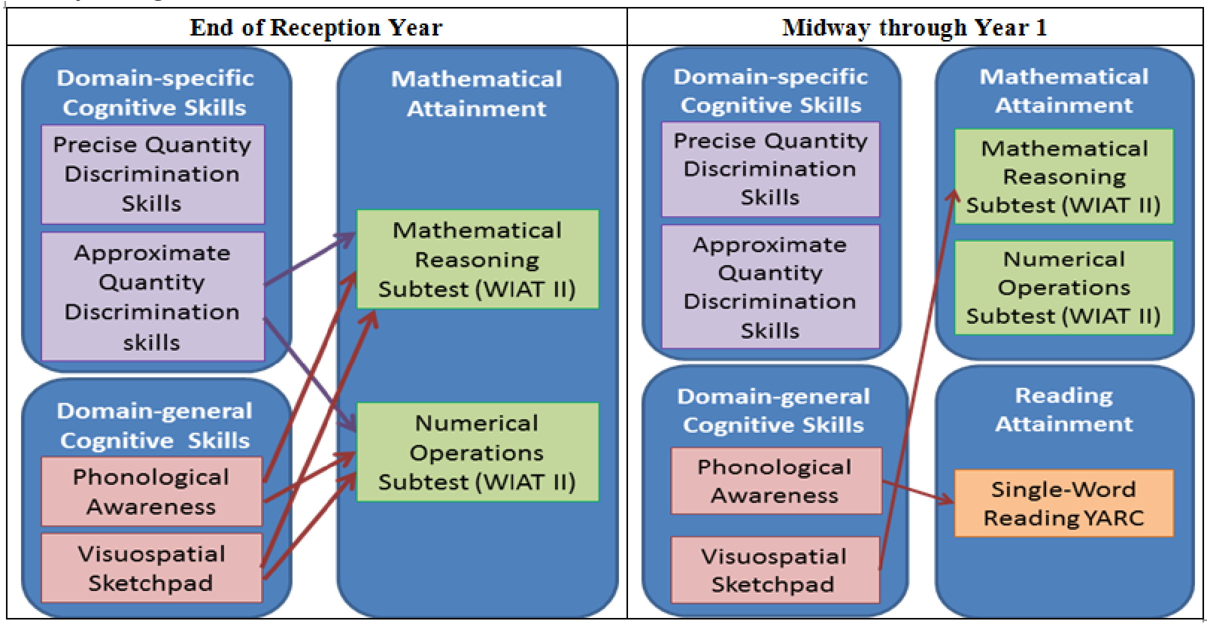
Approximate Quantity Discrimination skills' Accuracy only made unique longitudinal contributions to their mathematical attainment over a six-month period but not over an eighteen-month period. These results challenge those reported by Mazzocco et al. (2011) where pre-schoolers' approximate quantity discrimination skills predicted their performance on a standardised mathematical attainment test two years later. Mazzocco et al.'s (2011) significant findings could be an artefact of children's individual differences in their general cognitive abilities, which were not controlled for, or biased due to the very small sample recruited for their study (seventeen children). Nevertheless, the present results are consistent with those from Libertus et al. (2013) where pre-schoolers' ANS accuracy made significant longitudinal contributions over a six-month period after controlling for participants' age, vocabulary and math ability at the start of the study.

The number module hypothesis (Butterworth, 1999, 2005, 2010) postulates that a deficit in the representational system for analogue numerical magnitudes is a core aspect of early MLD. Although this hypothesis originally emerged from comparison studies where children presenting MLD had poor subitising skills (Landerl et al., 2004), it has been generalised to give an account of the relationships found between children's poor magnitude representation skills, whether precise or approximate, and their low mathematical performance. Therefore, all studies that have found children's performance on non-symbolic quantity discrimination tasks to be related to their concurrent or later mathematical attainment (Libertus et al., 2013; Libertus et al., 2011; Nordman et al., 2009, September) suggest that a deficit in the representational system for analogue numerical magnitudes might underpin children's poor mathematical attainment. In contrast, *the access deficit hypothesis* (Rousselle & Noel, 2007) postulates that mathematical learning disabilities are due to a deficit in accessing the analogue magnitude representations from symbolic numerical representations and is supported by studies that fail to find a relationship between children's performance on non-symbolic quantity discrimination tasks and their mathematical attainment but find a relationship between performance on symbolic quantity discrimination tasks and children's mathematical attainment (DeSmedt & Gilmore, 2011; Iuculano et al., 2008; Rousselle & Noel, 2007; Sasanguie et al., 2012; Soltész et al., 2010).

In view of the present results children's speed in making accurate and precise discriminations over small non-symbolic quantities seems to be a good long-term predictor of a specific early number skill (Counting Objects). Children's accuracy on non-symbolic approximate numerical judgements seems to be a good long-term predictor of their number-sequence reciting and a short-term predictor, but not a good long-term predictor, of their performance on standardised mathematical attainment measures. These findings support *the number module hypothesis* in that non-symbolic approximate quantity discrimination skills predicted children's mathematical attainment six months later. However, the present results also provide support to a very recent cross-sectional study which suggested that the relationship between children's non-symbolic approximate quantity discrimination skills with mathematical attainment weakens over early stages of schooling (Bonny & Lourenco, 2013).

The present study is the first to examine the relative and unique contributions that precise and approximate non-symbolic quantity discrimination skills make to the two mathematical attainment constructs in young children that have been proposed (Nunes et al., 2012) over a six-month and over an eighteen-month period and controlling for a children's age and baseline speed in making two-choice non-numerical discriminations. In addition by including the two domain-general cognitive predictors in the model, it is possible to be very confident that the relationship identified between Approximate Quantity Discrimination Skills' Accuracy and mathematical attainment cannot be explained by their visuo-spatial STM skills or phonological awareness. See figure 9.3 for a summary of the variables predicting mathematical attainment at the end of Reception Year and mathematical and reading attainment at midway through Year 1.

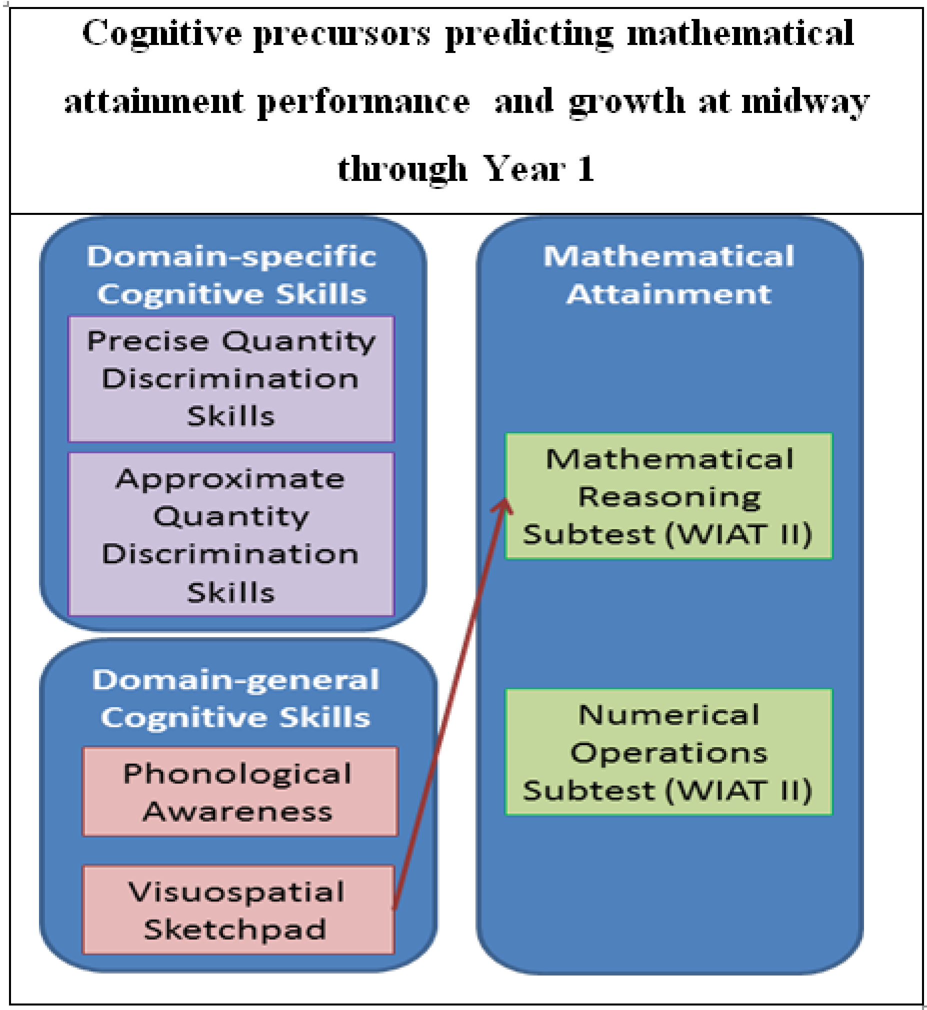
Figure 9.3: Independent contribution of the domain-specific and domain-general cognitive skills on children's mathematical attainment at the end of Reception Year and on children's mathematical attainment and reading at midway through Year 1.



9.6 GROWTH ANALYSES AND CAUSALITY: MATHEMATICAL ATTAINMENT MEASURES

Libertus et al. (2013) confirmed the role of approximate non-symbolic quantity discrimination skills' accuracy as causal predictors of pre-schoolers' mathematical attainment by conducting growth analyses over a six-month period. In the present study when all significant precursors of mathematical attainment over a six-month period were included in the growth analyses, only the regression model predicting Mathematical Reasoning at the end of Reception Year predicted a significant proportion of variance overall. However, no cognitive precursor was a unique predictor of Mathematical Reasoning growth over this time period. Therefore, the role of Approximate Quantity Discrimination skills' Accuracy, Phonological Awareness and VSSP functioning as causal predictors of mathematical attainment is equivocal. However, because growth analyses control for auto-regressor effects (the tendency of a variable to predict itself), they are very strict. For this reason, it is not wise to dismiss the role of cognitive precursors as causal predictors if they do not predict unique variance in growth because these analyses are very conservative (see Bowey, 2005). Whilst VSSP functioning predicted performance and *growth* on the Mathematical Reasoning subtest over an eighteen-month period, none of the other cognitive precursors included in the study predicted mathematical attainment over this time period. Thus, confirming the role of VSSP functioning as a causal predictor of mathematical attainment (see figure 9.4).

Figure 9.4: Variables predicting unique variance in Mathematical Attainment growth from the Start of Reception Year to midway through Year 1.



9.7 SPECIFICITY OF THE RELATIONSHIPS FOUND

Quantity discrimination skills are believed to be domain-specific because they emerge independently from other cognitive skills (Antell & Keating, 1983; Bijeljac-Babic et al., 1993; Wynn, 1996) and are supported by distinct brain networks (Arp et al., 2006; Fink et al., 2001; Pasini & Tessari, 2001; Piazza et al., 2003; Sathian et al., 1999) that systematically activate in the presence of tasks demanding specific numerical processing (Dehaene et al., 1998). Studies examining the relationships between quantity discrimination skills and mathematical development tend to control for children's individual differences in other cognitive skills to eliminate potential contamination from confounding variables. However, the possibility of quantity discrimination skills being broader precursors of scholastic attainment remains tenable because the tasks used to tap these systems could also unintentionally tap general cognitive skills that are not being controlled for. In the present study, when the independent contribution of domain-general and domain-specific cognitive precursors was explored simultaneously, children's Quantity Discrimination Skills predicted their later Early Number Skills and mathematical attainment but not their reading attainment. Thus, the relationships found between the cognitive precursors with the Early Number Skills and mathematical attainment in the present study seem to be specific to the number domain. In addition, children's reading performance was only predicted by their phonological awareness when all four cognitive precursors were included in the regression models and this relationship remained over and above children's General Conceptual Abilities. Therefore these results also corroborate the pivotal role that phonological awareness plays in early literacy development (Gathercole et al., 2005; Hecht et al., 2001)

9.8 LIMITATIONS AND FUTURE DIRECTIONS

The present thesis used Krajewski and Schneider's (2009) model as a general framework to study the relationships between domain-general and domain-specific cognitive skills and distinct early number skills. Krajewski and Schneider's (2009) model was chosen because they proposed three distinct developmental levels of early number skills instead of the two proposed by LeFevre et al. (2010). These three distinct developmental levels of early number skills include the distinction between sequential and cardinal number meaning proposed by previous developmental models of early mathematical competence (Fuson, 1988, 1992; Gelman & Gallistel, 1978; Wynn, 1992). However, there are a wide range of early number skills and future studies should consider including early number skills outside the ones considered in Krajewski and Schneider's (2009) model, such as children's ability to write or read numbers.

Two non-symbolic quantity discrimination skills tasks were designed to tap children's PNS and ANS respectively in the present study. These tasks aimed to meliorate previous methodology employed to tap these core systems for quantity representations by using non-symbolic stimuli, simultaneous presentation of the arrays to be compared and children having to respond by pressing a key. Thus, no verbal skills, memory skills or knowledge of the formal number system was needed to perform these tasks. RTs in the Precise Quantity Discrimination Skills' task showed a quadratic trend, suggesting that children were not employing a serial counting strategy to respond. Accuracy analyses in the Approximate Quantity Discrimination task showed the numerical ratio effect consistent with the idea that children were relying on numerical magnitudes represented in a continuum where vagueness increases as the numerical magnitude increases. However, recent studies have suggested that non-symbolic quantity discrimination tasks might in fact be tapping domain-general cognitive skills (Fuhs & McNeil, 2013; Gilmore et al., 2013; Soltész et al., 2010). In particular, Gilmore et al. (2013) and Fuhs and McNeil (2013) have proposed that the relationship between non-symbolic approximate quantity discrimination tasks and mathematical attainment in young children could be an artefact of their relationships with inhibition motor skills. Although this possibility cannot be completely dismissed because no measure of inhibition skills was included in the present study, it is highly unlikely that both non-symbolic Quantity Discrimination Skills' tasks designed for this study are indirect measures of inhibition motor skills for two main reasons. Firstly, children's Trimmed RT in the Precise Quantity Discrimination task and their Accuracy in the Approximate Quantity Discrimination task were not related to each other, suggesting that these measures tap different cognitive aspects. Secondly, Trimmed RT in the Precise Quantity Discrimination task and Accuracy in the Approximate Quantity Discrimination task related differently to the different Early Number Skills and predicted performance on these measures that could not be explained by domain-general cognitive skills or General Conceptual Abilities. Future studies may, however, want to explore whether the relationships identified in the present work between domain-specific cognitive skills and outcome measures are independent of children's inhibition skills.

In the current study, the unique proportion of variance predicted by phonological awareness, VSSP functioning and both Quantity Discrimination Skills is small. However, the number of predictors included in these models is larger than in most studies due to the inclusion of both Quantity Discrimination Skills and the two control measures (age in months and Baseline Trimmed RT). Therefore, after eliminating the variance that the control variables share with the outcome measures, the remaining unique variance that can be uniquely predicted by the cognitive precursors is smaller. Thus, the present methodological approach eliminates the influence of confounding variables to a greater extent than studies examining the relationships between either precise or approximate quantity discrimination skills and mathematical development. However, this is at the expense of reducing considerably the amount of remaining variance in the variable of interest that can be predicted by the cognitive precursors.

There is also a possibility that the proportions of variance predicted by the regression models in the present study are indirect. For instance the implication of central executive functioning in arithmetic tasks in six-year-olds has been previously reported (Bull et al., 2008; McKenzie et al., 2003; Rasmussen & Bisanz, 2005) and the possibility of approximate quantity discrimination tasks tapping central executive functioning has been raised (Fuhs & McNeil, 2013; Gilmore et al., 2013). Although the domain-general cognitive skills included in the present study are well-established precursors of mathematical attainment (Simmons et al., 2008), the relationships between the domain-specific cognitive skills and mathematical attainment are still controversial, especially for approximate quantity discrimination skills. Thus the relationships identified in the current study could be artefacts of the relationship between cognitive precursors and outcome measures with central executive functioning. The present study would have benefited from including a measure of central executive functioning measures to test this possibility. However, Krajewski and Schneider's (2009) model was used as a general framework for which the role of central executive functioning was not considered. Future studies examining the relationships of domain-general and domain-specific cognitive skills with early number skills and mathematical attainment may want to consider the inclusion of central executive functioning measures to clarify whether the relationships identified in the present work are direct.

While the four cognitive precursors included in the study were assessed at the start of Reception Year, General Conceptual Abilities were assessed at midway through Year 1. It is therefore likely that General Conceptual Abilities share greater variance with the outcome measures because they were both assessed at the same later time point. It could be that if they had been administered at the start of Reception Year General Conceptual Abilities and outcome measures would share less variance. However, due to the large number of participants recruited for the study and the large number of tasks administered at the start of Reception Year, the inclusion of General Conceptual Abilities tasks at this early stage of the study was not viable. Future studies might want to administer these control measures at the same time point as measures of the cognitive precursors are obtained, so that the control measures do not share greater variance with the outcome measures due to the fact that they had been administered at a later time point.

In summary, this study has identified key cognitive precursors of early number skills and mathematical attainment. It provides empirical evidence that each cognitive skill has a circumscribed role as a precursor of specific and distinct number skills. These findings are of key importance as they suggest that early assessments of the four cognitive skills included in this study could help with the identification of children that will struggle learning specific number skills and that targeted-interventions could be a straight forward solution to prevent later generalised mathematical low attainment in these children. For example one would predict that children with weak ANS may struggle with number-word sequence reciting and therefore early additional practice focussing on this early number skill could be very beneficial for these children. Similarly, children with weak PNS may find counting difficult and therefore encouraging them to engage in counting activities could help them to overcome these difficulties. Lastly, children with weak phonological awareness and/or weak VSSP functioning might find simple abstract arithmetic tasks particularly difficult, so early additional practice in arithmetic could be very useful. Whilst the study has a number of methodological strengths including its longitudinal design, it is recognised that there are weaknesses such as not including central executive measures. Future studies can extend and clarify whether the relationships identified in the current study are direct by including executive control measures.

GLOSSARY OF ABBREVIATIONS, ACRONYMS AND SPECIFIC TERMS

ANS:	Approximate Number System
AWMA:	Automatic Working Memory Assessment
BAS-II^{UK}:	British Ability Scales
DEMAT2+:	Modified version of the German Mathematics Test
Early number skills:	Skills that young children need to acquire to become competent users of the formal number system. This would include verbal skills such as counting as well as symbolic skills such as reading and writing numbers.
Trimmed RT:	Mean RTs for all correct responses taking less than two times the interquartile range from the median
ERPs:	Event-Related Potential
FINSTs:	Fingers of Instantiation
fMRI:	Functional Magnetic Resonance Imaging
Formal number system:	Conventional and culture-specific representations for number that are to be learnt through mathematical instruction
IQ:	Intelligence quotient
LTM:	Long-term memory
MLD:	Mathematics Learning Difficulties
NDE:	Numerical Distance Effect
PCA:	Principal Component Analysis
PIPA:	Preschool and Primary Inventory of Phonological Awareness
PET:	Positron Emission Tomography
PNS:	Precise Number System
QDS:	Quantity Discrimination Skills
QNC:	Quantity-number competence
RT:	Response Time
STM:	Short-term memory
TEMA:	Test of Early Mathematics Ability
VSSP:	Visuo-spatial sketch-pad
w parameter:	Minimum change needed in the numerical ratio presented by the two numerical magnitudes to be correctly discriminated
WASI:	Wechsler Abbreviated Scale of Intelligence
Weber JND:	Weber Just Noticeable Difference
WIAT-II^{UK}:	Wechsler Individual Achievement Test Second Edition
WJ:	Woodcock-Johnson Test
YARC:	York Assessment of Reading for Comprehension

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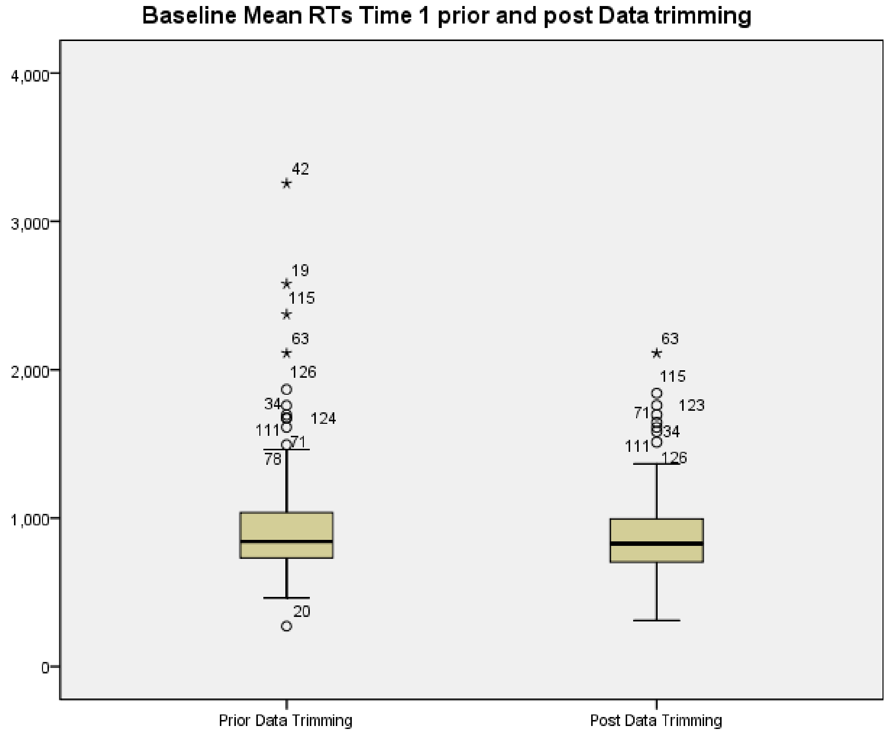
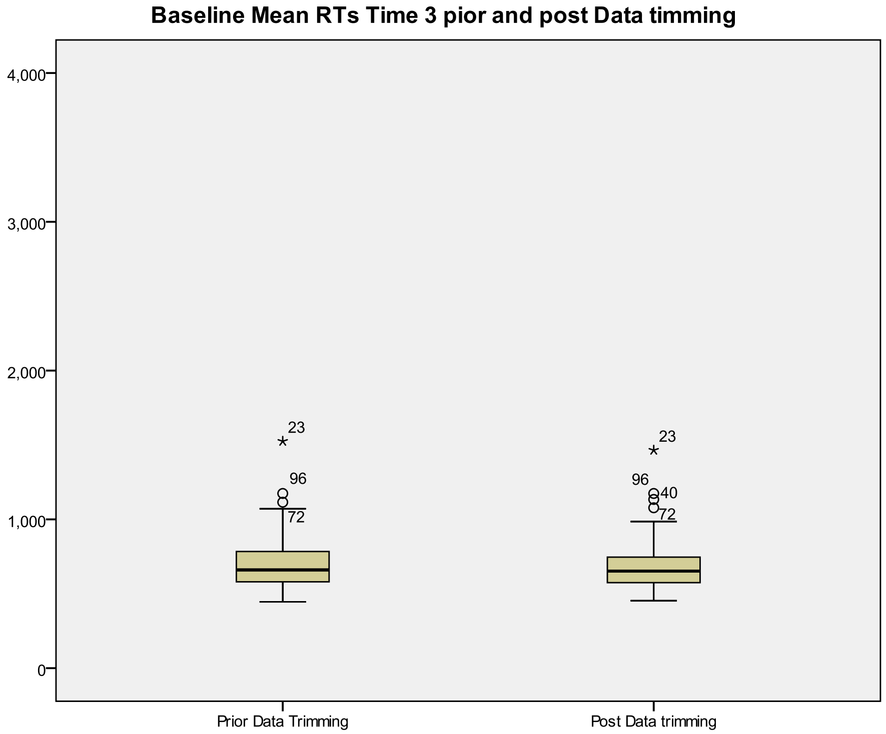
APPENDICES

Appendix 1

Box and whiskers graphs of Baseline task data distribution prior and post data trimming at T_1 (on the left) and at T_3 (on the right)





Appendix 2

Box and whiskers graphs for the Precise QDS data distribution prior and post data trimming at T_1 (on the left) and at T_3 (on the right)