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# Nonlinear Dynamics of Structures with Material Degradation

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**Abstract.** Structures usually experience deterioration during their working life. Oxidation, corrosion, UV exposure, and thermo-mechanical fatigue are some of the most well-known mechanisms that cause degradation. The phenomenon gradually changes structural properties and dynamic behaviour over their lifetime, and can be more problematic and challenging in the presence of nonlinearity. In this paper, we study how the dynamic behaviour of a nonlinear system changes as the thermal environment causes certain parameters to vary. To this end, a nonlinear lumped mass modal model is considered and defined under harmonic external force. Temperature dependent material functions, formulated from empirical test data, are added into the model. Using these functions, bifurcation parameters are defined and the corresponding nonlinear responses are observed by numerical continuation. A comparison between the results gives a preliminary insight into how temperature induced properties affects the dynamic response and highlights changes in stability conditions of the structure.

## 1. Introduction

From a macroscopic point of view, degradation can be defined as the changing of mechanical properties over the working life of a material component. Dynamical design of structures and machine components is usually performed based on the assumption of constant material properties. This is despite the fact that materials typically deteriorate gradually under their operating conditions and, as a result, their characteristics deviate from the original design assumptions.

Dynamic failure and fatigue are the most serious and well-known consequences of degradation, usually as a consequence of prolonged periods of cyclic loading. Other dynamic effects that lead to degradation include impact, rattle, freeplay and contact/friction effects. These occur in many applications relevant to the automotive industry. Among all different types of materials used in a vehicle, a high rate of change in properties is usually observed in elastomeric materials. Mechanical behaviour of elastomers can vary with external and working conditions over the operational life. As a wide range of parts with different functions are made from elastomeric materials, elastomer degradation is naturally a big concern for the automotive industry. However, the dynamic response of elastomers is nonlinear, which can make the degradation even more challenging to understand. In extreme cases, it can potentially change the stability behaviour of the system over the working life of the part and at different external conditions.

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The main goal of this paper is to investigate the changes in dynamic behaviours and stability of a nonlinear system with mechanical property changes and/or material degradation. An specific elastomer widely used in automotive industry is chosen and its experimental thermal degrading function is coupled to a 1-DoF nonlinear dynamic model. The numerical continuation technique is then used to mimic the nonlinear dynamics of the system and track the stability changes as the elastomer degrades with temperature.

## 2. Dynamic Behaviour of the system including ageing

To investigate the effects of the changes in material properties and/or degradation on the behaviour of a system, a one degree-of-freedom (1-DOF) dynamic model is considered with the following governing equation

$$M\,\ddot{X}(t) + C\,\dot{X}(t) + K\,X(t) + F_{nl} = F_0\,cos(\omega\,t), \tag{1}$$

where an overdot denotes derivative respect to time $t$, and $M$, $K$ and $C$ are the mass, stiffness, and damping matrices, respectively. The system is excited by a harmonic load of amplitude $F_0$ and frequency $\omega$ and a nonlinear restoring force

$$F_{nl} = \sum K_{i\,nl} X^i(t), \tag{2}$$

is assumed. Where $K_{i\,nl}$ s are the nonlinear stiffness coefficients of the corresponding nonlinear terms $X^i(t)$.

Now a non-dimensional form of Equation (1) is introduced

$$\delta^2\,x''(\tau) + \alpha\,x(\tau) + 2\,\delta\,\beta\,\zeta\,x'(\tau) + f_{nl} = f_0\,cos(\tau), \tag{3}$$

with dimensionless polynomial nonlinear restoring force $f_{nl}$ given by

$$f_{nl} = \sum \phi_i\,k_{i\,nl} x^i(\tau). \tag{4}$$

In Equation (3), prime now stands for the differentiation with respect to the dimensionless time ($\tau = \omega\,t$), and the other dimensionless parameters are defined as

$$\begin{aligned} &\delta = \tfrac{\omega}{\omega_0}, \qquad \omega_0 = \sqrt{\tfrac{K}{M}}, \quad x = \tfrac{X}{l}, \\ &\zeta = \tfrac{C}{2\,M\omega_0}, \quad f_0 = \tfrac{F_0}{K\,l}, \qquad k_{i\,nl} = \tfrac{l^{i-1}K_{i\,nl}}{K}. \end{aligned} \tag{5}$$

Equation (3)represents a generalized model for a nonlinear system under material degradation. $\alpha$, $\beta$ and $\phi_i$ are the dimensionless degrading parameters added to the equation , that control the variations of linear stiffness, damping, and nonlinear stiffness of the system, respectively.

In general, if $n$ different degradation variables are considered, the degradation parameters are written as

$$\alpha = \alpha(X_1, ...X_j, ...X_n)\,, \beta = \beta(X_1, ...X_j, ...X_n)\,, \phi_i = \phi_i(X_1, ...X_j, ...X_n), j = 1, 2.., n \tag{6}$$

with $X_j$ as the degradation variables such as time, mechanical loading, frequency, temperature,... . Variation of the degrading parameters with $X_j$ is usually determined by experimental tests. In this paper, as is described in Section (3), it is assumed that the material properties are changed only by temperature (i.e. $n = 1$ and $X_1 = \theta$). It is also assumed, for the sake of simplicity, that the nonlinearity of the system is degraded with a similar trend to the linear stiffness $\alpha = \phi_i$.

## 3. Material Degradation of the elastomer

The primary mechanism of degrading for elastomers is not yet fully understood. However, the most significant phenomena that affect an elastomeric component during its service life are temperature, UV exposure, oxidation, ionizing radiation, and mechanical loading [3]. Thermal degradation, which has already been thoroughly investigated in the literature [4, 5, 6], refers to the chemical and physical processes in elastomers that occur as a result of temperature changes at a microstructural scale. This will in turn will lead to a change in macroscale mechanical properties.

To capture the variation of mechanical properties of elastomeric materials Dynamic Mechanical Thermal Analysis (DMTA) is often carried out. By applying harmonic deformation, DMTA can measure the storage modulus $E_{st}$ and the loss modulus $E_{los}$ of elastomer coupons against different parameters such as time, temperature, and frequency. As a viscoelastic material, the complex form of the elastic modulus can be approximated for the rubber as $E_{st} + i\,E_{los}$. Where the storage modulus is associated with the equivalent stiffness and the loss modulus represents the damping characteristics of the structure.

The elastomer under analysis in this paper is a carbon black filled, sulphur-cured rubber widely used in engine mounts. Dynamic mechanical analysis of the elastomer was performed on a MK-II DMTA in the shear mode. The experiments were carried out at a frequency of $10Hz$, at a heating rate of $2^{\circ C}/min$ and a double strain amplitude of $64\mu\,m$ over a temperature range of $\pm 100^{\circ C}$. Two samples were used and the storage modulus $E_{st}$ and the loss modulus $E_{los}$ were measured for each sample in this temperature range.
It should be noted that Equation (3) presents a generalized model for structures with material degradation and/or with parameters-dependent materials. For material degradation, parameters in Equation (6) are usually defined as functions of *lifetime*, while for parameters-dependent materials, the degrading parameters are described as functions of *the specific parameters*. Elastomers are considered as parameters-dependent materials as their properties may vary with different parameters such as temperature, frequency, and displacement. They also represent a high rate of degradation during their working life. In this paper, and as a preliminary step in nonlinear dynamics of degrading elastomers, the rubber is considered as a temperature-dependent material, where the overall variations of the storage and loss stiffness with temperature are shown in Figures (1) and (2).

## 4. System characterization

To characterize the nonlinear dynamics of the system introduced in Equation (3), a numerical parameter continuation technique is used. In numerical continuation, the stable periodic solution is firstly calculated for the harmonically forced system, and the evolution of this solution will be tracked by changing the system parameters. Different types of instabilities and bifurcation points such as limit points (LP), Neimark-Sacker (NS) bifurcations, or period doubling (PDB) can be investigated by proper evolution of the system parameters. The bifurcation analysis for the system is carried out using the continuation software MatCont [2].

The model in Equation (3) has two important parameters; dimensionless forcing amplitude $f_0$ and, excitation frequency ratio $\delta$. Moreover, the characteristics of the system are also changed with temperature $\theta$, through parameters $\alpha$ and $\beta$. The practical temperature range of the elastomer in typical usage is in the range of $[-20\,,\,100]^{\circ C}$, where two different behaviors are observed. Variation of the parameters $\alpha$ and $\beta$ (normalized at $40^{\circ C}$) are shown in Figures (3 and 4) at two different profiles termed as cold $(-20 < \theta < 0)$ and hot $(0 < \theta < 100)$ scenarios .
Other system specifications and parameters are chosen from Table (1) to perform continuation. It is also assumed that the second and third order nonlinear restoring force are acting on the system with $k_{2nl} = k_{3nl} = 1$.

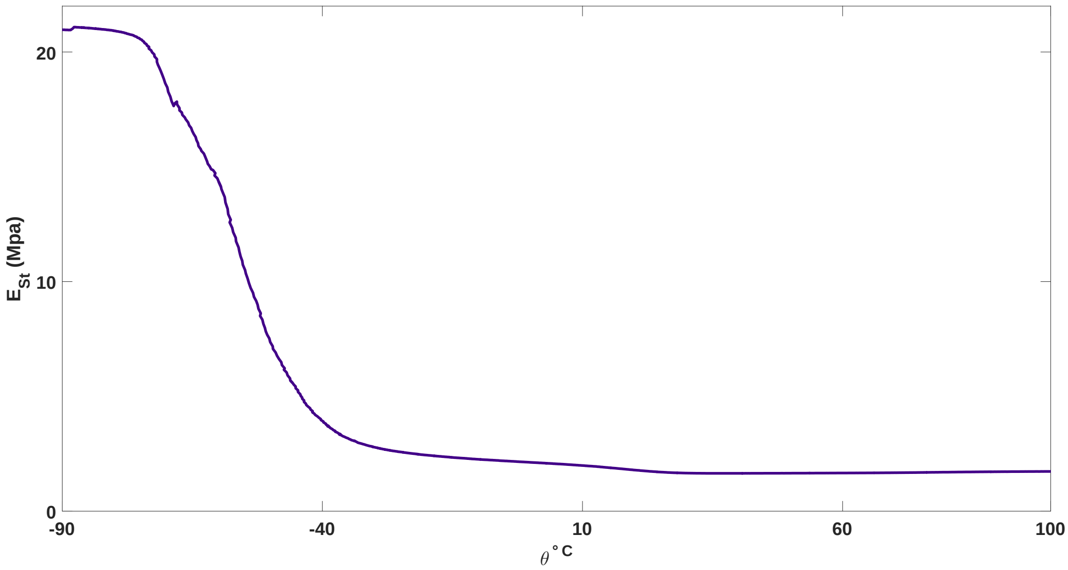



**Figure 1.** Temperature dependent storage modulus $E_{st}$ using DMTA data.

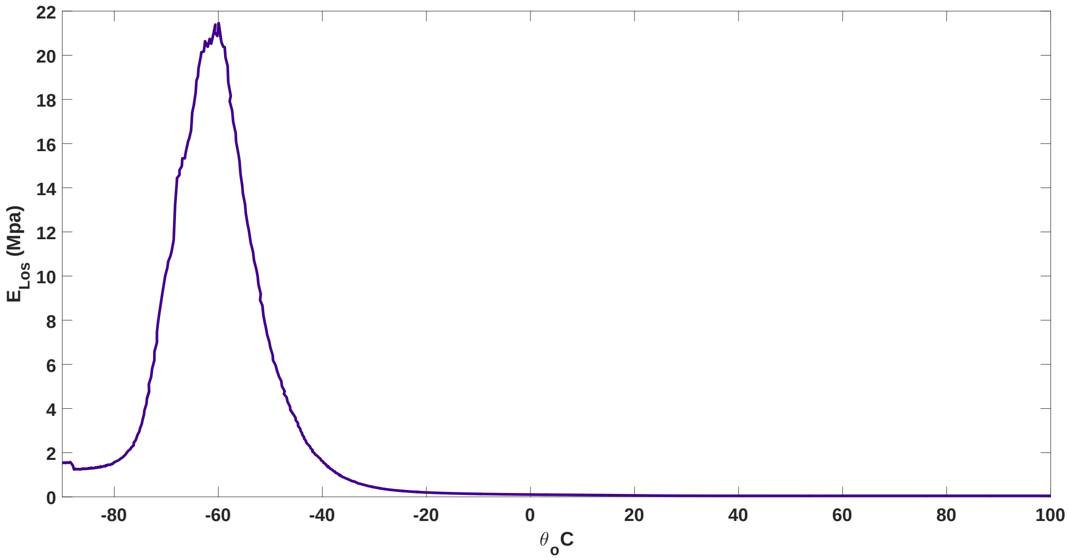
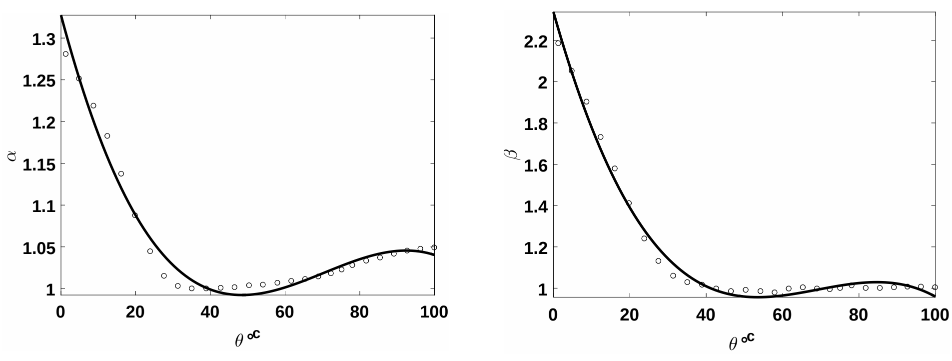
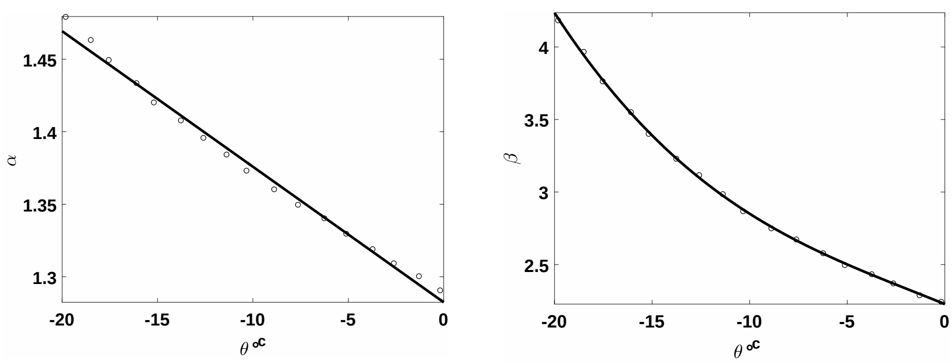



**Figure 2.** Temperature dependent loss modulus $E_{Los}$ using DMTA data.




(a) (b)

**Figure 3.** Variation of the degrading parameters (a) $\alpha$ (b) $\beta$ against temperature according to DMTA data (Hot scenario: $0 < \theta < 100$).




(a) (b)

**Figure 4.** Variation of the degrading parameters (a) $\alpha$ (b) $\beta$ against temperature according to DMTA data (Cold scenario: $-20 < \theta < 0$).

**Table 1.** System specifications.

| parameter | value |
|---|---|
| $\delta$ | 1 |
| $\zeta$ | 0.05 |
| $f_0$ | 3 |
| $l$ | 1 |

## 5. Results and Discussion

Two different temperatures from different scenarios are selected for analysis. The stable limit cycles at these temperatures are compared in Figure (5). By doing a continuation on the dimensionless forcing $f_0$ and from these stable limit cycles, the variation of the maximum amplitude $x_{max}$ is plotted for $\theta = 40^{oc}$ and $-1^{oc}$. Although the branch is stable in the cold scenario (Figure(6-b)); two LPs, two PDBs and an unstable branch are detected in Figure(6-a)

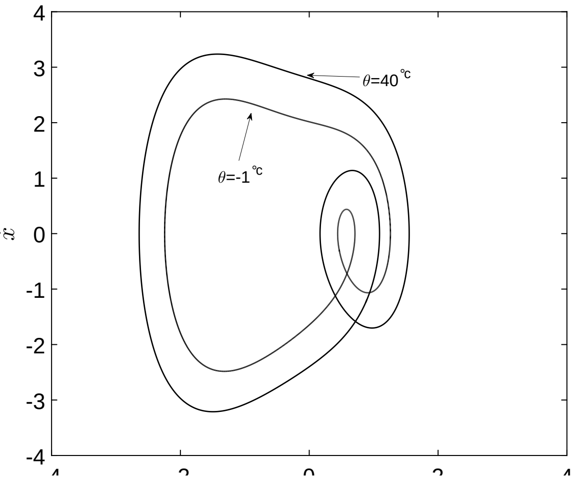



**Figure 5.** Stable limit cycles at to different temperatures.

for the hot scenario. The unstable solution is created between the LPs and having PDBs present is known to be an indication of more complex dynamic behaviour including chaos. Hence, as the system at the cold scenario behaves stably, different types of instabilities and unstable behaviour are observed at the hot scenario over a relatively wide range of $f_0$.

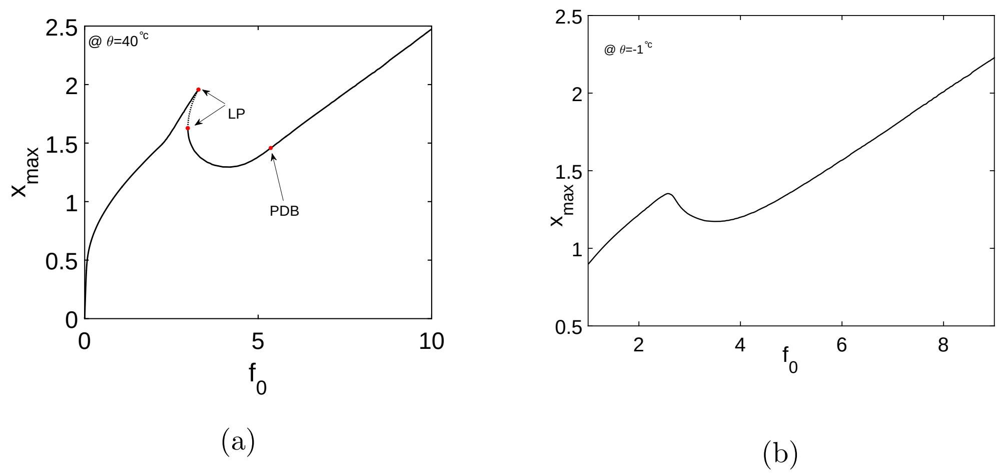
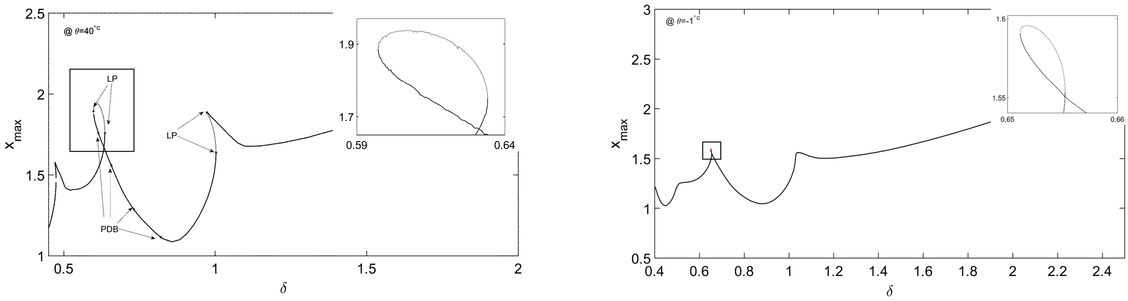



**Figure 6.** Variation of maximum amplitude $x$ against forcing amplitude $f_0$ at (a) hot scenario (b) cold scenario.

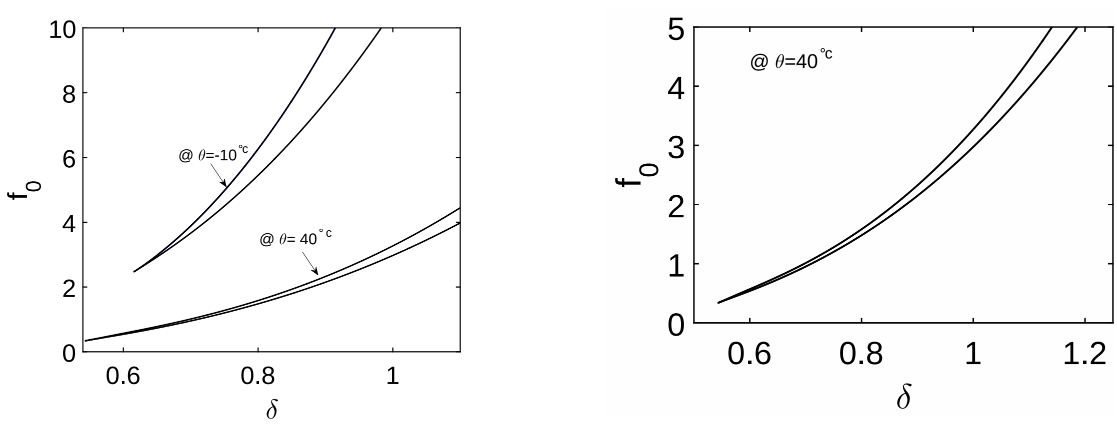
The influence of the exciting frequency on the dynamic behaviour is also investigated by sweeping $\delta$ around the resonant frequency using the continuation procedure. Results for the degraded system at the sample temperatures in the hot and cold scenarios are plotted in Figures (7). These figures, termed the nonlinear frequency responses (NL-FRF), disclose any instabilities that occur for both scenarios. Two LPs and an unstable branch are observed in both figures, while the unstable solution at the cold scenario occurs at a small range of $\delta$ and $x_{max}$ (Figure(7-b)).

As the exciting frequency $\delta$ changes, the unstable branch moves to different forcing amplitudes which should be clarified in nonlinear systems. Hence, in the last step, two-parameters




(a) (b)

**Figure 7.** Variation of maximum amplitude $x$ against dimensionless forcing frequency $\delta$ at (a) hot scenario (b) cold scenario.




(a) (b)

**Figure 8.** (a) Evolution of the unstable branches at different scenarios against the system parameters $f_0$ and $\delta$, (b) a zoom-in of the unstable area at the hot scenario .

continuation is conducted over a range of $\delta$ and $f_0$, initiated from the LPs. The sweeping frequency range is assumed as $0 < \delta < 1.5$. The two-parameters continuation reveals the evolution of the unstable branches and for the current model, it can be done at different temperatures. The locus of the LPs at different range of $\delta$ and $f_0$ are plotted and compared in Figures (8). Two different unstable ranges of $\delta$ and $f_0$ are observed. As it is expected, the unstable branch in the hot scenario happens at low forcing amplitudes ($f_0 < 0.4$) on a wide frequency range and with a narrow unstable area. Whereas, a thicker unstable region is seen for $\theta = -1^{oc}$ at a higher forcing range ($f_0 > 2.2$), far from the resonant frequency ($\delta < 1$).

## 6. Conclusion

This paper considers variation of the dynamic behaviour and stability conditions of a nonlinear structure with thermal induced material degradation. An elastomer is considered as the material under degradation.The experimental degrading function is coupled to a 1-DOF nonlinear dynamic model. Nonlinear and stability analysis of the system are performed using numerical continuation techniques in two different temperature degradation scenarios. The effects of the material degradation is highlighted when the different responses and instability conditions are observed as the system varies with temperature degradation.

## 7. Acknowledgments
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