Is Everything a Set? Quine and (Hyper)Pythagoreanism

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The story goes that today's orthodox ontological view—that there are physical objects, and abstract or mathematical objects, and nothing else, with mental entities at most reducing in some sense to physical objects—was firmly championed by Quine. True enough.   But he was the view's firm champion only for a comparatively short time. At the beginning of his career, Quine spent ten years or so resisting abstract objects, despite being the author of 'New Foundations for Mathematical Logic' and *Mathematical Logic*, works of no-nonsense abstract set theory. In 1946 he wrote 'I feel sure that nominalism can be executed, but I don't know in which sense' (CCE p. 23). He published a piece in 1947 with Nelson Goodman in support, if qualified, for nominalism. As late as *Word and Object* of 1960, to which one might well point as his most detailed statement of the 'orthodox ontological view', he gave a generous airing to nominalism, as if he wanted to examine it one last time, if only to show why in the end it cannot be 'executed'.   One recognises the commonsense appeal of the doctrine that there are only physical objects, or even that that is just what it means to speak of objects. But he also felt a tug in the opposite direction. In 1964 he published 'Ontological Reduction and the World of Numbers', in which he examines not the proposal to reduce the world of numbers to physical objects, but to reduce physical objects to the world of numbers. He explores the same idea in 'Ontological Relativity' of 1968, and revisits the idea time and time again throughout the rest of his life, notably in 'Whither Physical Objects?' of 1976.   In the 1976 piece he called the idea 'hyper-Pythagoreanism', which I will shorten in what follows to 'HPG' (he called it 'Pythagoreanism' in 'Ontological Reduction and the World of Numbers' of 1964 and 'Ontological Relativity' of 1968). Except in 'Whither Physical Objects?'—which apparently he saw as superseded by the view of 'Things and Their Place in Theories' of 1981 (and *Pursuit of Truth* 1992 and *From Stimulus to Science* of 1995), and which does not appear in any collection by Quine—it seems that he never quite embraced the doctrine, but he never categorically ruled it out, and it did figure in his later ontological calculations (notably in 'Assuming Objects' of 1994 and 'Naturalism; Or Living Within One's Means' of 1995; though not in 'Structure and Nature' of 1992).

The later works feature such statements as 'ontology as suffered a humiliating demotion' that it is 'defused' by ontological relativity, by which he means that although ontological questions remain real and substantive—contra Carnap— their scientific importance must be taken down a few pegs. But in this piece I will avoid for the most part the question of how Quine's estimate of ontology evolved from his early views to his classical view as expressed in *Word and Object*, to his view of it as relative and finally to his final 'structural' outlook (for the story see Kemp and Lugg, forthcoming). I will suggest that Quine could and perhaps should have accepted hyper-Pythagoreanism, without relinquishing any of his main commitments.

One preliminary point. Tarski proposed a criterion for a definition of truth in the form of T-sentences: Any (consistent) formal theory of truth for a language is adequate if it implies, for every sentence of the language in view, a sentence of the form 'S is true iff s', where s is replaced by a sentence of the language named by S (Tarski [1956] p. 404). This is Tarski's proposal for an *explicated* notion of truth. By implication the point is extended to the notion of reference (or satisfaction, 'true-of'): for example if truth is equated with *satisfaction by all sequences*, then an explicated scheme of reference is adequate if it implies the right 'truth-conditions', in effect the right extension

of the truth predicate. [1] Thus two assignments of reference which entail the same truth-conditions for all sentences of the language in question are semantically equivalent, are interchangeable from a scientific point of view (if one is 'materially adequate', then so is the other; ibid. p. 405).[2] Quine clearly accepts this.

1. I begin with Quine's idea of a 'proxy-function'. Two features distinguish the idea. First, in the simplest case, a proxy-function is a one-one function or bijection (later I consider cases of many-one proxy-functions) which for each object in the universe of the theory taken as argument, has a value which is distinct from the argument (ORWN pp. 217-8, OR p. 56, TT p. 19, PT p. 32; FSS p. 72). Illustrative examples are 'singleton set of' and 'spatio-temporal complement of' for physical objects, '+1' for integers.[3] Second, the general terms of the language of a theory are re-interpreted to be true of all and only the proxies of the objects they were originally true of, so that where we began with 'x is F' we now have 'the proxy of x is the proxy of an F' (the same is carried out for relational predicates). For example: 'Spot is spotted' now has the effect of 'The proxy of Spot is the proxy of a spotted thing'. The two sentences are materially equivalent: in general, a sentence is true just in case its proxy-transformation is true.

The point may be put as explicitly metalinguistic. Suppose a theory Γ is in view, and for ease of exposition assume that Γ has individual constants a, b, c and so on. If the Γ-example is 'Fa' then from a metalinguistic point of view we say "The referent of 'a' is something in the extension of 'F'"; given a proxy-function, we say "The proxy of the referent of 'a' is the proxy of something in the extension of 'F'". For dramatic effect we might abbreviate 'the proxy of ___' as '*___', writing the second option as "*The referent of 'a' is *something in the extension of 'F'". Using 'referent of' is materially equivalent to using (in every occurrence) '*referent of'.

Quine's conclusion is that reference is inscrutable: Given what we've said about an explicated notion of reference, a reference scheme is equivalent to any of its own proxy-schemes (FSS pp. 71-5; TT p. 19).

Quine does not consider seriously that the ordinary notion of reference might be subject to further constraints—causal or epistemic constraints for example—than what I am calling the explicated notion of reference. In 'Ontological Relativity' (pp. 29-54) it's clear that he thought that there are no semantic differences *in natural language* that do not show up in behavioural differences, and any such differences must show up at the level of assent and dissent to sentences, which roughly has the effect of an assignment of truth-values to those sentences. Suppose that we are speaking of one's home theory, one's total theory of everything. We accept such trivialities as "'London' refers to London"; more generally we implicitly accept the schematic rule (of thumb) "'t' refers to t" (or "'t' refers to x iff t=x") just as we implicitly accept the schema "'S' is true iff S" (PT p. 52, pp. 79-82). We 'acquiesce in the home tongue' as Quine puts it, as a way of putting our foot down to any purported regress of reference (OR p. 49). Among our fellows who speak the 'same language'—who use language more or less as we do—there is not normally any question of our speaking 'of' different things with our use of particular words. This normal smooth sailing is simply a point about linguistic behaviour, a matter of what we ordinarily say, the phonemes we tend to use in particular situations. There is nothing stopping us from interpreting 'the referent of' as '* the referent of', from 'rocking the boat', but the difference is immaterial. For Quine, we are 'free to choose' amongst equivalent

reference schemes since one's linguistic behaviour—in the form of assent and dissent from sentences of object language—would not be affected (TT p. 20). '[V]erbal behaviour proceeds undisturbed …. Nothing has really changed' (ibid).

Switching momentarily to radical translation, the idea is that nothing in linguistic behaviour could tell us whether the natives refer to things or *refer to things. As I see it, this just spells out the implications of the conception of reference described above: that so far as reference is concerned, different interpretations which make true all and only the same sentences say precisely the same thing. There are no immaterial referring rays connecting words to world, and no psychological mechanisms being posited. The illusion that there is more involves an over-estimation of our acquiescence in the home tongue, or what comes to the same thing, our habitual presupposition of a background, familiar scheme of reference (whose statements include "'London' refers to London", and so on).

2. I have considered what might look like ontology via one's monkeying around with semantics, with the interpretation of language. Quine often does so. But ontology does not have be considered semantically. In fact nothing in the argument to follow depends on the relativity of ontology or purported indeterminacy of semantics. Besides objections attacking the thesis that explicated conception of reference is scientifically all that can be salvaged from the commonsense notion of reference, one might be wary of theorising wholly from 'outside' one's theory or conceptual scheme, from a transcendental as opposed to an immanent point of view (bumping up on the limits of language, to add some Wittgensteinian spin; see TT pp. 21-2, 180; cf. PL pp. 19, 22). I'm not saying by any means that Quine himself is guilty of this. A central part of Quine's outlook is that philosophy, like any legitimate theoretical pursuit, must take place aboard Neurath's ship—the ship that is repaired while sailing the open seas. He said 'The world is as natural science says it is, insofar as natural science is right' (SN p. 405); in 'Things and Their Place in Theories', having just laid out the argument from proxy-functions, he writes:

> … I expressed … my unswerving belief in external things—people, nerve endings, sticks, stones….How is all this robust realism to be reconciled with the barren scene that I have been depicting? The answer is naturalism: The recognition that it is within science itself, and not in some prior philosophy, that reality is to be identified and described. (TT p. 21)

Semantics, in what for Quine is an empirically ill-founded sense, a theory that promises to tell us the uniquely right reference relation amongst empirically equivalent schemes, is an example of a would-be super-naturalist, pretending to a transcendent or god's eye enterprise, issuing from 'some prior philosophy'. Ontology, if it is to be a legitimate theoretical pursuit, must admit of being pursued within 'our theory' or language (in Kemp 2016 I claimed that this consideration shows that a Quinean can discount the spectre of the global underdetermination of theory). Proper ontological questions are at bottom just maximally general existence questions, related to ordinary existence questions in the way that 'Are there physical objects?' is to 'Are there wombats in New Zealand?'. Neither is especially a semantical question. Thus I propose to leave the issues of semantical determinacy aside in this section. I will speak occasionally of 'reference' but nothing I argue for depends on reference going inscrutable.

3. To the unwary, it often seems that Quine's contention is that the mere existence of proxy-functions forces one to relativise ontology wholesale. For example he writes: 'The conclusion is that there can be no evidence for one ontology as over against another, so long anyway as we can express a one-to-one correlation between them. Save the structure and you save all' (SN p. 405; see also TT p. 19, PT pp. 31-32). But a moment's reflection shows the application of a proxy-function needn't change the universe—the sum total of existing things—at all, as in 'cosmic complement of' (the universe minus the object): a theory must needs comprehend objects and their cosmic complements to begin with.   The extensions of particular terms trade places; the extensions of terms are swapped for their proxies. But the things which exist according to the theory as a whole may remain as they were; one can well-understand the reaction that an ontology and its proxy-alternative amount to the same, that they 'say the same thing'.   By contrast, HPG shows how certain proxy-functions pare down a given universe, i.e., how its range can be a proper subset of its domain. Again, the argument does not depend on any controversial semantic considerations. And by the way, for Quine, there is no paradox about one's having to recognise certain entities in order to disown them: the argument can be taken as a *reductio* of the supposition that there are certain entities, the ones the argument licenses one to disown (OR p. 58; cf. ORWN p. 215).

Now before finally discussing HPG itself it may help to have before us a quick sketch of Quine's view of the place of physics. Physics is 'basic' for Quine in that it 'discover[s] the ultimate constituents of the world and their regularities' (CCE p. 166). That is not to say that other sciences are reducible to physics, but they do not add to ontology: 'Other sciences that are likewise concerned to discover regularities in the world are the for the most part not derivable from physics, but only because their objects are excessively complex aggregates of physical elements' (ibid). Descriptions, predicates and laws of the special sciences may be irreducible '[b]ut the behavior of these gross objects *is* … the sum of the behavior of the ultimate physical constituents, however incalculable' (ibid; Quine's emphasis). Clearly the notion of 'the physical' is post-Newton, post-Einstein, and post-quantum theory: by definition it aims to encompass the 'ultimate constituents of the world and their regularities'; it includes the latest discoveries within physics but it is not beholden to physics as presently constituted, for '[t]he objective is just full coverage of the ultimate ingredients and forces of the world, whatever they may be' (ibid).

The takeaway is simply that for Quine, the question of the ontology of science just is that of the ontology of physics.

HPG appears in Quine's published work for the first time in 1964, in 'Ontological Reduction and the World of Numbers'. Responding to George Berry's idea that the Löwenheim-Skolem theorem—that any true theory has a model with a universe wholly within the natural numbers—implies that only 'common sense stands in the way of adopting an all-purpose Pythagorean ontology: natural numbers exclusively' (OWRN p. 214), Quine replies that on the contrary there is something standing the way: To say that *there is* such a model is not enough to specify an actual interpretation in the natural numbers (p. 215, p. 219; but see also PT p. 31-33). The problem is not that the predicates of a theory which has been subjected to Löwenheim-Skolem would not be effective or arithmetical (although that is true). The crucial consideration is simply that the Löwenheim-Skolem argument generates no recipe for identifying a function which takes any object from the original theory as input and yields a natural number as output; in fact Löwenheim-Skolem does not itself provide assurance that such a function exists (ORWN p. 219, OR p. 62; he pointed out later that we might

thanks to an 'extraordinary coincidence' be vouchsafed such a function, but the argument for the theorem provides no clue as to what it is; WPO? p. 503).[4] 'Ontological Relativity' adds further considerations, but the general picture and result are the same.

That is why 'Whither Physical Objects?' counts as progress, for now it appears that one *can* specify a proxy-function whose images lie within the realm of numbers, which can be understood via various means as abstract classes (Frege, Zermelo, von Neumann etc.); it therefore counts as a 'genuine reduction' by the lights of 'Ontological Reduction and the World of Numbers'. The argument is sometimes called the 'Argument from Physics', because it uses certain considerations from physics to specify a (now many-one) proxy-function.

We first make precise the notion of a physical object or body as 'as the aggregate material content of any portion of space-time, however ragged and discontinuous' (WPO? p. 497). But the idea of 'material content' is not itself well-behaved, because according to physics it makes no sense to speak of material content—for example an electron—as always being identified over time:

> It is sometimes wholly arbitrary to say whether two points events a and b are moments in the career of one continuing electron or moments rather in the careers of two different electrons. The point event a is followed by b, and meanwhile another nearby point event c is followed by d; and it can be a wholly arbitrary matter whether to assign a and b to one electron, c and d to another, or to assign them crosswise: a and d to one electron, c and b to another. What are objective are, at best, the point events. (p. 498).

Further considerations show that 'point events' remain too 'classical', in that they are likewise not free of arbitrariness:

> Thus consider … the east box and the west box, and two electrons x and y. Common sense recognises four ways in which x and y could occupy the boxes: they could both be in the east box, or both be in the west box, or x could be in the east box and y in the west, or vice versa. But statistical findings show … that these last two apparent possibilities have be counted as just one … the moral is that we should think not of individual electrons x and y at all, but of states of the boxes .. (pp. 498-9)

Point-events, inasmuch as they are infinitesimal temporal and spatial slices of particles, entail that the two possibilities remain two. Electrons are just one example; other particles—in particular other 'fermions' such as protons, neutrons, and leptons—are subject to the same indeterminacies.[5] 'We are left … with a field theory, a theory of the distribution of states over space-time' (p. 499). Quine is quick to add that the states or magnitudes themselves—temperature, length, charge, mass etc.— do not have to figure in the ontology, as entities themselves. The state-types figure in the theory merely as open sentences or predicates, which as a matter of longstanding principle Quine does not quantify over, does not explain as standing for properties or entities of any sort.[6] As long as we have numbers at our disposal, then there will be, for any measurable magnitude—temperature for example—relational predicates of the form 'the temperature of Φ is x', where Φ is a field and x is a real number (we could understand them as functions from fields to real numbers as at WPO? p. 503, but such functions would not add to ontology; see the next paragraph).[7]

By this time Quine has bid adieu to Nominalism; he is assuming that the 'full mathematical apparatus' must remain part of the ontological picture. Thus since mathematics reduces to pure set theory, one can identify space-time points themselves as quadruples of real numbers—*x, y, z, t* with *t* representing time—in the Cartesian fashion, and then space-time regions in turn as sets of points (p. 501). Thus 'physical objects have evaporated into mere sets of numerical coordinates' (p. 502; those sets are elsewhere called 'number tables': CCE p. 457, 470; FSS pp. 71-2). But that those are the objects does not imply that physics reduces to mathematics or set theory or let alone that statements of physics are provable; for it does not imply that the predicates we use to talk about the objects are themselves predicates of mathematics or set theory, or that they are recursive or arithmetically definable (WPO? p. 503). The twin lexicons 'stand stubbornly apart' (ibid).

Quine seems to have withdrawn his support for this argument almost immediately. He no longer positively asserts its conclusion after 1976, does not except for that one time assert that everything is a set. In 'Things and Their Place in Theories' of 1981, although he does mention in passing the possibility of HPG, he placed more emphasis on the argument from one-to-one proxy-functions to show that ontology is not so important as he once thought, that the real lesson is what he calls global structuralism. As hinted above, this is not to say that structures are all that exist. Speaking from within our theory or language, his view is that what exist rather are rabbits, electrons and numbers, even though one's evidence for saying so is equally evidence for espousing the existence of some proxy-equivalent alternative (SN pp. 405-6; though as above doubts can be raised about whether these alternatives are genuinely alternatives).

4. Quine says over and over that the notion of kind of object, and even the general notion of an object, are man-made, that objects are all posits (WO pp. 21-5; TT pp. 1, 8-15). All reification, all objects, properly speaking, are in a certain broad sense theoretical (TT p. 20). There is a continuum from the familiar 'posits' we gain the capacity to refer to as infants learning language to the more abstruse posits we refer to as relative grown-ups such as quasars, quarks and transfinite cardinals. They are all in the same boat epistemologically, differing only by the degree of conceptual sophistication involved in facility with the relevant language. We speak not just of quarks but also ordinary bodies such as rabbits thanks to our concepts, or our words, some of which are deliberately formulated but others handed down in at least rough form by evolution or culture. We might have had others, and some, or most, are renounced as science formulates its official doctrines. For the ontologist anyway, minded to regiment, it does not matter how they got to where they've got; all of it is on a level, and he desires to formulate the smoothest and most economical scheme he can. And Quine did not ever, so far as I know, forswear his allegiance to Occam's maxim. So why not go hyper-Pythagorean?

Two points that repeat material above, then two new points (though they are due to Quine himself).

First, one-to-one proxy-functions alone needn't move the ontologist: he might just go along with the proxies-idea, saying 'The proxy of x is a proxy of F' whenever he wanted to say what he used to say by 'Fx', so long as the overall ontology remains as it was. The overall ontology is the game he's interested in.

Second, one can make such a judgement as HPG recommends from an immanent point of view, from inside one's theory, from aboard one's Neurathean ship; there is no sense in which the judgement must be made from a transcendent point view, entirely bracketing all ontological

commitment. We are not considering alternative schemes of reference, where we have to consider our language as whole, and consider alternative ways of interpreting it from some other point of view. The Argument from Physics is no more reliant on a global metaperspective than any other question of what there is, like whether Venus has any moons. It's simply an attempt to answer the immanent and non-linguistic question 'What is there?'.

And now the two new points.

First, the place where I think Quine could have taken another line is where he characterises HPG as *renouncing* physical objects, of physical objects as *evaporating* into pure sets (p. 501-3). An important point that is stressed throughout the closing chapter of *Word and Object* but especially in §54, is that 'explication is elimination'. This means that if the relevant theoretical purposes filled by a certain otherwise troublesome or otherwise unwanted form of referential words are clear and capable of precise formulation, and those purposes can be fulfilled by other referential words drawn from an accepted vocabulary lacking in such troubles, then the troublesome questions can be dispensed with by accepting the explication:

> We have, to begin with, an expression or form of expression that is somehow troublesome. It behaves partly like a term but not enough so, or it is vague in ways that bother us, or it puts kinks in a theory or encourages one or another confusion. But also it serves certain purposes that are not to be abandoned. Then we find a way of accomplishing those same purposes through other channels, using other and less troublesome forms of expression. The old perplexities are resolved. (WO p. 260)

Exhibit A is the notion of the ordered pair, whose precise purpose is to satisfy the condition that if $\langle x, y \rangle = \langle z, w \rangle$, then $x=z$ and $y=w$. Various set theoretic surrogates can do the job, such as Wiener's $\langle x,y \rangle =_{df} \{\{x\}, \{y, \varnothing\}\}$ or Kuratowski's $\langle x,y \rangle =_{df} \{\{x\}, \{x, y\}\}$. Questions about the ordered pair are thus eliminated, for given such an explication we speak only of classes, not of classes-plus-pairs. Similar things hold of the natural numbers and the various schemes for explicating them as classes (Frege, Zermelo, Von Neumann etc.). Just as one is indifferent—beyond inessential pragmatic matters— between Wiener's method and Kuratowski's, one is indifferent between the various class theoretic construals of natural number.   They 'conflict with another only out among the don't cares' (WO p. 260), where a 'don't care' is question like whether $2 \in 3$. So long as the explication enables one to do arithmetic, there are according to any such explication no further scientific questions about numbers.

Quine applies the lesson of the ordered pair to the question of whether with Sir Arthur Eddington we should renounce bodies in favour of regions of space populated by swarms of molecules, or with Susan Stebbing identify the two (WO p. 265)   It's a don't care for Quine; he sees only a matter of decision. As far as I can see much the same line can be taken about the predicate 'x is a physical object' under HPG. Whatever serious mismatches between intuition and the HPG explication of physical object are already present in the initial sharpening of the intuitive notion of 'physical object or body' to the notion of a 'material content of a region of space-time, however discontinuous or scattered'. Under the envisaged HPG explication, there is no reason not to say that 'x is a physical object' is *true* of the things which will have replaced what on the old scheme were physical objects (and we can even, if we were so minded, regard 'is an abstract object' as false of those things).   'My left elbow is a physical object' will be true even if the deeper nature of my elbow is to be a set of

quadruples (but the number two will remain a non-physical object). We have no remaining questions that are not 'don't cares'. We have ontological savings through shrinking the domain, thus obeying Occam's maxim.

It is all one whether, from a strictly logical point of view, we choose to identify physical objects with their set theoretic surrogates or to eliminate them in favour of the surrogates. These are but two different ways of describing the same situation, as Quine insists. But just as Quine identifies *mental* states with their physical correlates (at the level of tokens), rather than eliminating or repudiating them, simply on the grounds that it is rhetorically less harsh and revisionary, so it is with physical objects and their set-theoretic surrogates.   '[T]he distinction between an eliminative and an explicative physicalism is unreal' but the former (in the case of mental states) is 'less drastic'; WO p. 265).

Second, as we have seen Quine stresses that proxy-functions make a point in the epistemology of ontology, not ontology proper: the evidence for a chosen ontology will support to an equal degree any number of proxy-equivalent schemes of reference.   But as above it isn't clear how this is an *ontological* difference, because nothing in the argument from proxy-functions alone requires that the overall domain of a given theory should change as we move from a given reference scheme to a proxy-equivalent scheme. Suppose then we required, in addition, that for substantive ontological import the range of values of the proxy-function must lie partly or wholly outside its range of arguments. We would move from our universe to another, partly or wholly new. But then we really would be in the unenviable position touched on earlier of having to countenance entities which are not real. Quine himself makes the point (where θ and θ' are the theory and its proxy):

> I must admit that my formulation suffers from a conspicuous element of make-believe. Thus, in the Carnap case I had to talk as if there *were* such things as $x°$C … My formulation belongs, by its nature, in an inclusive theory that admits the objects of θ, as unreduced, and the objects of θ' on an equal footing. (ORWN p. 219)

HPG, in embodying only a paring-down operation, is free of this 'imperfection' (ibid). As before, it is entirely an immanent procedure, in no tension with naturalism as Quine understands it. It is well in keeping with the view of ontology expressed in *Word and Object* but also with the somewhat watered down views of ontology expressed later.

So although he came to take it back soon after, Quine may have been right when he said the following in the final sentence of 'Whither Physical Objects?': 'We might come to look to pure mathematics as the locus of ontology as a matter of course, and consider rather that the lexicon of natural science, not the ontology, is where the metaphysical action is' (p. 504).

5. Speaking of metaphysics, it must be allowed that the metaphysical situation is marginally transformed as matter of ideology under the hypo-pythagorean shift, somewhat contrary to the spirit of 'save the structure and you save all'.[8] It is not a case of holding all the sentences steady in truth-value through varying interpretations. Although as I've said one can save the extensions of certain expressions such as 'x is a physical object' if one likes, nevertheless it will be an inescapable truth, for example, that each rabbit will contain set-theoretic members. But that is not obviously more repellent to untutored intuition than that rabbits are mostly empty space or that some

infinities are greater than others. And it is not so far away from rabbits' being mereological fusions, as indeed many think.

Speaking of Naturalism in the *Roots of Reference*, Quine writes 'we gain access to the resources of natural science', but 'we accept the methodological restraints of natural science' (p. 34). We might add that, granted that we are all sailing on the Neurathean ship of knowledge, the philosopher is by no means the captain; he or she must go where the ship is going. [9] On the other hand one might see where the ship is headed and jump off. One philosopher's Modus Ponens is another's Modus Tollens: granted the validity of what I've argued here, one might still be so wedded to rabbits not being sets as reason to cast aspersions on the premises in play. One might point to the idea that all objects are posits.

References

Frege, Gottlob. 1964 [1893]. *The Basic Laws of Arithmetic.* Translated and Introduced by M. Furth. Berkeley: University of California Press.

Goodman, Nelson and Willard Van Orman Quine. 1947. 'Steps Toward a Constructive Nominalism'. *The Journal of Symbolic Logic* 12:4: 105-122.

Janssen-Lauret, Frederique and Gary Kemp. 2016. *Quine and His Place in History*. New York, NY: Palgrave-Macmillan.

Kemp, Gary. 2016. 'Underdetermination, Realism, and Transcendental Metaphysics in Quine'. In Janssen-Lauret and Kemp: 168-88.

Kemp, Gary and Andrew Lugg. 'Quine on Ontology: Chapter 7 of *Word and Object*'. Unpublished.

Quine, Willard Van Orman. 1951a [1940]. *Mathematical Logic*, revised edition. Cambridge, Mass.: Harvard University Press.

-- 1960. *Word and Object.* Cambridge, Mass.: M. I. T. Press. Noted as WO.

-- [1963]. 'Necessary Truth'. In Quine 1976a: 68-76.

-- [1964]. 'Ontological Reduction and the World of Numbers'. In Quine 1976a: 212-20. Noted as

ORWN.

-- 1969. *Ontological Relativity and Other Essays*. New York: Columbia University Press.

-- [1968] 'Ontological Relativity', in Quine 1969a. Noted at OR.

-- 1974. *The Roots of Reference*. La Salle, Ill.: Open Court. Noted as RR.

-- 1976a. *Ways of Paradox*, revised edition. Cambridge, Mass.: Harvard University Press.

-- 1976b. 'Whither Physical Objects?'. Studies in the Philosophy of Science 39: 303-10. Noted as WPO?.

-- 1981. *Theories and Things.* Cambridge, Mass.: Harvard University Press.

-- [1981] 'Things and Their Place in Theories'. In Quine 1981: 1-21. Noted as TT.

-- 1986. *Philosophy of Logic*, second edition. Cambridge, Mass.: Harvard University Press. Noted as PL.

-- 1992. *Pursuit of Truth*, second edition. Cambridge, Mass.: Harvard University Press. Noted as PT.

-- [1992]. 'Structure and Nature'. In Quine 2008: 401-6. Noted as SN.

-- [1994]. 'Assuming Objects'. In Quine 2008: 449-60.

-- 1995a. *From Stimulus to Science*. Cambridge, Mass.: Harvard University Press. Noted as FSS.

-- [1995b]. 'Naturalism; Or, Living within One's Means'. In Quine 2008: 461-72.

-- 2008. *Confessions of a Confirmed Extensionalist and Other Essays*, ed. by D. Føllesdal and D. Quine. Cambridge, Mass.: Harvard University Press. Noted as CCE.

Tarski, Alfred. 1983 [1933]. 'The Concept of Truth in Formalized Languages'. In *Logic, Semantics, Meta-Mathematics*. Alfred Tarski, trans. by J. H. Woodger, edited and introduced by J. Corcoran. Indianapolis, IN: Hackett. 152-278.

Tarski, Alfred. 1983 [1955]. 'The Establishment of Scientific Semantics'. In *Logic, Semantics, Meta-Mathematics*. Alfred Tarski, trans. by J. H. Woodger, edited and introduced by J. Corcoran. Indianapolis, IN: Hackett. 401-408.

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[1] Frege played with what appears to be the same idea in his *Grundgesetze*—Frege 1892 §10, although unlike Quine does not restrict the notion to the first-order case.

[2] I am skipping over Tarski use of 'satisfaction' for a relation between sentences, open or closed, and infinite sequences of objects in his [1933]. In [1956] Tarski speaks only of a simplified notion of a relation between open sentences or predicates and objects. Quine would often speak of 'denotation' for that relation. And for Quine the relation between a proper name and its bearer—which one might suppose is the paradigm of reference—is not needed for serious purposes. These are technical matters, which don't as far as I can see affect what I'm saying here.

[3] The proxy-function can be a mere 'virtual function', not an entity within the range of quantifiers. The key idea is the notion of a 'virtual class' as in *Set Theory and Its Logic* §2: Instead of 'Fa' we write 'a is a member of {x: Fx}'. This enables one to speak 'of' certain entities in a way that can be paraphrased into language without any actual commitment to those entities.

[4] I thank Bruno Whittle for clarifying Quine's thinking on this point. In further detail: It is not sufficient that for any selection of sets $s_1, …, s_n$, we can show that there is a function p such that for any formula G, $G(s_1, …, s_i)$ iff $G^*(p(s_1),…,p(s_i))$, where G^* is suitably related to p. Rather, we would have to establish that there is a function p such that for any sets $s_1, …,s_i$, and any formula G, $G(s_1, …, s_i)$ iff $G^*(p(s_1),…,p(s_i))$.

[5] Thanks to Kenny Wraight for setting me straight on this.

[6] For Quine, there is reason to contend that speaking of properties is more than a convenient way of speaking—and not just to speak of the predicates themselves or extensions of predicates and open sentences—only if there is a compelling reason to employ second-order quantification that somehow does an

essential job beyond what extensions of predicates do. There is for Quine no theoretical gain in doing so, and it would commit one to entities with no clear criteria of identity.

[7] I thank Stephan Leuenberger and Adam Rieger for forcing me to be clear in this paragraph.

[8] I thank Nathan Kirkwood for making me stress this.

[9] Overall I owe a great deal of what I say here to Andrew Lugg, who of course is not to be blamed for it.