

Receiver Design for DCT based Multicarrier Signals

Adnan Zafar*, Chang He*, Pei Xiao*, Muhammad Ali Imran*, Aijun Cao[†]

*

{a.zafar, c.he, p.xiao, m.imran}@surrey.ac.uk

: Institute for Communication Systems (ICS), University of Surrey, Guildford, UK

†

{cao.aijun}@zte.com.cn

: ZTE Wistron Telecom AB, 19F, Farogatan 33, Kista Science Tower, 164 51 Kista, Sweden

Abstract—DCT based multicarrier system also known as fast orthogonal frequency division multiplexing (FOFDM) is a promising multicarrier transmission technique that requires half the subcarrier spacing compared to conventional OFDM technique. The signal processing complexity and power consumption of such system is also less due to its real arithmetic operations compared to DFT based system (OFDM) that require complex arithmetic operations. However, unlike OFDM, FOFDM requires a finite impulse response (FIR) front-end pre-filter at the receiver to achieve single-tap equalization for simplifying the receiver design. The receiver design can be further improved using the fact that FOFDM system transmits real valued symbols compared to complex valued symbols in conventional OFDM. This fact enabled us to improve the system performance by exploiting the improperness of such DCT based multicarrier signals using widely linear processing (WLP). In this paper, a novel equalization technique using WLP is proposed to effectively improve the system performance, and it is shown that the proposed FOFDM receiver can provide better estimate of the transmitted symbols and outperforms its OFDM counterpart.

Index Terms—Widely linear processing, Fast OFDM, OFDM

I. INTRODUCTION

FAST orthogonal frequency division multiplexing (FOFDM) is a promising multicarrier technique that can provide twice the data rate as compared to conventional OFDM technique [1]. FOFDM utilizes one-dimensional symbols (real symbols) for transmission. This enables it to reduce the required subcarrier spacing to half of that of the conventional OFDM, leading to a promising multicarrier system with high data rate capability [2].

Unlike conventional OFDM system that employs discrete Fourier transform (DFT), FOFDM adopts discrete cosine transform (DCT) for multiplexing the symbols on the subcarriers. This also reduces the complexity of the system as DCT only uses real arithmetic as opposed to the complex valued DFT in OFDM. This reduces the transmitter complexity and power consumption for

FOFDM. Additionally, the intercarrier interference (ICI) coefficients in DCT-based multicarrier system are more concentrated around the main coefficient than in DFT based multicarrier system, resulting in better improved robustness against frequency offsets. Since only one dimensional modulation is used for FOFDM the phase estimation in the coherent detection at the receiver is also simplified [3]. Some successful experiments have demonstrated that 14.348 Gbit/s data rate is achievable in optical FOFDM system [4]. Due to such merits, it is expected that FOFDM will obtain more attentions in near future. However, one major challenge for FOFDM is the equalization issue under frequency-selective channels due to lack of circular convolution property in DCT transform. As a consequence, channel cannot be easily compensated by single-tap equalization in FOFDM unless the channel impulse response (CIR) is symmetric. Fortunately, two effective methods have been proposed in literatures that enables FOFDM to support single-tap equalization at the receiver [3], [5]. The first simpler approach involves zero-padding instead of cyclic prefix but leads to intercarrier interference. The second method involves inserting prefix and suffix into each data symbol block at the transmitter whilst a so called prefilter is imposed at the front side of the receiver to achieve symmetric channel impulse response (CIR).

To the best of our knowledge all the research related to FOFDM employs linear processing techniques for the estimation of the transmitted symbols. But as FOFDM utilizes single dimensional modulation schemes like amplitude shift keying (ASK) that generates real valued/improper signal constellation. This improperness can be exploited using the widely linear processing (WLP) technique to improve the performance of the system. In this paper we have focused on exploiting the improperness of FOFDM signals using widely linear filtering and the major contribution in this regard is related to the investigation of how widely linear receiver affects the FOFDM system performance. The perfor-

mance is evaluated by measuring the mean square error (MSE) and bit error rate (BER) of FOFDM system under frequency selective channel and the results are compared with conventional linear processing.

The remaining paper is organized as follows. In section II, a brief discussion on improper signals and widely linear processing is presented along with the system models of FOFDM. The simulation parameters are discussed in section III. Results are given in section IV and the conclusion is given in section V.

II. PRELIMINARIES

A. Improper signals and widely linear processing

Classical linear signal processing techniques are widely used in wireless communication systems that employs circular (or proper) signals e.g. M-ary Phase Shift Keying (MPSK) and M-ary Quadrature Amplitude Modulation (MQAM) etc. But in various cases, the transmitted signals are non-circular (or improper) e.g. Amplitude Shift Keying (ASK) and Offset Quadrature Amplitude (OQAM) etc. In such cases, the linear processing techniques do not take into account all the second order statistics of the received signal and therefore the estimation at the receiver is suboptimal. Widely linear processing (WLP) takes advantage of the impropriety of these signals, by processing the signal together with its conjugate version to obtain a more precise estimate at the receiver [6]

To understand improperness, let us define a complex valued random vector \mathbf{s} as $\mathbf{s} = \mathbf{s}_I + j\mathbf{s}_Q \in \mathbb{C}^{(N \times 1)}$, where \mathbf{s}_I and \mathbf{s}_Q are real valued random vectors i.e. $\mathbf{s}_I, \mathbf{s}_Q \in \mathbb{R}^{(N \times 1)}$ with zero mean. The second order statistics of \mathbf{s} can be defined using the autocorrelation matrix (\mathbf{R}_{ss}) and pseudo-autocorrelation matrix (\mathbf{R}_{ss^*}) as $\mathbf{R}_{ss} = E\{\mathbf{s}\mathbf{s}^H\}$ and $\mathbf{R}_{ss^*} = E\{\mathbf{s}\mathbf{s}^T\}$ respectively. Where $E(\cdot)$ is the expectation operator. In order for \mathbf{s} to be proper or circular, the complete second order statistics of \mathbf{s} should be completely defined by \mathbf{R}_{ss} only. But if the second order statistics are described by both \mathbf{R}_{ss} and \mathbf{R}_{ss^*} , then the complex random vector \mathbf{s} will be improper/non-circular [6]. The improperness of such random vectors can be exploited using widely linear processing (WLP) at the receiver.

The receiver with WLP consists of a widely linear minimum mean square error (WL-MMSE) estimator. The estimator make use of the received data vector $\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$ and its conjugate version \mathbf{r}^* to estimate the transmitted symbol \mathbf{s} . Where \mathbf{H} is the channel matrix and \mathbf{n} is the Gaussian noise. The structure of the WL-MMSE estimator is given in Fig. 1.

From Fig. 1, we can write the expression for the WL estimator as (1)

$$\hat{\mathbf{s}} = \mathbf{f}_1^H \mathbf{r} + \mathbf{f}_2^H \mathbf{r}^* \quad (1)$$

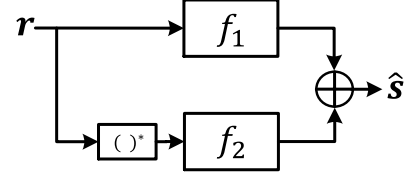


Fig. 1: Widely Linear Estimator

where \mathbf{f}_1 and \mathbf{f}_2 are two receive filter vectors and are designed in order to minimize the mean square error between the transmitted symbol vector \mathbf{s} and the estimated symbol vector $\hat{\mathbf{s}}$. The filter vectors \mathbf{f}_1 and \mathbf{f}_2 are obtained using (2)

$$\begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{rr} & \mathbf{R}_{rr^*} \\ \mathbf{R}_{rr^*}^* & \mathbf{R}_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{r}_s \\ \mathbf{r}_v^* \end{bmatrix} \quad (2)$$

where $\mathbf{R}_{rr} = E\{\mathbf{r}\mathbf{r}^H\} = \mathbf{H}\mathbf{R}_{ss}\mathbf{H}^H + \mathbf{N}_o\mathbf{I}$ is the auto-correlation matrix, $\mathbf{R}_{rr^*} = E\{\mathbf{r}\mathbf{r}^T\} = \mathbf{H}\mathbf{R}_{ss}^*\mathbf{H}^T$ is the pseudo-correlation matrix and finally $\mathbf{r}_s = E\{\mathbf{s}^*\mathbf{r}\} = \mathbf{H}\mathbf{R}_{ss}^*$ and $\mathbf{r}_v = E\{\mathbf{s}\mathbf{r}\} = \mathbf{H}\mathbf{R}_{ss}$. The solution to (2) is given as (3) and (4)

$$\mathbf{f}_1 = [\mathbf{R}_{rr} - \mathbf{R}_{rr^*}(\mathbf{R}_{rr}^*)^{-1}\mathbf{R}_{rr^*}]^{-1} [\mathbf{r}_s - \mathbf{R}_{rr^*}(\mathbf{R}_{rr}^*)^{-1}\mathbf{r}_v^*] \quad (3)$$

$$\mathbf{f}_2 = [\mathbf{R}_{rr}^* - \mathbf{R}_{rr}^*(\mathbf{R}_{rr})^{-1}\mathbf{R}_{rr}]^{-1} [\mathbf{r}_v^* - \mathbf{R}_{rr}^*(\mathbf{R}_{rr})^{-1}\mathbf{r}_s] \quad (4)$$

The widely linear filters \mathbf{f}_1 and \mathbf{f}_2 together with the received vector \mathbf{r} and its conjugate version \mathbf{r}^* provides more precise estimate of the transmitted signal \mathbf{s} compared to linear processing technique as the difference $\Delta\mathcal{J}$ given as (5) between mean square error of a linear estimator \mathcal{J}_{L-MMSE} and widely linear estimator $\mathcal{J}_{WL-MMSE}$ is always non-negative [6]

$$\Delta\mathcal{J} = \left[\mathbf{r}_v^* - \mathbf{R}_{rr}^*(\mathbf{R}_{rr})^{-1}\mathbf{r}_s \right]^H \left[\mathbf{R}_{rr}^* - \mathbf{R}_{rr}^*(\mathbf{R}_{rr})^{-1}\mathbf{R}_{rr} \right]^{-1} \left[\mathbf{r}_v^* - \mathbf{R}_{rr}^*(\mathbf{R}_{rr})^{-1}\mathbf{r}_s \right] \quad (5)$$

It is because the matrix $[\mathbf{R}_{rr}^* - \mathbf{R}_{rr}^*(\mathbf{R}_{rr})^{-1}\mathbf{R}_{rr}]$ is positive definite and $\Delta\mathcal{J} = 0$ only when the matrix $[\mathbf{r}_v^* - \mathbf{R}_{rr}^*(\mathbf{R}_{rr})^{-1}\mathbf{r}_s] = 0$. Therefore we can say that $\mathcal{J}_{L-MMSE} \geq \mathcal{J}_{WL-MMSE}$ and that the WL estimator gives more precise estimation than linear estimators.

B. System Model

A DCT-based FOFDM is proposed through the use of a front end filter at the receiver to keep ICI and intersymbol interference (ISI) free transmission while achieving simpler equalization at the same time [3]. The

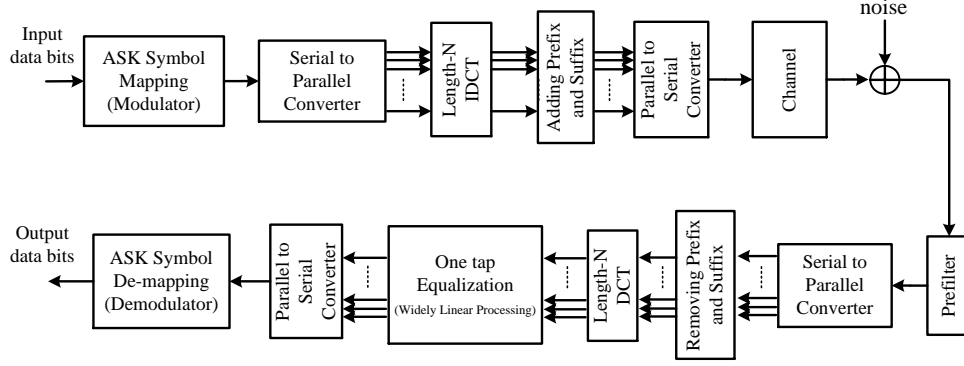


Fig. 2: DCT based multicarrier system

block diagram of this method is illustrated in Fig.2 The complete DCT based multicarrier system shown in figure Fig. 2 can be modeled as (6)

$$\mathbf{y} = \frac{1}{\gamma} \mathbf{DRPHCD}^H \mathbf{s} + \mathbf{DRPn} \quad (6)$$

where \mathbf{y} is the signal received at the input of the equalizer, $\mathbf{s} \in \mathbb{R}^{N \times 1}$ is the transmitted real symbol vector with normalized power. $\mathbf{D} \in \mathbb{R}^{N \times N}$ is power normalized DCT matrix. $\mathbf{C} \in \mathbb{R}^{L_1 \times N}$ is the matrix implementation of adding prefix (L_p) and suffix (L_s) and is represented as follows:

$$\mathbf{C} = [\mathbf{I}_{L_p} \mathbf{J}_{L_p}, \mathbf{0}_{L_p \times (N-L_p)}; \mathbf{I}_N; \mathbf{0}_{L_s \times (N-L_s)}, \mathbf{I}_{L_s} \mathbf{J}_{L_s}]$$

where \mathbf{I}_{L_p} is an identity matrix and \mathbf{J}_{L_p} is a reversal matrix each of dimension L_p , $\mathbf{0}_{L_p \times (N-L_p)}$ is a zero matrix of size $L_p \times (N-L_p)$ and $L_1 = N + L_p + L_s$. $\mathbf{H} \in \mathbb{C}^{L_1 \times L_1}$ is the channel convolution matrix, which is a Toeplitz matrix with the first row and first column defined as $[\mathbf{g}, \mathbf{0}_{1 \times (L_1-L)}]$ and $[h_L, \mathbf{0}_{1 \times (L_1-1)}]^T$ respectively. where $\mathbf{h} = [h_1, h_2, \dots, h_L]$ is the channel impulse response and $\mathbf{g} = [g_1, g_2, \dots, g_L] = [h_L, h_{L-1}, \dots, h_1]$. $\mathbf{P} \in \mathbb{C}^{L_1 \times L_1}$ is the Toeplitz matrix with first row and column defined as $[h_L, \mathbf{0}_{1 \times (L_1-1)}]$ and $[\mathbf{g}, \mathbf{0}_{1 \times (L_1-L)}]^T$ respectively. $\mathbf{R} \in \mathbb{R}^{N \times L_1}$ is the matrix implementation form of removing the prefix and suffix and is defined as follows.

$$\mathbf{R} = [\mathbf{0}_{N \times L_p}, \mathbf{I}_N; \mathbf{0}_{N \times L_s}]$$

γ is the power normalization factor defined as (assuming \mathbf{s} is normalized) follows.

$$\gamma = \sqrt{\frac{1}{N} \text{trace}(\mathbf{DC}^H \mathbf{CD}^H)} = \sqrt{\frac{L_1}{N}}$$

From the system model defined in (6), we can write the effective channel matrix $\mathbf{H}_{eff} \in \mathbb{C}^{N \times N}$ as follows.

$$\mathbf{H}_{eff} = \mathbf{DRPHCD}^H \quad (7)$$

The noise variance of the system is also changed because of the prefiltering operation. The prefiltering of the noise

is represented as (8)

$$\mathbf{v} = \mathbf{DR}(\mathbf{P}_r \mathbf{n}_r + j \mathbf{P}_i \mathbf{n}_i) \quad (8)$$

where \mathbf{P}_r and \mathbf{P}_i are the real and imaginary parts of the prefiltering matrix \mathbf{P} and \mathbf{n}_r and \mathbf{n}_i are the real and imaginary parts of the noise vector \mathbf{n} . This \mathbf{n} is the actual additive white Gaussian noise (AWGN) with variance σ_n^2 . This original σ_n^2 depends upon the modulation type (m), code rate (R_c), length of prefix (L_p) and length of suffix (L_s) as they directly affect the average bit energy and consequently the E_b/N_o of the system. The original σ_n^2 is calculated using (9).

$$\sigma_n^2 = \frac{E_s}{\alpha m R_c} 10^{\frac{-E_b/N_o}{10}} \quad (9)$$

Where E_s is the average symbol energy which is assumed to be unit i.e. $R_{ss} = E[|s_k|^2] = 1$, α is the SNR reduction factor and its value is $\alpha = \frac{N}{L_p + N + L_s}$. The effective noise variance $E\{\mathbf{vv}^H\}$ after the prefilter can be expressed as follows.

$$\begin{aligned} E\{\mathbf{vv}^H\} &= E\{\mathbf{DR}(\mathbf{P}_r \mathbf{n}_r + j \mathbf{P}_i \mathbf{n}_i)(\mathbf{P}_r \mathbf{n}_r + j \mathbf{P}_i \mathbf{n}_i)^H \mathbf{R}^H \mathbf{D}^H\} \\ &= E\{\mathbf{DR}(\mathbf{P}_r \mathbf{n}_r + j \mathbf{P}_i \mathbf{n}_i)(\mathbf{n}_r^H \mathbf{P}_r^H - j \mathbf{n}_i^H \mathbf{P}_i^H) \mathbf{R}^H \mathbf{D}^H\} \\ &= E\{\mathbf{DR}(\mathbf{P}_r \mathbf{n}_r \mathbf{n}_r^H \mathbf{P}_r^H + \mathbf{P}_i \mathbf{n}_i \mathbf{n}_i^H \mathbf{P}_i^H) \mathbf{R}^H \mathbf{D}^H\} \\ &= E\{\mathbf{DR}(\mathbf{P}_r E\{\mathbf{n}_r \mathbf{n}_r^H\} \mathbf{P}_r^H + \mathbf{P}_i E\{\mathbf{n}_i \mathbf{n}_i^H\} \mathbf{P}_i^H) \mathbf{R}^H \mathbf{D}^H\} \end{aligned} \quad (10)$$

As $E\{\mathbf{n}_r \mathbf{n}_r^H\} = E\{\mathbf{n}_i \mathbf{n}_i^H\} = \frac{\sigma_n^2}{2}$. We can write (10) as (11)

$$\begin{aligned} E\{\mathbf{vv}^H\} &= \mathbf{DR} \left[\frac{\sigma_n^2}{2} E\{\mathbf{P}_r \mathbf{P}_r^H\} + \frac{\sigma_n^2}{2} E\{\mathbf{P}_i \mathbf{P}_i^H\} \right] \mathbf{R}^H \mathbf{D}^H \\ &= \frac{\sigma_n^2}{2} \mathbf{DR} \left[E\{\mathbf{P}_r \mathbf{P}_r^H\} + E\{\mathbf{P}_i \mathbf{P}_i^H\} \right] \mathbf{R}^H \mathbf{D}^H \\ &= \frac{\sigma_n^2}{2} \mathbf{DR} E\{\mathbf{PP}^H\} \mathbf{R}^H \mathbf{D}^H \end{aligned} \quad (11)$$

As the elements of \mathbf{P} consist of channel impulse response h_i with $E\{h_i h_j^H\} = 0$ when $i \neq j$. So we can define $E\{\mathbf{PP}^H\} = \mathbf{T} = \text{diag}(\mathbf{t})$. The elements of vector $\mathbf{t} =$

$[t_1, t_2, \dots, t_{L_1}]$ are calculated as follows.

$$t_m = \begin{cases} \sum_{i=1}^M E|g_i|^2 & 1 \leq m \leq L \\ \sum_{i=1}^L E|g_i|^2 & L < m < L_1 \end{cases} \quad (12)$$

So (11) can be written as follows.

$$E\{\mathbf{v}\mathbf{v}^H\} = \frac{\sigma_n^2}{2} \mathbf{D}\mathbf{R}\mathbf{T}\mathbf{R}^H\mathbf{D}^H$$

The effective noise variance \mathbf{N}_{eff} at the N different subcarriers can be reframed as the following diagonal matrix.

$$\mathbf{N}_{eff} = \frac{\sigma_n^2}{2} \text{diag}(\mathbf{D}\mathbf{R}\mathbf{T}\mathbf{R}^H\mathbf{D}^H) \quad (13)$$

The \mathbf{H}_{eff} from (7) and \mathbf{N}_{eff} from (13) will be used for the designing of the widely linear receive filters \mathbf{f}_1 and \mathbf{f}_2 as discussed in Section II-A.

III. SIMULATION PARAMETERS

The simulation parameters are given in Table. I

Table I: Simulation Parameters

FFT Size (N)	64	Blocks/Frame (N_{sym})	10
Prefix & Suffix (L_p, L_s)	12	Modulation Type (m)	ASK
Channel Type: 802.11 Multipath Channel			

IV. RESULTS

The results have shown that WL filtering can significantly improve the BER performance of the DCT based multicarrier system due to its inherent property of generating improper signals. The bit error rate (BER) performance of the system can be seen from Fig. 3

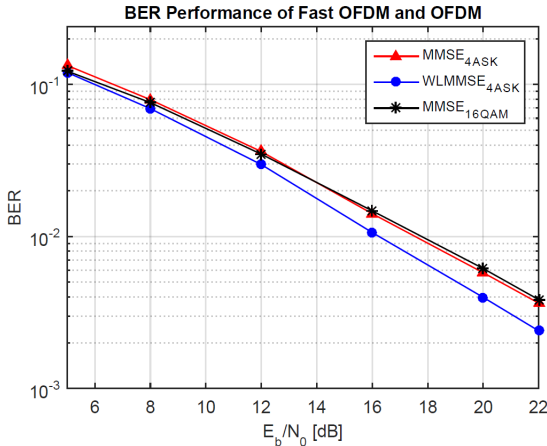


Fig. 3: BER Performance of FOFDM and OFDM

It can be observed from Fig. 4 that the mean square error (MSE) performance is also better in WL case compared to its linear counterpart. Which leads to a more precise estimate of the transmitted symbols. The comparison between OFDM and FOFDM is based on the assumption that two systems achieve the same transmission rate. Therefore we use 4ASK for FOFDM and 16QAM for OFDM.

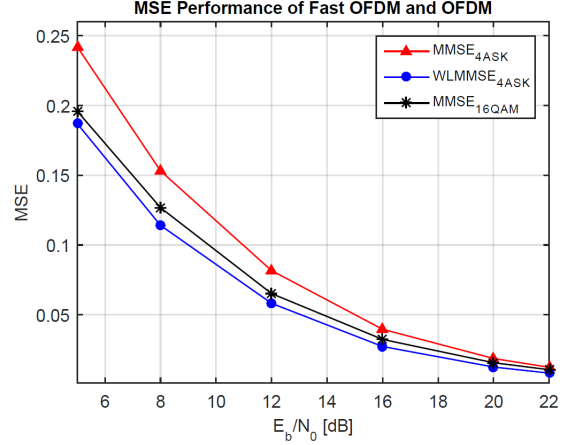


Fig. 4: MSE Performance of FOFDM and OFDM

V. CONCLUSIONS

The paper presents the improvement in the BER and MSE performance of a DCT based multicarrier system also known as fast OFDM by exploiting the improperness of its real valued symbols. The widely linear filtering significantly improves the performance of the FOFDM system by processing the received symbols along with their conjugate version to provide a more precise estimate of the transmitted symbols at the receiver as compared to its linear counterpart.

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ABOUT THE AUTHORS

A Zafar received the B.S. and M.Sc. degrees from Institute of Space Technology, Islamabad, Pakistan and University of Surrey, Guildford, United Kingdom in 2007 and 2010, respectively. He joined Institute of Space Technology, Islamabad, Pakistan in 2007, where he worked as Assistant Professor since 2011. He is currently working toward the Ph.D. degree from the Institute for Communication Systems (ICS), University of Surrey, Guildford, United Kingdom. His current research interests include satellite link design, signal processing for wireless communication systems and physical layer for 5G networks.

C He is a PhD candidate in Electronic Engineering in the institute for Communication Systems (ICS) at University of Surrey, UK. He completed his Master’s degree in Wireless Communications from University of Southampton. His current research interests are mainly DCT (Discrete Cosine Transform) based multi-carrier transceiver design and CMT (Cosine Modulated multitone) based filter bank systems.

P Xiao received the PhD degree from Chalmers University of Technology, Sweden in 2004. Prior to joining the University of Surrey in 2011, he worked as a research fellow at Queens University Belfast and had held positions at Nokia Networks in Finland. He is a Reader at University of Surrey and also the

technical manager of 5G Innovation Centre (5GIC), leading and coordinating research activities in all the work areas in 5GIC (<http://www.surrey.ac.uk/5gic/research>). Dr Xiaos research interests and expertise span a wide range of areas in communications theory and signal processing for wireless communications.

M A Imran is currently a Reader (Associate Professor) in the Institute for Communication Systems (ICS - formerly known as CCSR) at the University of Surrey, UK. His research interests include physical layer of 5G networks, energy efficient design of cellular system and machine learning/selforganising techniques for optimisation of cellular system. He is a senior member of IEEE and a Senior Fellow of Higher Education Academy (SFHEA), UK. He is associate editor of IEEE Communications Letters, IEEE Access, IET Communications, guest editor of several special issues in IEEE journals and chair for several tracks in highly reputed international conferences and workshops. He has been awarded IEEE Comsocs Fred Ellersick award 2014 and FEPS Learning and Teaching award 2014. He is a shortlisted finalist for The Wharton-QS Stars Awards 2014 for innovative teaching and VCs learning and teaching award in University of Surrey.

A Cao is a principal architect in ZTE R& D center in Sweden (ZTE Wistron Telecom AB). He has over 17 years of experiences in wireless communications research and development from baseband processing to network architecture, including design and optimization of commercial UMTS/LTE base-station and handset products, HetNet and small cell enhancement, etc. He has also been involved in standardization works and contributed to several 3GPP technical reports. In addition, he holds more than 50 granted or pending patents. His current focus is 5G technologies related to the new energy-efficient unified air-interface and network architecture, e.g., new waveform design, non-orthogonal multiple access schemes, random access challenges and innovative signaling architecture for 5G networks, etc.