Human capital accumulation and transition to skilled employment*

Konstantinos Angelopoulos
University of Glasgow, and CESifo

James Malley
University of Glasgow, and CESifo

Apostolis Philippopoulos
Athens University of Economics & Business, and CESifo

November 22, 2016

Abstract

This paper assesses the impact of investment- and education-specific technical change on occupational transition and the skill premium in a model with human capital. In this framework, human capital augments labour productivity and also facilitates the transition to skilled employment. In line with empirical evidence, this setup predicts that an increase in the productivity of physical capital (investment-specific change) leads to very small increases in the relative supply of skilled workers and to significant and rising increases in the skill premium. Additionally, reforms that improve the productivity of resources used in education (education-specific change) reduce wage inequality and increase mobility.

Keywords: skill acquisition, wage premium, human capital, education reforms

JEL Classification: E24, J31, J62

Corresponding author: Jim.Malley@glasgow.ac.uk

*We would like to thank the editors, Isaac Ehrlich and Stephen Turnovsky, an anonymous referee, Richard Agènor, Stefan Bühler, Guido Cozzi, Amanda Gosling, Wei Jiang, Miguel León-Ledesma, Jochen Mankart, Kyriakos Neanidis, Tatiana Kirsanova, Ulrich Woitek and participants at the CESifo Macroeconomics Area Conference, February 2014, and at the Universities of Kent, Manchester, St. Gallen and Zürich for helpful comments and suggestions. We are also grateful for financial support from the ESRC, Grant No. RES-062-23-2292, but the views expressed here are entirely our own.

1 Introduction

Following reductions for most of the 20th century, earnings inequality in the U.S. has increased since 1980 such that the wage premium for skilled workers has been in recent years at its highest level since 1910. Concurrently, educational attainment, measured by numbers of high school and college graduates has stagnated or increased at a very slow pace since the late 1970s. These patterns have been thoroughly documented and analysed in the literature (see e.g. Goldin and Katz (2008)). This literature has drawn attention to rising labour productivity differentials and lower productivity-enhancing human capital accumulation as the key forces driving the reduced economic growth and rising wage inequality experienced in the U.S. since the last quarter of the past century.

Given the importance of these developments, an extensive literature has studied the division of the labour force between college and high school graduates and the resulting wage premium to skilled workers (see e.g. Acemoglu and Autor (2011), Goldin and Katz (2008) and Hornstein *et al.* (2005) for reviews). Building on Tinbergen (1974, 1975) and Katz and Murphy (1992), the main drivers of the skill/wage premium are widely accepted to be skill-biased technical change and the relative supply of skilled versus unskilled labour. Rises in the former tend to increase the demand for skilled labour and hence the skill premium, while increases in the latter contribute to falls in the wage premium.

As historical evidence for the 20th century demonstrates, these factors operate via a production sector which is characterised by capital-skill complementarity (see Goldin and Katz (2008)). Models incorporating investment-specific technological progress have been shown to match several key aspects of the dynamic behaviour of the skill premium in the U.S. data (see e.g. Krusell *et al.* (2000) and He (2012)). Skill-enhancing education reforms, in the form of the mass high school movement at the beginning of the 20th century and the mass higher education movement in the middle of the last century have contributed to the rise in the relative skill supply and the important decline in wage inequalities observed in the first three quarters of the 20th century (see e.g. the evidence documented in Goldin and Katz (2008)).

There is also a comprehensive literature which has examined the occupational choice of economic agents, usually focusing on the distinction between entrepreneurs and workers, and its implications for skill acquisition (see e.g. Quadrini (2000), Matsuyama (2006) and Kambourov and Manovskii (2009)). A small strand of this literature has studied the occupational choices of skilled and unskilled workers, thus endogenising the relative skill supply in setups that may allow for skill-biased technical change and a skill premium paid to college-educated workers (see e.g. Galor and Moav (2000), Maoz and Moav (2004), Guvenen and Kuruscu (2010, 2012) and He (2012)).

Historical evidence shows significant returns to education for workers remaining in unskilled employment in periods of increased educational attainment (see e.g. Goldin and Katz (2008)). This suggests that human capital accumulation augments the productivity of workers, in addition to driving the transition to skilled employment. Despite this evidence, the productivity augmenting role of human capital accumulation of the unskilled workers, while in their unskilled occupations, has not been considered in this research jointly with occupational transition under capital-skill complementarity.[1] Omitting the labour augmenting effects of skill accumulation on the current skill-type job can lead to biased assessments of the effect of investment and education specific technical change. Consequently, this bias can be transmitted, in general equilibrium, to output, transition to skilled employment, wage inequality, etc.

In light of the above, our aim is to reassess the aggregate dynamic implications of skill-biased and education specific technical change on occupational transition and wage inequality. To this end we model the joint determination of the skill premium, transition to skilled employment, and labour and mobility augmenting human capital under capital-skill complementarity. We build on the standard infinite-horizon human capital model of aggregate dynamics (see e.g. Uzawa (1965) and Lucas (1988) and Barro and Sala-i-Martin (2004) for reviews)[2] and extend it to include workers differentiated by skill type, capital-skill complementarity on the production side, and transition to skill employment driven by human capital accumulation on the labour supply side. We assume that skilled and unskilled workers are members of a representative dynasty, so that, although they face different time constraints, provide distinct labour services and earn different returns for their labour supply, the household that they belong to guarantees their consumption irrespective of their labour market status.

Human capital accumulated by the unskilled is assumed to equip them with the knowledge to become skilled workers and thus to improve their position on the professional ladder. In particular, the switch to the skilled

[1] See also e.g. Heathcote *et al.* (2009) on the distinction between educational choice and skill formation over time.

[2] Our focus is on aggregate dynamics relating to human capital accumulation, rather than the educational choice. Infinite horizon models are commonly employed to study aggregate dynamics with human capital accumulation (see e.g. Barro and Sala-i-Martin (2004)). They have also been employed to study dynamics of occupational transitions (see e.g. Quadrini (2000) and Kambourov and Manovskii (2009)) and of the skill premium (see e.g. Lindquist (2004), Pourpourides (2011) and Angelopoulos *et al.* (2015)).

type is achieved once a threshold for the level of skill is reached. Importantly, and consistent with the evidence in Goldin and Katz (2008), we acknowledge that the human capital stock accumulated by unskilled workers, who have not yet reached the critical threshold to find a skilled occupation, augments the efficiency of their labour effort. This is modeled using the standard labour augmenting productivity channel in the literature of human capital and growth.

To highlight the importance of allowing human capital to be both labor productivity and mobility augmenting, versus mobility augmenting only, we compare the model's predictions regarding the short- and long-run effects of investment-specific technical change on wage inequality and skill supply to the data. In particular, we first estimate the impact and long-run elasticities of the skill premium and the share of skill in the population with respect to investment-specific technical change, using data from Acemoglu and Autor (2011) and Cummins and Violante (2002) and find that, while these elasticities are statistically insignificant for the skill share, they are significant for the skill premium. In particular, the impact elasticity of the skill premium is low (about 0.09), while the long-run is high (about 0.33).

The above results are consistent with the previous findings of a large literature, but nonetheless allow us to quantify the dynamic relationships between the skill premium, skill supply and investment-specific technical change in a way that facilitates direct comparisons with the model's predictions. In particular, we find that the model matches these predictions only when it includes the labour augmenting channel for human capital that leads to skill transition. When this channel is missing, the model predicts higher elasticities for the skill share and is overly optimistic regarding the long-run response of skill supply to investment-specific technical change, which in turn contributes to a lower increase in the model predicted skill premium in longer horizons. The role of labour augmenting human capital is further analysed below when we examine in more detail the general equilibrium effects of technical change.

We evaluate the dynamic implications of permanent changes to investment- and education specific technical change. We focus on skill-biased technical change and education since, despite the contribution of additional factors, they are widely considered to be the main drivers of the transition to skilled employment and the skill premium.[3] We quantify these dynamic effects

[3] Other factors that have been proposed to contribute to the observed patterns of the skill premium and relative skill supply include: (i) internationalisation in product and labour markets (see e.g. Acemoglu (2003) and Autor *et al.* (2003)); (ii) demographic and other factors related to immigration and war periods (see e.g. Card and Lemieux (2001) and Goldin and Margo (1992)); and (iii) institutional changes unfavourable to unionised

in a unified framework that allows us to examine the required increase in education-specific technical change to alleviate the wage inequality effects of investment-specific technical change.

We first find that a permanent increase in investment-specific technical change leads to improvements in aggregate outcomes, but implies sizeable increases in the skill premium, while having negligible effects on skill supply. The rising skill premium incorporates incentives for human capital creation to facilitate the transition to skill. However, as human capital increases, the effective return to unskilled work time also increases, given the labour augmenting role of human capital, and this effectively mediates the desirability of transition to skill. In turn, the relatively low increase in the skill supply is not sufficient to offset the skill-biased effects of increased capital accumulation, leading to an equilibrium with low mobility and high wage inequality. These results provide the intuition behind the role of labour augmenting human capital in helping the model to match the elasticities of the skilled share and skill premium in the data, as was noted earlier.

Second, we consider the effects of a permanent increase in education-specific technical change, which improves the effectiveness of resources allocated to education. As expected, this increases mobility and decreases wage inequality. Moreover, we find that despite a short-run decrease in the wages for skilled, in the medium to long-run education-specific technical change raises the returns to labour for both types of workers. This is due to the positive effects for skilled workers from increased capital accumulation, which in turn is associated with a more productive and wealthier economy.

Third, we find that education-specific technical change must increase by about half of the increase in investment-specific technical change to eliminate the rise in the skill premium in the long-run. This combined technical change also implies a smaller increase in wage inequality in shorter time horizons, to about or less than a third of the increase observed if only investment-specific technical change takes place. Moreover, it increases average consumption and income in the economy by about 50%, compared with the situation where only investment-specific technical change takes place.

Finally, we find that the model's predictions with respect to the dynamic responses of different measures of earning inequality and the skill premium, for the above scenarios, are generally very similar. This is broadly consistent with the analysis in Ehrlich and Kim (2007) and with patterns found in the

labour and minimum wage protection (see e.g. Di Nardo *et al.* (1996)). While these factors contribute to understanding specific episodes in the skill-premium skill-supply nexus, they work to complement the race between education and technology, which is qualitatively the key driver of the main stylised facts (see e.g. Goldin and Katz (2008) for a detailed analysis).

U.S. data (see e.g. Heathcote *et al.* (2010)).

The rest of this paper is organised as follows. Section 2 sets out the theoretical model and Section 3 discusses its quantitative implementation and evaluates its predictions regarding short- and long-run elasticities of skill supply and skill premium with respect to investment-specific technical change. Section 4 analyses the effects of investment- and education-specific technical change and Section 5 concludes.

2 The model

In this section, we develop a model with a vertical division of labour into two professional types, skilled and unskilled, allowing for occupational transition from unskilled to skilled. To increase their level of skill, unskilled workers accumulate human capital, which is also augmenting their productivity while unskilled. Transition to skilled employment is thus incorporated by allowing unskilled members to switch to skilled status once their level of skill reaches a threshold level. We use a closed-economy setup where a representative household is an infinitely-lived dynasty with both types of workers/members.[4] These members differ in the type of labour services they offer. Unskilled workers accumulate human capital as a function of time and expenditure allocated to education.

The firms use capital, skilled labour and unskilled labour to produce a homogeneous product. Since skilled labour is more complementary to capital than unskilled, capital accumulation, as well as technological developments that are capital augmenting, favour the skilled wage premium. In contrast, increases in the relative supply of skilled labour tend to reduce the skill premium.

Compared to the relevant literature, our modeling highlights labour and mobility augmenting human capital accumulation and occupational transition under capital-skill complementarity. All these ingredients are an important part of the experience of the 20th century. In particular, capital-skill complementarity is consistent with empirical and historical evidence (see e.g. Goldin and Katz (2008) and Krusell *et al.* (2000)). Moreover, skilled and

[4]The modelling assumption that the population is made up by a representative household composed of members which differ in their labour market position is a standard modelling device in dynamic macroeconomic models with two-state heterogeneity in the labour market since Merz (1995). Examples include the models incorporating search and matching labour market frictions and unemployment (see e.g. Rogerson and Shimer (2011) and Arseneau and Chugh (2012)). We adapt this modelling device here to capture a two-state heterogeneity in the labour market consisting of skilled and unskilled workers.

unskilled workers are different entities, and human capital accumulation related to college education leads to transition to skilled employment (see e.g. the data discussed in Acemoglu and Autor (2011)). Finally, human capital is labour-augmenting, in addition to its indirect productivity-augmenting role (see e.g. Goldin and Katz (2008)).

2.1 Human capital and skill acquisition

The numbers of skilled and unskilled members at the start of time t for the representative household are denoted as N^s_{t-1} and N^u_{t-1} respectively. Thus, the total size of the household (population) is $N_{t-1} = N^s_{t-1} + N^u_{t-1}$.[5] The respective population shares are defined as $n^s_{t-1} = N^s_{t-1}/N_{t-1}$ and $n^u_{t-1} = N^u_{t-1}/N_{t-1}$, where $\{N_t\}_{t=0}^{\infty}$ is assumed to grow at an exogenous net rate, $\theta = \frac{N_t}{N_{t-1}} - 1$, to allow new unskilled members to join the household.

The head of the household makes all decisions and treats all members symmetrically, so that unskilled household members are identical regarding their human capital accumulation. In particular, we assume that an unskilled household accumulates knowledge in the form of human capital, which requires the use of both time and goods (see e.g. Ben-Porath (1967) and Trostel (1993)):

$$h_t = (1 - \delta^h)h_{t-1} + g\left(e_t, z_t\right) \tag{1}$$

where $0 < \delta^h < 1$ is the human capital depreciation rate; e_t is time devoted to education; z_t is private education spending; and the function $g\left(e_t, z_t\right)$ is differentiable, increasing and strictly concave. In addition to its productivity augmenting role, human capital accumulation via primary, secondary and college education also increases unskilled labour's potential for becoming skilled.

More specifically, when the effective skill level of an unskilled individual, defined as S^i_t, $i = 1, 2, ...N^u$, is higher than an exogenous level, S^*, he/she becomes skilled. We assume that S^i_t is the outcome of a combination of endogenous human capital or knowledge, h_t, which is common to all households, with exogenous factors that are specific to each individual, ϕ^i_t. The latter are assumed to be uncorrelated across individuals and capture idiosyncratic preference and aptitude shocks for transition to skilled employment, due to e.g. individual differences in aspirations, life-style, neighborhood environment, school quality, health, etc, and implies that only a proportion of unskilled members will be able to work as skilled in the next time period.[6]

[5] The total size of the population is assumed to be large. This allows us to approximate below the distribution of abilities in skill acquisition by a continuous function.

[6] This idiosyncratic term generates a type of heterogeneity in preferences and/or apti-

These relationships can be defined more formally as follows.

Definition 1. at the end of period t, an unskilled member i becomes skilled if:

$$S_t^i \equiv \phi_t^i q(h_t) > S^* \tag{2}$$

where the function $q(h_t)$ is differentiable, increasing and strictly concave in h_t; ϕ_t^i is drawn from a uniform distribution and is identically and independently distributed over time with a probability density function (pdf), f_ϕ;[7] and $S^* > 0$ denotes the skill threshold. The pdf for ϕ is given by:

$$f_\phi = \frac{1}{\phi^h - \phi^l}, \ \phi^h > \phi > \phi^l \tag{3}$$

where ϕ^h and ϕ^l are the maximum and minimum ability levels within the members of the household.

The condition defined in (2) can be rewritten as:

$$\phi_t^i > \frac{S^*}{q(h_t)} \equiv \phi_t^* \tag{4}$$

which suggests that an increase in the human capital of unskilled agents lowers the critical level of idiosyncratic abilities required for becoming skilled, ϕ_t^*.

Since all decisions are taken by the household, this model cannot capture educational choices made by individuals. Our focus instead is on aggregate dynamics that allow for between-group wage and earnings inequality, as opposed to within-group labour income inequality, and we consider the case where all unskilled members are identical in each time period before the realisation of the idiosyncratic ability shock. This requires the assumption that new household members share a common value of human capital with existing unskilled members at the period t when they join the household.[8] This

tude towards education, within the class of unskilled labour, without which the proportion of unskilled members that become skilled in each period would be either zero or one (see also e.g. Heckman *et al.* (1998) and He (2011, 2012) who use "idiosyncratic disutility costs" for this purpose). This preference/aptitude heterogeneity is modelled so that the idiosyncratic shock received by an individual can differ between periods (see e.g. Krusell and Smith (1998) for time-varying preference heterogeneity).

[7] Since non-linearities in the skill transition function (2) are captured by the functional form of $q(h_t)$, we assume for simplicity a uniform distribution for ϕ_t^i. As shown below, this allows for an analytic expression for the equation summarising occupational transition.

[8] See also e.g. Curdia and Woodford (2009) for similar assumptions regarding within-group initial conditions.

feature implies that the model does not permit the study of heterogeneity in human capital accumulation, and thus the study of earnings inequality within unskilled workers, but allows for tractability when analysing the dynamics of wage and earnings differentials between skilled and unskilled workers. The role of the distribution over ϕ_t^i is to determine the proportion of unskilled workers who become skilled at the aggregate level.

The proportion of unskilled household members who become skilled, π, is given by:

$$\pi = 1 - \int_0^{\phi^*} f_\phi d\phi. \tag{5}$$

Evaluating the definite integral of (5) gives the occupation transition function:

$$\pi_t = 1 - F_\phi \left[\phi^* \left(h_t; S^*\right)\right] \tag{6}$$

where, $F_\phi = \frac{\phi - \phi^l}{\phi^h - \phi^l}$ is the cumulative distribution function (cdf) of ϕ.

Given (6), the numbers of skilled and unskilled members change over time as follows:

$$N_t^s = N_{t-1}^s + \pi_t N_{t-1}^u \tag{7}$$

and

$$N_t^u = N_{t-1}^u + \theta N_{t-1} - \pi_t N_{t-1}^u \tag{8}$$

where,

$$\pi_t = 1 - \frac{\frac{S^*}{q(h_t)} - \phi^l}{\phi^h - \phi^l}.$$

Equivalently, the population shares of skilled and unskilled members are given by:[9]

$$n_t^s = \frac{n_{t-1}^s + \pi_t n_{t-1}^u}{1 + \theta} \tag{9}$$

and

$$n_t^u = \frac{n_{t-1}^u + \theta - \pi_t n_{t-1}^u}{1 + \theta} \tag{10}$$

where $n_t^s + n_t^u = 1$. As long as $0 < \pi < 1$, and $\theta > 0$ equation (9) implies a well defined steady-state, where the share of skilled converges to a stationary quantity, given by $n^s = \pi / \left(\pi + \theta\right)$, where $0 < n^s < 1$.

[9]The evolution of the share of skilled (9), has, in fact, analogies with the basic employment evolution equation in search and matching models (see e.g. equation (7) in Rogerson and Shimer (2011)). In our case, the occupational transition function, π_t, plays a role similar to the job finding probability in the search literature.

2.2 The problem of the household

In this setup, the head of the household makes all the choices on behalf of its members by maximising aggregate welfare. The household guarantees the same level of consumption and welfare to all its members, irrespective of individual labour market status. Formally, the household maximises the discounted lifetime utility of its members:

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{11}$$

where c_t is per capita consumption; and $0 < \beta < 1$ is the time preference rate.[10] The period utility function, $u(\cdot)$, is increasing and strictly concave.

The household's budget constraint is:

$$c_t + i_t + n_{t-1}^u z_t = n_{t-1}^s w_t^s + n_{t-1}^u w_t^u h_{t-1} l_t + r_t k_{t-1} \tag{12}$$

where k_{t-1} is the per capita stock of physical capital at the start of period t; i_t is per-capita investment in physical capital; $h_{t-1}l_t$ is the effective labour supply of unskilled workers; w_t^s and w_t^u are the returns to skilled labour and effective unskilled labour supply respectively; and r_t is the return to physical capital. Hence, the return to one unit of unskilled-labour time in the job market is given by $w_t^u h_{t-1}$. Each unskilled worker allocates one unit of his/her effort time to work and education:

$$l_t + e_t = 1. \tag{13}$$

While in the literature to date the accumulation of knowledge from unskilled workers as a mechanism for them to become skilled has been considered, the labour augmenting role that this human capital may have for the unskilled labour input, as captured in this framework in (12) and in the market clearing conditions below, has not been explored.

Finally, the motion of physical capital is given by:

$$k_t = (1 - \delta^k) k_{t-1} + B i_t \tag{14}$$

where $0 < \delta^k < 1$ is the physical capital depreciation rate and $B > 0$ is investment specific technical change (see, e.g. Greenwood *et al.* 2000).[11]

[10] Note that at the household level, there is no uncertainty regarding the proportion of agents who become skilled. Also, for simplicity, there is no aggregate uncertainty. There is, of course, uncertainty, for each individual within the class of unskilled workers, regarding their progression to skilled employment. However, by guaranteeing the same consumption for all members, the household provides complete income insurance to its members (see also e.g. Rogerson and Shimer (2011) and Arseneau and Chugh (2012) for examples with shocks to employment).

[11] In this setup, skill-biased technical change can be captured by changes in B.

The household, taking prices and initial conditions as given, chooses the time paths of $\{c_t, i_t, z_t, k_t, l_t, e_t, h_t, n_t^s\}_{t=0}^{\infty}$ to maximize (11) subject to constraints (1), (9), (12) – (14).

2.2.1 Occupational transition and human capital

It is useful for the understanding of the quantitative results which follow below, to examine the trade-offs relating to the household's choice of human capital and skill acquisition in more detail. The Euler-equation for human capital illustrates the trade-off associated with choosing h_t:

$$\begin{aligned} \lambda_t^h &= \beta\lambda_{t+1}^b \left(1 - n_t^s\right) w_{t+1}^u l_{t+1} + \beta\lambda_{t+1}^h (1 - \delta^h) + \\ &+ \lambda_t^s \left(\frac{1 - n_{t-1}^s}{1 + \theta}\right) \frac{\partial \pi_t}{\partial h_t} \end{aligned} \tag{15}$$

where λ_t^h, λ_{t+1}^b and λ_t^s are the Lagrange multipliers attached to the unskilled human capital, budget constraint and skilled employment share equations respectively.

This Euler states that the representative household equates the opportunity cost of education-time to produce an extra unit of human capital valued in utility terms, λ_t^h, to the benefits, which are the sum of the discounted future increase in net labour income and human capital, valued respectively by λ_{t+1}^b and λ_{t+1}^h and the increase in the share of household members who become skilled due to a marginal increase in h_t.

Equation (15) demonstrates the importance of labour augmenting human capital in unskilled jobs when the choice of unskilled human capital is considered. In particular, in a model with mobility augmenting human capital only, the first term on the right-hand-side of (15) would disappear, so that all positive benefits attributed to h_t would only arise because of the latter two terms, which relate to the benefits from skill acquisition.

Moreover, the Euler-equation for the share of skilled labour illustrates the trade-off associated with choosing n_t^s:

$$\begin{aligned} \lambda_t^s &= \beta\lambda_{t+1}^b \left(w_{t+1}^s - w_{t+1}^u h_t l_{t+1} + z_{t+1}\right) + \\ &+ \beta\lambda_{t+1}^s \left(\frac{1}{1 + \theta}\right) \left(1 - \pi_{t+1}\right). \end{aligned} \tag{16}$$

Here the costs of foregone unskilled work-time required to increase the share of members that find employment in skilled jobs, evaluated in utility terms via λ_t^s, is equal to, in equilibrium, the benefits from increasing n_t^s. The latter are given by the discounted future increase in net labour income of being employed in skilled versus unskilled jobs plus the discounted future savings from not having to spend goods in the next period to transition to the skilled pool, valued in utility terms by λ^b_{t+1}. The benefits are further augmented by the fact that a higher share of skilled increases the discounted future stock of skilled, and this is valued in utility terms by λ^s_{t+1}.

Note that, by comparing the returns to labour in the above condition for the share of skilled labour, we can see that an increase in the skill premium tends, *ceteris paribus*, to increase the benefits of employment in the skilled sector and is thus expected to increase job mobility. However, at the same time, an increase in human capital (which is required to increase the transition to skill) acts to decrease the benefits of skilled employment, by closing the gap between the returns to skilled and unskilled workers. Hence, while being the engine of occupational transition, human capital also incorporates effects that work to mediate the desirability of this transition.

2.3 The firm

There are N^f_{t-1} identical firms at the start of time t. For simplicity, we assume that the number of firms equals the number of household members which implies $N^f_t = (1+\theta) N^f_{t-1}$. Each firm produces output, y^f_t, using physical capital, k^f_t, and the two distinct types of labour, unskilled, $l^{u,f}_t$, and skilled, $l^{s,f}_t$, where the latter is relatively more complementary to capital than unskilled labour. The production function is given by a constant returns to scale technology with respect to its three inputs:

$$y^f_t = Y\left(l^{s,f}_t, l^{u,f}_t, k^f_t\right). \tag{17}$$

Each firm acts competitively by choosing inputs, k^f_t, $l^{s,f}_t$ and $l^{u,f}_t$, to maximise profits:

$$\Pi^f_t \equiv y^f_t - r_t k^f_t - w^s_t l^{s,f}_t - w^u_t l^{u,f}_t \tag{18}$$

subject to the technology constraint, (17). In equilibrium, profits, Π^f_t, are zero.

2.4 Market-clearing conditions

The market-clearing conditions for capital, skilled labour, unskilled labour and goods are respectively:

$$k^f_t = k_{t-1} \tag{19}$$

$$l^{s,f}_t = n^s_{t-1} \tag{20}$$

$$l^{u,f}_t = n^u_{t-1} h_{t-1} l_t \tag{21}$$

$$y_t^f = c_t + k_t - (1-\delta)\, k_{t-1} + n_{t-1}^u z_t \tag{22}$$

where (22) also gives the resource constraint of the economy.

2.5 Decentralized competitive equilibrium

The decentralized competitive equilibrium (DCE) is defined as follows.

Definition 2. The DCE is an allocation sequence $\{c_t, i_t, z_t, k_t, l_t, e_t, h_t, n_t^s\}_{t=0}^{\infty}$, a price sequence $\{w_t^s, w_t^u, r_t\}_{t=0}^{\infty}$ and initial conditions for k_0, h_0, and n_0^s such that:

1. the representative household and firm undertake their respective optimization problems assuming that all household members of a specific skill type are symmetric;
2. all budget constraints are satisfied; and
3. all markets clear.

3 Quantitative specification and evaluation

In this section we implement the model quantitatively by specifying functional forms for utility, education, skill acquisition, and production. We calibrate the model so that it is consistent with U.S. data averages over the 1963-2000 period, for which data for the key variables, i.e. skill premium, skill share and investment-specific technical change are available. To proceed, we first estimate the elasticities capturing the effect of investment-specific technical change on the skill premium and the skill share in the data. We then calibrate and solve the model and evaluate the importance of the labour augmenting channel of human capital in allowing the model predictions regarding these elasticities to cohere with the data.

3.1 Data analysis: skill premium, skill share and investment specific technical change

We use annual data for the U.S. economy from 1963-2000. Data on the skill premium and the skill share are from Acemoglu and Autor (2011). In particular, the skill premium refers to the ratio of the college to high school wages (as used in e.g. Figure 1 in Acemoglu and Autor (2011)), while the skill share refers to the share of college educated workers in effective units (used to construct e.g. the ratio of college to high school supply in Figure

2 in Acemoglu and Autor (2011)). We plot these two series in Figure 1 below together with the series of investment-specific technical change from Cummins and Violante (2002).[12] Cummins and Violente (2002) calculate the series for investment-specific technical change as the ratio of the official NIPA price index of consumption (non-durables and services) to the Cummins-Violante quality adjusted price index of total investment (equipment and structures).[13]

[Figure 1 here]

The series in Figure 1 summarise the main stylised facts regarding the evolution of wage inequality in recent decades that have been extensively documented and analysed in the literature. In particular, much discussion has concentrated on the relative stability and decline of the skill premium up to the early 1980s, and its increase thereafter. The latter is typically attributed to the combination of the slowdown of skilled supply and the rapid increase in investment-specific technical change, which, under capital-skill complementarity, acts as skill-biased technical change. Hence, investment-specific technical change is positively related to the skill premium in the literature.[14] At the same time, the evolution of the skill supply and investment-specific technical change point to a weak relationship between the growth in the two quantities, and why it is the case is indeed a pertinent question in the literature, given that skill-biased technological change should create economic incentives to educate and thus increase the supply of skill (see e.g. the detailed analysis and review of the literature in Goldin and Katz (2008)).

To evaluate the model's potential to capture the effect of investment-specific technical change on the skill premium and the skill share, we first need to quantify the elasticities implied in Figure 1, with respect to both short- and long-run effects of investment-specific technical change on the skill premium and the skill share. To this end, we use the data in Figure 1 to estimate dynamic econometric models of the skill premium and the skill share, which are presented, together with the estimation results and the

[12] Note that in Acemoglu and Autor (2011) the log of the ratios is shown in Figures 1 and 2, whereas we plot the level of the ratio of wages (skill premium) and the proportion of skilled workers as a share of the skilled and unskilled labour force (skill share), so that the units of the data series correspond to the model relevant variables.

[13] The series in Cummins and Violante (2002) starts in 1947, so we re-normalise the constructed index for investment-specific technical change to start in 1963.

[14] For instance, to quantify the effect of investment specific technical change and skill supply on the skill premium using historical data, Goldin and Katz (2008) consider a regression of the skill premium on relative skill supply and linear and non-linear trends that are intended to capture the effects of investment-specific technical change. The trend is estimated to be positive, while the effect of skill supply is negative.

implied elasticities, in Table 1. All variables in the econometric models are denoted following the notation used for the corresponding model quantity, but feature a tilda to make clear that they refer to data and not model quantities.

Table 1: Long-run elasticities of the skill premium and skill share (1963-2000)

	$\ln\left(\frac{\widetilde{w}_t^s}{\widetilde{w}_t^u}\right)=\alpha_1+\beta_1\ln\left(\widetilde{B}_t\right)+$ $+\gamma_1\ln\left(\frac{\widetilde{w}_{t-1}^s}{\widetilde{w}_{t-1}^u}\right)+\delta_1\ln\left(\widetilde{n}_t^s\right)$	$\ln\left(\widetilde{n}_t^s\right)=\alpha_2+\beta_2\ln\left(\widetilde{B}_t\right)+$ $+\gamma_2\ln\left(\widetilde{n}_{t-1}^s\right)+\delta_2\ln\left(\frac{\widetilde{w}_t^s}{\widetilde{w}_t^u}\right)$
	OLS	
	Impact elasticity	Impact elasticity
$\widehat{\beta}_i$	0.0949	0.0125
$s.e.$	0.0415	0.0359
H_0: $\widehat{\beta}_i=0$; F_{stat}	5.2260	0.1222
$p-val$	0.0288	0.7288
	Long-run elasticity	Long-run elasticity
$\frac{\widehat{\beta}_i}{1-\widehat{\gamma}_i}$	0.3287	0.4833
$s.e.$	0.0538	0.9451
H_0: $\frac{\widehat{\beta}_i}{1-\widehat{\gamma}_i}=0$, F_{stat}	37.4055	0.2615
$p-val$	0.0000	0.6125
	Equation fit	Equation fit
DW	1.7051	1.6954
$\overline{R}^2$	0.9629	0.9967
	GMM	
	Impact elasticity	Impact elasticity
$\widehat{\beta}_i$	0.0904	-0.0030
$s.e.$	0.0320	0.0526
H_0: $\widehat{\beta}_i=0$; F_{stat}	7.9700	0.0032
$p-val$	0.0080	0.9556
	Long-run elasticity	Long-run elasticity
$\frac{\widehat{\beta}_i}{1-\widehat{\gamma}_i}$	0.3256	-0.1767
$s.e.$	0.0723	3.4925
H_0: $\frac{\widehat{\beta}_i}{1-\widehat{\gamma}_i}=0$, F_{stat}	20.255	0.0026
$p-val$	0.0001	0.9599
	Equation fit	Equation fit
DW	1.7155	1.7201
$\overline{R}^2$	0.9629	0.9967

The models include the lagged dependent variable as an explanatory variable to capture persistence in the data and remove serial correlation from the residuals. We start with least squares results and, in addition, given potential endogeneity of the skill share in the skill premium regression and of the skill premium in the skill share regression, we also present generalised method of moments estimation results, where the endogenous right-hand side variable is instrumented using its predetermined lagged value.

Regarding first the effect of investment-specific technical change on the skill premium, the results in Table 1 indicate that the short-run elasticity is low, at about 0.09, but the long-run is high at about 0.33. Both are statistically significant and the results do not vary much between OLS and GMM estimation. To save on space, we do not report the estimates for the effect of the skill share, since the p-values are between 0.104 to 0.124 for the OLS regressions and 0.374 to 0.778 for the GMM regressions. Turning next to the effect of investment-specific technical change on the skill share, the results in Table 1 indicate that for both short- and long-run elasticities we cannot reject the null that the effect is zero, hence confirming the expectation based on the previous discussion of a non-significant relationship.

3.2 Functional forms for the model

To solve the model and obtain quantitative results, we need to employ specific functional forms. The form we employ for the period utility function is:

$$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \tag{23}$$

where $\sigma > 0$ is the coefficient of relative risk aversion.

The education and skill acquisition functions for unskilled workers respectively, are:

$$g(e_t, z_t) = B^e \left[(e_t)^{\gamma} (z_t)^{1-\gamma}\right]^{\xi} \tag{24}$$

where B^e is an education productivity parameter;[15] $0 < \gamma < 1$ is the elasticity of new human capital with respect to education time; $0 < \xi < 1$ measures returns to scale; and

$$q(h_t) = (h_t)^{\psi} \tag{25}$$

where $0 < \psi < 1$ is the elasticity of skill acquisition with respect to human capital.

[15] For example, B^e can be thought of as being determined by exogenous factors such as: (i) technology developments which facilitate learning; (ii) education research to improve teaching methods; (iii) societal changes involving greater parental participation in the education of offspring; and (iv) government education policies.

Finally, following e.g. Krusell *et al.* (2000) and He (2012), the production technology is given by:

$$\begin{gathered} Y\left(l_t^{s,f}, l_t^{u,f}, k_t^f\right) = \\ = A\left\{\mu\left(l_t^{u,f}\right)^{\alpha} + (1-\mu)\left[\left(k_t^f\right)^{\nu} + (1-\rho)\left(l_t^{s,f}\right)^{\nu}\right]^{\frac{\alpha}{\nu}}\right\}^{\frac{1}{\alpha}} \end{gathered} \tag{26}$$

where $A > 0$ is the level of total factor productivity; $\alpha, \nu < 1$, are the parameters determining the factor elasticities, i.e. $1/(1-\alpha)$ is the elasticity of substitution between capital and unskilled labour and between skilled and unskilled labour, whereas $1/(1-\nu)$ is the elasticity of substitution between capital and skilled labour; and $0 < \mu, \rho < 1$ are the factor share parameters. The functional form employed implies that the marginal products of both types of labour are increasing in the capital stock. The DCE obtained using these functional forms is summarised in Appendix A.

3.2.1 Skill premium

The above specific functional forms can help us to analytically examine the factors that drive the skill premium in our setup, which is defined as the ratio of the return to one unit of skill-labour time, w_t^s, over the return to one unit of unskilled-labour time, $h_{t-1}w_t^u$. The competitive equilibrium obtained using the functional forms (23) to (26) implies that the skill premium is given by:

$$\begin{gathered} \frac{w_t^s}{h_{t-1}w_t^u} = \frac{(1-\mu)}{h_{t-1}\mu}(1-\rho)\left(n_{t-1}^s\right)^{\nu-1}(\Omega)^{1-\alpha}\left[\rho\left(k_{t-1}\right)^{\nu} + \right. \\ \left. + (1-\rho)\left(n_{t-1}^s\right)^{\nu}\right]^{\frac{\alpha}{\nu}-1} \end{gathered} \tag{27}$$

where $\Omega \equiv \left(1-n_{t-1}^s\right)h_{t-1}l_t$. Consistent with the literature (see e.g. Krusell *et al.* (2000) and He and Liu (2008)), the skill premium is, *ceteris paribus*, increasing in the capital stock, as long as $\alpha > \nu$ and $0 < \mu,\rho,\alpha < 1$. To see this, note that:

$$\begin{gathered} \frac{\partial\left(\frac{w_t^s}{h_{t-1}w_t^u}\right)}{\partial k_{t-1}} = \frac{(1-\mu)}{h_{t-1}\mu}(1-\rho)\left(n_{t-1}^s\right)^{\nu-1}(\Omega)^{1-\alpha} \times \\ \times\left[\rho\left(k_{t-1}\right)^{\nu} + (1-\rho)\left(n_{t-1}^s\right)^{\nu}\right]^{\frac{\alpha}{\nu}-2}(\alpha-\nu)\rho\left(k_{t-1}\right)^{\nu-1} > 0. \end{gathered} \tag{28}$$

Moreover, regarding the effect of skill acquisition, it can be shown that in our model, the skill premium is, *ceteris paribus,* decreasing in the share of skilled labour, for values of $\nu < 1$ and $0 < \mu,\rho,\alpha < 1$. To see this, note that:

$$\begin{aligned}
&\frac{\partial\left(\frac{w_t^s}{h_{t-1}w_t^u}\right)}{\partial n_{t-1}^s} = \tfrac{(1-\mu)}{h_{t-1}\mu}\left(1-\rho\right)\left(\nu-1\right)\left(n_{t-1}^s\right)^{\nu-2}\left(\Omega\right)^{1-\alpha}\left[\rho\left(k_{t-1}\right)^{\nu}+\right.\\
&\left.+\left(1-\rho\right)\left(n_{t-1}^s\right)^{\nu}\right]^{\frac{\alpha}{\nu}-1}+\tfrac{(1-\mu)}{h_{t-1}\mu}\left(1-\rho\right)\left(n_{t-1}^s\right)^{\nu-1}\left(1-\alpha\right)\left(\Omega\right)^{-\alpha}\times\\
&\times h_{t-1}l_t\left(-1\right)\left[\rho\left(k_{t-1}\right)^{\nu}+\left(1-\rho\right)\left(n_{t-1}^s\right)^{\nu}\right]^{\frac{\alpha}{\nu}-1}+\tfrac{(1-\mu)}{h_{t-1}\mu}\left(1-\rho\right)\times\\
&\left(n_{t-1}^s\right)^{\nu-1}\left(\Omega\right)^{1-\alpha}\left[\rho\left(k_{t-1}\right)^{\nu}+\left(1-\rho\right)\left(n_{t-1}^s\right)^{\nu}\right]^{\frac{\alpha}{\nu}-2}\left(\alpha-\nu\right)\times\\
&\times\left(1-\rho\right)\left(n_{t-1}^s\right)^{\nu-1}<0.
\end{aligned} \tag{29}$$

Finally, regarding the *ceteris paribus* effect of human capital, it can be shown that the skill premium is decreasing for values $0 < \mu,\rho,\alpha < 1$. To see this, note that:

$$\begin{aligned}
&\frac{\partial\left(\frac{w_t^s}{h_{t-1}w_t^u}\right)}{\partial h_{t-1}} = -\alpha\tfrac{(1-\mu)}{\mu}\left(1-\rho\right)\left(n_{t-1}^s\right)^{\nu-1}\left(\left(1-n_{t-1}^s\right)l_t\right)^{1-\alpha}\left[\rho\left(k_{t-1}\right)^{\nu}+\right.\\
&\left.+\left(1-\rho\right)\left(n_{t-1}^s\right)^{\nu}\right]^{\frac{\alpha}{\nu}-1}h_{t-1}^{-\alpha-1}<0.
\end{aligned} \tag{30}$$

To summarise, as in the literature, the skill premium increases when the capital stock increases and when the labour supply of skilled decreases. In our model, given that effective labour supply is driven by the share of skilled workers in the labour force and by the human capital of the unskilled, increases in either tends, other things equal, to lower the skill premium.

3.3 Calibration, solution and evaluation

We calibrate the model by first using values that are standard in the literature and then choose the remaining parameters so that the model's steady-state is consistent with the relevant annual data for the U.S. economy from 1963-2000. The parameters for the model in Section 2 are shown in Table 2.

With respect to commonly employed values in the literature, we set the: (i) rate of time preference, $\beta = 0.98$; (ii) depreciation rate on physical capital, $\delta = 0.10$; (iii) coefficient of relative risk aversion, $\sigma = 2$; and (iv) net population growth rate, $\theta = 0.01$. We follow Perli and Sakellaris (1998) and set the depreciation rate on human capital as $\delta^h = 0.10$. We normalise the exogenous productivity parameters, i.e. A, B, B^e and the exogenous skill threshold, S^*, to unity. Finally, we normalise the range of the distribution of the idiosyncratic abilities, ϕ_t, $[0, 1]$.

The elasticities in the production function are parameterised using the estimates in Krusell *et al.* (2000), i.e. $\nu = -0.495$ and $\alpha = 0.401$, implying elasticities of substitution between capital and skilled labour and between capital (or skilled labour) and unskilled labour of about 0.67 and 1.67 respectively.[16]

Table 2: Parameter values

parameter	value	definition
$\frac{1}{1-\alpha}> 0$	1.667	capital to unskilled labour elasticity
$0 < \beta < 1$	0.980	rate of time preference
$A > 0$	1.000	total factor productivity
$B> 0$	1.000	productivity of physical capital
$B^e> 0$	1.000	productivity of education, unskilled
$0 \leq \delta \leq 1$	0.100	depreciation rate on physical capital
$0 \leq \delta^h\leq 1$	0.100	depreciation rate on human capital
$0 < \gamma < 1$	0.912	human capital to education elasticity, unskilled
$0 < \mu < 1$	0.230	labour weight in composite input share
$\frac{1}{1-\nu}> 0$	0.669	capital to skilled labour elasticity
$\phi^l\geq 0$	0.000	minium ability bound
$\phi^h\geq 0$	1.000	maximum ability bound
$0 < \rho < 1$	0.590	capital weight in composite input share
$S^*> 0$	1.000	skill threshold
$\sigma > 1$	2.000	coefficient of relative risk aversion
$\theta > 0$	0.010	net population growth rate
$\xi > 0$	0.455	returns to scale in unskilled human capital
$0 <\psi < 1$	0.005	skill to human capital elasticity

The steady-state variables we concentrate on matching to the data include the: (i) skill premium (1.628); (ii) share of skilled workers (0.429); (iii) labour share in income (0.694); (iv) education-time share (0.286); and (v) private education spending to GDP (0.01). The data on skill premium and skill share were explained in detail above and the targets are averages over the time period. We use data on GDP and private education spending from the Federal Reserve Bank of St. Louis FRED database to calculate the average private education spending to GDP ratio and data from NIPA Table 1.10 to set the target for the labour share in income. Finally, to obtain the target for the education time share, we assume that for the unskilled labour force total time (in years) consists of approximately 35 years of work and 14 years of schooling. The latter number is obtained under the assumption that workers who are high school graduates but do not have a college degree have at least 12 years of basic education and perhaps some years of college or other

[16]As discussed in Krusell *et al.* (2000) and Hornstein *et al.* (2005), these estimates cohere well with the microeconometric evidence reported in the literature.

training, but less than 16 years of education.[17] This implies that on average 14 years out of 49 years of total time are spent in education, giving the value of 0.286. To achieve these five targets we set the following five parameters as follows: $\mu = 0.230$, $\psi = 0.005$, $\rho = 0.59$, $\gamma = 0.912$ and $\xi = 0.455$.

The results for the steady-state solution are reported below in Table 3. Given that we do not have government consumption and investment spending in the model, the consumption and investment to output ratios reflect total consumption and investment in the economy.

Table 3: Steady-state

$\frac{c}{y}$	$\frac{i}{y}$	$\frac{k}{y}$	$\frac{z}{y}$	$\frac{wl}{y}$	e	$\frac{w^s}{hw^u}$	n^s	$r-\delta$
0.736	0.254	2.54	0.010	0.694	0.286	1.628	0.429	0.0204

To solve for the dynamic paths, we work as follows. We assume that the economy is at its steady-state at the time of a permanent shock which takes the form of one-off, permanent changes in the relevant productivity parameter. We then obtain the transitional paths of the endogenous variables of the system as they evolve towards the new steady-state. To solve for the dynamics of the model, we impose the permanent shock in period-zero and obtain the dynamic solution of the non-linear DCE system of equations in Appendix A for T periods. Therefore, the initial conditions for the model's state variables are given by the initial steady-state. For the terminal values of the forward looking variables, we assume that after T years the dynamic system has converged to its new steady-state. This implies that the appropriate terminal conditions are obtained by setting the values for these variables equal to those of the preceding period. After appropriate substitutions at the level of the DCE, the final system is reduced to $(6 \times T)$ equations, which is solved non-linearly using standard numeric methods (see, e.g. Adjemian *et al.* (2011)). This gives the dynamic transition to the new steady-state for the model's endogenous variables. We set $T = 500$ to ensure that convergence is achieved. Our results show that this occurs for all endogenous variables for both models within 300 years.[18]

To evaluate the contribution of the labour augmenting channel, in Figure 2 below, we plot the transition paths, in terms of % deviations from the original steady state, for $\{n_t^s, \frac{w_t^s}{h_{t-1}w_t^u}\}_{t=0}^{T^*}$, $T^* = 300$, after a permanent, 1% change

[17] See, for example, Goldin and Katz (2008) on the difference between college enrolment and college graduation for the U.S. labour force. These data imply that many unskilled workers have attended college for a few years.

[18] By initialising the economy from below or above its steady state and calculating the transition paths as described above, we can confirm that the economy does not converge to another steady state, when the parameters in Table 2 remain unchanged.

in B, implemented as described above. These paths are denoted as LMA on the figure and thus capture the model predicted elasticities for the skill premium and the skill share with respect to investment-specific technical change. Moreover, we also plot the corresponding paths for a version of the model where although unskilled human capital contributes to skill accumulation, it is not augmenting the productivity of unskilled workers in unskilled jobs. This version of the model therefore only considers the mobility augmenting role of human capital and thus is termed as MA in Figure 2. This is obtained by setting $h_{t-1}l_t = l_t$ in (12) and (21) and it is re-calibrated to ensure that it implies the same long-run solution for the targets of the model's steady state as described for the LMA model above.

[Figure 2 here]

As can be seen in Figure 2, the elasticities predicted by the LMA model presented in Section 2 cohere with the empirical evidence presented in Table 1. In particular, the model predicts an impact elasticity of investment-specific technical change on the skill premium of about 0.1 and a long-run elasticity of about 0.34.[19] Moreover, both the impact and long-run elasticity of the skill share with respect to investment-specific technical change are about zero, consistent with the findings in Table 1.[20] These results are in sharp contrast with those obtained from the MA version of the model (note that the initial values for the skill share and the skill premium for both versions are the same and consistent with the data averages as discussed above). In particular, in this case, the impact elasticity of the skill premium in response to a change in B is more than 0.5, whereas the long-run elasticity is significantly lower, at about 0.1. In the medium run, the predictions of both models regarding the effect of investment-specific technical change on the skill premium are more similar, although generally the MA version gives rise to results which imply a smaller increase in wage inequality. Furthermore, this version predicts an unrealistically high response of the skill supply in the medium to long-run, as the elasticity of the skill share is constantly higher than the LMA version and is about 0.27 in the long-run. In the short-run, instead, the MA version

[19] The skill premium in the MA version of the model is defined as $\frac{w_t^s}{w_t^u}$, since the return to one unit of labour is w_t^u and not $h_{t-1}w_t^u$, as in the LMA version. If we define the skill premium in the LMA version as $\frac{w_t^s}{w_t^u}$, then the impact and long-run elasticities are 0.095 and 0.382 respectively, which is very similar to the results reported here (see also the analysis in section 4.1).

[20] The economic mechanism in the model that gives rise to these results is explained in detail below when we analyse the effects of different forms of technological changes on the endogenous variables of the model.

predicts a decrease in the skill share, compared with very small changes predicted by the LMA version and estimated in Table 1.

To summarise, the predictions of the model as presented in Section 2, allowing for the labour augmenting role of human capital in addition to transition to skilled employment, are very close to the data regarding the short- and long-run effect of investment-specific technical change on skill supply and the skill premium. Partial and general equilibrium models in the literature have been successful in matching many properties of the patterns of the skill premium in the data (see e.g. Goldin and Katz (2008), Acemoglu and Autor (2011) and He (2012)). Here we focused on the impact and long-run elasticities of the effect of investment-specific technical change on wage inequality and skill supply. Our analysis suggests that the workhorse model of human capital and aggregate dynamics can be modified to study the joint determination of skill transition and the skill premium, and this approach highlights the labour augmenting role of human capital that is required for the transition to skilled jobs. Therefore, we continue with the version of model with labour augmenting human capital, to evaluate the dynamic effects of productivity changes.

4 Technical change and transition to skill employment

We next calculate the dynamic effects of permanent increases in investment- and education-specific technical change, which are directly affecting demand for and supply of skilled labour and thus have a particular importance in the analysis of the skill premium. We assume that the economy is at its steady-state at the time of a one-off, permanent 1% change in respectively B and B^e. We then obtain the transitional paths of the endogenous variables of the system as they evolve towards the new steady-state. To solve for the dynamics of the model we work as described above. The results are plotted in Figures 3-5.

4.1 Investment-specific technical change

We first examine the effects of a permanent change in investment-specific technical change (B), which is expected to lead to increases in the capital stock and thus in the skill premium under capital-skill complementarity (see section 3.2.1). However, under endogenous skill supply these wage differentials act as an incentive for increased human capital accumulation and transition to skilled employment (see section 2.2.1). The latter, in turn, tends to increase the relative skill supply which acts to reduce the skill premium (see section 3.2.1). Hence, the net effect of investment-specific technical change on wage inequality can only be determined when the forces that define the above trade-off are taken into account quantitatively.

We plot, in Figure 3, the transition paths of the key endogenous variables of the system in percent deviations from the initial steady-state (as summarised in Table 3). As expected, investment-specific technical change increases consumption and output. It also increases the accumulation of capital and thus increases the marginal products of both skilled and unskilled labour, although, by equation (28) in section 3.2.1, it favours the returns to skilled hours more than unskilled, leading to increased labour income inequality, as captured by the rising skill premium. The increased skill premium creates an incentive, via equation (16) as discussed in Section 2.2.1, to increase occupational transition in the form of increases in the share of skilled labour force. However, this requires resources (work time and goods), for which the opportunity cost has risen given the rise in the capital stock. In particular, goods investment in physical capital is relatively cheaper than investment in human capital, given the rise in B, hence the household finds it optimal to re-allocate resources from education to physical capital. At the same time, since the marginal product of the unskilled labour input has increased following the increase in capital (see the response of w_t^u), the incentives to increase the effective supply for unskilled labour are higher.

[Figure 3 here]

In the short-run, the optimal resolution of this trade-off for unskilled labour implies a small decrease in human capital and a larger increase in work time, which jointly imply an increase in effective labour supply, $h_{t-1}l_t$. The small decrease in human capital is achieved by releasing time for work and goods for investment in physical capital, but also implies in the short-run a very small reduction in the skill share. In the medium- to long-run, the incentive to create skilled labour dominates the incentive to save resources to invest in physical capital, so that human capital is increasing (implying increases in time and goods investment in human capital) and this leads to increases in the skill share. However these remain quantitatively small, so that they do not work to reduce in effect the skill premium, which remains at an increased level. The reason is that as human capital increases, the effective return to unskilled work time, $h_{t-1}w_t^u$, also increases, thus working against the reduction in unskilled work time and in effect mediating the desirability of transition to skill (recall equation (16) and the discussion in Section 2.2.1).

4.2 Education-specific technical change

We next examine the effects of a 1% permanent increase in B^e, capturing an education reform that increases the effectiveness of time and goods invested in education, in the process of unskilled human capital creation. This can be achieved by improving education provision via, for example, changes in the curriculum, smaller class size, improvements in teaching methods, pre-school support for children from less privileged backgrounds, etc. Such reforms are proposed by e.g. Goldin and Katz (2008) as a key policy recommendation to improve the high wage inequality - low skill acquisition stalemate observed in recent decades. Improvements in the education technology that creates unskilled human capital are expected to increase human capital accumulation of the unskilled and thus occupational transition. These in turn should put pressure to lower the skill premium. However, it is less clear whether such a policy will create benefits for all agents, or whether the reduction in wage inequality will be created by improvements in the returns to the unskilled which do not imply increased benefits for the skilled. To answer this question, we plot, in Figure 4, the transition paths of the key endogenous variables of the system in percent deviations from the pre-reform steady-state.

[Figure 4 here]

As expected, this reform increases human capital accumulation, which leads to increased occupational transition, as captured by the increased share of skilled workers in the population. These changes imply falls in the skill premium (see equations (29) and (30))). However, the implications of this form of technical change for the earnings accruing to the skilled and unskilled groups differ between the short-run and the long-run. The difference is due to the dynamics of capital accumulation. In particular, in the short-run, investment in physical capital decreases to create resources for investment in education, given the increase in the relative productivity of the latter. Hence, physical capital is temporarily reduced, and given the strong complementarity between the capital stock and skilled labour, the returns to the latter also temporarily fall. However, the increase in economic resources following education-specific technical change implies that in the medium- to long-run investment and the stock of physical capital increase, and with them the returns to skill labour. Hence, education-specific technical change leads to temporary losses to earnings for skilled workers, but it in the medium- to long-run this type of reform benefits both skilled and unskilled. This is consistent with historical evidence from earlier educational reforms, which were generally broadly supported by the population irrespective of occupation (see e.g. Galor and Moav (2006) and Goldin and Katz (2008)).

4.3 Investment- and education-specific technical change

We next ask the question: how much must education-specific technical change increase, in the face of exogenous increases in investment-specific technical change, to neutralise the negative effects of the latter on the skill premium? The motivation is the following. Investment-specific technical change not only is a constant feature of production structures in modern society, but also an important contributor to improvements in aggregate economic outcomes and productivity (as was captured, for instance, in the context of this model, by the dynamic paths in Figure 3). However, it does increase wage inequality and is biased against the unskilled, who fail to capitalise on a large part of the new opportunities that it creates, as is captured in recent experience by the increases in wage inequality (see again Figure 3). A policy that has often been proposed to mediate the negative wage inequality effect of investment-specific technical change is to improve the effectiveness of the education system, to provide opportunity and means to the unskilled to improve their skills. The model developed here, by combining the previous two experiments, i.e. considering a simultaneous increase in B and B^e, allows us to quantify the relationship between the two types of technical change that would achieve the aim stated in the opening question.

[Figure 5 here]

Obviously, the answer depends on the time horizon set. As shown in Figure 5, if the target is eliminating long-run wage inequality, then education-specific technical change must increase by about half of the increase in investment-specific technical change, implying that reforms and improvements of the education system must be such that the productivity of the inputs is increased by half of the increase in technology that favours capital accumulation. Such a policy would effectively eliminate the increase in the skill premium in the long-run, but would also mediate its increase in shorter time horizons, to about or less than a third of the increase observed if only investment-specific technical change took place. Moreover, it would increase aggregate gains to average consumption and income in the economy by about 50%, compared with the situation where the improvement in skill-biased technical change was not met by any improvement in education.

4.4 Welfare and earnings inequality implications

We summarise the welfare and inequality implications of the above changes. First, to rank order the above scenarios in terms of aggregate welfare, Table

4 reports the welfare gains along the transition path and at the steady-state based on a constant compensating consumption supplement calculated as:

$$\xi \simeq \left| \frac{1}{(1-\sigma)} \right| \times \ln \left[\frac{U^s}{U} \right] \times 100 \tag{31}$$

where U^s refers to either post-shock discounted lifetime welfare (see equation (11)) or steady-state welfare and U refers to pre-shock steady-state welfare. The welfare gains reported in Table 4 range from a bit over 1/3 of a percent to nearly 3/4's of a percent. These gains are generally lower along the transition path than at the steady-state for each permanent change. Finally, both lifetime and steady-state welfare gains are highest when education-specific technical change, B^e, and investment-specific technical, B, change are shocked with the aim of neutralising the negative effects of B on the skill premium.

Table 4: Welfare gains ξ (%)

shocks	lifetime	steady-state
B	0.344	0.422
B^e	0.336	0.489
B & B^e	0.512	0.667

Finally, in Figure 6, we plot the dynamic paths of the model under the three cases of permanent changes considered above for various inequality indicators. In particular, we plot the earnings inequality ratio between the two groups, E_t, the proportion of total labour income that accrues to skill labour relative to unskilled labour, S_t, and also a Gini measure of earnings inequality, G_t. The definitions for these measures follow Ehrlich and Kim (2007, p. 144). In particular,

$$E_t = \frac{w_t^s}{w_t^u h_{t-1} l_t} \tag{32}$$

$$S_t = E_t \frac{n_{t-1}^s}{n_{t-1}^u} \tag{33}$$

and

$$G_t = \frac{S_t - P_t}{(1+S_t)(1+P_t)} \tag{34}$$

where $P_t = \frac{n_{t-1}^s}{n_{t-1}^u}$.

The general message from Figure 6 is that earnings inequality between the two classes: (i) increases after investment-specific technological shocks; (ii) decreases after education-specific changes; and (iii) when both shocks are combined, earnings inequality follows an inverted-U pattern, at least with respect to the earnings inequality ratio and Gini index. Moreover, in all cases, the patterns for earnings inequality are generally similar to the dynamic pattern of the basic inequality in wages in Figures 3-5, especially for the earnings inequality ratio and the Gini index. This similarity of dynamic paths between measures of wages and earnings inequality has also been noted in another context in Ehrlich and Kim (2007) and is also broadly consistent with the American data (see e.g. Heathcote *et al.* (2010)) and cross-sectional country data (see e.g. Ehrlich and Kim (2007)).

[Figure 6 here]

5 Conclusions

This paper extended the literature concentrating on the endogenous joint determination of transition to skilled employment and the skill premium by treating the former as a dynamic process depending on labour augmenting human capital accumulation by unskilled workers. We employed a tractable dynamic general equilibrium framework, which predicts short- and long-run effects of investment-specific technical change on skill supply and the skill premium that are very close to the data when the labour augmenting role of human capital for transition to skilled employment is taken into account. In particular, this model produces a small impact and a larger long-run elasticity of the skill premium with respect to investment-specific technical change, of magnitudes similar to those in the data. It also predicts that the elasticity of skill supply with the respect to the data is very small, which is also consistent with empirical evidence.

We then evaluated the dynamic general equilibrium effects of permanent increases in technical change. Investment-specific technical change leads to very small increases in the relative supply of skilled workers and to significant and rising increases in the skill premium. The rising skill premium incorporates incentives for human capital creation to facilitate the transition to skilled employment. However, as human capital increases, the effective return to unskilled work time also increases, given the labour augmenting role of human capital. This in turn works to reduce the desirability of transition to skill. Education-specific technical change reduces wage inequality and increases mobility. Although there is a fall in the wages of the skilled in the short-run, in the medium to long-run, returns to both skilled and unskilled labour increase. This is because a more productive and wealthier economy, resulting from the education reforms, is associated with higher capital accumulation which has positive effects for skilled workers. We also find that when education-specific technical change permanently increases by about half of the 1% permanent increase in investment-specific technical change, wage inequality is reduced between half and two thirds in the short-run and is eliminated in the long-run. Moreover, output and consumption increase by about 50% in the long-run.

Our analysis focused on the effects of investment- and education-specific technical change that are fully accounted for by the market mechanism. In other words, we did not consider market failures that may result from e.g. human capital externalities and/or credit constrained unskilled workers. Such market failures are important in explaining persistent inequality and in justifying education interventions (see e.g. Galor and Zeira (1993), Galor and Moav (2006) and Galor *et al.* (2009)) and would further increase the desirability of education reforms to accompany investment-specific technical change.

References

[1] Acemoglu, D. (2003). 'Cross country inequality trends,' *Economic Journal*, 113, F121-F149.

[2] Acemoglu, D. and D. Autor (2011). 'Skills, tasks and technologies: implications for employment and earnings,' in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, 4b, 1043-1171, Amsterdam: Elsevier.

[3] Adjemian, S., Bastani, H., Karamé, F., Juillard, M., Maih, J., Mihoubi, F., Perendia, G., Pfeifer, J., Ratto, M. and S. Villemot (2011). 'Dynare: Reference Manual Version 4', Dynare Working Papers 1, CEPREMAP, revised Jan 2014.

[4] Angelopoulos, K., Asimakopoulos, S., and J. Malley (2015). 'Tax smoothing in a business cycle model with capital-skill complementarity', *Journal of Economic Dynamics and Control*, 51, 420-444.

[5] Arseneau, D. and S. Chugh (2012). 'Tax smoothing in frictional labor markets,' *Journal of Political Economy*, 120, 926-985.

[6] Autor, D., Katz, L. and M. Kearney (2008). 'Trends in U.S. wage inequality: Revising the revisionists,' *The Review of Economics and Statistics*, 90, 300–323.

[7] Autor, D., Levy, F. and R. Murmane (2003). 'The skill context of recent technological change: An empirical investigation,' *The Quarterly Journal of Economics*, 118, 1279-1333.

[8] Barro, R. and X. Sala-i-Martin (2004). Economic Growth (2nd ed), MIT Press.

[9] Ben-Porath, Y. (1967). 'The production of human capital and the life cycle of earnings', '*The Journal of Political Economy*', 75, 352-365.

[10] Card, D. and T. Lemieux (2001). 'Going to college to avoid the draft: The unintended legacy of the Vietnam war,' *American Economic Review*, 91, 97-102.

[11] Cummins, J. and G. Violante (2002). 'Investment-Specific Technical Change in the US (1947-2000): Measurement and Macroeconomic Consequences,' *Review of Economic Dynamics*, 5, 243-284.

[12] Cúrdia, V. and M. Woodford (2009). 'Credit frictions and optimal monetary policy', BIS Working Papers 278, Bank for International Settlements.

[13] Di Nardo, J., Fortin, N. and T. Lemieux (1996). 'Labor market institutions, and the distribution of wages, 1973-1992: A semiparametric approach, *Econometrica*, 64, 1001-1044.

[14] Ehrlich, I. and J. Kim (2007). 'The evolution of income and fertility inequalities over the course of economic development: A human capital perspective', *Journal of Human Capital*, 1, 137-174.

[15] Galor, O., Moav, O. and D. Vollrath (2009). 'Inequality in land ownership, the emergence of human-capital promoting institutions, and the great divergence,' *Review of Economic Studies*, 76, 143-179.

[16] Galor, O. and O. Moav (2006). 'Das human-kapital: A theory of the demise of the class structure,' *Review of Economic Studies*, 73, 85-117.

[17] Galor, O. and O. Moav (2000). 'Ability-biased technological transition, wage inequality, and economic growth,' *The Quarterly Journal of Economics*, 115, 469-497.

[18] Galor, O. and J. Zeira (1993). 'Income distribution and macroeconomics,' *Review of Economic Studies*, 60, 35-52.

[19] Goldin, C. and L. Katz (2008). The Race between Education and Technology, Harvard University Press.

[20] Goldin, C. and R. Margo (1992). 'The great compression: The wage structure in the United States at mid-century,' *The Quarterly Journal of Economics*, 107, 1-34.

[21] Greenwood, J., Hercowitz, Z. and P. Krusell (2000). 'The role of investment-specific technological change in the business cycle', *European Economic Review*, 44, 91-115.

[22] Guvenen, F. and B. Kuruscu (2012). 'Understanding the evolution of the US wage distribution: A theoretical analysis,' *Journal of the European Economic Association*, 10, 482-517.

[23] Guvenen, F. and B. Kuruscu (2010). 'A quantitative analysis of the evolution of the US wage distribution, 1970–2000,' In D.Acemoglu, K. Rogoff, and M. Woodford, editors, NBER Macroeconomics Annual, 24, 227–276.

[24] He, H. (2011). 'Why have girls gone to college? A Quantitative examination of the female college enrollment rate in the United States: 1955-1980', Annals of Economics and Finance, 12, 43-66.

[25] He, H. (2012). 'What drives the skill premium: Technological change or demographic variation?' *European Economic Review*, 56, 1546-1572.

[26] He, H. and Liu, Z. (2008). 'Investment-specific technological change, skill accumulation, and wage inequality,' *Review of Economic Dynamics*, 11, 314-334.

[27] Heathcote, J., Storesletten, K. and G. Violante (2009). 'Quantitative macroeconomics with heterogeneous households,' NBER Working Paper 14768.

[28] Heckman, J., Lochner, L., and C. Taber (1998). 'Explaining rising wage inequality: explanations with a dynamic general equilibrium model of labor earnings with heterogeneous agents,' *Review of Economic Dynamics*, 1, 1-58.

[29] Hornstein, A., Krusell, P. and Violante, G. (2005). 'The effects of technical change on labor market inequalities,' in P. Aghion and S. Durlauf (eds.), *Handbook of Economic Growth*, vol. 1, 1275-1370. Amsterdam: Elsevier.

[30] Kambourov, G. and I. Manovskii (2009). ‘Occupational mobility and wage inequality,’ *Review of Economic Studies*, 76, 731-759.

[31] Katz, L. and K. Murphy (1992). ‘Changes in relative wages, 1963-1987: Supply and demand factors,’ *The Quarterly Journal of Economics*,107, 35-78.

[32] Krusell, P., Ohanian, L., Ríos-Rull J. and Violante, G. (2000). ‘Capital–skill complementarity and inequality: A macroeconomic analysis,’ *Econometrica*, 68, 1029–1053.

[33] Krusell, P. and A. Smith (1998). Income and wealth inequality in the macroeconomy, *Journal of Political Economy*, 106, 867-896.

[34] Lindquist, M. (2004). ‘Capital–skill complementarity and inequality over the business cycle’, *Review of Economic Dynamics*, 7, 519-540.

[35] Lucas, R. (1988). ‘On the mechanics of economic development,’ *Journal of Monetary Economics*, 22, 3-42.

[36] Maoz, D. and O. Moav (2004). ‘Social stratification, capital skill complementarity, and the non-monotonic evolution of the education premium,’ *Macroeconomic Dynamics*, 8, 295-309.

[37] Matsuyama K. (2006). ‘The 2005 Lawrence R. Klein Lecture: Emergent class structure,’ *International Economic Review*, 47, 327-360.

[38] Merz, M. (1995). ‘Search in the labor market and the real business cycle,’ *Journal of Monetary Economics*, 36, 269-300.

[39] Perli, R. and P. Sakellaris (1998). ‘Human capital formation and business cycle persistence,’ *Journal of Monetary Economics*, 42, 67–92.

[40] Pourpourides, P. (2011). ‘Implicit contracts and the cyclicality of the skill-premium’, *Journal of Economic Dynamics and Control*, 35, 963-979.

[41] Quadrini, V. (2000). ‘Entrepreneurship, saving, and social mobility,’ *Review of Economic Dynamics*, 3, 1-40.

[42] Rogerson, R. and R. Shimer (2011). ‘Search in macroeconomic models of the labor market,’ in *Handbook of Labor Economics*, vol. 2, edited by O. Ashenfelter, R. Layard, and D. Card, Amsterdam: Elsevier.

[43] Tinbergen, J. (1974). 'Substitution of graduate by other labor,' *Kyklos*, 27, 217-226.

[44] Tinbergen, J. (1975). 'Income distribution: Analysis and policies, Amsterdam: North-Holland.

[45] Trostel, P. (1993). 'The effect of taxation on human capital', *Journal of Political Economy*, 101, 327-350.

[46] Uzawa, H. (1965). 'Optimum technical change in an aggregative model of economic growth,' *International Economic Review*, 6, 18-31.

6 Appendix A

6.1 Household's optimality conditions

Consumption:

$$\frac{1}{c_t^{\sigma}} - \lambda_t^b = 0 \tag{A1}$$

Unskilled labour supply:

$$\begin{aligned} & w_t^u h_{t-1}(1-n_{t-1}^s)\lambda_t^b - \gamma B^e \xi \left(z_t\right)^{1-\gamma} \left[\left(z_t\right)^{1-\gamma}\left(1-l_t\right)^{\gamma}\right]^{\xi-1} \times \\ & \times \left(1-l_t\right)^{\gamma-1} \lambda_t^h = 0 \end{aligned} \tag{A2}$$

Unskilled human capital:

$$\begin{aligned} -\lambda_t^h + \beta \left[\left(1-\delta^h\right)\lambda_{t+1}^h + w_{t+1}^u l_{t+1}\left(1-n_t^s\right)\lambda_{t+1}^b\right] + \\ + \frac{S^*\left(1-n_{t-1}^s\right)\psi\lambda_t^s}{\left(h_t\right)^{1+\psi}\left(\phi^h-\phi^l\right)(1+\theta)} = 0 \end{aligned} \tag{A3}$$

Physical capital:

$$\frac{\beta\left(r_{t+1}+1-\delta\right)\lambda_{t+1}^b}{B_{t+1}} - \frac{\lambda_t^b}{B_{t+1}} = 0 \tag{A4}$$

Skilled labour share:

$$\begin{aligned} \beta\left[\left(w_{t+1}^s + z_{t+1} - w_{t+1}^u h_t l_{t+1}\right)\lambda_{t+1}^b - \frac{\lambda_{t+1}^s\left(\phi^l - \frac{S^*}{\left(h_{t+1}\right)^{\psi}}\right)}{\left(\phi^h-\phi^l\right)(1+\theta)}\right] - \\ -\lambda_t^s = 0 \end{aligned} \tag{A5}$$

where λ_t^b, λ_t^h, and λ_t^s refer to the time t Lagrange multipliers attached to the budget, unskilled human capital and skilled labour share constraints respectively.

6.2 Dynamic constraints

Unskilled human capital:

$$B^e (1 - l_t)^\gamma (z_t)^{1-\gamma} + (1 - \delta^h) h_{t-1} - h_t = 0 \tag{A6}$$

Skilled labour share:

$$\frac{n^s_{t-1} + \left(\frac{\phi^l - \frac{S^*}{(h_t)^\psi}}{\phi^h - \phi^l} + 1 \right) \left(1 - n^s_{t-1}\right)}{1 + \theta} - n^s_t = 0 \tag{A7}$$

6.3 Firm's optimality conditions

Physical capital:

$$\begin{aligned} &-r_t - A\rho \left(k^f_t\right)^{\nu-1} \left[\rho \left(k^f_t\right)^\nu - \left(l^{s,f}_t\right)^\nu (\rho - 1)\right]^{\frac{\alpha}{\nu}-1} (\mu - 1) \times \\ &\times \left\{ \left(l^{u,f}_t\right)^\alpha \mu - \left[\rho \left(k^f_t\right)^\nu - \left(l^{s,f}_t\right)^\nu (\rho - 1)\right]^{\frac{\alpha}{\nu}} (\mu - 1) \right\}^{\frac{1}{\alpha}-1} = 0 \end{aligned} \tag{A8}$$

Skilled labour:

$$\begin{aligned} &A \left(l^{s,f}_t\right)^{\nu-1} \left[\rho \left(k^f_t\right)^\nu - \left(l^{s,f}_t\right)^\nu (\rho - 1)\right]^{\frac{\alpha}{\nu}-1} (\mu - 1)(\rho - 1) \times \\ &\times \left\{ \left(l^{u,f}_t\right)^\alpha \mu - \left[\rho \left(k^f_t\right)^\nu - \left(l^{s,f}_t\right)^\nu (\rho - 1)\right]^{\frac{\alpha}{\nu}} (\mu - 1) \right\}^{\frac{1}{\alpha}-1} - \\ &-w^s_t = 0 \end{aligned} \tag{A9}$$

Unskilled labour:

$$\begin{aligned} &A \left(l^{u,f}_t\right)^{\alpha-1} \mu \{ \left(l^{u,f}_t\right)^\alpha \mu - \left[\rho \left(k^f_t\right)^\nu - \left(l^{s,f}_t\right)^\nu (\rho - 1)\right]^{\frac{\alpha}{\nu}} \times \\ &\times (\mu - 1) \}^{\frac{1}{\alpha}-1} - w^u_t = 0 \end{aligned} \tag{A10}$$

6.4 Market clearing conditions

Capital market:

$$k^f_t - k_{t-1} = 0 \tag{A11}$$

Unskilled labour market:

$$l^{s,f}_t - n^s_{t-1} = 0 \tag{A12}$$

Skilled labour market:

$$l^{u,f}_t - \left(1 - n^s_{t-1}\right) h_{t-1} l_t = 0 \tag{A13}$$

Aggregate resource constraint:

$$\begin{aligned} &A\{\mu \left[h_{t-1} l_t \left(1 - n^s_{t-1}\right)\right]^\alpha + \left[\rho (k_{t-1})^\nu + (1 - \rho) \left(n^s_{t-1}\right)^\nu\right]^{\frac{\alpha}{\nu}} \times \\ &\times (1 - \mu) \}^{\frac{1}{\alpha}} - k_t - c_t + (1 - \delta) k_{t-1} - \left(1 - n^s_{t-1}\right) z_t = 0 \end{aligned} \tag{A14}$$

Figure 1: Skill Premium, Relative Skill Supply and Productivity (1963-2000)

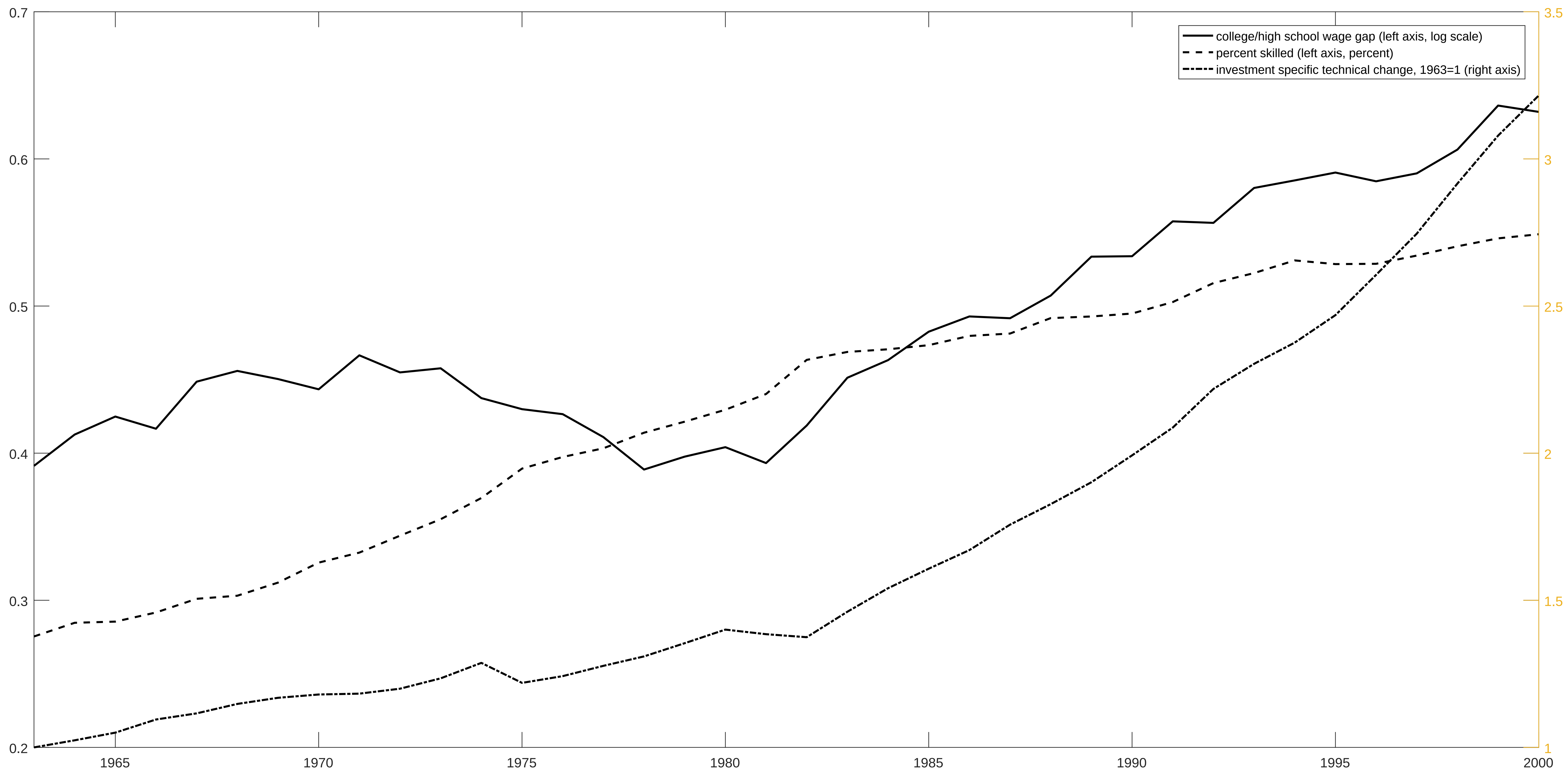

Figure 2: One-Percent Permanent Increase in B

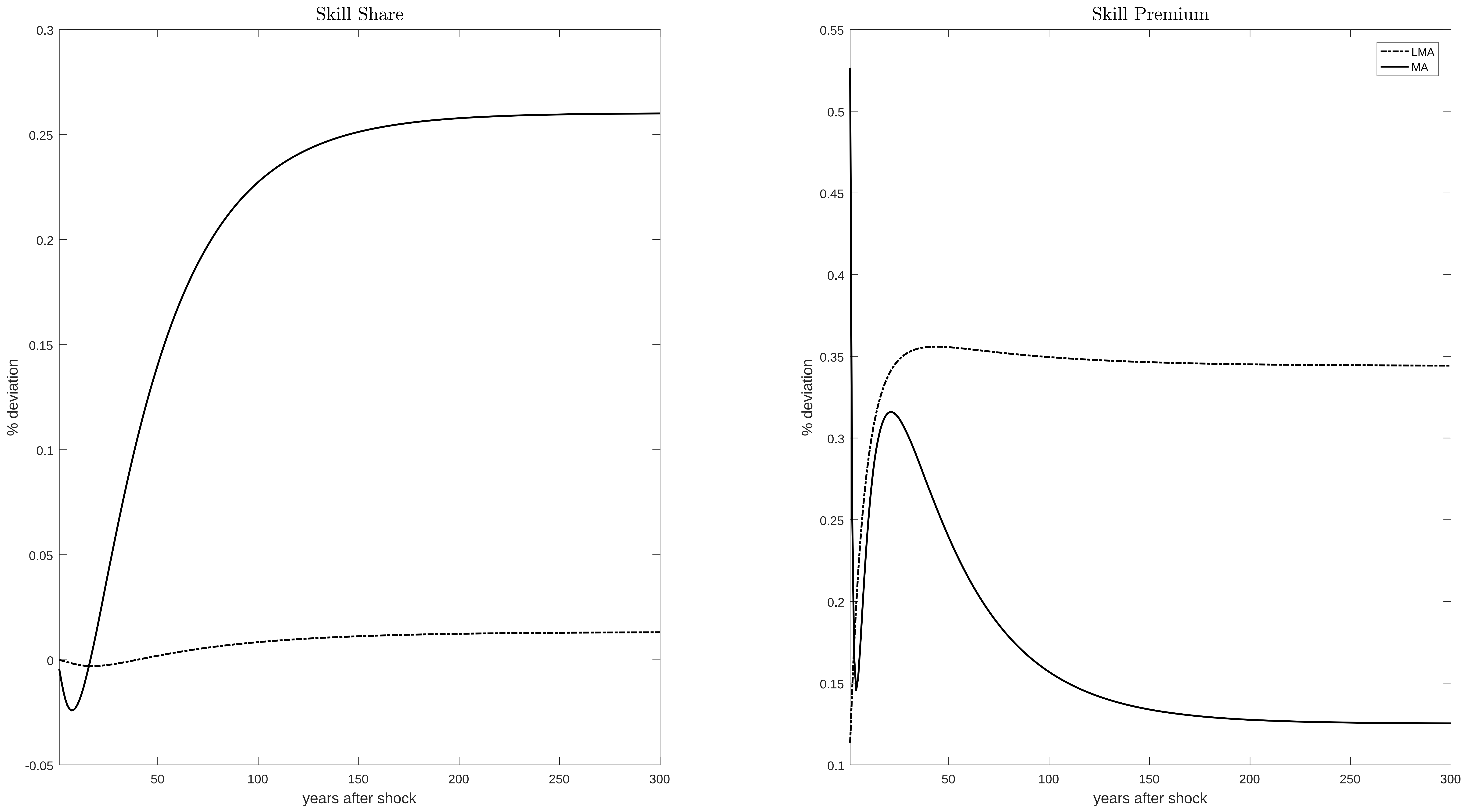

Figure 3: One-Percent Permanent Increase in B

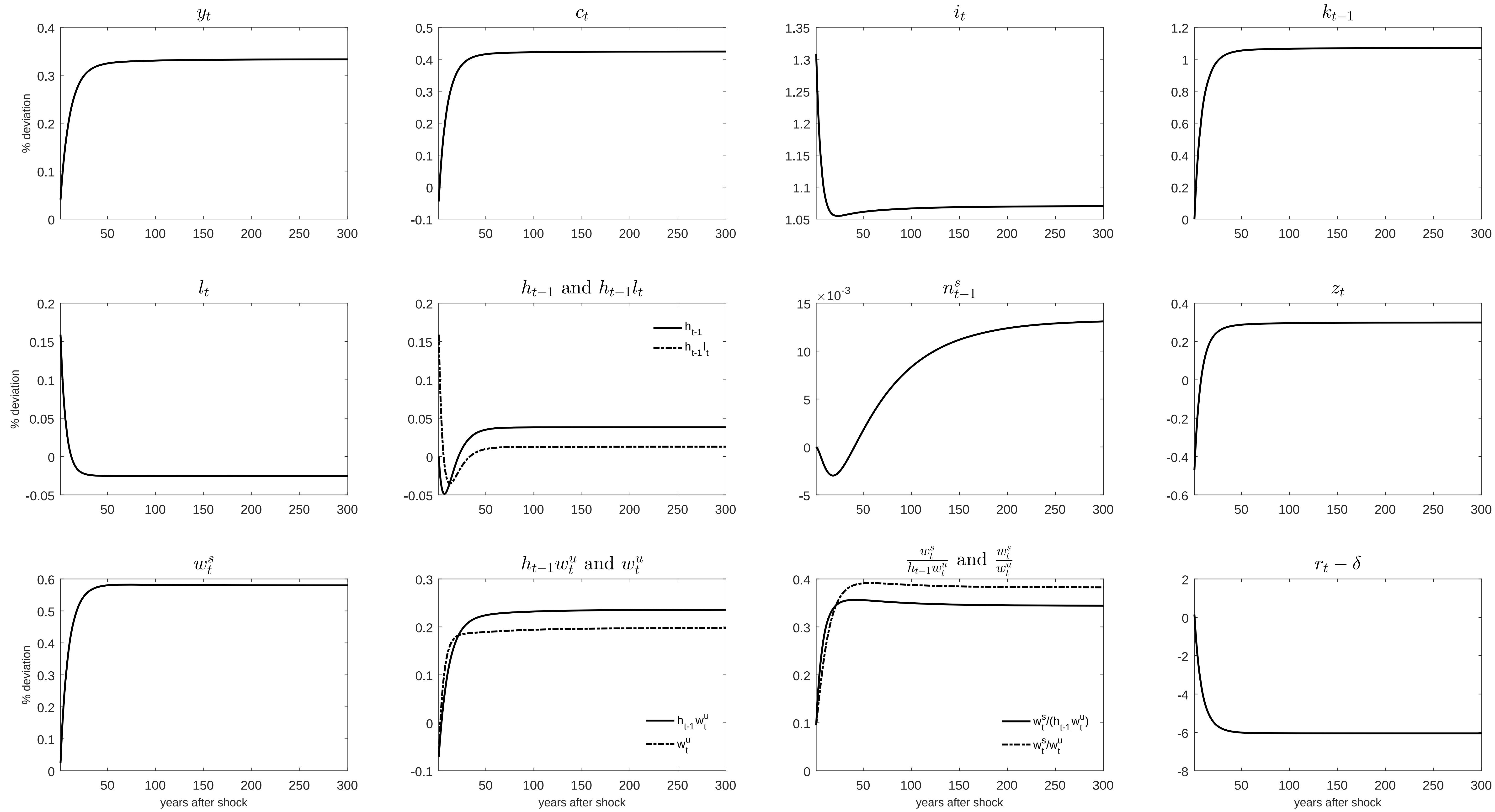

Figure 4: One-Percent Permanent Increase in B^e

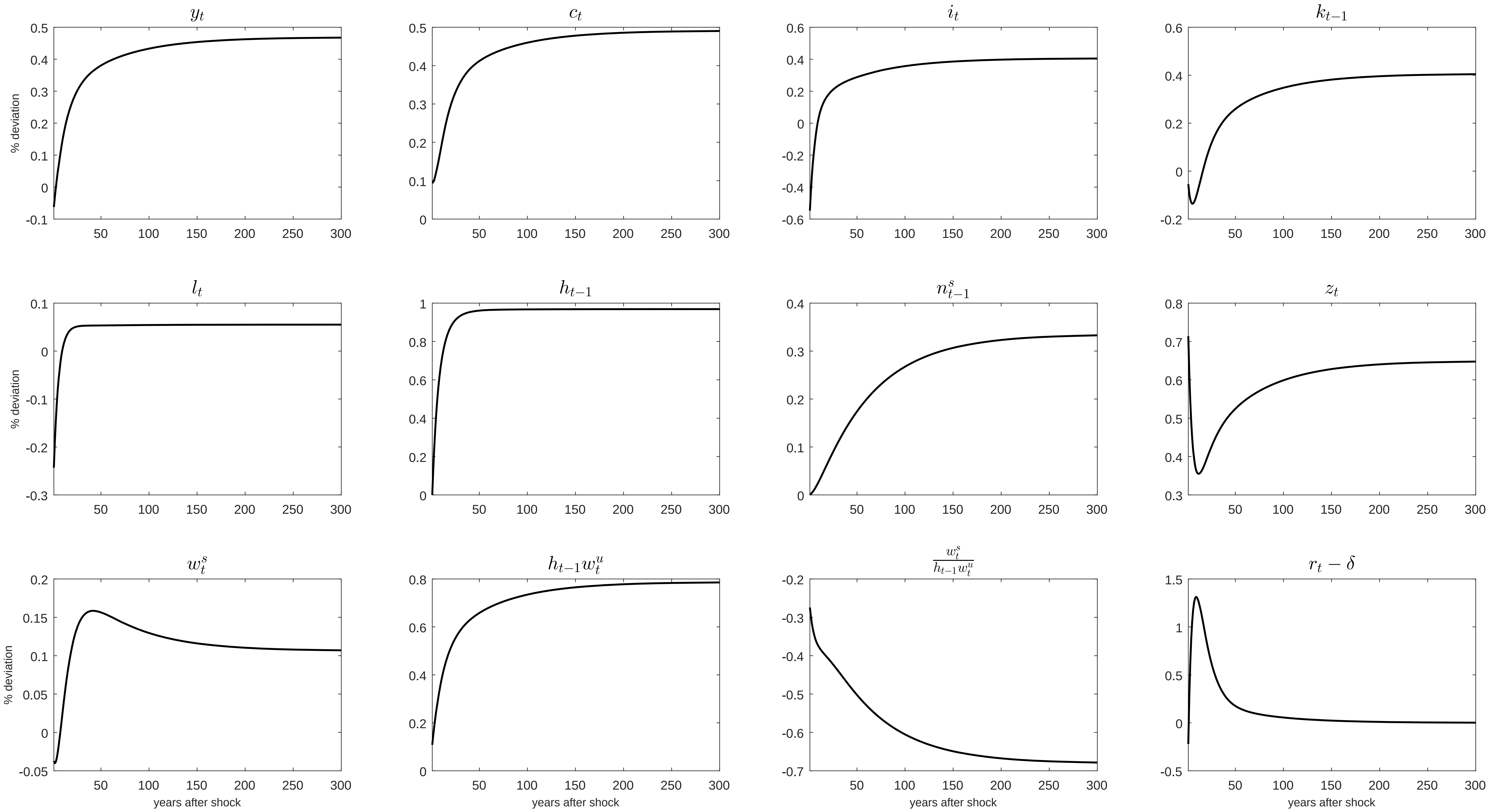

Figure 5: One Percent Permanent Increase in B and Half Percent Permanent Increase in B^e

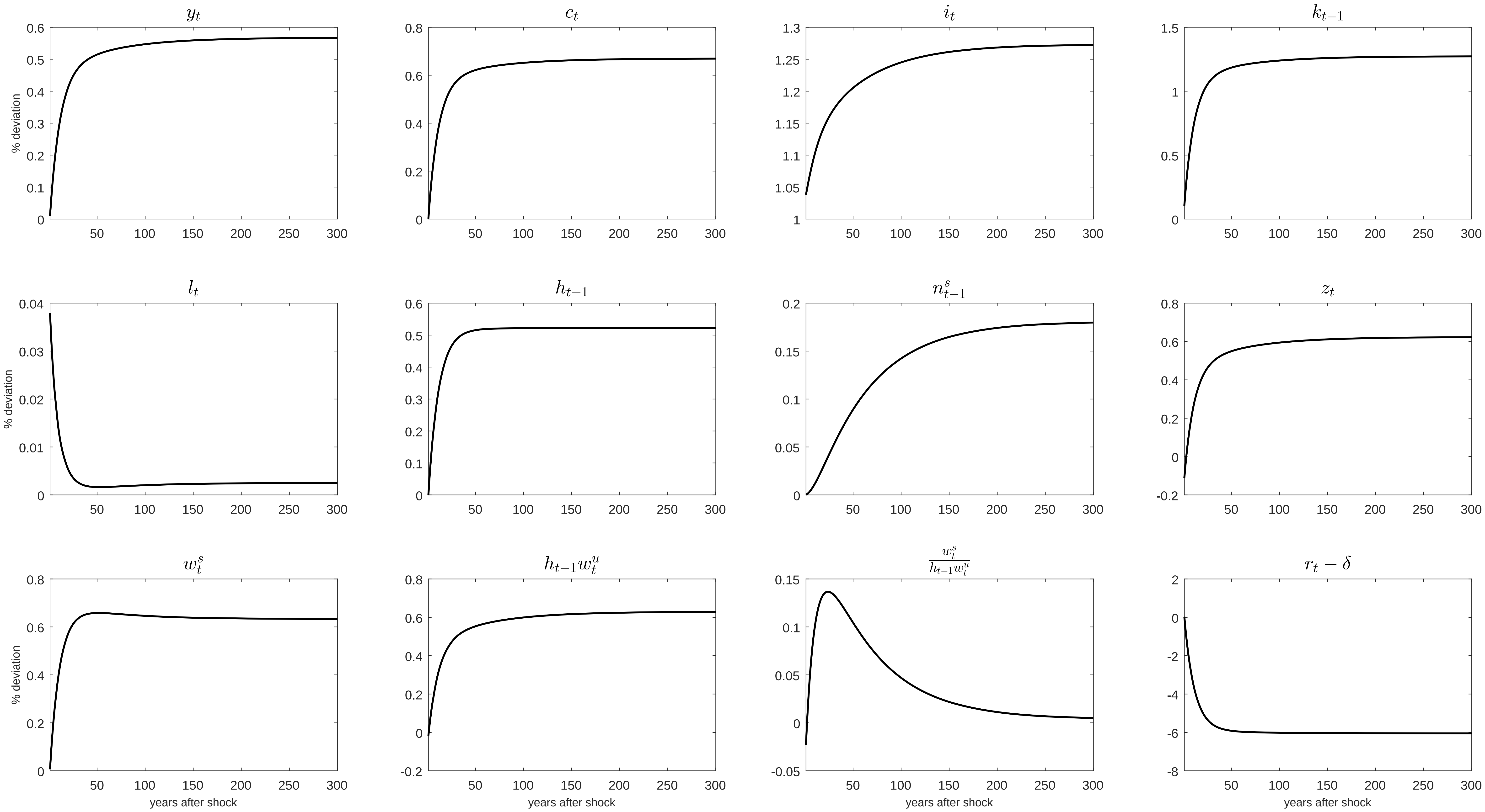

Figure 6: Inequaltity Measures: B Shock (row 1), B^e Shock (row 2), B and B^e Shocks (row 3)