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# Improved machine tool linear axis calibration through continuous motion data capture 

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#### Abstract

Machine tool calibration is becoming recognised as an important part of the manufacturing process. The current international standards for machine tool linear axes calibration support the use of quasi-static calibration techniques. These techniques can be time consuming but more importantly a compromise in quality due to the practical restriction on the spatial resolution of target positions on the axis under test. Continuous motion calibration techniques have the potential to dramatically increase calibration quality. Through taking several measurement values per second while the axis under test is in motion, it is possible to measure in far greater detail. Furthermore, since machine tools normally operate in dynamic mode, the calibration data can be more representative if it is captured while the machine is in motion. The drawback to measuring the axis while in motion is the potential increase in measurement uncertainty. In the following paper, different methods of continuous motion calibration are discussed. A time-based continuous motion solution is proposed as well as a novel optimisation and correlation algorithm to accurately fuse the data taken from quasi-static and continuous motion measurements. The measurement method allows for minimal quasi-static measurements to be taken while using a continuous motion measurement to enhance the calibration process with virtually no additional time constraints. The proposed method does not require any additional machine interfacing, making it a more readily accessible solution for widespread machine tool use than other techniques which require hardware links to the CNC. The result of which means a shorter calibration routine and enhanced results. The quasi-static and continuous motion measurements showed correlation to within $1 \mu \mathrm{~m}$ at the quasi-static measurement targets. An error of $13 \mu \mathrm{~m}$ was detailed on the continuous motion, but was missed using the standard test. On a larger, less accurate machine, the quasi-static and continuous motion measurements were on average within $3 \mu \mathrm{~m}$ of each other however, showed a standard deviation of $4 \mu \mathrm{~m}$ which is less than $1 \%$ of the overall error. Finally, a high frequency cyclic error was detected in the continuous motion measurement but was missed in the quasi-static measurement.


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## 1. Introduction

Measurement of the geometric errors of machine tools is becoming a more important part of maintaining the part accuracy of manufactured components. The measurement data can be used to evaluate capability when buying a machine and to monitor its performance during its operational lifetime. Furthermore, by measuring the errors it is possible to perform diagnostic tasks and remedial caution in the form of mechanical adjustment or numerical compensation. In order for machine tool owners to be able to manufacture with confidence, they must have assurance that their machines are working within a required tolerance. For high

[^0]quantity, low value production, the tolerances are likely to be larger than that of low quantity, high value production such as aerospace where tolerances are at a micrometre level.

In order to ensure that parts manufactured are within tolerance (right first time production) a high quality calibration process must be in place to ensure that the production machine is capable. The calibration process will encompass a range of different techniques. For example, a laser interferometer measurement system can be used to measure many of the geometric errors shown in Fig. 1 [1], while linear displacement sensors and artefacts are used for others. The telescopic ball bar system [2] provides a popular way of assessing machine tool performance during contouring.

There are a total of 21 geometric errors (Fig. 1) for a three axis machine tool such as the c-frame machine which are found extensively in industry (Fig. 2). Machine tool owners will systematically calibrate their machines based on manufacturing requirements.


Fig. 1. Six-degrees-of-freedom and squareness error.

The current international standards for machine tool calibration of linear positioning deviations defined by ISO [3] supports the use of quasi-static calibration. Quasi-static measurement (QS-M) involves commanding the machine to move from point to point along the length of the axis under test. The machine is commanded to move to a target position, where it will remain stationary for a nominal time period while a measurement is taken.

Commanding the machine to move from point to point and dwelling until the machine has settled is time consuming, a problem that is exacerbated if a fine spatial target interval is selected, an example of which is provided in Section 3. Manufacturers want their machines to be in operation, minimising machine down-time for the measurement process. In order to achieve this, a coarse target interval might be selected. However, this could result in some errors of the axis remaining undetected due to the effects of signal aliasing; the true error form will not be sufficiently represented if there is insufficient spatial sample rate. Furthermore, when a machine tool is in operation, it is likely that the axis will


Fig. 2. Three-axis machine tool.
not be stationary. Due to this, quasi-static calibration could be seen as an inadequate method for measuring the performance of machine tools during their intended operation; it is a reasonable compromise to indicate machine tool capability but will miss some potentially vital information.

Precision machining can be improved by applying numerical compensation, for which two requirements must be met: precise error prediction and accurate compensation (correction of error in the machine's controller) [4]. As QS-M techniques do not measure the machine tool in its usual mode of operation it is reasonable to argue that these techniques do not entirely meet the first requirement, making the second not possible.

By taking several readings per second on a time basis while the axis is in motion, continuous motion calibration has the potential to significantly reduce the overall calibration time while increasing calibration quality by removing the effects of signal aliasing.

During quasi-static calibration, the CNC part program provides the nominal positions for the measurement. The error is then simply calculated as the difference between the nominals from the part program and the actuals measured by the laser. The technical challenge for continuous motion measurements is in the ability to convert the measurement, which is typically in the form of a displacement measurement in the time domain (linear positioning measurement), to a geometric error in the position domain, since there is no explicit nominal position when using this method.

It is the aim of this paper to demonstrate the correlation between the quasi-static and continuous motion error of machine tools and how a continuous motion measurement can be used to enhance the calibration process while reducing the required number of targets for a quasi-static measurement. Different techniques for calculating the geometric error from a linear positioning measurement have been developed and will be discussed in the following section.

## 2. Background

CNC machine tools contain a range of different errors. These errors can be defined as: kinematic errors; thermo-mechanical errors; errors induced by loads on the machine; and continuous motion forces. All of these contribute to the overall geometric performance of the machine [5,6].

Kinematic errors are due to the machine's imperfect geometry such as axis misalignment and errors in the machine's measuring system [5]. Barakat [7] describes kinematic errors in relation to a coordinate measuring machine (CMM) as the error appearing in the ability of the CMM to reach the exact specified position by the controller. Due to the similarities in the kinematic chain and homogeneous coordinate system between machine tools and coordinate measuring machines the same definition can be applied to machine tools.

Thermo - mechanical errors are caused by internal and external heat sources resulting in thermo-elastic deformations of the machine tool causing geometrical inaccuracies [8]. Relative movement between the various elements of the machine causes heat to be generated at the contact zones and it is this heat that leads to the deformation of the machine elements [9]. Thermo-mechanical errors are said to be the cause of $70 \%$ of machine tool geometric inaccuracy [10]

Loads on the machine are caused by internal and external forces. For example, the location and weight of the workpiece could affect the machine's angular profile and so impair the overall accuracy [5,11].

The effect of the three error sources mentioned exist when the machine is stationary. Dynamic errors are the additional errors occurring when the machine is moving at a programmed feedrate. The dynamic errors result from varying alterations of the machine
components under dynamic forces [12] as well as electronic and control-related errors.

As industry strives for higher accuracies and faster production cycles, the importance of an efficient calibration process becomes even more critical [13,14]. Machine tool owners will systematically calibrate their machines based on their individual requirements and the calibration process is supported by a range of literature [15,3,16-18].

In order to reduce calibration times, new methods of measurement have been introduced. A displacement measurement approach was introduced to combat the length of time it would take to identify the 21 possible geometric errors of a 3-axis machine. The method involves taking position measurements along 15 lines of the machine working zone, allowing the individual axis errors to be inferred $[19,20]$. This method is faster than traditional techniques but is a quasi-static measurement method.

A novel method of artefact probing was also introduced in order to rapidly verify the accuracy of machine tools [21]. The method works by fixing a reference artefact somewhere within the machine's working volume. After a full calibration has been performed, a baseline routine is performed where the artefact is probed and sets machine variables for each point. After this, whenever a rapid verification is needed, the machine can perform a verification routing where the artefact is probed again and the deviation from the baseline result is reported. This method is effective in measuring a small area of the machine's working volume. However, artefact methods do not measure the entire machine, a problem particularly for large machines, or give spatially detailed characterisation of all the individual errors. Furthermore, artefact measurement at different speeds to evaluate dynamic effects is not a trivial extrapolation.

The "Chase-the-ball" technique [22] delivers a quick calibration for five axis machine tools. The measurements are taken while the machine is stopped to avoid backlash influences caused by a reversal error when the machine changes direction of motion. A dynamic R-test was also introduced for the calibration of rotary axes [23]. These methods all use multiple axis movements as a way to reduce calibration time.

Continuous motion calibration techniques for individual machine tool axis calibration have also been explored. Postlethwaite [24] developed a time-based approach. This approach does not require knowledge of the machine's nominal position since it infers it from the change in feedrate. The system works by fitting an end point straight line through the measurement data when the machine was estimated to be at a nominally constant velocity. The process hinges on the fact that when a machine is programmed to move at a constant feedrate, any error will be manifested in small fluctuations in axis velocity. This method does not provide a fitting technique and the technique proposed will potentially lose the gradient component of the axis error.

A position based measurement system was developed by Castro [25]. The method of calibration uses the machine's encoder signal pulses as the trigger reference, thus enabling to take measurements "on the fly". The system works by selecting targets at fine intervals. Provided that the interval is an exact multiple of the encoder, each time the machine gets to the interval position the encoder will send a pulse to the attached computer informing the measurement system to take a reading.

This work has limitations which affect its use. For example, in order to identify cyclic errors, targets must be chosen at a fine enough increment. However, if the target interval is too small the system will go into error. It is worth noting that in the tests carried out, a target interval of 1 mm was sufficient to pick up cyclic errors.

On the fly calibration has also been achieved through the use of a tracking interferometer. Schwenke [26] used the analogue signals from the measurement system (encoder/scales), where the
analogue data from all five axes are digitized by counter cards and processed in a real time operating system.

This technique has the ability to speed up the calibration process as well as significantly increasing the measurement point density. The technical challenge with this method of measurement is the synchronous acquisition of the machine's axis position and the measurement signal [26]. Furthermore, such techniques rely on specialist hardware and software as well as invasive modification of the machine's feedback loop, this is not always practical and requires a potentially high time-investment to initiate.

The development of an auto-alignment laser interferometer, in which multiple machine axes could be moved simultaneously, allows for faster calibration times [27]. However, such systems that rely on measurements and feedback from multiple devices have an increased uncertainty on the overall measurement due to timing error between the different measurement devices or the individual inaccuracy of system components [28-30].

When calibrating a machine tool, it is also important to consider the relationship between the different errors on the machine. The kinematic errors of a machine tool influence the uncertainty of measurement of the other errors. For example, straightness deviations of a linear axis can compromise the result of a squareness measurement. Furthermore, when measuring linear positioning of the $X$-axis, the results will be dependent on that of the chosen position of the other stationary axes [31] due to the inherent angular error motions of the slides and the Abbé offsets involved [15].

As confirmed by the on the fly calibration technique [26], measuring continuously will provide significantly more measurement points and produce a detailed measured signal. This has the potential to allow for a far greater analysis of the relationships between the different geometric errors. Additionally, the identification of any error caused by electrical compensation within the machine's controller will be clearly visible, as will the effects of the compensation on the axis.

The focus of this paper is to identify an accurate and reliable fitting technique to continuous data. It is well known that continuous data acquisition provides a far higher concentration of measurement points. However, the technical challenge is ensuring that the measurement data acquired can be reliably converted into error in the position domain without secondary signals.

## 3. Quasi-static calibration

The total number of target positions selected for a quasi-static measurement has a direct influence over the quality of the measurement signal. ISO 230:2:2014 [3] requires a minimum of 5 targets per meter for an axis up to 2 m in length. Due to commercial pressure, machine tool owners want to maximize machine tool availability. It is therefore possible that many machine tool owners will calibrate their machines aiming to meet the minimum permitted standard.

Fig. 3 shows the results of such a measurement performed on a three axis machine tool moving from 0 mm to 450 mm . When a coarse target resolution is chosen ( 90 mm step size) the measurement signal shows bidirectional positioning error of $8 \mu \mathrm{~m}$.

Increasing the total number of targets to 18 and reducing the spatial resolution from 90 mm to 25 mm improves the measurement signal and shows bidirectional positioning error of $16 \mu \mathrm{~m}$. Not only does the range in error increase, but the measurement signal starts to show a cyclic error that could not been seen in the previous low resolution measurement due to signal aliasing.

Selecting a fine spatial resolution ( 5 mm step size) increases the number of target positions dramatically ( 200 targets per meter). Selecting such a fine spatial resolution gives a highly detailed measurement signal that shows another small increase in error


Fig. 3. Quasi-static measurement resolution comparison.
range, however shows a detailed picture of the machine's cyclic behaviour.

A machine tool calibration engineer could, for the sake of expediency, take only the test at 90 mm step size. From this, they may conclude that the axis under test is performing well and decide not to compensate. When measuring at a 25 mm step size what looks to be a ball screw pitch or cyclic error is becoming evident, machine tool owners may choose to try and compensate this error or even replace the ball screw. It is not until a measurement at a 5 mm step size is selected that we can clearly see that the error is unlikely to be mechanical, but is due to numerical compensation in the machine controller. Because mechanical errors are usually gradual, sudden jumps in axis position as seen in Fig. 3 indicates an effect from an electrical control loop. In this case it was found to be the results of poorly applied numerical compensation. Such behaviour of the axis could not readily be identified using a traditional quasi-static measurement, reinforcing the value of the proposed techniques for diagnostic purposes.

In fact Fig. 3 is from a popular type of machine tool which has a very common (industry standard) controller which is sold in the many thousands. From extensive testing on the machine, the shape of the plot is not a "malfunction" as such, but rather a problem with the way the numerical compensation is being applied on this series of machines; there is a resolution limitation within the controller coupled with the responsiveness of the control loop, that causes quantisation of the compensatory moves. This phenomenon illustrates a fairly common, real-world example of compensation causing steps which are not normally picked up by quasi-static measurement. The new ISO/TR 16907:2015 [32] recognises the importance of the "least increment step" and while the presented figure may be an extreme example, it is by no means unusual. The issue highlighted by this example is that machine tool users can be presented with QS-M graphs showing excellent performance. It is not until a high target resolution is selected that the true performance of the axis is uncovered.

Fig. 3 highlights the limitations of choosing a coarse target resolution. Choosing a 5 mm step size provides a highly detailed signal, however took over 1 h to complete. To repeat such a measurement for all components of each axis of the machine would not be feasible to many machine tool owners due to time constraints.

The length of time to complete a quasi-static measurement can be estimated using the following formula
$t=\left(\frac{l}{f r}\right)+(n \times d w)$
where $l$ is the total length of the measurement, $f r$ is the programmed feedrate, $n$ is the total number of targets and $d w$ is the dwell time. This function gives an approximate time duration for a measurement as it does not take into account acceleration or deceleration for each target.

However, based on the above formula (Eq. (1)) selecting a 5 mm step size at a programmed feedrate of $500 \mathrm{~mm} / \mathrm{min}$ for an axis 500 mm in length with a dwell time of 4 s gives a total time of over 7 min per direction, completing five bi-directional runs takes the total time to 76 min compared to 23 min and 13 min for 25 mm and 90 mm respectively. Repeating this test for the two angular measurements (pitch and yaw) means that for a single axis a conservative estimate for the total test time is nearly 4 h .

It is unlikely that such a fine spatial resolution will be chosen for a quasi-static measurement. If the results of a measurement show an unusual pattern, it is possible that calibration engineers will do another measurement at a finer resolution in order to diagnose a cyclic effect. However, testing the entire machine at such a high resolution will not be feasible for many machine tool owners due to the length of time that the machine would be out of operation. Furthermore, ambient and internally generated thermal effects will have significantly more influence on measurement over such a long cycle.

## 4. Continuous motion calibration

In this paper, continuous motion measurements refer to measuring the machine tool axis displacement continuously over time. The machine's nominal position can be defined as the desired position of the axis with zero error.

Although highly effective, position-based measurement systems that rely on specialist hardware and software as well as modification of the machine's feedback loop are undesirable as a general approach due to the cost of implementation, implications on warranties if feed drive systems are interrupted and the need for a machine specific solution. The focus of this investigation was therefore developing a time-based approach that can be used universally across all machines with no modifications to the machine feedback loop.

When measuring continuously only displacement is measured. The measured signal will contain geometric error, however this error is not visible from a displacement against time graph (Fig. 4)


Fig. 4. Continuous motion measurement.
since it is dominated by the change in nominal position. The error is calculated by firstly removing periods of acceleration and deceleration from the displacement signal. This ensures that the fitted line only applies to the part of the signal where the axis under test was travelling at a nominally constant velocity. With acceleration rates on modern CNC's, the loss of data is normally less than 5 mm at each end of travel. Although a limitation, it can be argued that as it is unlikely for parts to be machined in this region of the working volume therefore removing this period of the signal will have minimal effect on the calibration quality. The overall process is as follows;

1. Identify the section of data captured where the axis under test is nominally moving with constant velocity.
2. Calculate the constant velocity by linear least squares (the gradient of the measurement with respect to time).
3. Remove the velocity to convert to the position domain (removes progressive error).
4. Reconstruct the progressive (scale) error lost during the least square fitting using a coarse quasi-static measurement.
5. Apply a small (sub-micron) shift to the continuous motion data in order to datum the data with the quasi-static measurement.

### 4.1. Velocity algorithm

The algorithm developed in order to identify the period of the signal where the axis under test was travelling at a nominally constant velocity presented a non trivial task. The process hinges on the fact that the machine tool is traversing the length of the axis at a nominally constant velocity. The first step of the algorithm was to differentiate the displacement data to produce a velocity graph in the time domain.
$\vec{v}=\frac{\delta \vec{x}}{\delta t}$
The mean velocity as well as the direction is calculated. If the axis under test is travelling at a positive velocity the first series of samples with values less than the mean of the whole signal are removed. Likewise, the last series of samples whose value is less than the mean are removed. If the axis under test is moving with negative velocity then the same process is applied for samples greater than the mean.

This is a recursive process, after the first iteration and the first sets of samples are removed. A new mean value is calculated as well as the standard deviation. Due to the noise in the signal, the machine is presumed to be at a constant velocity when all the samples are within two standard deviations of the mean. If this condition is not met, the process is repeated, continually removing parts from the beginning and end of the signal until the test condition is met. Once the test condition is met, data captured for a further 1 ms is removed from the start and end of the signal to ensure that any spikes caused by changes in acceleration are removed.

The process hinges on the fact that the machine is traversing the length of the axis at a nominally constant velocity. Furthermore, in some cases the test condition could be changed to make the algorithm more robust. For high precision machines, the test condition could have a tighter tolerance than two standard deviation of the mean.

### 4.2. Continuous motion error

Once the part of the signal where the axis was travelling at a nominally constant velocity has been identified, the gradient of the
best fit line can then be calculated using a linear least squares fit shown in the following formula;
$m=\frac{\sum_{i=1}^{n}\left(x_{i}-\bar{X}\right)\left(y_{i}-\bar{Y}\right)}{\sum_{i=1}^{n}\left(x_{i}-\bar{X}\right)^{2}}$
where $\bar{X}$ and $\bar{Y}$ is the mean of the time data and displacement vector respectively. This function evaluates to the mean velocity.

The variable $m$ is the gradient and mean velocity calculated using $x$ representing the time domain data and $y$ representing the measurement data.

Once the coefficients have been found they are evaluated and a best fit vector $(\vec{y})$ that can be used as the axis nominal position for calculating continuous motion error (Eq. (4)).
$\vec{\xi}_{d}=\vec{d}-\vec{y}$
where $\vec{y}$ is the best fit vector, $\vec{d}$ is the continuous motion measurement data and $\vec{\xi}_{d}$ is the calculated continuous motion error.

### 4.3. Scale error

Although effective, using least square fitting to generate a reference vector will remove any progressive (scale) error from within the continuous motion measurement as shown in Section 5.

In order to reapply the scale error a quasi-static measurement is required. From the continuous motion data, it is possible to extract two possible target positions at the start and end of the signal as the commanded start and stop positions are known.

Although it may seem intuitive to use these two points to reapply the scale error, and initially does work for the simple case, it is not a robust method because effectively the reconstruction is based upon only two measurement points. This introduces a weakness, which would then be amplified by choosing two points which may not correlate to the time-based data set. In many cases machine tool axes have rapidly changing error profiles at the extremes of travel. The effect of this phenomenon can be demonstrated with simulated data.

Fig. 5 shows a simulated error signal which contains a large end effect. As periods of acceleration and deceleration are being removed, the continuous measurement data will not detect the end effect that is being detected in the quasi-static measurement. For simplicity, acceleration to constant velocity took one second, removing the end effect completely from the continuous motion


Fig. 5. Simulated signal.


Fig. 6. Simulated signal.
signal. The actual axis error during the constant velocity period is shown in Fig. 6.

It is clear that applying the simulated end point from Fig. 5 would be incorrect. As the end point fit represents the amount of error over the full axis length rather than the error over the constant velocity period. Performing the least square fit on the constant velocity path and applying the gradient from the end point fit will give the results shown in Fig. 7.

It is possible to reconstruct the scale error using just two quasistatic measurement points providing that the measurement points are taken within the constant velocity path. This will give a more accurate fit to the end point method however, it will introduce uncertainty due to potential aliasing. The result of this method of fitting is shown in Fig. 8.

In order to reapply the scale error to more accurately reconstruct the signal an optimisation algorithm has been developed. The algorithm searches for the minimum root mean square error between the quasi-static and continuous motion measurement signal using the simplex search method [33].


Fig. 7. Reconstruction with incorrect gradient.


Fig. 8. Reconstruction through end point fit.

The goal of this process is to converge on the best coefficients for the slope $(m x+c)$ that minimises the root mean square error between the quasi-static and dynamic error values. Once the coefficients have been found and the gradient calculated, it can be easily applied.
$\vec{\xi}_{d}=\vec{\xi}_{d}+\xi_{\text {sgradient }}$
Reconstruction of the signal to reintroduce the scale error requires QS-M to perform the above operation. If the axis under test is being measured according to standards, a good-quality fit can be achieved using the data being captured as demonstrated in Fig. 9.

Using just two quasi-static measurement points has greater uncertainty than using multiple points for correlation. Whereas when using the optimization technique proposed in the paper, uncertainty is minimised as more data is being used to reconstruct the signal.

Comparing the reconstructed continuous motion error with the actual error showed that the end point method shown in Fig. 7


Fig. 9. Reconstruction through optimisation technique.
has a root mean square error of $3.03 \mu \mathrm{~m}$. Using two QS-M points from within the constant velocity path shown in Fig. 8 had a root mean square error of $0.48 \mu \mathrm{~m}$. Whereas the optimization technique shown in Fig. 9 the root mean square error was $0.14 \mu \mathrm{~m}$.

### 4.4. Axis location

The final step to convert to the position domain is to find the datum point which allows for the reconstruction of the nominal discrete "command" values.

Firstly, the actual position of the axis at the first (non-zero) target position (quasi-static measurement) is calculated simply by
$\rho_{1}=\tau_{1}+\delta_{1}$
where $\tau_{1}$ is the target position at index 1 and $\delta_{1}$ is corresponding error value for the target. Note that this is true for all target positions where $\delta_{n}$ and $\tau_{n}$ would be for target position and measured value at the $n$th target number.

The next stage involves iterating through the continuous motion displacement data and locating the index of the sample closest to $\rho_{1}$, we will call the index $\rho_{i}$. The quasi-static target position $\rho_{1}$ has to be in an area of the continuous motion measurement signal when the axis under test was travelling at a nominally constant velocity, as any period of acceleration has been removed. If $\rho_{i}$ falls within a period of acceleration for the continuous motion measurement, the next static measurement is selected and the index calculated.

It is presumed that because the axis under test was commanded to move to position $\tau_{1}$ but was actually at position $\rho_{1}$ (quasi-static measurement), that the sample that corresponds to $\rho_{1}$ in the continuous motion signal ( $\rho_{i}$ ) corresponds to the nominal position $\tau_{1}$. The deviation from the actual position and the nominal target position is then calculated for the continuous motion measurement signal.
$\Delta_{l}=\rho_{i}-\tau_{i}$
A new shifted continuous motion target vector $\tau^{\prime}$ can be generated by applying the small shift $\Delta_{l}$ to the continuous motion displacement data.
$\tau^{\prime}=\tau-\Delta_{l}$
This fully defines the nominal command value in the position domain for each measurement point.

## 5. Validation

In order to validate the technique described above, continuous motion measurements were compared to quasi-static measurements with a small step size. However, the method is also equally effective when comparing against quasi-static measurement with a much larger step size, so that it can be used on machines where a fine spatial resolution is not necessary (i.e. the error of the axis is not rapidly changing with respect to position.)

The algorithm and mathematical processing was developed in MATLAB for practicality. However, the solution can easily be developed for any programming language.

In order to validate the technique presented in Section 4, the method was validated against a range of quasi-static measurements. The quasi-static measurements comply with the ISO standard for targets per meter, however no random targets were selected. A random element is sometimes used to help indicate any cyclic error. The first test was performed on a three axis C-body machine tool. The axis under test was commanded to move from 0 mm to 450 mm at $500 \mathrm{~mm} / \mathrm{min}$. The continuous motion data was sampled at a frequency of 100 Hz .

Periods of acceleration and deceleration were removed from the signal leaving only the part of the signal where the axis under test


Fig. 10. Continuous motion error.
was travelling at a nominally constant velocity. The least squares line was calculated and the continuous motion error calculated through Eq. (4) (Fig. 10).

The axis was then measured using quasi-static techniques. Three different spatial resolutions were selected (as shown in Fig. 3).

Comparing the high resolution quasi-static measurement ( 5 mm step size) showed correlation to within $2 \mu \mathrm{~m}$ (Fig. 11). However this deviation is due to the gradient being removed from the continuous motion error signal. The quasi-static measurement contains 5 bidirectional runs as recommended by ISO [3]. This, as well as the small step size is causing thermal drift of approximately $4 \mu \mathrm{~m}$.

By applying the gradient $(m x+c)$ calculated through the optimisation algorithm to the continuous motion error shows submicrometer correlation between the continuous motion error and the quasi-static measurement (Fig. 12).

High resolution quasi-static measurements are time consuming. The quasi-static measurement, shown in Fig. 12 took approximately 1 h to complete. Due to the length of time it took to complete the measurement, the effects of thermal drift are clearly distorting the measurement. This is another limitation to the quasi-static measurement technique. The algorithm developed gives the user


Fig. 11. Comparison between continuous motion and quasi-static (gradient removed).


Fig. 12. Comparison between continuous motion and quasi-static measurement error.
the option to fit the continuous motion measurement taking into account thermo-mechanical effects by averaging the optimised coefficients for each quasi-static bi-directional run. In this case, just the first bi-directional run was chosen to eliminate thermal drift (Fig. 12).

Additionally, the user can select just the first bi-directional run to observe the effect of thermo-mechanical errors as demonstrated in Fig. 13.

So far, for the requirement of demonstrating the correlation between the quasi-static and continuous motion errors of machine tools the quasi-static measurements have been of a high spatial resolution. It is unlikely that machine tool owners will use such a high spatial resolution due to the length of time the machine tool will be out of production.

### 5.1. Enhancing the calibration process

Continuous motion measurements are not yet supported by the international standard for machine tool calibration [3]. The proposed method can work as a valuable enhancement tool for


Fig. 13. Comparison between continuous motion and quasi-static measurement on the first bi-directional run.


Fig. 14. Comparison between continuous motion and low resolution quasi-static measurement.
machine tool calibration engineers. Performing a quasi-static measurement that meets the minimum ISO standard provides enough information to fit and fuse a continuous motion measurement as demonstrated in Fig. 14.

Additionally, it is worth remembering that a continuous motion measurement for a medium sized machine tool will take less that a minute to complete. Therefore performing a calibration that exceeds the ISO standard ( 10 points per meter) can be easily enhanced using the proposed method as shown in Fig. 15 with virtually no time constraints to the calibration process.

The more quasi-static measurements are taken the more accurate the continuous motion measurement will fit. However, as seen by comparing Figs. 14 and 15, the continuous motion measurement fitted using the low resolution quasi-static measurement is accurate to within approximately $2 \mu \mathrm{~m}$ of the fitted continuous motion measurement for a higher resolution measurement.

Comparing the continuous motion measurement signals that have been fitted against quasi-static measurements of different target resolutions also shows the robustness of the method. The continuous motion signal fitted against the high resolution


Fig. 15. Comparison between continuous motion and medium resolution quasistatic measurement.

Table 1
RMSE values.

|  | 25 mm fit | 90 mm fit |
| :--- | :--- | :--- |
| 5 mm Benchmark | $0.6 \mu \mathrm{~m}$ | $2.5 \mu \mathrm{~m}$ |



Fig. 16. Comparison between fitted continuous motion signals.
quasi-static measurement is the bench mark for this test as it demonstrates the best fit.

By calculating the root mean square error (RMSE) of the high resolution fit continuous motion error with the low resolution fit continuous motion error shows the impact of choosing a coarse target resolution on fitting the continuous motion signal.

The signals used to calculate the values in Table 1 are shown below in Fig. 16.

### 5.2. Further validation

The continuous motion measurement technique proposed was also validated on a high speed gantry machine. The longer axis ( 2500 mm ) and the reduction of machine rigidity provided an increased challenge for continuous motion measurement techniques. The test conditions were as follows.

| Feedrate | $5000 \mathrm{~mm} / \mathrm{min}$ |
| :--- | :--- |
| Start position | 0 mm |
| End position | 2450 mm |
| Sample rate | 1 kHz |
| Overrun | 1 mm |

The machine traversed the length of the axis before coming to a halt, the reversal error was removed by applying the overrun before a reverse run could be performed.

Two quasi-static measurements were also performed for correlation. The test conditions were as follows.

| Feedrate | $5000 \mathrm{~mm} / \mathrm{min}$ |
| :--- | :--- |
| Start position | 0 mm |
| End position | 2450 mm |
| Step size | $25 \mathrm{~mm} \& 175 \mathrm{~mm}$ |
| Num of targets | $99 \& 15$ |
| Num of runs | 1 |

A continuous motion linear positioning measurement was taken. The continuous motion error was calculated using the least square fitting technique, the gradient was fitted using the optimisation algorithm and shifted to correlate with the quasi-static measurement.


Fig. 17. Comparison between dynamic and medium resolution quasi-static.

Once more, a lower resolution quasi-static measurement ( 175 mm step size) was selected to validate the measurement method against a more realistic quasi-static measurement. Fig. 17 shows how the proposed method can be used to enhance the calibration process, removing the effects of aliasing with minimal additional time constraints

Once the gradient has been re-applied (Fig. 17), the continuous motion error and quasi-static error were on average within $3 \mu \mathrm{~m}$ of each other. A high resolution quasi-static measurement was also taken and used for the continuous motion fit (Fig. 18).

The difference between the two signals shown in Fig. 18 (high resolution quasi-static and continuous motion error) shows a standard deviation of $4 \mu \mathrm{~m}$. This is shown in Fig. 19. The large difference at the start of the signal is caused by a cyclic error in the continuous motion measurement error which soon disappears. It is likely that this is related to post-acceleration of the optic and may be trimmed from the signal.

The difference between the two continuous motion signals shown in Fig. 20 was $1 \mu \mathrm{~m}$ (RMSE). Considering the overall error


Fig. 18. Comparison between continuous motion and quasi-static measurement.


Fig. 19. Difference between continuous motion and quasi-static error values.


Fig. 20. Fitted continuous motion signals.
of the axis is in excess of $700 \mu \mathrm{~m}$, this result shows the capabilities of the technique proposed in this paper and its effectiveness in enhancing the current calibration process supported by international standards.

## 6. Reproducibility

To validate the method further, a set of repeatability tests were performed. The tests were performed on a gantry machine, the test parameters were as follows.

| Feedrate | $5000 \mathrm{~mm} / \mathrm{min}$ |
| :--- | :--- |
| Start position | 0 mm |
| End position | 2450 mm |
| Sample rate | 1 kHz |
| Overrun | 1 mm |

A total of 20 tests were performed, 10 tests were performed with the axis under test travelling in a positive direction, 10 tests were performed with the axis travelling in a negative direction. The tests were performed using the Renishaw XL-80 Laser Interferometer, the data was post-processed using the method described in this paper.


Fig. 21. Repeatability test (forward motion).


Fig. 22. Repeatability test (forward motion).

Zooming into the figure over a 30 mm period and datuming the error value to zero (Fig. 21) shows the results in better detail. Zooming in over a 5 mm (Fig. 22) period shows repeatability to be in the region of $6 \mu \mathrm{~m}$. Considering the length of the axis (over 2000 mm ) as well as the length of time taken to perform 10 measurements in each direction (approximately 20 min ) increases the probability of thermal expansion. The results are satisfactory in validating the measurement technique.

The quasi-static measurement taken on the same axis showed unidirectional repeatability of the machine's axis in the positive direction to be $11 \mu \mathrm{~m}$. The reproducibility of the continuous motion error measurement signal is well within the repeatability of the axis.

## 7. Uncertainty analysis

The greatest uncertainty in the majority of laser measurement systems arises from deviations in environmental conditions such as air temperature and humidity. The measurement device used in this investigation is sensitive to environmental change. A $0.26^{\circ} \mathrm{C}$ increase in air temperature results in $0.25 \mu \mathrm{~m}$ uncertainty over a distance of 1 meter. The environmental deviations form the ideal
$20^{\circ} \mathrm{C}$ can be compensated however this requires additional sensors which have uncertainty associated with them also. The laser device and environmental compensation system used in this analysis has an expanded uncertainty of $0.49 \mu \mathrm{~m}$ over 1 meter with a coverage factor of $k=2$ [34]. Additionally, ISO 230:2 (Appendix A) [3] provides formulas for calculating the uncertainty of measurement when using a laser interferometer and examples for different environments.

The uncertainty with regard to feedrate has been assessed experimentally. A typical three axis machine (Fig. 2) was used for this analysis. A quasi-static measurement was performed, followed by a range of continuous motions tests taken at different feedrates. The first continuous test was performed at $500 \mathrm{~mm} / \mathrm{min}$ and the results can be seen in Fig. 23. The results show sub-micrometre correlation between the quasi-static and continuous motion error. However, the continuous motion signal detected a high frequency cyclic error which would have remained undetected using a traditional quasi-static measurement technique.

In order to assess how increasing the velocity effected the fitting process the root mean square error between the fitted continuous motion data and quasi-static measurement was calculated. The results of which can be seen Table 2.

The continuous motion error was reconstructed using both the proposed optimisation technique and by using just two QS-M points in the constant velocity region. The reconstructed continuous signals were compared against the first bi-directional run of a QS-M. Where the optimisation routine was used, the RMSE at the QS-M points was sub-micrometre, whereas when just two QS-M points were used the RMSE was on average $1.5 \mu \mathrm{~m}$.


Fig. 23. Quasi-static and continuous motion measurement.

Table 2
RMSE values.

| Feedrate $\mathbf{m m} / \mathrm{min}$ | RMSE <br> (optimised) | RMSE (Two <br> QS-M points) |
| :--- | :--- | :--- |
| $500 \mathrm{~mm} / \mathrm{min}$ | $0.5 \mu \mathrm{~m}$ | $1.5 \mu \mathrm{~m}$ |
| $2000 \mathrm{~mm} / \mathrm{min}$ | $0.5 \mu \mathrm{~m}$ | $1.5 \mu \mathrm{~m}$ |
| $4000 \mathrm{~mm} / \mathrm{min}$ | $0.6 \mu \mathrm{~m}$ | $1.4 \mu \mathrm{~m}$ |
| $6000 \mathrm{~mm} / \mathrm{min}$ | $0.6 \mu \mathrm{~m}$ | $1.4 \mu \mathrm{~m}$ |
| $8000 \mathrm{~mm} / \mathrm{min}$ | $0.7 \mu \mathrm{~m}$ | $1.5 \mu \mathrm{~m}$ |
| $10,000 \mathrm{~mm} / \mathrm{min}$ | $0.9 \mu \mathrm{~m}$ | $1.5 \mu \mathrm{~m}$ |

## 8. Conclusion

Measurement of machine tool axis geometric errors for assessing performance is an important part of precision manufacture. The measurement data can be used to evaluate capability when buying a machine and to monitor its performance during its operational lifetime. Furthermore, by measuring the errors it is possible to perform diagnostic tasks and remedial action in the form of mechanical adjustment or numerical compensation.

The accepted standard quasi-static measurement (QS-M) method for positioning accuracy directly measures the error between a nominal (programmed) and the actual (measured) position by moving to discrete targets, waiting for the axis to settle and then measuring before moving to the next target. Such an approach provides a good indication of the axis performance, but does not represent a machine tool in its normal mode of operation where the axes are often in motion. QS-M techniques also provide calibration engineers with the challenge of selecting the optimal number of target positions; the number of target positions is a compromise between the calibration quality and the manufacturing downtime required for the calibration process. As manufactures strive for increased machine availability, an effective, rapid measurement process becomes more important.

This paper introduces a measurement technique to establish axis accuracy at a high spatial resolution without incurring a time penalty. The position is captured while the axis is moving at a nominally constant velocity. This method does not require additional hardware or interfacing with the controller and relies only on universally available motion commands, but requires data processing to convert from the time-based measurement into an error measurement in the position domain. The initial conversion by least-squares fitting has the unwanted effect of removing any progressive error of the axis (scale elongation) from the measurement. A QS-M is required to reconstruct the overall profile, but a coarse step-size using as few as the ISO minimum 5 targets is shown to be sufficient, therefore has little impact upon the overall measurement time.

Measuring continuously provides calibration engineers with far greater knowledge of the machine's performance. Any small cyclic errors or discontinuities from mechanical damage or electronic control parameters can be detected by the high spatial sampling; the QS-M method is relatively very under-sampled so would miss the effects due to signal aliasing. The better sampling resolution allows an engineer to interpret whether an axis is progressive, cyclic, likely to be mechanical or electrical, etc. This allows a diagnostic function which is highly beneficial to machine tool users. Any corrections to the machine's geometry by repair can be made with far more informed decisions than would be possible with a quasi-static measurement. The data produced using this method can also be converted into axis compensation tables in the same way as from a traditional QS-M measurement. The benefit of the new method being that, where the numerical controller has the capacity, a much finer compensation can be achieved, reducing the residual error.

Three case studies using separate machines with large variations in performance and axis length are provided in the paper. In the first case study, the tests performed showed correlation to within one micro-metre between the reconstructed measurement from the proposed technique with a typical QS-M step-size and a benchmark measurement of the error using a very fine step-size for validation. The fitting technique was still accurate to within $2.5 \mu \mathrm{~m}$ (RMSE) when selecting the minimum target positions as recommended by international standards. On the second, less accurate machine, when a coarse 175 mm step size was selected the fitting technique was accurate to within $1 \mu \mathrm{~m}$ of the high resolution fit. Finally, a third case study is introduced to
evaluate the uncertainty of the method. The continuous motion signal and quasi-static measurement were accurate to within $0.5 \mu \mathrm{~m}$ at $500 \mathrm{~mm} / \mathrm{min}$ and $0.9 \mu \mathrm{~m}$ at $10000 \mathrm{~mm} / \mathrm{min}$. In addition to this, a high frequency cyclic error was detected in the continuous motion signal which would have otherwise been missed. This good correlation shows that the proposed technique is an effective way of measuring the geometric performance of a machine tool to enhance measurements taken according to the current supported standard

The proposed technique is limited to linear measurement of the machine only. It can therefore be used to evaluate or compensate for measurement parallel to an individual axis, or two or more linear axes that are programmed with linear interpolation. This approach is commensurate with the tests specified in the prevailing standards, such as ISO 230-2:2014 [3]. This technique is not readily applicable to other tests where a curved trajectory is desired so should be supplemented with other methods, such as the double ballbar, if evaluation of circularity is required (ISO 230:4:2005 [16]). The measurement method will lead to further research into speed-related errors of machine tool axes.

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