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Domain Decomposition Approach for Near-wall Turbulence Modeling

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1 Introduction to Near-wall Turbulence

2 Near-wall Domain Decomposition



Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall Turbulence

The turbulent boundary layer



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Sergei Utyuzhnikov DOMAIN DECOMPOSITION

Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall Turbulence

• near-wall sublayer significantly affects mean flow

The turbulent boundary layer



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Near-wall Turbulence

The turbulent boundary layer



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- near-wall sublayer significantly affects mean flow
- resolution of near-wall area requires up to 90% of CPU time

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- domain decomposition should be efficient

Near-wall Turbulence

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- resolution of near-wall area requires up to 90% of CPU time
- it is a multiscale problem
- domain decomposition should be efficient
- standard approach: domain decomposition based on wall functions + HRN

Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall Turbulence

 Low Reynolds Number models (LRN): governing equations include all terms of the Reynolds Averaged Navier-Stokes Equations (RANS)

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Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall Turbulence

- Low Reynolds Number models (LRN): governing equations include all terms of the Reynolds Averaged Navier-Stokes Equations (RANS)
- High Reynolds Number models (HRN): governing equations ignore all near-wall damping terms
- Wall functions:

Dirichlet boundary conditions are set for $\ensuremath{\mathsf{HRN}}$ at the nearest to the wall cell

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non-overlapping domain decomposition:

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- non-overlapping domain decomposition:
- the b.c can be transferred
 from the wall to y = y*

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The turbulent boundary layer



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- non-overlapping domain decomposition:
- the b.c can be transferred
 from the wall to y = y*
- the interface near-wall b.c. (INBC) is nonlocal

general formulation of b.c.

$$\frac{\partial u}{\partial n}_{|y^*} = u(y^*)S_{y^*}(1) + f_y^*,$$

 S_{y^*} is the Steklov-Poincaré operator Utyuzhnikov, 2009

Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall domain decomposition. Sketch (linear problem)

• Original BVP (possible statement)



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• Transfer of b.c. to an interface boundary



Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall domain decomposition. Sketch (linear problem)

• Original BVP (possible statement)



• Transfer of b.c. to an interface boundary



• R.b.c. does not depend on the outer region!

Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall domain decomposition. Sketch (cont.)

• Solution of BVP in the outer region



Near-wall Turbulence Modelling Non-overlapping Domain Decomposition

Near-wall domain decomposition. Sketch (cont.)

• Solution of BVP in the outer region



• Solution of BVP in the inner region (if needed)

The near-wall domain decomposition Interface Near-wall Boundary Condition

The near-wall domain decomposition (NDD)

Consider equation

$$(\mu u_y)_y = R(y)$$

in $D = [0 \ y_e]$ with boundary condition at y = 0:

$$u(0)=0.$$

INBC is set at y^*

$$0 < y^* < y_e, \ D^- := [0 \ y^*].$$

The near-wall domain decomposition Interface Near-wall Boundary Condition

Interface Near-wall Boundary Condition (Utyuzhnikov, 2005)

Consider

 $(\mu u_y)_y = R(y),$ u(0) = 0.

INBC at $y = y^*$:

$$u(y^*) = u'(y^*) \int_0^{y^*} \frac{\mu(y^*)}{\mu(y)} dy - \frac{1}{\mu(y^*)y^*} \int_0^{y^*} (\frac{\mu(y^*)}{\mu(y)} \int_y^{y^*} R(y') dy') dy.$$

The near-wall domain decomposition Interface Near-wall Boundary Condition

1D Equation. Wall flux

The wall flux can be obtained without resolution of the inner layer

$$C_f = (\mu u_y)_{|y=y^*} - \int_0^{y^*} R(y) dy,$$

where $C_f = (\mu u_y)_{|y=0}$.

The near-wall domain decomposition Interface Near-wall Boundary Condition

Near-wall non-overlapping domain decomposition



The near-wall domain decomposition Interface Near-wall Boundary Condition

Interface boundary conditions (IBCs)

• The interface boundary condition is exact even in non-linear case

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Advantages of NDD

• Interface boundary *y*^{*} provides a clear trade-off between accuracy and time consumption

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- Meshes in the inner and outer regions completely independent
- No free parameters
- Implementation into industrial codes easy since IBCs computed with external subroutine
- Robin boundary condition is robust, converges fast since both Φ and ∂_νΦ taken at the same iteration:

$$\Phi|_{y^*} = f_1 \left. \frac{\partial \Phi}{\partial y} \right|_{y^*} + f_2$$

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Low-Re Velocity Profile. *Re* = 3950



 $y^{+*} = u_\tau y^* / \nu = 1$

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Low-Re Velocity Profile. *Re* = 3950



 $y^{+*} = u_{\tau} y^* / \nu = 10$

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Low-Re Velocity Profile. *Re* = 3950



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Low-Re Velocity Profile. *Re* = 3950



$$y^{+*} = u_{ au} y^* / \nu = 100$$

Channel Flow. $k - \epsilon$ (Chien) model **Turbulent asymmetric diffuser** Ribbed channel flow

Turbulent asymmetric diffuser, $Re = 1.8 * 10^4$

• Compute inlet conditions with a separate LRN calculation



• Cut boundary layers off both walls

Compute C_f along the inclined wall (Jones, Utyuzhnikov, 2015)

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Turbulent asymmetric diffuser, $Re = 1.8 * 10^4$. Different y^*/H



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• Clear convergence to LRN solution as $y^* \rightarrow 0$ for SA model

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Turbulent asymmetric diffuser, $Re = 1.8 * 10^4$. Different y^*/H



- Clear convergence to LRN solution as $y^* \rightarrow 0$ for SA model
- DD produces recirculation region with both models

Channel Flow. $k - \epsilon$ (Chien) model **Turbulent asymmetric diffuser** Ribbed channel flow

Turbulent asymmetric diffuser, $Re = 1.8 * 10^4$. $y^*/h = 0.03$



• Only Robin boundary condition for k predicts the separation

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Turbulent asymmetric diffuser, $Re = 1.8 * 10^4$. Velocity profile



Spalart-Almaras model. $10U/U_b$

Channel Flow. $k - \epsilon$ (Chien) model **Turbulent asymmetric diffuser** Ribbed channel flow

Comparison of simulation time for SST model



Max error compared to LRN solution is shown

$$e = \frac{\max \left| C_f^{\text{LRN}}(x) - C_f^{\text{NDD / HRN}}(x) - C_f^{\text{NDD / HRN}}(x) - C_f^{\text{NDD / HRN}}(x) \right|}{C_{f0}}$$

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Plane ribbed channel flow (2D). SA model (Jones, Utyuzhnikov, 2015)



- Periodic flow, both walls heated with constant heat flux
- h/H = 0.1
- Remove same amount from upper and lower walls
- Interesting limit is $y^*/h = 1$
- For $y^*/h < 1$, some of rib tops is included in mesh





Friction factor for different y^* , the rib height is fixed

• For SA, error is less than 2.5% until $y^*/h = 0.9$

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Plane ribbed channel flow (2D). *f* for different *h*



Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

Plane ribbed channel flow (2D). *f* for different *h*



Varying the rib height is trivial since only one parameter needs adjusting:

$$h = h_{resolved} + y^*$$

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

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Conclusions

• NDD proved to be an efficient and quite accurate method for simulating near-wall regions in turbulent flows

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

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- NDD represents a clear trade-off between accuracy and computational resources that is easily managed

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- Changing the rib height is simple and the results are reasonably accurate
- NDD can be especially efficient in optimal engineering design

Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

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Channel Flow. $k - \epsilon$ (Chien) model Turbulent asymmetric diffuser Ribbed channel flow

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