A General Theory of Colour Calibration

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SUMMARY

A new 'General Theory of colour transform development' has been evolved. The theory uses the principles and predictive power of vector-spaces to guide transform development.

The investigation arose from the feeling that the central laws of additive colour mixing might form a fundamental basis on which to build transforms. The intent was to create a system capable of guiding and proving transform structure. The principles of vector-space modelling, vector algebra, and vector analysis are used. The resulting transforms are detailed predictive models of the additive effects of all the variables concerned in the properties of the phenomenon being modelled. The predictions are based on the properties of Euclidean vector-spaces. Mathematical arguements have been developed, which show that vector algebraic methods can be used to both develop and evaluate the success of any candidate calibration transform.

General mathematical principles are used, to show that any transform may be constrained to be a theoretically exact vector-based calibration model of a colour mixing system. If this is done, then evaluation of correctness is possible. This is shown to cover all instances of additive colour mixing. It is predicted that an exact model is also possible for subtractive colour mixing, with full predictive power for the result of mixing colorants on a substrate. While this is thought to be mathematically provable, time has not allowed practical substantiation, in the case of subtractive mixing.

The Basis of the Theory

The principles on which the theory rests are the 'Law's of Additive Colour Mixing', first developed by Newton and Grassman. These were first restated in terms of vector notation, and then vector-based predictions of colour mixing results was compared experimentally with the properties of additive mixing systems. The first system for comparison, and for successful prediction, was the structure used by the CIE in 1931 to establish the CIE Colorimetric Transform.

It is well known that the central principle of both Colorimetry and Photometry is the strict additivity of CIE Tri-stimulus Values. This property of multi-stimulus, or multi-variable additivity is also the central property of a Euclidean vector-space. In consequence, all aspects of CIE Colorimetry and Photometry are shown to be in essence vector-space models of the human colour sensation, although they were not conceived as such.

The full argument for the basis of the theory is presented by Oulton & Bowen in the first of five detailed theoretical papers entitled, 'Newton, Grassman, the CIE System, and Additive Colour Mixing'. In this paper, vector-algebraic reasoning is applied, to the modelling of the properties of light-stimulus mixtures. Paper two is entitled ' The Generality of Vector-Space Modelling, as a Proven Implementation of the Principles of Additive Colour Mixing'. It validates by algebraic reasoning, the use of additive vectors in a Euclidean space, to express the additivity of spectral stimuli implied in the 'Laws of additive colour mixing'. Follow-up papers cover a range of potential applications in various types of transform development..

Conclusions

Vector-algebraic reasoning has been used to create and prove colour transforms. It has been applied in particular, to prove the theoretical accuracy of a non-linear transform. Following evaluation of theoretical correctness, the predictions of the model were confirmed by experimental evidence from CRT screen calibration.

The effect of the General Theory, given full peer acceptance, will be to make exhaustive interpolation based device characterisation unnecessary. The gain from such a development has been demonstrated empirically in a previously published paper by Oulton and Porat, in which it was shown that an accuracy of calibration is available which is typically a factor of ten more accurate, compared with that delivered by typical interpolation based methods.

The experimental programme examined the detailed characteristics of both additive and subtractive colour mixing systems, and helped to confirm the validity of the principles on which the theory is based. Full experimental confirmation of the predictive power of the theory however, awaits its use across a wider field applications. It is predicted that the development of transforms will be materially aided in the of colour calibration of most types of colour capture and colour reproduction device. Potentially, it may also aid colour difference modelling, colour appearance modelling, and subtractive colorant recipe formulation as well.

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' A General Theory of Colour Calibration'

Final Report

Introduction

The investigation entitled 'A General Theory of Colour Calibration' arose from the feeling that there was much circumstantial evidence that all pointed toward some as yet unidentified central principle of colour calibration. Two approaches have been used to uncover this central principle, one relatively unsuccessful, one apparently completely successful. The original aims of the project were framed round an empirical approach, which seeks to discover the central principles by detailed practical and theoretical investigation. The characteristics and phenomena of devices which work by both additive and subtractive colour mixing have been investigated in detail, and compared with possible explanatory theories.

While the empirical investigative programme was in hand, opportunity was taken to re-visit the central principles of Colorimetry itself. The objective was to see if they could be extended to account for the non-linear cross dependencies typical of colour calibration problems. It is this second approach that has borne fruit. The result is a set of at least three detailed theoretical papers, in which the central principles of CIE Colorimetry and photometry are unified with the new theory, stated in vector algebraic terms, and extended to cover new types of transform.

Starting from the fundamental laws of additive colour mixing, the investigators have developed a new approach to the development of colour transforms. The new approach applies the principles of vector-space modelling, vector algebra, and vector analysis to the definition of colour calibration transforms. The resulting transforms are detailed predictive models of the effects of all the variables concerned in the properties of the phenomenon being modelled. Mathematical arguements have been developed, which show that there is a sound central guiding principle that can be used to both develop and evaluate the success of any candidate calibration transform.

The implications for the 'General Theory' are potentially relevant to a much wider area of Colour Science than the specialist field of colour calibration originally addressed. A comprehensive exercise of peer group review and discussion has been initiated to explore both the central principles of the General Theory, and its applications in the wider context of Colour Science as a whole.

The process of peer evaluation started with the submission of the key papers to the Journal Color Research and Applications (copies attached). Direct interactive discussion with the main scientists currently active in the CIE takes place at the CIE/NPL International Conference 'Visual Scales : Photometric and Colorimetric Aspects' at the end of March. The Principal Investigator awaits with keen interest the response of the Colour Science Community.

The Practical Investigation

In the case for support, the investigator identified as the problem to be addressed, the general principles that might underlie and guide the development of colour calibration transforms. Such transforms are predictive multi-dimensional models that allow precise colour to be reproduced on an given sample of colour capture or colour output device, by characterising the detailed features of its performance.

Investigations were specified, which would seek to :-

1. Predict a set of general principles for colour calibration.
2. Test the applicability of calibration principles in model systems.
3. Predict and test calibration strategies for a range of colour systems.

A full programme of investigative experimental work has been carried out into to colour reproduction of subtractive colour mixing devices such as dye-sublimation and half-tone colour printers. The colour capture characteristics of several different types of electronic and conventional camera have been investigated, and a detailed analysis of existing and new calibration results for CRT screen monitors was carried out. A brief review of the experimental work is given in APPENDIX I.

The General Theory of Colour Calibration

The starting point for the successful investigation, which lead to the new theory was CRT screen calibration. The published algorithm (1) developed prior to the project appeared to have theoretically exact predictive properties, for the additive mixing of screen phosphor colour. As better measurement techniques were employed, and wider coverage of the device colour-space was used, the calibration became accurate to the third decimal place.

Any General Theory of colour calibration must however do more than achieve predictive accuracy of a single instance of calibration.

It should demonstrate at least the majority of the following :-

1. That it does not contradict any of the established features and results of either Colorimetry or Photometry.
2. That it explains why CIE Colorimetry, and transforms of CIE XYZ colour-space are a suitable basis for colour calibration.
3. That it correctly predicts both the structure, and detailed nature of successful calibration algorithms.
4. That it can be used to generate new successful calibration transforms for both additive and subtractive colour mixing systems.
5. That it successfully explains empirical techniques of colour calibration, including 'Gamma correction' and 'Grey Balancing'.
6. That the predictions of the theory are susceptible to either mathematical reasoning, or experimental substantiation, (or indeed both).

A General Theory of colour calibration has been developed which meets all the above criteria.

It turns out that it can be proved from general mathematical principles (2,3), that any transform based on the theory is a theoretically exact calibration model of any type of colour mixing system. This certainly covers all known instances of additive colour mixing. The General Theory also predicts that an exact model is possible for subtractive colour mixing, with full predictive power for the result of mixing colorants on a substrate. While this is supported by mathematical arguements, time has not allowed practical substantiation, in the case of subtractive mixing.

The Basis of the Theory

The principles, from which a start was made, are the 'Law's of Additive Colour Mixing' first developed by Newton and Grassman. These are the bed-rock and core of all modern Colorimetry. They are seldom quoted, and taken as axiomatic to Colour Science. Reliable literature sources were however found, and the central principles of additive colour mixing were then enumerated in detail. These were then restated in terms of vector notation. A vector-based approach having previously proved useful in analysing the calibration algorithms previously developed. The next step was to compare vector-based prediction of colour mixing results, with the properties of additive mixing systems. The first system for comparison, was the structure used by the CIE in 1931 to establish the CIE Colorimetric Transform.

It is a well known and established fact that all aspects of both Colorimetry and Photometry are based on the strict additivity of colour stimuli, expressed by additivity of CIE Tri-stimulus Values. This property of multi-stimulus, or multi-variable additivity is also the central property of a Euclidean vector-space. In consequence, all aspects of CIE Colorimetry and Photometry can be represented by, and are in essence, vector-space models of the human colour sensation.

An 'n' dimensional Euclidean vector-space is widely used in many areas of science to model the effect of multiple contributing variables to some overall phenomenon. It is on this that the general theory is based. In consequence, the resulting model has theoretically exact predictive power for all combinations of value of the variables modelled.

The correctness of the theory is in a large part demonstrated by its ability to predict the complete structure of both CIE Colorimetry and Photometry, and also predict and prove a theoretical basis for non-linear three-dimensional CRT monitor calibration (1).

The Output of the ROPA Project

While the project has generated a significant volume of detailed experimental results, the investigators wish to highlight as the main output, a series of inter-related theoretical papers. These papers are essentially mathematical in inspiration, and contain a series of arguments whose only prior assumption are the basic principles or 'Laws' of additive colour mixing. In most cases where experimental backing rather than theoretical proof is required, it has been found sufficient to cite existing, often quite long standing experimental evidence.

The investigators show in a series of papers :-

1. That the established principles of additive colour mixing predict that coloured light stimuli can be treated as vector quantities, with meaningful magnitude and direction of effect in a vector-space.

2. That the resulting vector-space is a predictive model for the combined effects of the variables modelled.

3. That the CIE 1931 2° Standard Observer Colorimetry system is a correct partial model of the human colour sensation. Although not designed as such, it is in essence a vector-space model which predicts colour identity by multi-dimensional vector additivity.

4. That all instances of additive colour mixing can be modelled by a vector-space that is Euclidean with respect to the phenomenon being modelled. Such models can then be used to predict the individual effects of those variables identified as contributing to the phenomenon.

5. That additivity of effect can be projected across a non-linear transform, allowing the predictive modelling of non-linear phenomena such as CRT output calibration.

6. That the calibration method previously developed for CRT monitors (1) is an instance of such a projection.

7. That all types of additive colour-capture and colour reproduction device including cameras and scanners are instances of the general principles of additive mixing. Any calibration transform describing them must necessarily be constrained to be an additive-model, if it is to be a theoretically correct model. If this is done, the only constraints on the accuracy of calibration are those associated with defining the contributing variables.

8. That an additive space does not require exhaustive evaluation, and can be characterised by as few as 30 calibration points which characterise the contributing basis-vectors.

9. That subtractive colour-mixing may well prove to be an 'n' dimensional instance of the more general case of additive mixing, and that the CIE Colorimetric Transform is a proven practical example of this hypothesis, whose features may be generalised to cover other instances.

10. That both Neugebauer and Kubelka-Munk models of subtractive mixing are mechanism specific sub-classes, of a more general class of model, which is predictive of all types of subtractive colour mixing.

Conclusions

There are many detailed and far reaching claims in the above list, and the task of proving them to the peer community is only just beginning. Many of the hypotheses are however substantiated directly by mathematical reasoning, invoking only the central principles of Colorimetry and Photometry as prior assumptions. Substantiation is also provided by reference to existing well established experimental results in the literature. These include the output of workers in the fields of Vision Science, Colorimetry, and Photometry. The central and most important substantiation comes from the detailed and long lasting success of CIE Colorimetry itself, which is shown to be an instance of the general theory.

The full argument for the basis of the theory is developed by Oulton & Bowen in the paper "Newton, Grassman, the CIE system, and Vector-Based Description of Colour Mixing" (2) a copy of which is attached.

The major arguments will be presented, in a paper entitled "The Role of Vector Analysis, and Vector-Based Modelling, in Colour Science."(4) on March 25th 1997 to The NPL/CIE- UK International Conference, 'VISUAL SCALES: Photometric and Colorimetric Aspects'. At this conference there will be a number of scientists whose work is quoted, and an in-depth preliminary peer review is expected.

The published paper " The Control of Colour Using Measurement and Feed-back" by Oulton and Porat (1), copy attached, gives direct practical substantiation of the theory. In particular it validates the projection of three-dimensional additivity across a two-stage non-linear transform, and shows the predictive power of correctly defining the characteristic basis-vectors of a CRT monitor.

The paper 'The Generality of Vector-Space Modelling, as a proven Implementation of the Principles of Additive Colour Mixing' (3) by Oulton and Bowen (Synopsis attached), introduces the necessary arguements to substantiate the General Theory.

The paper "Vector-Space Modelling of the Mechanisms in the Human Visual System" (5) by Oulton and Bowen, (synopsis attached) extends the application of the General Theory beyond the confines of colour calibration. In particular it applies the constraints of vector additivity to the evaluation of proposed mechanisms for the colour-opponency characteristics of the human visual system. Significant insights into this related field of transform development may prove possible. This paper has been submitted along with (2) to the major source Journal ' Color Research and Applications' for early publication.

A sixth paper entitled "Subtractive Colour Mixing, as a Vector-Additive Mixing Phenomenon" (6) by Oulton and Bowen is in preparation. This addresses the extension of Vector-additive modelling to subtractive mixing. A mathematical arguement, based on the laws of additive mixing will be given that shows in principle that all instances of subtractive mixing are a sub-class of the general class of the phenomena of additive mixing. The ability to define both device independent, and subtractive-mechanism independent calibration transforms may prove to be significant in the practical fields of colorant recipe formulation and device characterisation.

Vector-algebraic reasoning has been applied for the first time, to prove the theoretical accuracy of a non-linear transform.

The predictions of the proven model were confirmed by experimental evidence from CRT screen calibration. (see Appendix I), and extend theoretical predictive accuracy derived from vector-space modelling, to include a non-linear colour calibration transform.

As and when the General Theory has achieved full peer acceptance, its effect will be to make exhaustive interpolation based device characterisation unnecessary. The gain from such a development has been demonstrated empirically in (1), where just 30 calibration patches deliver an accuracy of calibration, which is typically a factor of ten more accurate than that delivered by widely used exhaustive three-dimensional evaluation across a 9*9*9 cube of interpolation points.

Bibliography

1 'The Control of Colour Using Measurement and Feedback' D.P.Oulton & I.Porat J Text. Inst. Vol 83 No.3 pp. 454 – 461.

2 "Newton, Grassman, the CIE system, and Vector-Based Description of Colour Mixing" D.P.Oulton & A.W Bowen submitted for Publication to the Journal. 'Color Research and Applications' 1997

3 "The Generality of Vector-Space Modelling, as a proven Implementation of the Principles of Additive Colour Mixing" D.P.Oulton & A.W.Bowen submitted for Publication to the Journal. 'Color Research and Applications' 1997

4 "The Role of Vector Analysis, and Vector-Based Modelling, in Colour Science." D.P.Oulton & A.W.Bowen. Proceedings of the NPL / CIE Int. Conference 'Visual Scales Colorimetric and Photometric Aspects' March 97. To be Published by the CIE.

5 "Vector-Space Modelling of the Mechanisms in the Human Visual System" D.P.Oulton & A.W.Bowen submitted for Publication to the Journal. 'Color Research and Applications' 1997

Two further papers are in preparation :-

6 "Subtractive Colour Mixing, as a Vector-Additive Mixing Phenomenon" D.P.Oulton & A.W.Bowen, in preparation. to be submitted for Publication to the Journal. 'Color Research and Applications' 1997

7 " A vector analysis of multi-stage transforms which maintain vector additivity". in preparation. to be submitted to the Journal 'Vision Research' 1997

APPENDIX I. THE EXPERIMENTAL INVESTIGATION

Introduction

Following the original aims and objectives of the project, an experimental plan was set in place to investigate the characteristics of devices using both additive and subtractive coloration principles. This programme, reviewed briefly below, was informative, and set the scene for the main theoretical conclusions. However attempts to deduce the principles empirically were unsuccessful.

Printer Calibration

There are many different models which attempt to describe specific cases of subtractive mixing, of which Kubelka-Munk and Neugebauer are two. The first is used mainly for textile dyeing applications and the latter for half tone systems. Both systems fail in our particular application primarily because of paper translucency, ink diffusion, and the consequent interaction of multiple subtractive mechanisms.

Our subsequent studies took two approaches, the first utilising the properties of the CIE system, the second looking at ink and dither characteristics on a more microscopic level.

By adapting the linearisation and grey balance mechanisms normally applied to additive systems such as CRT's, we hoped to characterise the transformation of CMYK printer primaries to CIE XYZ primaries. Initial attempts to print a linear (with Y), balanced D65 greyscale by manually altering CMY values (K to be incorporated later) proved difficult. **Fig (i)** demonstrates the non-systematic nature of the relationship. **Fig (ii)** shows how the printer primaries undergo a hue shift as chroma increases, and **fig (iii)** gives us the reason for this, namely that each primary progressively filters out a proportion of the visible spectrum, causing a profile change in the curve.

After further experiments and attempts to create virtual primaries it was considered futile to pursue this approach. The printing of colour by the placing of dots in a dither pattern also produced unpredictable effects when the ink diffused, causing dots to overlap (illustrated in **fig (iv)**).

Further investigations followed the idea that each of the printer primaries could be considered independent and in a mathematical sense orthogonal. From another project the idea was taken to represent each of the primaries at 3 monochromatic wavelengths only. For a calibration system to be viable however, measurements had to be taken of different dither orders of **C+M**, **C+Y**, **M+Y** and **C+M+Y**, i.e. the binary and tertiary mixtures.

Difficulties arose in developing an accurate iteration method. The method eventually reached a stage of development at which it was not only machine specific but also dither pattern specific. In consequence the investigation was not followed to a logical conclusion.

Camera Calibration

Cameras are an additive system and were therefore subjected to the same treatment as was successful for developing the CRT screen transform, i.e. linearisation and grey balancing. The presence of a complicating factor with cameras became apparent. There are usually mechanisms within the camera which attempt to compensate for the ultimate lack of dynamic range available. It was these mechanisms that we either wanted to bypass or mathematically deduce.

The three cameras we investigated were

1...A very small analogue camera which is supplied with a Silicon Graphics Indy.
2...A Panasonic VHS video camera.
3...A Leaf Lumina scanning digital camera.

The results quoted here are for the Silicon Graphics camera on the grounds that the principles for all the cameras were the same. A calibrated monitor was used as the colour source because it is produces accurate and reproducible results.

On attempting to analyse the software mechanisms, which were altering the cameras raw responses, we found that the software could not be overridden and that mathematical deduction was impossible. Efforts were made to create a situation where although the full mathematical characterisation was not available, we could find a means to counteract its effect.

Fig (v) shows the non-linear RGB camera response to a linear D65 greyscale.

Fig (vi) illustrates how the camera responds differently to single monitor primaries.
Fig (vii) depicts the cross dependence of the camera primaries. If the camera is initially presented with maximum monitor blue for example, then, as in the graphs is presented with an increasing amount of green, the cameras blue response decreases.

In our studies with the Leaf Lumina scanning digital camera the dynamic range proved an insurmountable problem. We could not get the relevant information from the manufacturer to write our own control software. Combined with the relatively poor dynamic range response characteristics of CCD technology, the software problem prevented any sensible attempt to calibrate the camera.

Screen Calibration

The original 1992 paper 'The control of colour using measurement and feedback' (1) announced and substantiated a very accurate method of CRT screen calibration. Its success was ascribed to the use of CIE T-Unit colour specifications, and feedback optimisation of non-linear functions. The method has subsequently been refined, and now gives predictive accuracy up to the digital quantisation limit available from gun-drive values defined on a 0 – 256 scale. An arguement has been devised that demonstrates the level of calibration accuracy achieved is predictable from the General Theory. This is the first application of a non-linear transform, that is also backed by detailed experimental evidence.
It confirms that the fundamentals of CIE Colorimetry can indeed be extended to colour calibration transforms as hypothesised in this report.

Conclusions

In line with the subsequently deduced general theory, attempts to generate predictive vectors empirically for CMY subtractive primaries was doomed to failure. A re-run, will require working in an n-dimensional space (n = 16 or more) representing a full spectral treatment of the response vectors, not the three dimensional visual response vector-space. We are confident that the problem can then be solved for perhaps all classes of subtractive mixing.

It is the ability to direct investigation from first principles which is the major contribution of the theory.

The problem of image capture calibration is potentially simpler, but practical implementation is seriously hampered by limitations of camera design.

We identify the following constraints on practical calibration of typical low-cost cameras:-

1. Lack of dynamic range, and the consequent internal compensation mechanisms used to avoid general problems of distorted appearance in picture reproduction.
2. Variable response characteristics in individual CCD array elements.
3. Non-linearity of response to input stimulus.
4. Lack of published data on 'negative lobe' compensation for rendering out-of –gamut colour specifications.
5. Response instability due to thermal effects.

There is however a new and evolving class of precision image capture systems, which was investigated briefly at the end of the project. These are precision instruments rather than 'picture taking' cameras. As time and funding allows, we are confident that the new general theory will allow definitive characterisation of such image capture devices. They must however be both stable and linear over a reasonable range to become practical 'imaging colorimeters'.

Theoretical accuracy of transforms :-

The first mathematical evaluation of theoretical accuracy of a non-linear transform has been achieved. It is backed by experimental evidence, and extends theoretical predictive accuracy derived from vector-space modelling, to include a non-linear colour calibration transform.

The evidence includes detailed experimental work published in reference (1), and its analysis and testing in this project, during the development of the proof. It confirms that the fundamentals of CIE Colorimetry can indeed be extended to non-linear colour calibration transforms as hypothesised in this report.

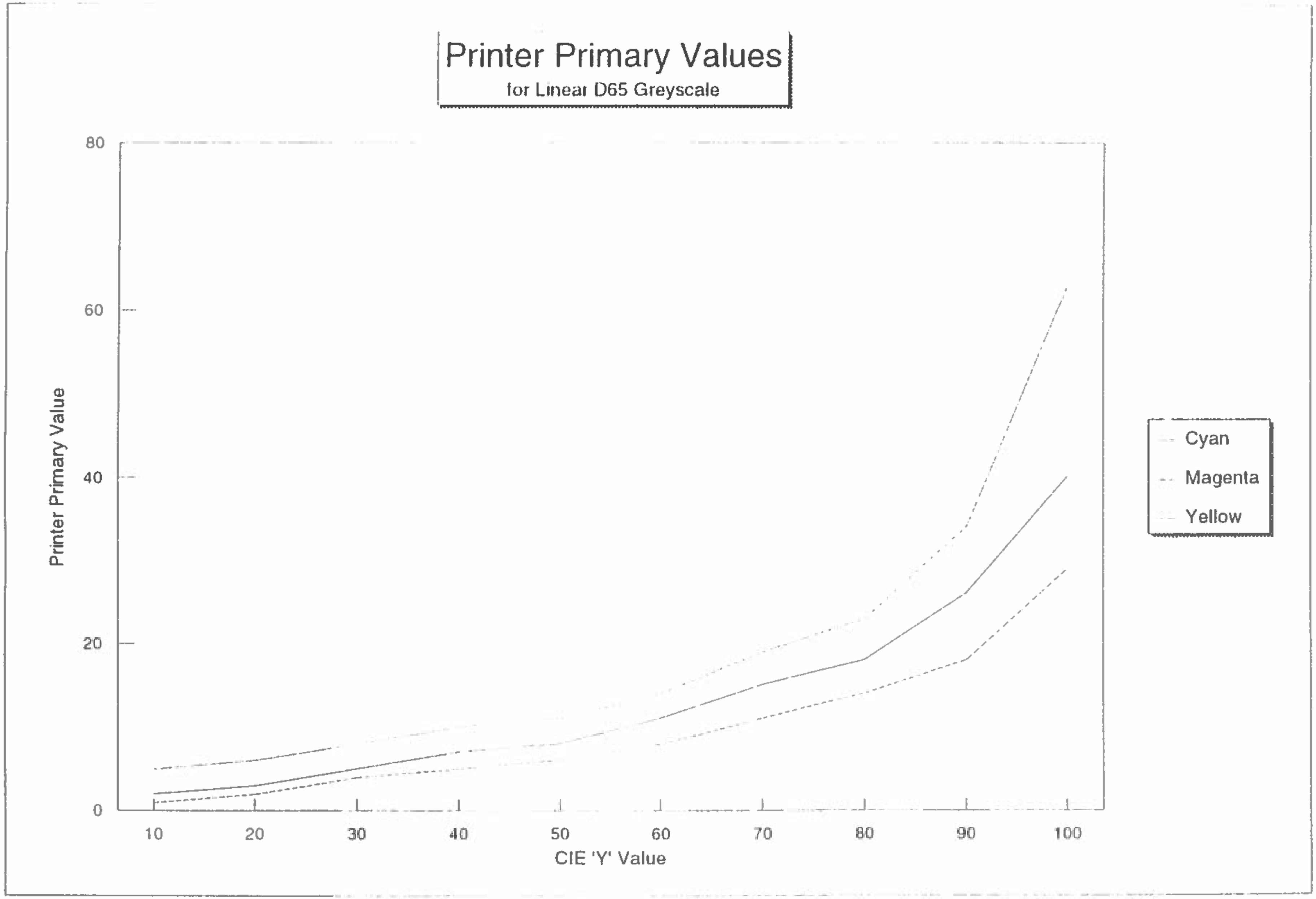
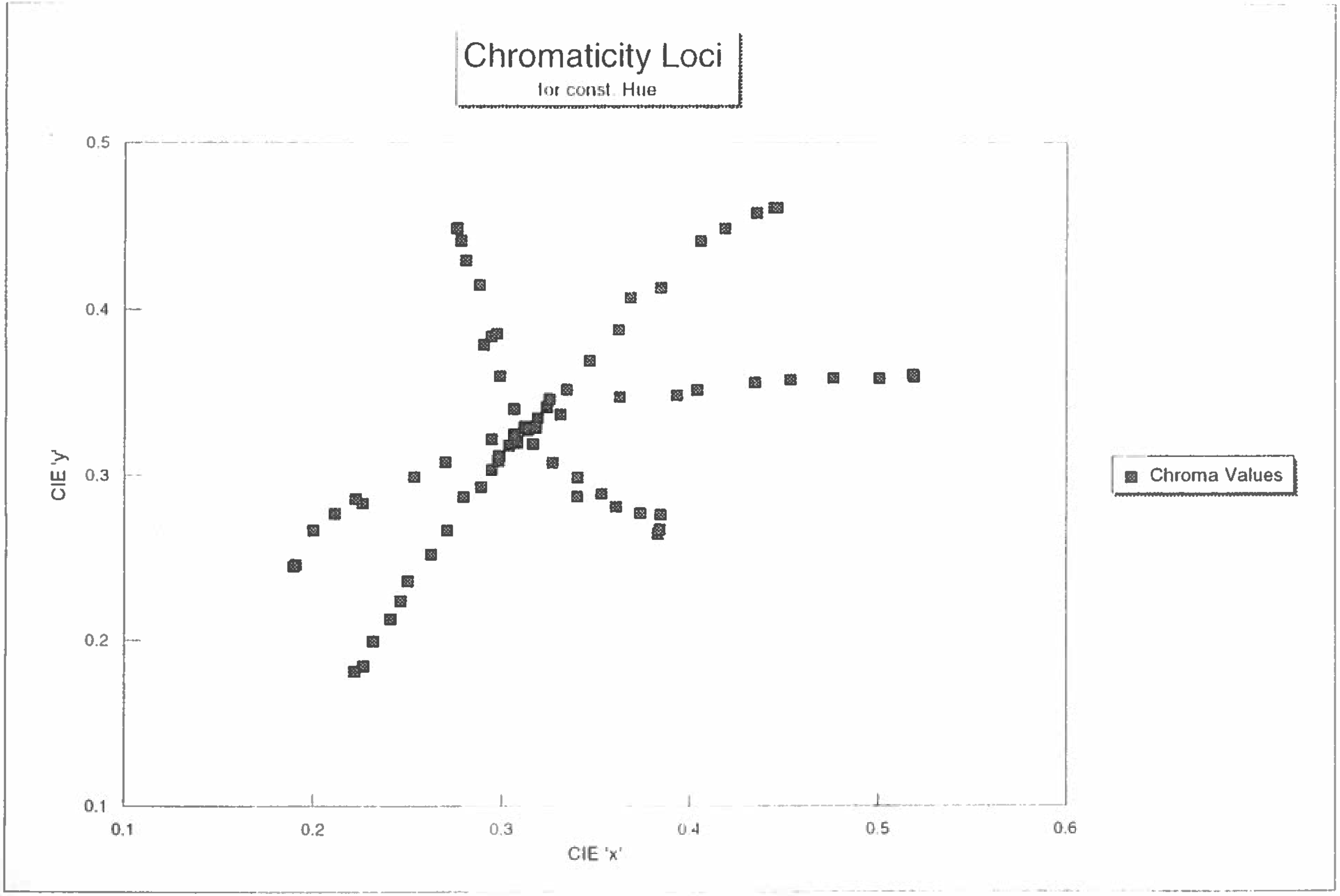
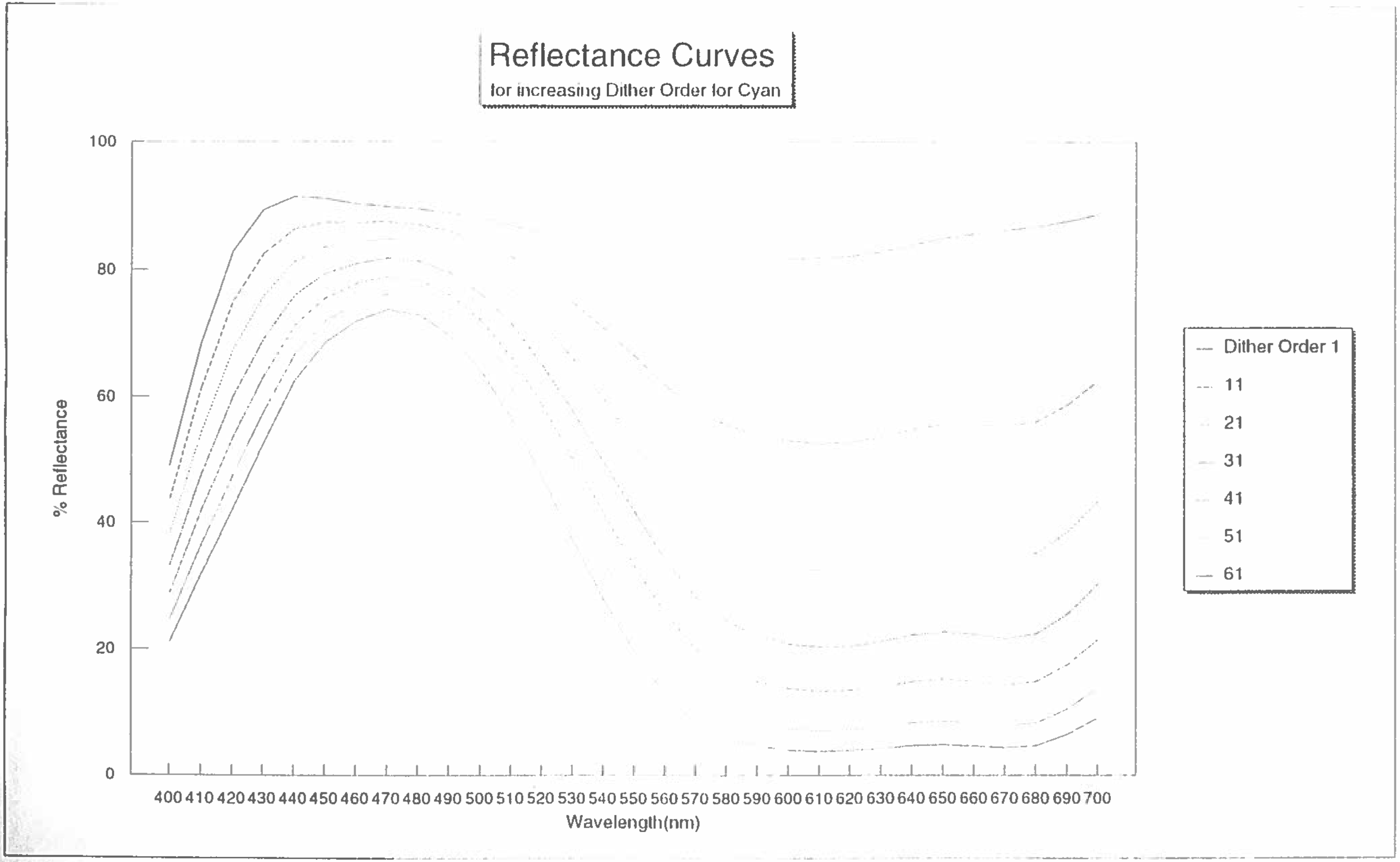

Printer Primary Values
for Linear D65 Greyscale
Printer Primary Value
80
60
40
20
0
10
20
30
40
50
60
70
80
90
100
CIE 'Y' Value
Cyan
Magenta
Yellow

Fig (i)



Chromaticity Loci
for const. Hue
Chroma Values
CIE 'y'
CIE 'x'
0.5
0.4
0.3
0.2
0.1
0.1
0.2
0.3
0.4
0.5
0.6

Fig (ii)


Reflectance Curves
for increasing Dither Order for Cyan
100
80
60
40
20
0
% Reflectance
400 410 420 430 440 450 460 470 480 490 500 510 520 530 540 550 560 570 580 590 600 610 620 630 640 650 660 670 680 690 700
Wavelength(nm)
Dither Order 1
11
21
31
41
51
61

Fig(iii)

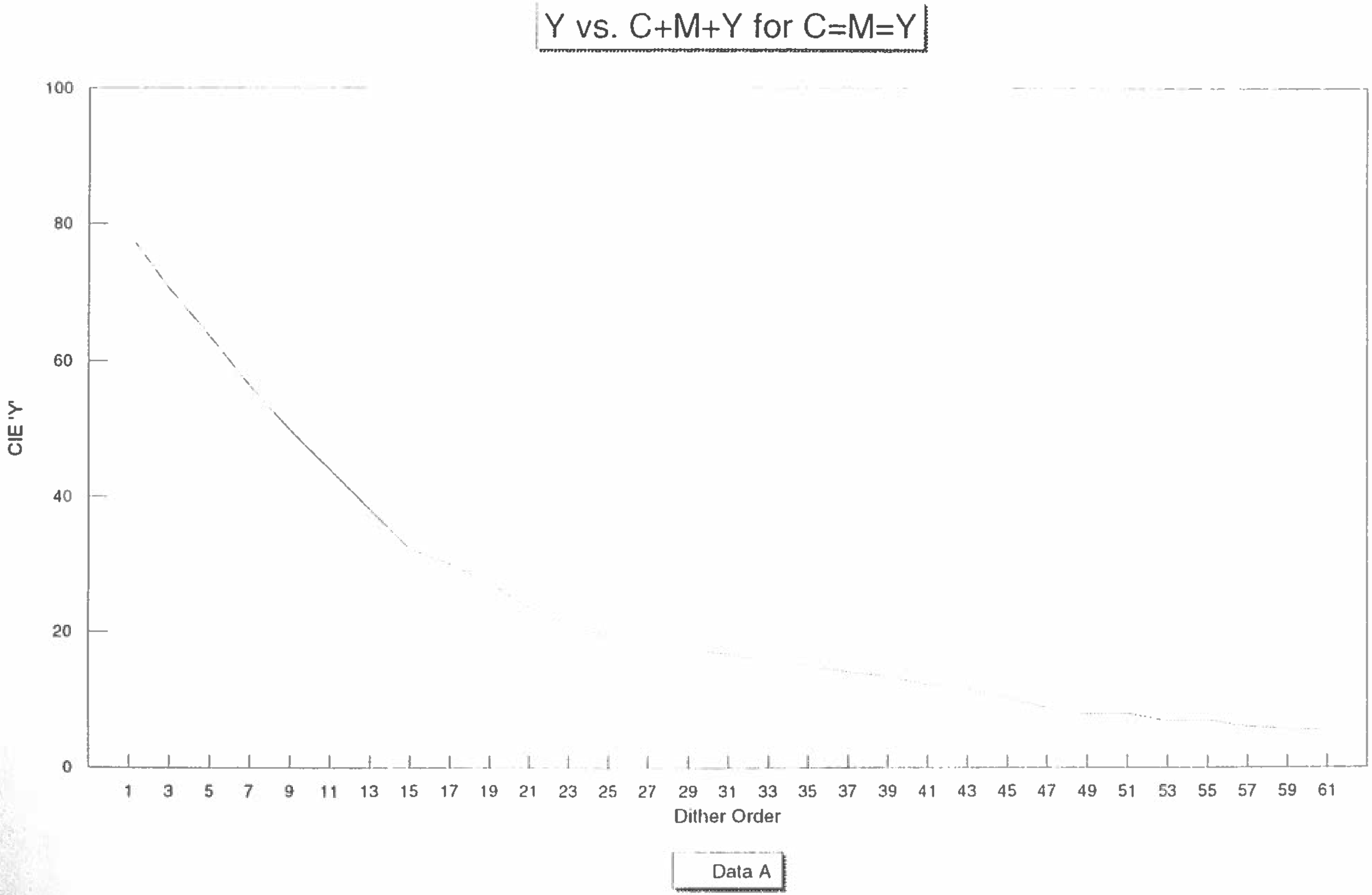

Y vs. C+M+Y for C=M=Y
100
80
60
40
20
0
CIE 'Y'
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61
Dither Order
Data A

fig (iv)

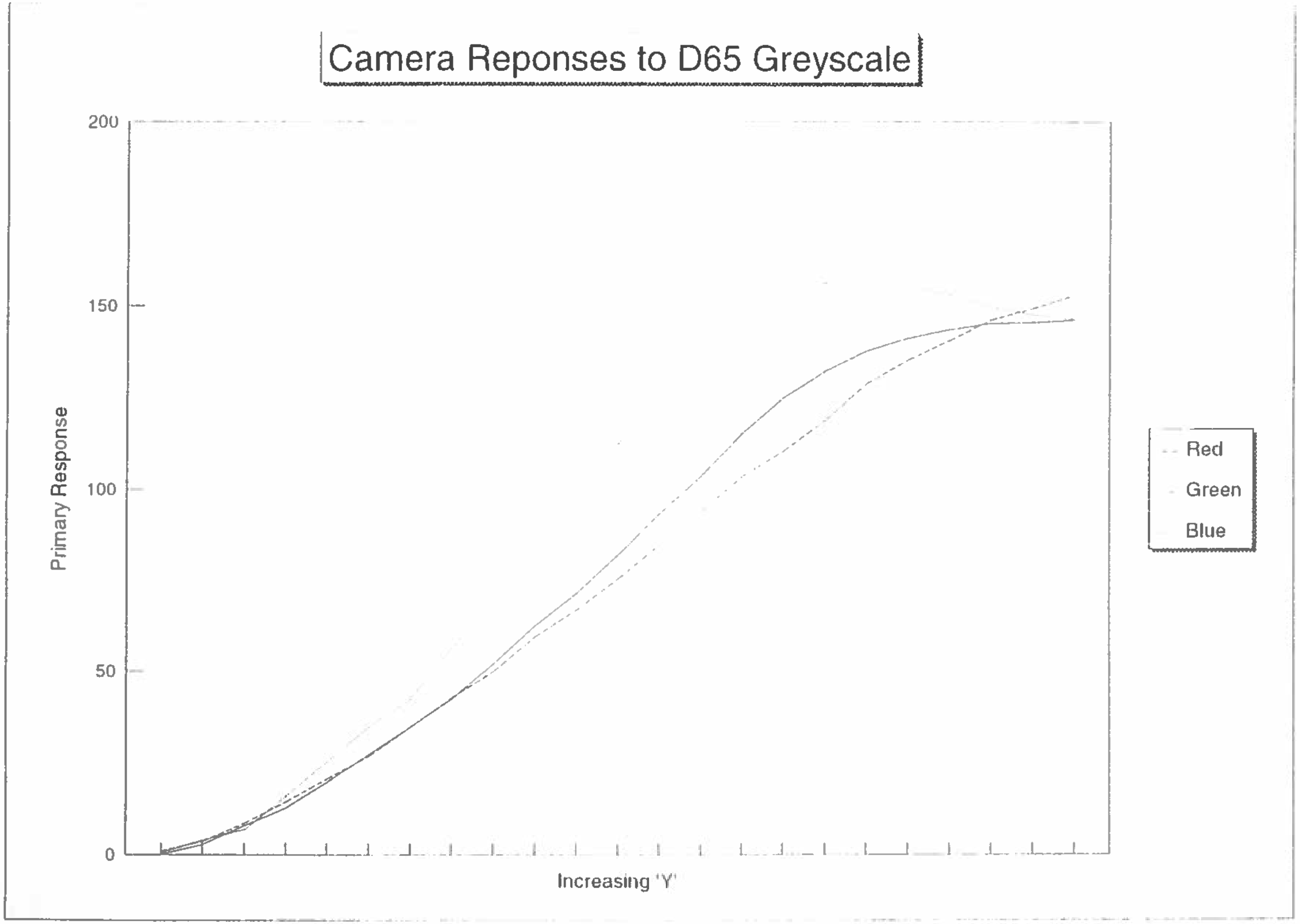
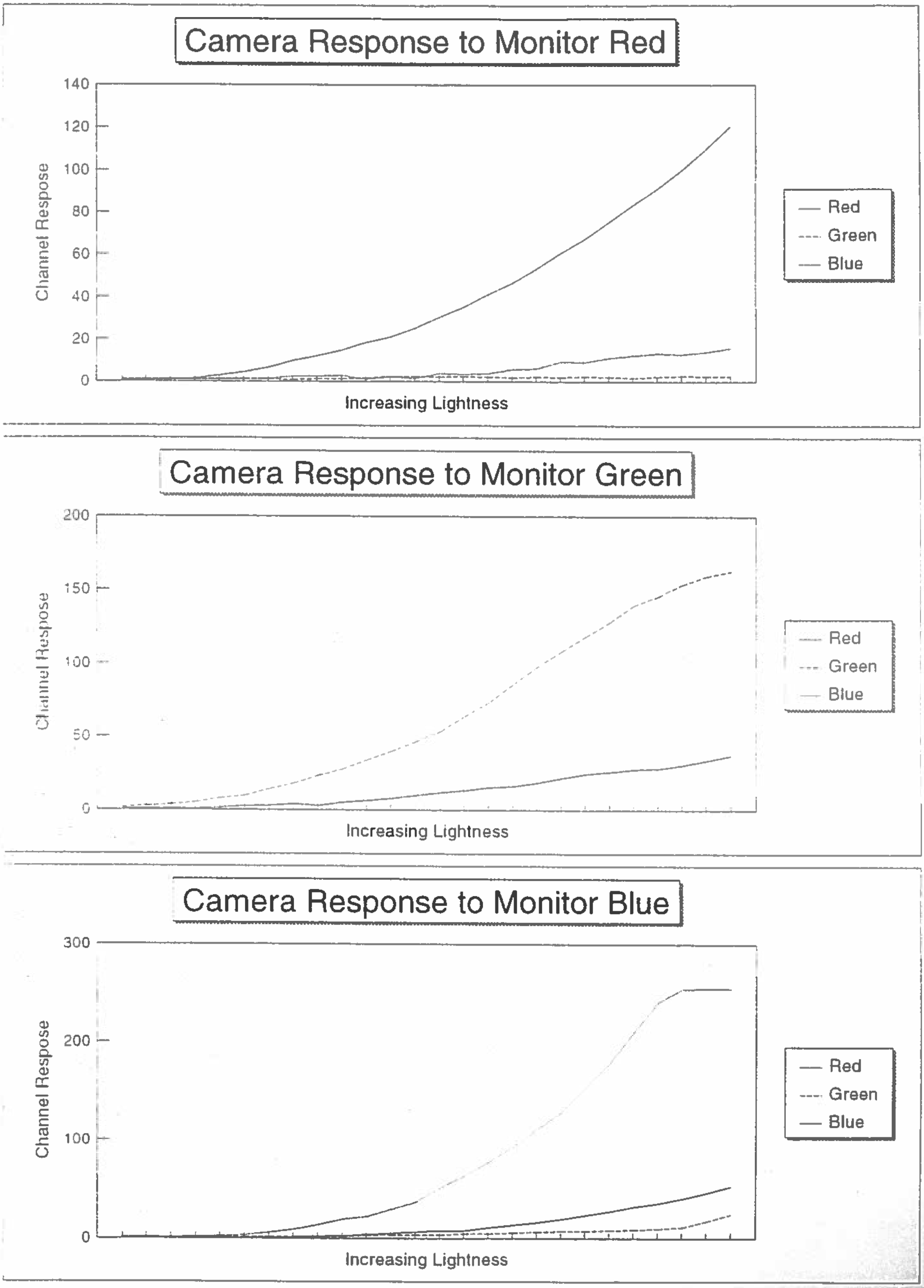

Camera Reponses to D65 Greyscale
200
150
100
50
0
Primary Response
Increasing 'Y'
Red
Green
Blue

Fig (v)



Camera Response to Monitor Red
Channel Respose
140
120
100
80
60
40
20
0
Increasing Lightness
Red
Green
Blue
Camera Response to Monitor Green
Channel Respose
200
150
100
50
0
Increasing Lightness
Red
Green
Blue
Camera Response to Monitor Blue
Channel Respose
300
200
100
0
Increasing Lightness
Red
Green
Blue

Fig (vi)

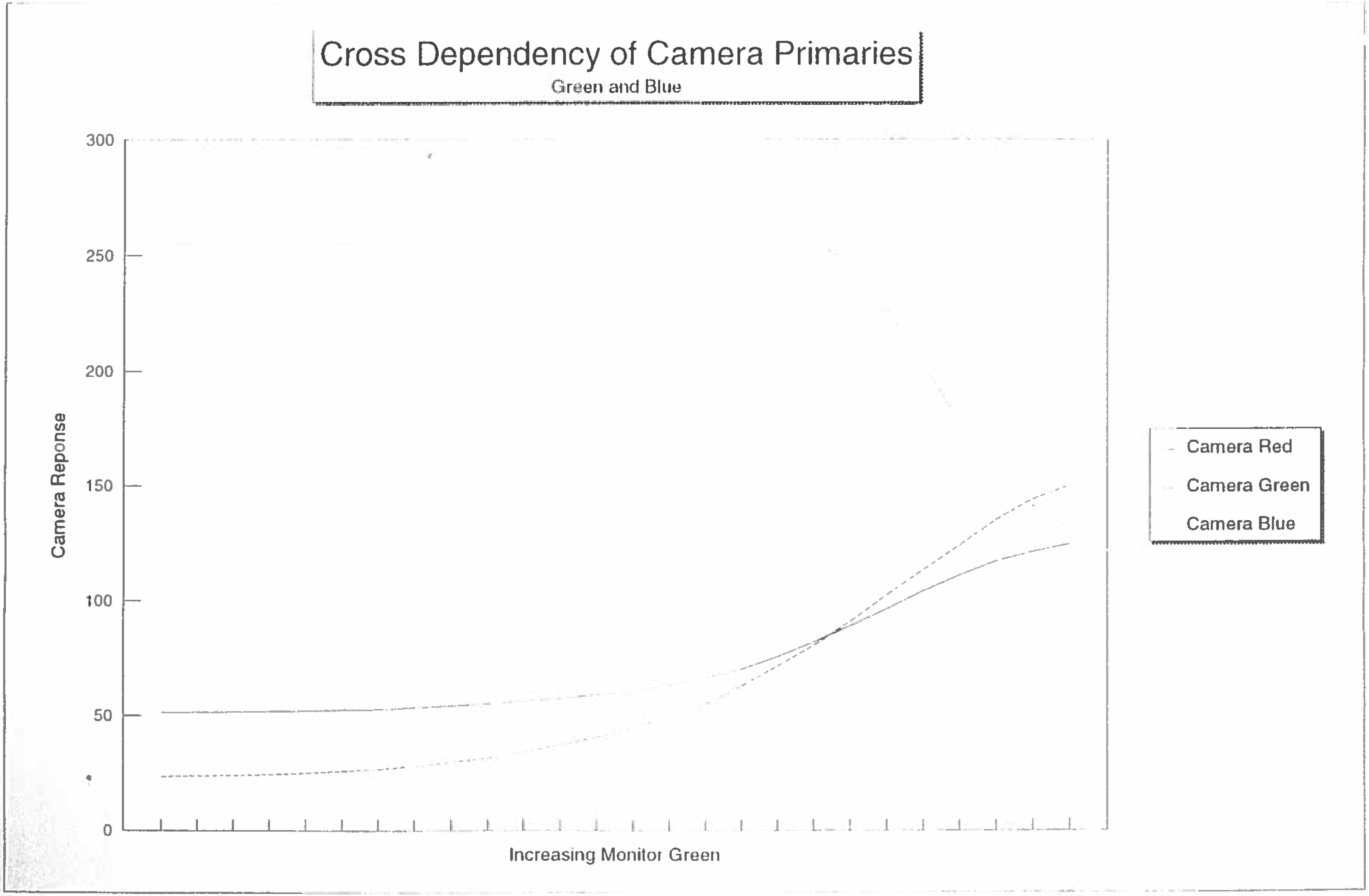

Cross Dependency of Camera Primaries
Green and Blue
300
250
200
150
100
50
0
Camera Reponse
Increasing Monitor Green
Camera Red
Camera Green
Camera Blue

Fig (vii)