Iterative Learning Controller - Rate of Convergence Analysis

$x(i+1,k)=Ax(i,k)+Bu(i,k)$

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ITERATIVE LEARNING CONTROLLER RATE OF CONVERGENCE ANALYSIS

INTRODUCTION: Iterative Learning Control (ILC) is one of the *Intelligent Control Systems* which improves the transient performance of systems operating in a repetitive manner. After each repetition or batch, the controller learns to produce the best possible control input signal. Convergence analysis of the first order D - type ILC algorithm has been investigated. It generates the control input update for the next batch based on the derivative of the error between the desired and actual outputs during the current batch.

SYSTEM: Discrete-time, sampled-data system, relative degree one (state space form):

$$\mathbf{x}(i+1,k)=\mathbf{A}\mathbf{x}(i,k)+\mathbf{B}u(i,k) \quad (1)$$

$$y(i,k)=\mathbf{C}\mathbf{x}(i,k)+\mathbf{D}u(i,k) \quad (2)$$

D-ILC ALGORITHM:

$$u(i,k+1)=u(i,k)+\mathbf{K}_d\{e(i+1,k)-e(i,k)\} \quad (3)$$

Where

k = the iteration or batch number

i = Sampling instant inside a batch

$u(i,k)$ = input at instant i with in batch k

$u(i,k+1)$ = input at i for next batch $k+1$

$\mathbf{K}_d$ = Gain matrix

$e(i,k)=y^*(i)-y(i,k)$ = residual error

$e(i+1,k)-e(i,k)$ = forward difference

CONVERGENCE CONDITION:

For repeated initial conditions at the start of each batch, the monotonic convergence for output as well as control input is achieved if:

$\|\mathbf{I}-\mathbf{K}_d\mathbf{CB}\|_p<1$

Where $\|\bullet\|_p$ is the pth operator norm and the product $\mathbf{CB}\neq 0$

CONTROL INPUT RESIDUAL RELATIONSHIP:

$$\begin{bmatrix}\Delta u(1,k+1)\\ \Delta u(2,k+1)\\ \Delta u(3,k+1)\\ \vdots\\ \Delta u(M,k+1)\end{bmatrix}=\begin{bmatrix}(\mathbf{I}-\mathbf{K}_d\mathbf{CB}) & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{K}_d\mathbf{C}(\mathbf{I}-\mathbf{A})\mathbf{B} & (\mathbf{I}-\mathbf{K}_d\mathbf{CB}) & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{K}_d\mathbf{CA}(\mathbf{I}-\mathbf{A})\mathbf{B} & \mathbf{K}_d\mathbf{C}(\mathbf{I}-\mathbf{A})\mathbf{B} & \ddots & \mathbf{0} & \vdots\\ \vdots & \vdots & \ddots & \ddots & \mathbf{0}\\ \mathbf{K}_d\mathbf{CA}^{M-2}(\mathbf{I}-\mathbf{A})\mathbf{B} & \cdots & \cdots & \cdots & (\mathbf{I}-\mathbf{K}_d\mathbf{CB})\end{bmatrix}\begin{bmatrix}\Delta u(1,k)\\ \Delta u(2,k)\\ \Delta u(3,k)\\ \vdots\\ \Delta u(M,k)\end{bmatrix} \quad (4)$$

RATE OF CONVERGENCE:

$$\frac{\|\Delta u(i,k)\|_\infty}{\|\Delta u(i,k-1)\|_\infty}=|\lambda|\sum_{j=0}^{i-1}\frac{\left|\binom{k-1}{j}\left(\frac{1}{\lambda}\right)^j\right|}{\left|\binom{k-2}{j}\left(\frac{1}{\lambda}\right)^j\right|}$$

$$\text{where } \binom{k-1}{j}=\frac{(k-1)!}{(k-1-j))!\,j!} \quad (5)$$

SIMULATION & RESULTS:

For a simple 1st order system, the batch to batch control inputs evolution is governed by the Toeplitz matrix in Eq. (4). See figure 1.

Rate of convergence in figure 2 has bounds:

Upper bound = Maximum singular value

Lower bound = Eigenvalue

Finally all component control inputs have convergence rate equal to the eigenvalue of Toeplitz matrix in Eq. (4).

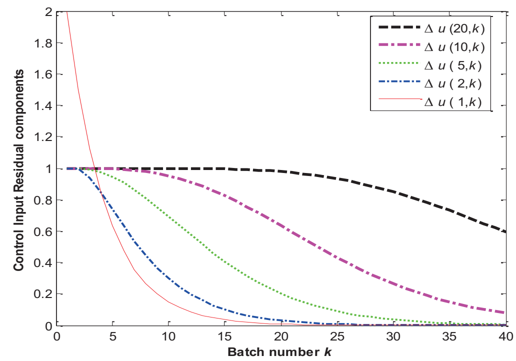

Figure.1

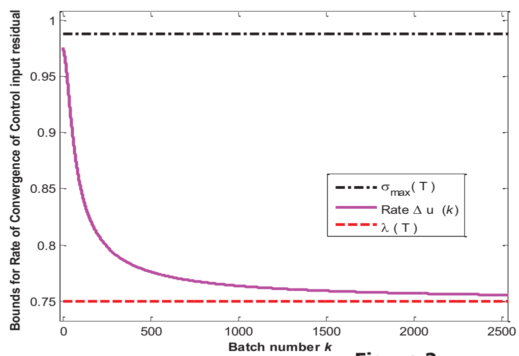

Figure 2

CONTROL SYSTEMS CENTRE

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