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Bound-State Pairing Singularities in the ^3He Galitskii–Feynman T -Matrix: Temperature Dependence

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The temperature-dependent Galitskii–Feynman T -matrix, which sums the two-body scattering series, allowing any number of pairs of either particles outside or holes inside the Fermi sea in intermediate states, for a two-body potential appropriate to liquid ^3He , is shown to exhibit a bound-state singularity. The two-body binding energy within the ^3He background is shown to be highly sensitive to both the temperature and the density of the system. A critical density below which the binding disappears is observed to be a function of the temperature and of the total momentum of the interacting pair. Detailed numerical computations of this structure are presented using the modified Frost–Musulin potential. To our knowledge, these computations represent the first attempt at a temperature-dependent many-body calculation based on a realistic ^3He potential. Possible consequences are drawn for more detailed and realistic calculations of the properties of liquid ^3He .

1. INTRODUCTION

In a previous communication¹ (hereafter referred to as I) it was reported that the zero-temperature Galitskii–Feynman (GF) T -matrix² possesses a first-order pole in the bound-state region for two separate strongly interacting fermion systems of interest, whereas the corresponding Bethe–Goldstone³ (BG) T -matrix does not exhibit such a pole in either case. Both T -matrices sum the ladder diagrams for repeated two-body interactions in the medium, the difference being that in the GF case the intermediate states are either two-particle or two-hole states, whereas in the BG case the interactions are restricted to be between two particles outside the Fermi sea. In the language of diagrammatic perturbation theory the BG T -matrix sums the two-body

ladder diagrams reckoned as Goldstone or time-ordered diagrams, and the GF T -matrix sums the same ladder diagrams but reckoned as Feynman or non-time-ordered diagrams. The diagrams summed by the GF T -matrix thus correspond to the set of Goldstone diagrams summed by the BG T -matrix after all allowable time orderings have been performed, whereas the BG T -matrix sums the diagrams with just that particular time ordering that permits only two particles outside the Fermi sea in intermediate states.

The BG and GF T -matrices arise quite naturally in the respective contexts of time-independent and time-dependent many-body perturbation theory, when one attempts to describe two-body scattering inside the many-body background. Both formalisms provide an equally valid starting point for any microscopic calculation of a many-body system, and must provide identical results for any physical observable when carried to infinite order, i.e., when all diagrams have been summed in both formalisms. In practice, of course, this aim is never realizable, and one has to be content with summing that class of diagrams in either formalism that is physically most important. In particular it has been known since the pioneering work of Brueckner⁴ that for any many-body system where the two-body interaction contains a strong short-range repulsion, it is necessary to at least sum the ladder diagrams as a complete set in order to obtain convergence of any physical observable. Both the BG and GF T -matrices serve this purpose in the context of their respective formalisms.

It is clear that the GF T -matrix sums many more terms in the perturbation series than does the corresponding BG T -matrix, and that the BG T -matrix can be regarded as the low-density limit of the corresponding GF quantity, since in this limit hole propagation becomes negligible. It was shown in I that for at least two many-fermion systems of interest the singularity structure of the two respective T -matrices is different. In particular it was shown that for realistic interparticle potentials appropriate to both neutron matter and liquid ³He the S -wave GF T -matrix possesses a bound-state pole whereas the corresponding BG T -matrix does not. The structure of the singularity was documented in detail, and in particular it was shown to be present at all densities above a certain critical density in both cases. In the case of ³He the critical density was somewhat less than that of the actual liquid. For this reason particular attention was focused on the case of liquid ³He.

It was shown in I that a pole in the T -matrix corresponds to a bound pair in the medium. In the case of the GF T -matrix, which treats both particles and holes symmetrically, the bound pair corresponds necessarily to binding between quasiparticles that are a linear combination of particle and hole states, and the absence of a pole in the BG T -matrix indicates that the hole states are vital to obtain this binding. It is possible that in a more

realistic theory of liquid ${}^3\text{He}$, which starts from the simple T -matrix (in either formalism) and is then extended to include other diagrams, this pairing is not destroyed. By analogy to the BCS theory of superconductivity,⁵ where the Cooper pairs⁶ are directly responsible for the physical phenomenon of superconductivity, it is to be expected that the pairing in this case may have physical consequences. In any case, we argued in I that it is of interest to compare two different lowest order microscopic theories applied to liquid ${}^3\text{He}$ and to focus attention on a distinct difference between them. It is clear that if the pairing that occurs in the lowest order GF T -matrix persists in a more realistic calculation and is not destroyed by the addition of higher order perturbation terms, then this formalism is to be preferred to the BG formalism, which would probably have to be extended to infinite order to reproduce a singularity that is absent in the lowest order.

The calculations in I were restricted to the case of zero temperature, which is not necessarily a good approximation for systems of interest. The zero-temperature approximation is presumably only justified if a typical microscopic energy (such as the energy $\hbar^2 k_F^2/2M$ of a free particle of mass M at the Fermi momentum k_F corresponding to the physical density) is considerably less than the thermal energy $k_B T$. Taking the two cases considered in I as examples, the microscopic energy scale defined above corresponds to a temperature of the order of 5×10^{11} K for neutron matter at ordinary nuclear matter density, and to a temperature of the order of 5 K for liquid ${}^3\text{He}$. Thus the zero-temperature approximation is probably well justified for neutron stars, which are believed to exist at temperatures⁷ about 10^8 K, whereas temperature dependence should be explicitly included in calculations of liquid ${}^3\text{He}$.

The zero-temperature many-body perturbation theory employing non-time-ordered Feynman Green's functions has been extended to finite temperatures in a completely consistent way by several authors,⁸ and results in a graph-theoretic perturbation series in complete analogy with the zero-temperature theory. The diagrams enumerated at finite temperature can be put into a one-to-one correspondence with those in the zero-temperature theory, the only difference being that the one-body propagator is explicitly temperature dependent.* Thus the same reasoning that dictates the necessity of summing the two-body ladder diagrams at zero temperature for a potential with a strong short-range repulsive core requires that a similar summation be performed at finite temperatures. Our philosophy in summing the ladder diagrams at zero temperature has been that it is a necessary precursor in

*There are actually two equivalent but distinct temperature-dependent many-body perturbation theories—the so-called real-time and imaginary-time formulations. The GF T -matrices obtained in the two formulations are simply related to one another by an analytic continuation in the CM energy variable of the interacting pair.

determining the properties of the many-body system, although it is probably not sufficient for any physical system of interest. However, even were sufficiency of summing the two-body ladder diagrams guaranteed for the zero-temperature case, one could not necessarily guarantee their sufficiency for calculations of properties of the system at finite temperatures.

For these reasons, in the present work we have extended the calculations of I for the GF T -matrix obtained from the same two-body potential appropriate to liquid ^3He to include temperature dependence. In complete analogy with the zero-temperature theory, it is possible to obtain a temperature-dependent GF T -matrix, from which one can again determine (to this approximation of keeping only the ladder diagrams) a corresponding temperature-dependent self-energy function. The self-energy function in turn can then be used to self-consistently obtain the single-particle excitation spectrum and the thermodynamic functions at finite temperature. We adopt the same dual philosophy as in I, namely that these results are both interesting in their own right and form a necessary first step for a detailed investigation, within the formalism outlined, of the thermodynamic functions for liquid ^3He , in particular, and other many-fermion systems in general. Furthermore, to within our knowledge this work presents the first detailed microscopic calculations for a many-body system at finite temperatures employing realistic interparticle potentials.

The temperature-dependent GF formalism adopted in this work is described in the next section, and the details of the numerical results obtained for the bound-state pole appropriate to liquid ^3He are presented in Section 3. Detailed numerical results are presented for the binding energy of a pair (in a relative S -wave state) as a function of each of their total linear momentum, the density of the system, and its temperature. The results are summarized in Section 4, where possible consequences for a more detailed and realistic calculation are drawn.

2. THE TEMPERATURE-DEPENDENT T -MATRIX FORMALISM

Using finite-temperature many-body theory in either the real-time or imaginary-time Green's function formalism,⁸ the temperature-dependent GF T -matrix, which sums the ladder diagrams in the many-body medium, evolves exactly as that for the zero-temperature case.^{2,9} An infinite number of non-time-ordered Feynman diagrams are summed, in which the virtual intermediate states comprise either pairs of particles outside or pairs of holes inside the Fermi sea, propagating as free states except for the effects of the exclusion principle. Considering the GF T -matrix equation, one can show in either the real-time or imaginary-time formalism that the introduction of finite temperatures affects only the two-body propagator of the intermediate

scattering states. Thus the single-particle occupation probabilities [which at zero temperature are given by the unit-step function $\theta(k - k_F)$ in momentum space] appearing in the intermediate propagator are replaced by the temperature-dependent Fermi-Dirac distribution functions. In the CM frame of the interacting pair, the resulting GF T -matrix is the solution of the operator equation

$$T(s, \mathbf{P}; \beta) = u - u\{Q(\mathbf{P}, \beta)g_0(s) - \bar{Q}(\mathbf{P}, \beta)g_0^\dagger(s)\}T(s, \mathbf{P}; \beta) \quad (1)$$

The T -matrix depends parametrically* on s and \mathbf{P} , respectively the total available energy of the interacting pair in the CM frame and their total momentum, and also the temperature β^{-1} . The function u is proportional to the two-body interaction potential V , $u = M\hbar^{-2}V = \frac{1}{2}V$, and the free two-body Green's function $g_0(s)$ is specified in terms of the relative kinetic energy operator H_0 of the pair as $g_0(s) = (\frac{1}{2}H_0 - s - i\eta)^{-1}$, with η a positive infinitesimal in the scattering region ($s > 0$) and zero otherwise. In the momentum-space representation of Eq. (1), the operators Q and \bar{Q} are diagonal in the relative momentum variable \mathbf{k} of the intermediate scattering states,

$$\langle \mathbf{k}|Q(\mathbf{P}, \beta)|\mathbf{k}' \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}')Q(\mathbf{k}, \mathbf{P}, \beta)$$

and similarly for the operator \bar{Q} , with

$$\begin{aligned} Q(\mathbf{k}, \mathbf{P}, \beta) &= n_0(\mathbf{P} + \mathbf{k})n_0(\mathbf{P} - \mathbf{k}) \\ \bar{Q}(\mathbf{k}, \mathbf{P}, \beta) &= [1 - n_0(\mathbf{P} + \mathbf{k})][1 - n_0(\mathbf{P} - \mathbf{k})] \end{aligned} \quad (2)$$

defined in terms of the Fermi-Dirac distribution function

$$n_0(\mathbf{k}) = \{1 + \exp[\beta(k_F^2 - k^2)]\}^{-1}$$

We find it most convenient to study Eq. (1) in the relative momentum representation, where

$$\langle \mathbf{p}|T(s, \mathbf{P}; \beta)|\mathbf{p}' \rangle = T(\mathbf{p}, \mathbf{p}'; s, \mathbf{P}; \beta)$$

and where operator multiplication is defined by

$$\langle \mathbf{p}|AB|\mathbf{p}' \rangle = (2\pi)^{-3} \int d\mathbf{k} \langle \mathbf{p}|A|\mathbf{k} \rangle \langle \mathbf{k}|B|\mathbf{p}' \rangle$$

As in I, the dependence of Eq. (1) on the angular orientation of the momenta of the virtual scattering states through the operators Q and \bar{Q} is a formidable complication to its numerical study, and we make the same approximation of angle-averaging these operators, whence this dependence

*We use throughout units $2M = \hbar = k_B = 1$, where M is the particle mass and k_B is the Boltzmann constant.

on the angular variables vanishes. The averaging reduction over the orientation of the CM momentum can be performed analytically, with the result

$$\begin{aligned}
 Q(k, P, \beta) &\equiv \frac{1}{2} \int_{-1}^1 d\mu Q(\mathbf{k}, \mathbf{P}, \beta) \\
 &= [2\beta k P \{1 - \exp[-2\beta(P^2 + k^2 - k_F^2)]\}]^{-1} L(k, P) \\
 \bar{Q}(k, P, \beta) &\equiv \frac{1}{2} \int_{-1}^1 d\mu \bar{Q}(\mathbf{k}, \mathbf{P}, \beta) \\
 &= [2\beta k P \{\exp[2\beta(P^2 + k^2 - k_F^2)] - 1\}]^{-1} L(k, P) \\
 L(k, P) &= \ln \frac{\cosh \{\frac{1}{2}\beta[(P+k)^2 - k_F^2]\}}{\cosh \{\frac{1}{2}\beta[(P-k)^2 - k_F^2]\}}
 \end{aligned} \tag{3}$$

where $\mu = \hat{k} \cdot \hat{P}$. In the limit as β^{-1} approaches zero, the functions in Eqs. (3) smoothly and continuously approach their zero-temperature counterparts defined in I. Using this approximation, Eq. (1) can be written in an uncoupled partial-wave series for the functions $T_l(p, p'; s, P; \beta)$,

$$T(\mathbf{p}, \mathbf{p}'; s, P; \beta) = \sum_l (2l+1) T_l(p, p'; s, P; \beta) P_l(\hat{p} \cdot \hat{p}')$$

exactly as in I. Defining $\mathbf{p}_1, \mathbf{p}_2$ and $\mathbf{p}'_1, \mathbf{p}'_2$, respectively, as the initial and final momentum states of the interacting pair in the many-body background, and taking matrix elements in the CM frame, with the conventions

$$\mathbf{p} = \frac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2); \quad \mathbf{p}' = \frac{1}{2}(\mathbf{p}'_1 - \mathbf{p}'_2), \quad \mathbf{P} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2) = \frac{1}{2}(\mathbf{p}'_1 + \mathbf{p}'_2)$$

Eq. (1) becomes, after the angle-averaging approximation has been made,

$$\begin{aligned}
 T_l(p, p'; s, P; \beta) &= u_l(p, p') - \int_0^\infty \frac{k^2 dk}{2\pi^2} u_l(p, k) \\
 &\times \left[\frac{Q(k, P, \beta)}{k^2 - s - i\eta} - \frac{\bar{Q}(k, P, \beta)}{k^2 - s + i\eta} \right] T_l(k, p'; s, P; \beta) \tag{4}
 \end{aligned}$$

The study of the numerical solutions to the one-dimensional integral equation (4) for a potential $u_l(p, p')$ appropriate to liquid ^3He forms the basis of the remainder of this work. In principle, the solution to Eq. (4) leads directly to the ladder contribution to the proper self-energy in either the real-time or imaginary-time formalism, whence contact is immediately made with the excitation spectrum and the thermodynamic functions, respectively.⁸

3. NUMERICAL RESULTS

In the current study the results reported in I are extended to include finite temperatures by performing a numerical investigation of Eq. (4), and

the behavior of the bound-state singularity as a function of temperature is examined in detail. Using the same S -wave modified Frost–Musulin (MFM) potential as described in I, the GF T -matrix defined by Eq. (4) has been investigated using numerical techniques identical to those used in I, and for a range of temperatures from zero to over 100 K. Equation (4) is basically solved by discretization on a 96-point Gaussian grid and directly inverting the resulting matrix equation. The T -matrix in the S -wave channel was found to be everywhere a continuous, slowly varying function of the temperature except for values of its other arguments near those responsible for the singularity structure reported in I. The basic structure of the singularity as a first-order pole with factorizable residue,

$$T_0(p, p'; s, P; \beta)_{s \rightarrow s_0} - \left[\frac{f(p, P, \beta)f(p', P, \beta)}{s - s_0} \right]$$

is unchanged at finite values of the temperature. The position of the singularity s_0 , which corresponds to the binding energy of the pair, is a function of the magnitude of the total momentum P of the pair, the density (or equivalently the Fermi momentum k_F), and the temperature β^{-1} of the many-body system in which they are imbedded. The singularity s_0 is independent of any particular representation of the T -matrix; i.e., independent of p and p' in the relative momentum representation.

The function $s_0(P, k_F, \beta)$ exhibits a rather complicated dependence on the temperature. The surface described by choosing the total momentum P to be zero is characteristic of the entire hypersurface, and is shown in Fig. 1, where the position of the singularity is shown plotted as a function of temperature for 11 values of k_F . For a given value of k_F there is seen to exist a critical temperature $\beta^{-1} = \beta_c^{-1}(k_F)$ above which the singularity vanishes, or inverting this relation, we can define a critical Fermi momentum $k_F = k_{Fc}(\beta^{-1})$ as a function of temperature. For values of k_F greater than the zero-temperature critical Fermi momentum [$k_{Fc}(0) = 0.7146 \text{ \AA}^{-1}$ at $P = 0$] and for temperatures less than $\beta_c^{-1}(k_F)$, the pole exists, and as the temperature approaches zero the pole smoothly and continuously approaches the shelllike structure described in I. For values of k_F less than $k_{Fc}(0)$ the singularity in the T -matrix does not exist at zero temperature, by definition. However, for k_F in the range $k_{Fc}^{\min} < k_F < k_{Fc}(0)$, the pole reappears at finite temperature above a certain lower critical temperature, say $\beta_c^{\prime -1}(k_F)$, and again disappears as the temperature is increased above the upper critical temperature $\beta_c^{-1}(k_F)$ as described above for the case $k_F > k_{Fc}(0)$. The minimal critical Fermi momentum, which is found to occur at a value $k_{Fc}^{\min} = 0.70674 \text{ \AA}^{-1}$, is characterized as being that value of k_{Fc} at which the upper and lower critical temperatures become equal, in this case at a value $\beta_c^{\prime -1}(k_{Fc}^{\min}) = \beta_c^{-1}(k_{Fc}^{\min}) = 6.38 \text{ K}$. For densities

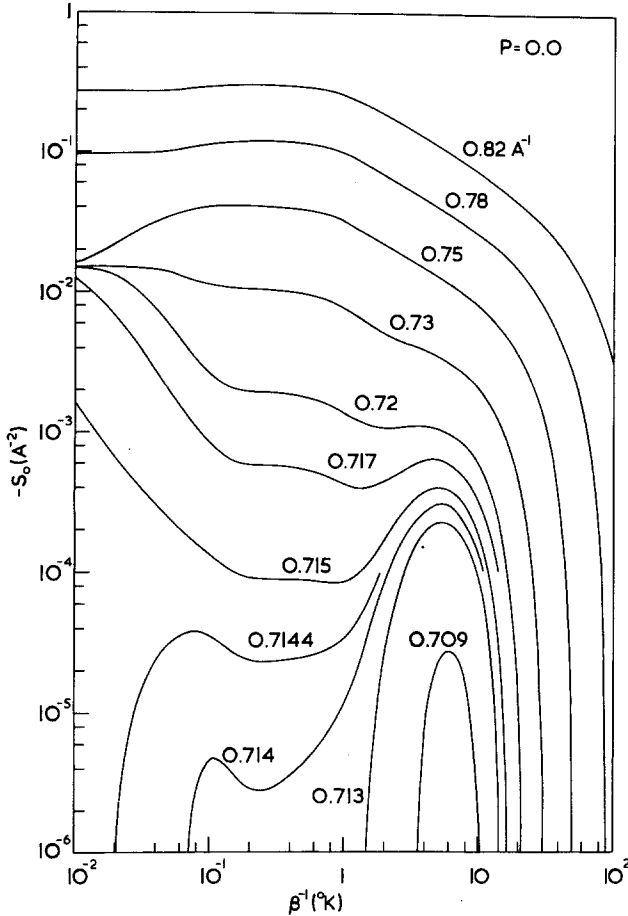


Fig. 1. The position of the singularity s_0 in the S -wave GF T -matrix for the MFM potential at zero CM momentum of the pair as a function of temperature β^{-1} for 11 values of k_F .

corresponding to $k_F < k_{Fc}^{\min}$, the pole is absent for all values of the temperature. This entire structure for the case of zero CM momentum of the pair ($P = 0$) is detailed in Fig. 1, and the bottom curve of Fig. 2 shows the function $k_{Fc}(\beta^{-1})$ for the case $P = 0$. From this last curve, one readily sees that the inverse function $\beta_c^{-1}(k_F)$ at $P = 0$ is double-valued in the range $k_{Fc}^{\min} < k_F < k_{Fc}(0)$, and the second branch in this region corresponds to what was identified as $\beta_c^{\prime-1}(k_F)$ from observation of Fig. 1.

The general structure of the function $s_0(P, k_F, \beta)$ discussed above for the case $P = 0$ persists at nonzero values of the CM momentum P . As the CM momentum is increased from $P = 0$, it is found that the critical quantity

$k_{F_c}(\beta^{-1}, P)$ varies smoothly as a function of P , some typical curves being plotted in Fig. 2, where an arbitrary constant momentum (equal to 0.70650 \AA^{-1}) has been subtracted from k_{F_c} , purely for ease of display. One observes that the number of relative maxima and minima of the family of curves, considered as functions of temperature, depends on the value of P , although the positions of these extrema are themselves smooth functions of the parameter P . The number of extrema is obviously directly related to the number of branches of the multivalued inverse functions $\beta_c^{-1}(P, k_F)$. Thus each relative minimum in a given member of the family determined by its value of the parameter P is responsible for a separate "seedlike" structure, centered at the minimal value of the temperature, in a contour plot for the function $s_0(P, k_F, \beta)$, similar to that shown in Fig. 1 centered at a temperature of 6.38 K for the case $P = 0$. Similarly, each relative maximum for a member of the family represents the demarcation contour in k_F which separates two disconnected seedlike structures. That the relative extrema in members of the family move smoothly as functions of the parameter P can be seen clearly from Fig. 2. Thus, for example, the relative minimum at 6.38 K in the $P = 0$

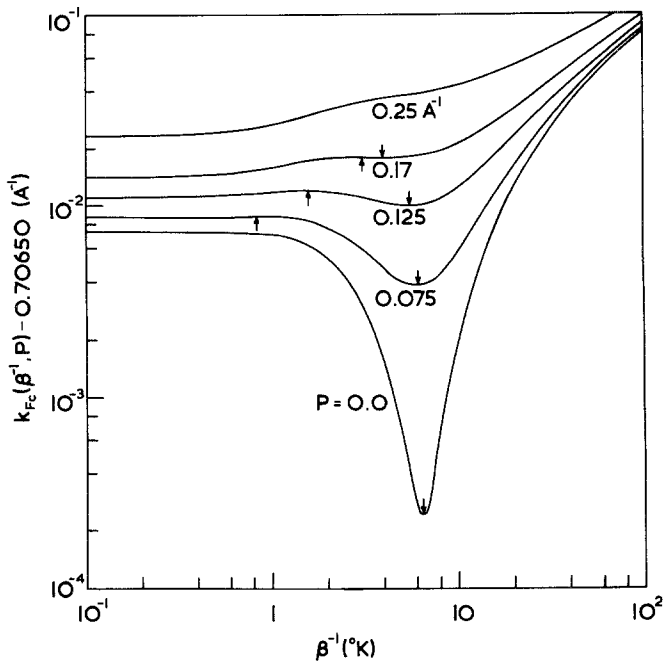


Fig. 2. The critical Fermi momentum k_{F_c} for the S -wave MFM potential as a function of temperature β^{-1} for five values of the CM momentum P of the pair. The arrows indicate the positions of the relative extrema of the curves.

member of the family moves smoothly to lower temperatures as P is increased, until at a value $P = 0.172 \text{ \AA}^{-1}$ it has merged continuously with a relative minimum (to form an inflection point in this member of the family) at a value of the temperature of 3.5 K. For all values of $P > 0.172 \text{ \AA}^{-1}$, the functions $k_{\text{Fc}}(\beta^{-1}, P)$ are everywhere increasing functions of the temperature β^{-1} .

The detailed numerical results presented in this section are summarized in Section 4, and are discussed with particular reference to their pertinence for realistic microscopic calculations of liquid ^3He .

4. DISCUSSION AND SUMMARY

To summarize, the singularity structure of the GF T -matrix described in I and in the present work manifests itself as a first-order pole in the CM energy variable, with factorizable residue at the pole. The position s_0 (< 0) of the (bound-state) pole is a function of the CM momentum of the pair, of the density of the system, and of the temperature. The value s_0 measures the binding energy of the bound-state pair interacting in the presence of the many-fermion background.

The function $s_0(P, k_{\text{F}}, \beta)$ was shown to exhibit a critical Fermi momentum $k_{\text{Fc}} = k_{\text{Fc}}(\beta^{-1}, P)$, at which density the pair has zero binding energy and below which binding does not occur. For the MFM potential, which appears to be a good approximation to the interaction between ^3He atoms, the critical values k_{Fc} for temperatures in the range from zero to about 100 K and for most values of the CM momentum are close to and below the value 0.79 \AA^{-1} appropriate to real liquid ^3He .

Such a bound-state singularity in the GF T -matrix will almost certainly qualitatively affect, and in certain instances possibly dominate, the behavior of the (lowest order) proper self-energy function calculated from it, and consequently influence both the single-particle energy spectrum and the quasiparticle lifetimes, as well as the thermodynamic functions. The results of this study indicate that the position of the pole, or indeed its presence or absence, is strongly dependent on both temperature and density in the range of interest for these parameters, and would seem to indicate in particular that a proper study of this system should include temperature dependence.

The two-body binding within the many-body ^3He background that we have documented for the relative S -wave channel of the pair occurs only for the GF T -matrix. A similar investigation of the corresponding BG T -matrix with the same two-body potential reveals the absence of any binding for this case for a comparable range of temperatures and densities. The absence of a pole in the BG T -matrix highlights the importance of including the intermediate hole states accessible to the pair imbedded in the many-body medium. The pairing in the GF formalism corresponds to pairing between

quasiparticles that are linear combinations of hole and particle states, or equivalently to pairing between pairs of particles dressed by hole states within the ^3He background.

The most interesting open question at this point is whether or not the remaining many-body interactions in the medium will destroy the two-body pairing obtained with the GF T -matrix. Both the BG and GF T -matrices sum exactly all of the two-body interactions between a pair in the medium, where the pair in the former case is restricted to be two particles, while in the latter the possibility of the pair being two holes is also included. If it were known that all explicitly three- or more-body effects* were small, then presumably the GF T -matrix would adequately describe the system, whereas the BG T -matrix would not. That is to say, in the time-dependent perturbation theory one would have to include many more diagrams than are summed by just the BG T -matrix. Unfortunately, no such guarantee exists for real liquid ^3He . While either T -matrix presumably accounts for the strong short-range repulsion, neither is likely to be sufficiently weak that higher order terms in either the time-dependent or time-independent perturbation theory may be safely neglected. The three-body (and possibly the four-body) cluster terms are likely to be of importance, just as in the case of nuclear matter. Worse than this is the fact that even though summing the ladder diagrams to obtain a T -matrix takes care of "softening" the short-range repulsion, the ^3He interparticle potential also contains a long-range attractive tail, the strong effects of which are liable to remain in *all* n -body clusters.

In order to investigate the effects of the first of these two complications on the pairing in the GF T -matrix, we are currently investigating the structure of the three-body terms using the GF T -matrix. In order to determine the effect of the long-range part of the interaction in higher orders, we are investigating the effect of screening the bare interaction with the so-called bubble diagrams that correspond to the random-phase approximation. Should the bound-state singularity in the GF T -matrix turn out not to be destroyed (but merely shifted) by these higher order effects, the question remains of how the binding will manifest itself in the framework of the physical observables for liquid ^3He , and this, too, is currently being investigated. As a final point we note that the current work has been restricted to the MFM potential, whereas there exist in the literature numerous alternative potentials which fit the available ^3He data. We are presently comparing the properties of the GF T -matrices obtained from these various bare potentials. Results of these investigations will be published elsewhere. It should be stressed at this point

*By an n -body contribution, we mean in this context contributions from an n -body cluster (in the non-time-ordered or Feynman diagram sense) interacting together in the medium through the same two-body potential. The effect of three- or more-body forces is an added complication which we presently ignore.

that the S -wave pairing investigated by us for the bare potential appears to be absent in the P - and higher-wave channels, and in its present form thus bears no resemblance at all to the pairing presumably responsible for the superfluid phases of ^3He . Whether this situation will change as higher order effects are taken into consideration and as the bare potential is screened by particle-hole pairs due to its long-range tail remains to be seen.

It should be finally emphasized that the results reported in this study can easily be extended to other dense Fermi systems. In particular, neutron matter was shown in I to have a similar singularity structure in the GF T -matrix, the temperature dependence of which is expected to be comparable to that reported here. However, the internucleon potential has added complications, such as a tensor component (and other explicit state dependence), and at densities high enough for binding to occur, mesonic degrees of freedom (suppressed in a static potential formalism) can substantially alter the binding mechanism. We are also studying such effects.

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REFERENCES

1. R. F. Bishop, M. R. Strayer, and J. M. Irvine, *Phys. Rev. A* **10**, 2423 (1974).
2. V. M. Galitskii and A. B. Migdal, *Soviet Phys.—JETP* **7**, 96 (1958); V. M. Galitskii, *Soviet Phys.—JETP* **7**, 104 (1958).
3. H. A. Bethe and J. Goldstone, *Proc. R. Soc. (London)* **A238**, 551 (1957).
4. K. A. Brueckner, *Phys. Rev.* **97**, 1353 (1955).
5. J. Bardeen, L. Cooper, and J. R. Schrieffer, *Phys. Rev.* **108**, 1175 (1957).
6. L. Cooper, *Phys. Rev.* **104**, 1189 (1956).
7. D. Pines, in *Proc. 12th Int. Conf. Low Temp. Phys.* (Kyoto, Japan, 1970), p. 7.
8. A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971), Chapters 7-9.
9. R. F. Bishop, *Ann. Phys. (N.Y.)* **77**, 106 (1973).