Lattice Hamiltonian approach to the Schwinger model

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K. Cichy[1,2], A. Kujawa-Cichy[3], M. Szyniszewski[4,5]

[1]NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany
[2]Department of Physics, Adam Mickiewicz University, 61-614 Poznań, Poland
[3]Institut für Theoretische Physik, Goethe-Universität, 60438 Frankfurt am Main, Germany
[4]Physics Department, Lancaster University, Lancaster, LA1 4YB, UK
[5]NoWNano DTC, University of Manchester, Manchester, M13 9PL, UK

Outline

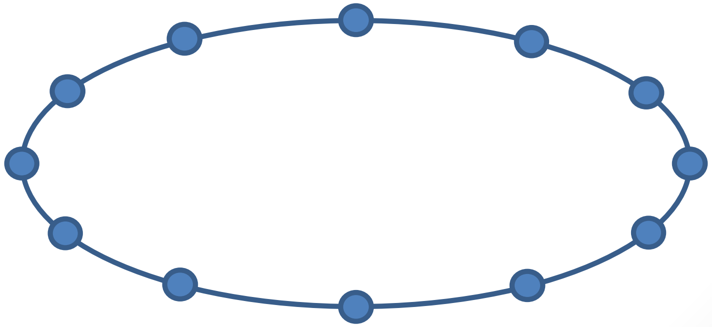
1. The Schwinger model on lattice
2. Strong coupling expansion (SCE)
3. Ground state energy
4. Mass gaps
5. Chiral condensate
6. Oscillations of chiral condensate
7. Summary & outlook

The Schwinger model

Hamiltonian of the Schwinger model in the Kogut-Susskind staggered discretization [1,2]:

$$\mathcal{H} = -\frac{i}{2a}\sum_{n=1}^{M}\left(\phi^{\dagger}(n)e^{i\theta(n)}\phi(n+1) - \phi^{\dagger}(n+1)e^{-i\theta(n)}\phi(n)\right) + m\sum_{n=1}^{M}(-1)^{n}\phi^{\dagger}(n)\phi(n) + \frac{ag^{2}}{2}\sum_{n=1}^{M}L^{2}(n)$$

- $\phi(n)$ – single-component fermion field on a circle with M sites
- $\theta(n) = agA_1(n)$ – gauge field variable related to the Abelian vector potential
- $L(n) = E(n)/g$ – variable related directly to the electric field
- m – fermion mass
- a – lattice spacing
- g – gauge coupling constant



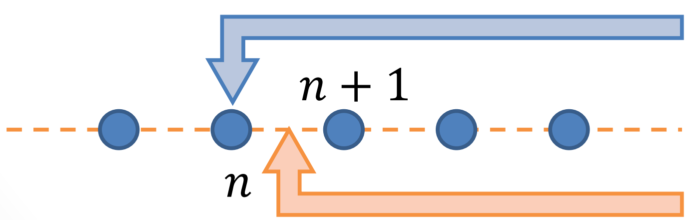
[1] Kogut and Susskind, Phys. Rev. D 11 (1975) 395
[2] Banks, Susskind and Kogut, Phys. Rev. D 13 (1976) 1043

The Schwinger model

Hamiltonian of the Schwinger model in lattice representation after the Jordan-Wigner transformation [3]:

$$\mathcal{H}_{JW} = -\frac{1}{2a}\sum_{n=1}^{M}\left(\sigma^{+}(n)e^{i\theta(n)}\sigma^{-}(n+1) + \text{h.c.}\right) + \frac{m}{2}\sum_{n=1}^{M}\left(1 + (-1)^{n}\sigma^{3}(n)\right) + \frac{ag^{2}}{2}\sum_{n=1}^{M} L^{2}(n)$$

- $\sigma^{i}(n)$ – Pauli matrices residing on the sites
- $L(n)$ – gauge field excitations defined between sites n and $n+1$
- $e^{\pm i\theta(n)}$ – ladder operators for gauge field excitations



Spin matrices $\sigma^{3}(n), \sigma^{\pm}(n)$
$\sigma^{3}(n) = \pm 1$

Gauge field excitations $L(n), e^{i\theta(n)}$
$L(n) = 0, \pm 1, \pm 2, \ldots$

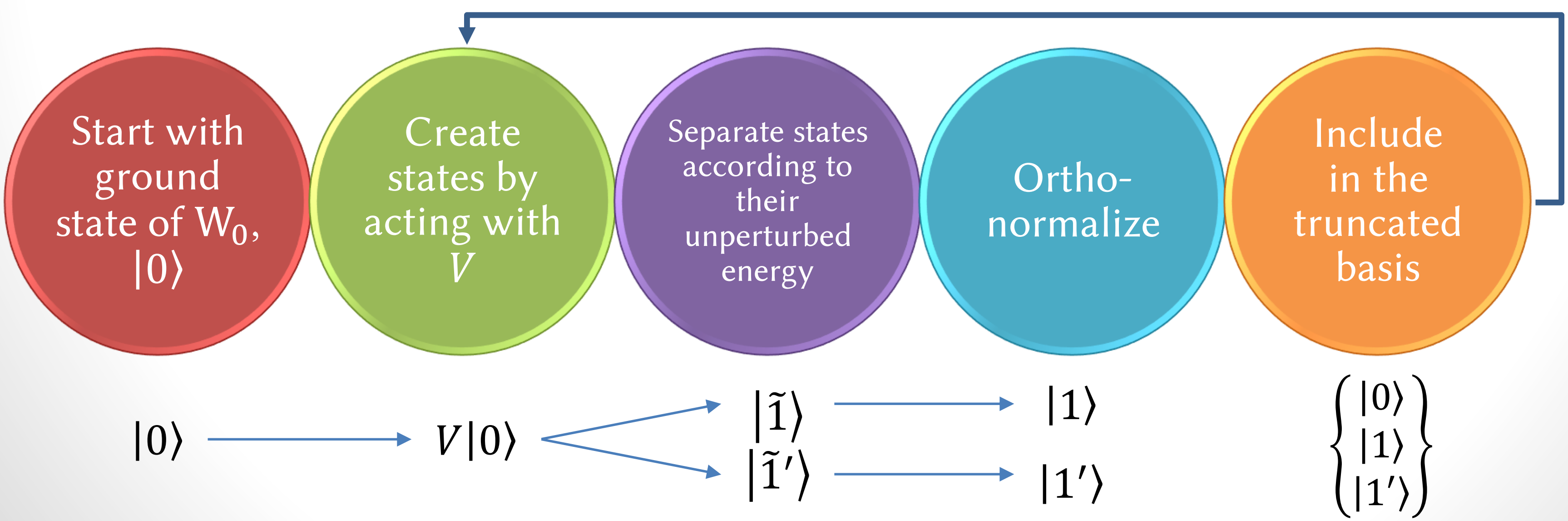
[3] Jordan, Wigner, Z. Phys. 47 (1928) 631.

Strong coupling expansion on the Schwinger model

Rewrite the Hamiltonian in a dimensionless form:

$$W = \frac{2}{ag^2}\mathcal{H}_{JW} = \underbrace{\boxed{W_0}}_{\frac{m}{ag^2}\sum_{n=1}^{M}(1+(-1)^n\sigma_n^3)+\sum_{n=1}^{M}L^2(n)} + x\overbrace{\boxed{V}}^{\sum_{n=1}^{M}(\sigma_n^+ e^{i\theta(n)}\sigma_{n+1}^- + \text{h.c.})}$$

- If $x \equiv \beta = \frac{1}{a^2g^2}$ is small, we can treat W_0 as an unperturbed Hamiltonian and V as a perturbation.
- SCE creates the truncated basis of W



Observables

- Ground state energy:

$$E_0 = \frac{\omega_0}{2Mx} \xrightarrow[M\to\infty]{a\to 0} -\frac{1}{\pi}$$

- Scalar mass gap ($m = 0$):

$$\frac{M_S}{g} = \frac{\omega_1 - \omega_0}{2\sqrt{x}} \xrightarrow[M\to\infty]{a\to 0} \frac{2}{\sqrt{\pi}}$$

- Vector mass gap ($m = 0$):

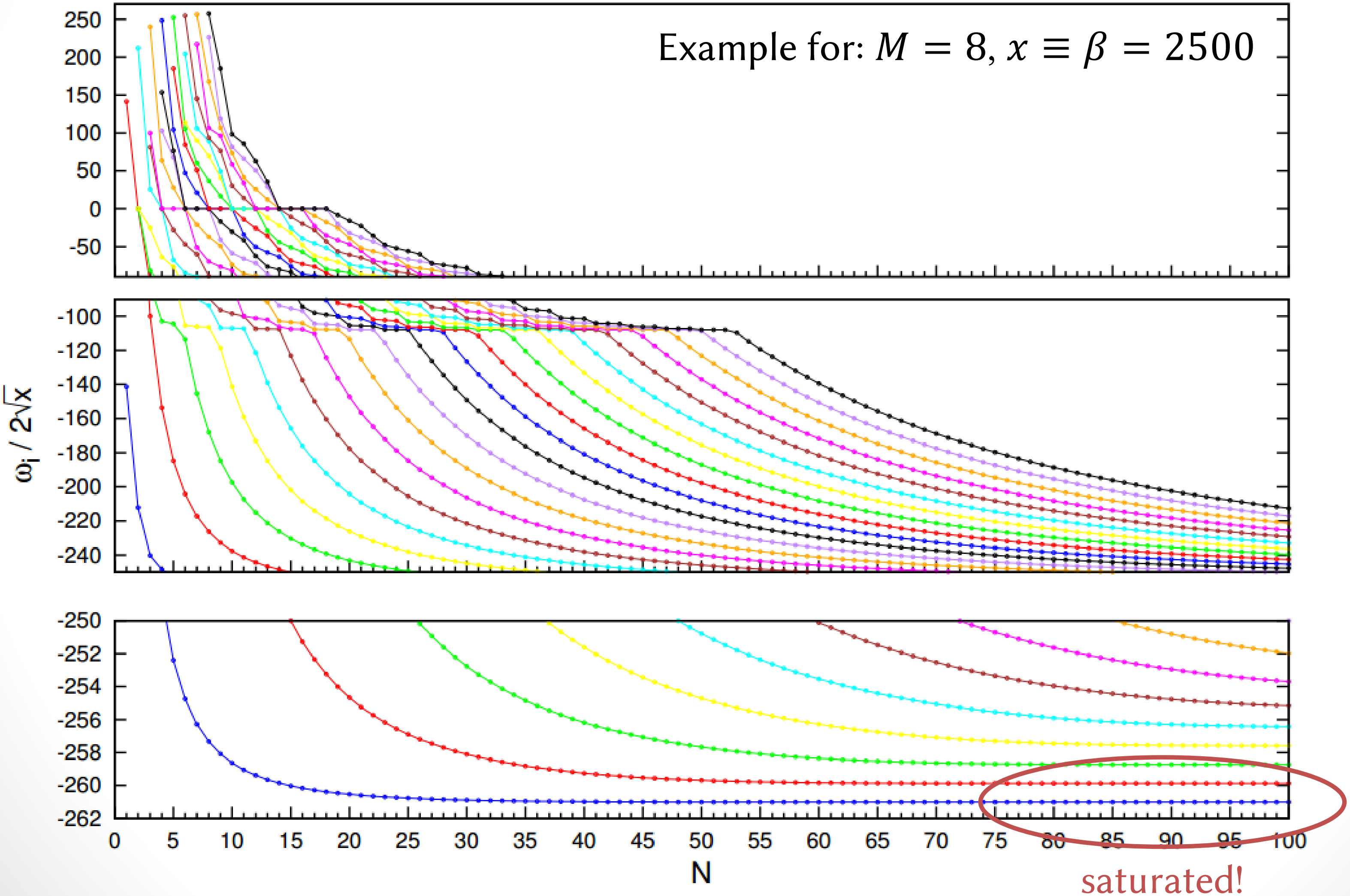
$$\frac{M_V}{g} = \frac{\omega_0^V - \omega_0}{2\sqrt{x}} \xrightarrow[M\to\infty]{a\to 0} \frac{1}{\sqrt{\pi}}$$

- Chiral condensate (chiral order parameter):

$$\frac{\langle\bar{\psi}\psi\rangle_0}{g} = \frac{\sqrt{x}}{2M}\langle 0|\sum_{n=1}^{M}(-1)^n\sigma^3(n)\,|0\rangle$$

- ω_i – eigenvalues of W_0
- ω_i^V – eigenvalues of vector Hamiltonian created using SCE with first state $V^-|0\rangle$.

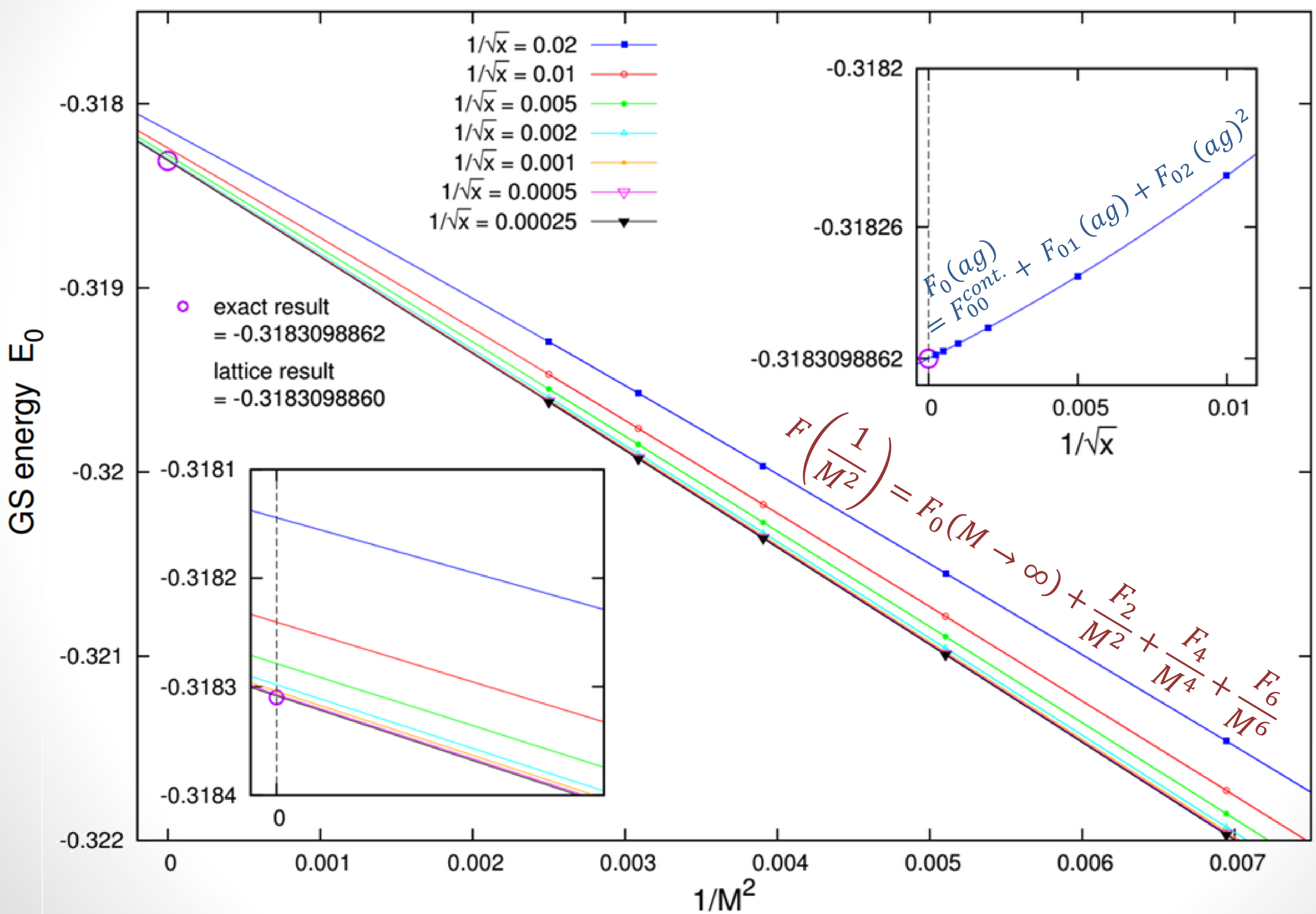
Eigenvalue flow with the order of strong coupling expansion, *N*



Ground state energy

$$E_0 = \frac{\omega_0}{2Mx}$$

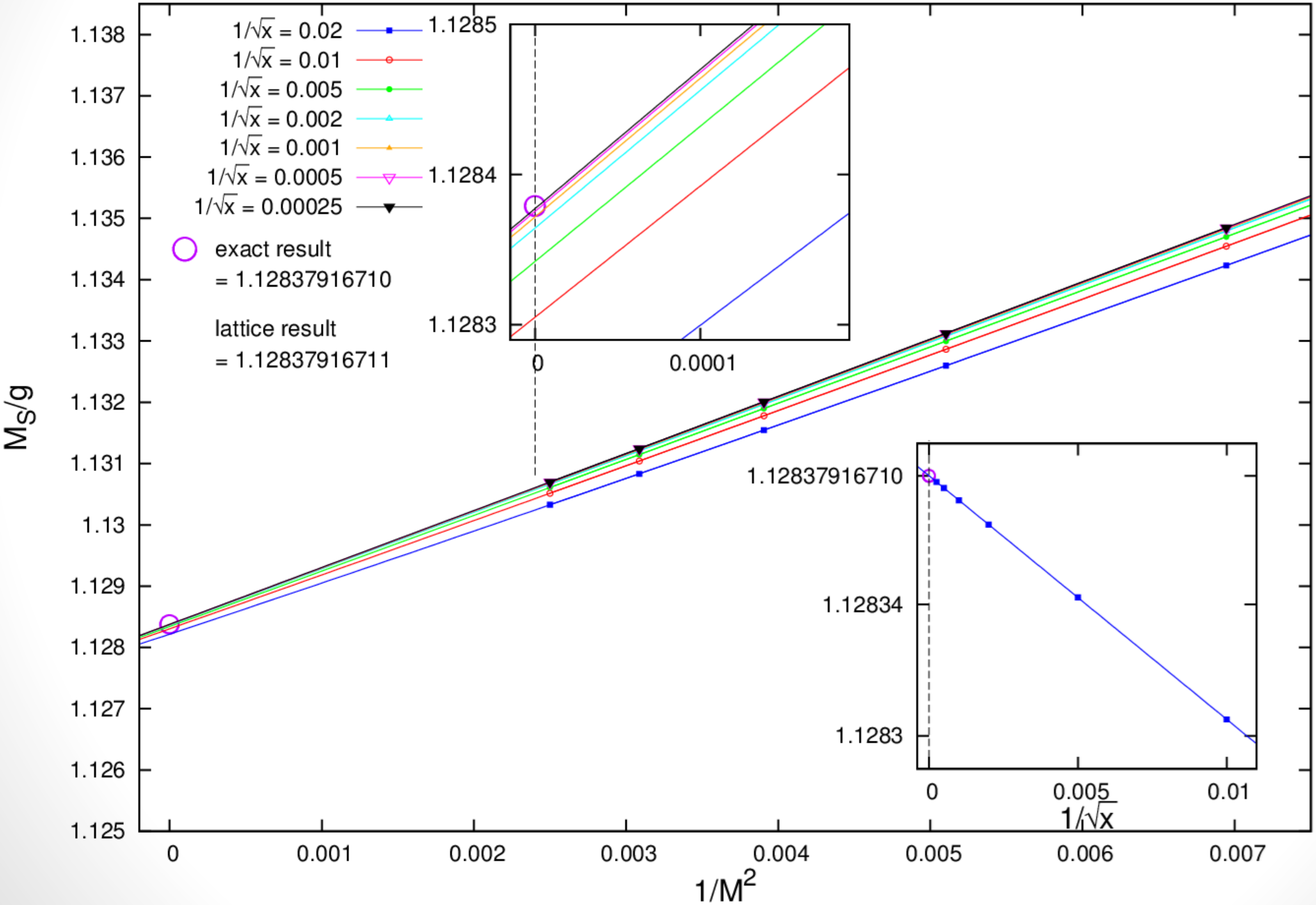
$m = 0$



Scalar mass gap

$$\frac{M_S}{g} = \frac{\omega_1 - \omega_0}{2\sqrt{x}}$$

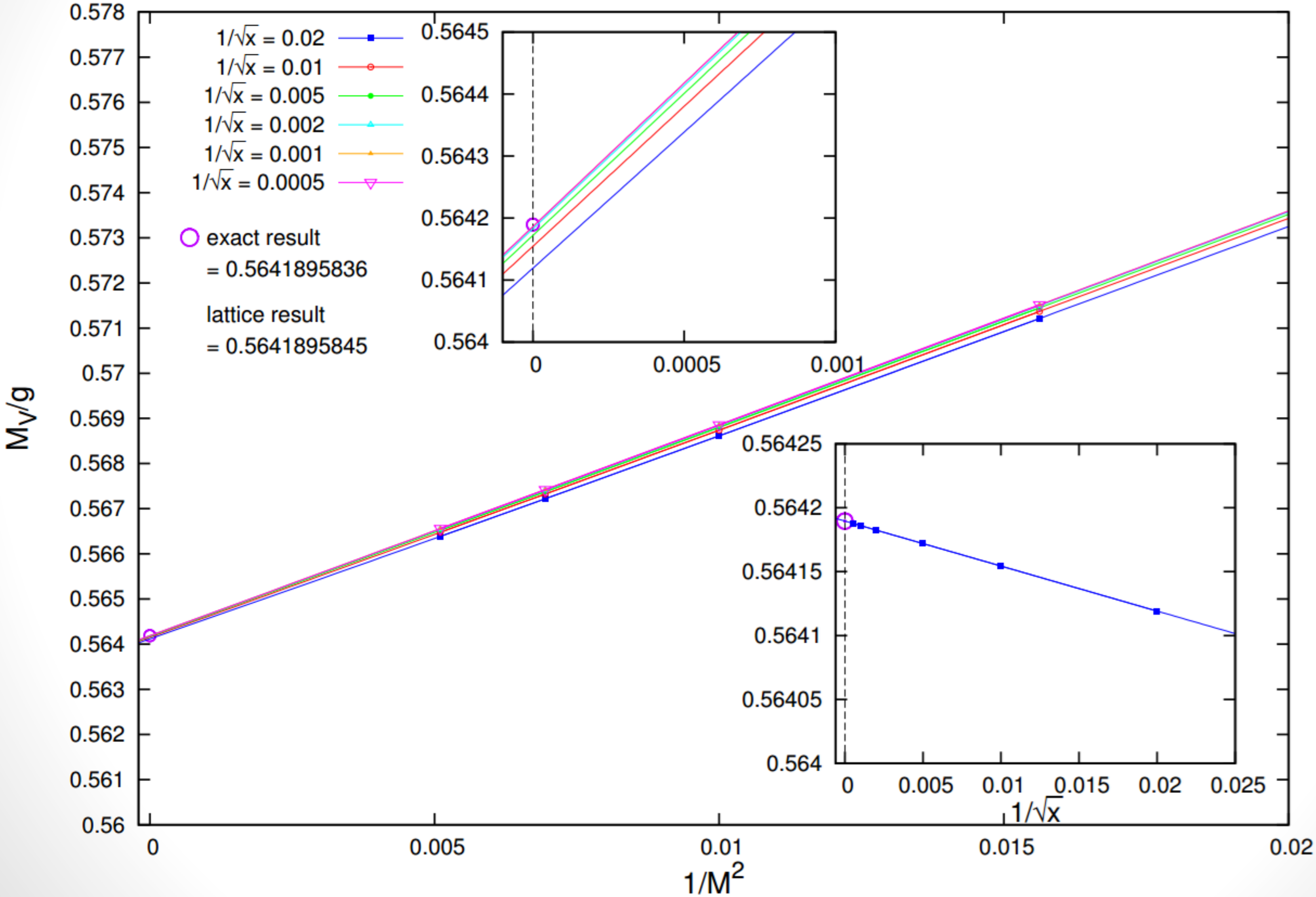
$m = 0$



Vector mass gap

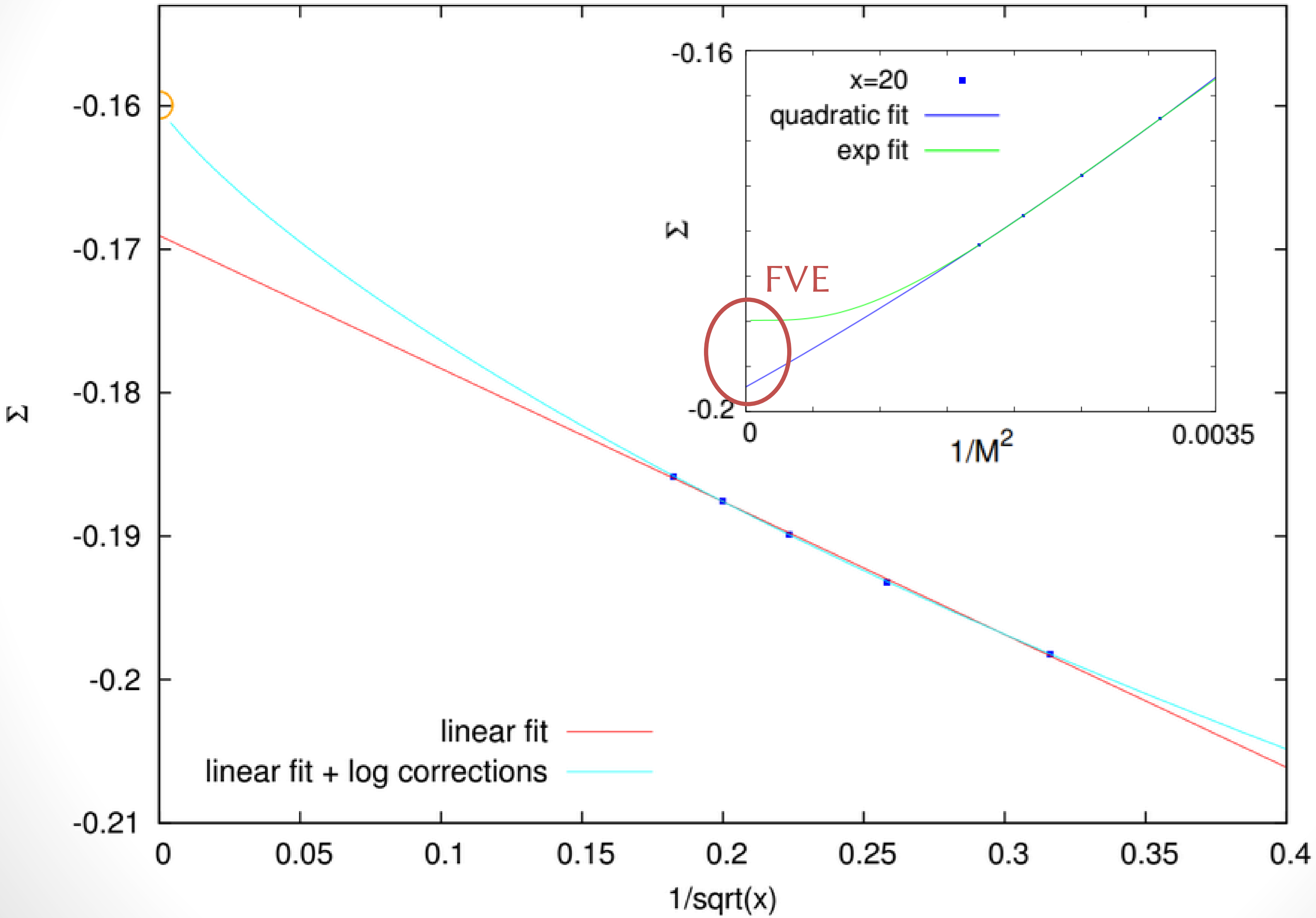
$$\frac{M_V}{g} = \frac{\omega_0^V - \omega_0}{2\sqrt{x}}$$

$m = 0$



Chiral condensate

$m = 0$



Comparison with MPS results

- Ground state energy and mass gaps – massless model:

Observable	SCE+ED	MPS [4]
E_0	-0.3183098860(2)	-0.318338(24)
M_S/g	1.12837916711(1)	1.1279(12)
M_V/g	0.5641895845(9)	0.56421(9)

- Chiral condensate - massless case:

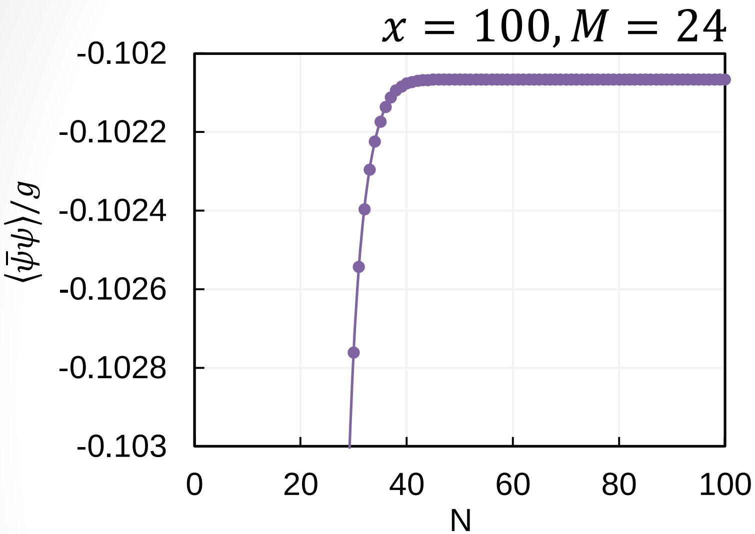
x	SCE+ED	MPS [5]	Difference
20	-0.189878819389204	-0.19025255847009401	0.00037
25	-0.187519020840406	-0.18796879340592226	0.00045
30	-0.185829589660617	-0.18620821935803569	0.00038
cont.	-0.16(1)	-0.159930(8)	

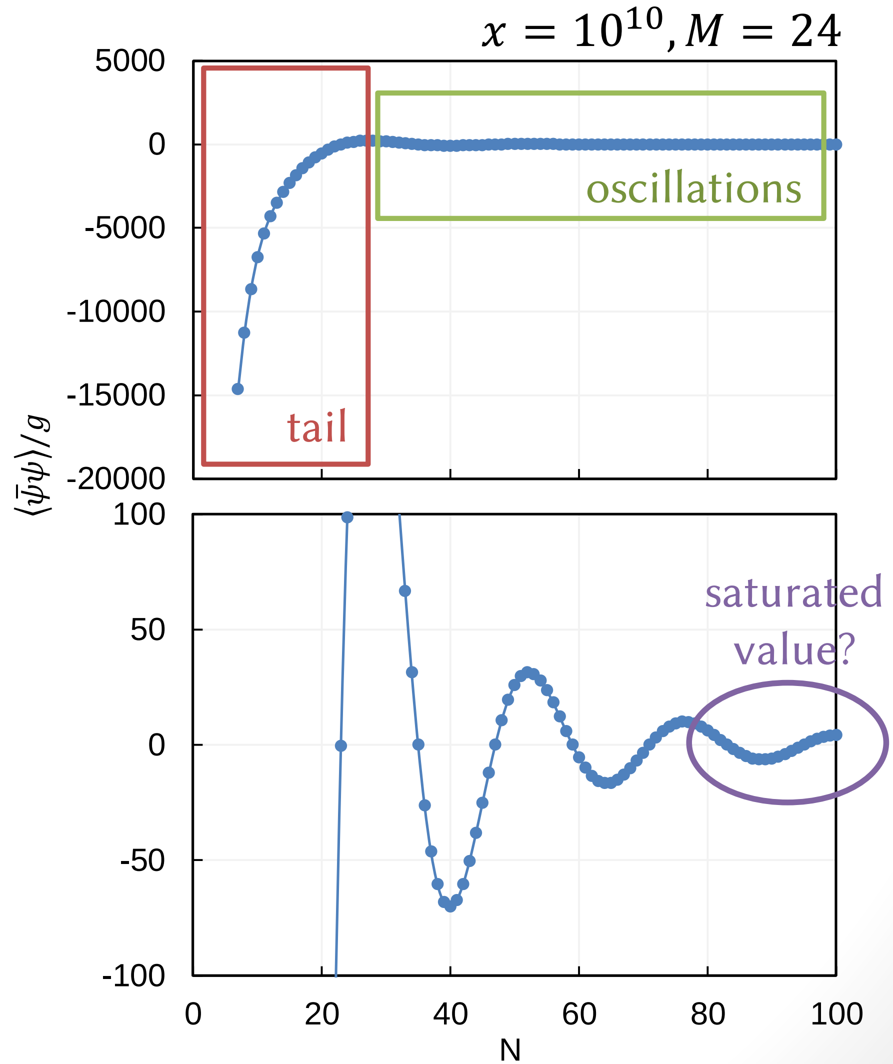
- C.c. - massive $m = 0.125$ (this is after subtracting log divergence [6]):

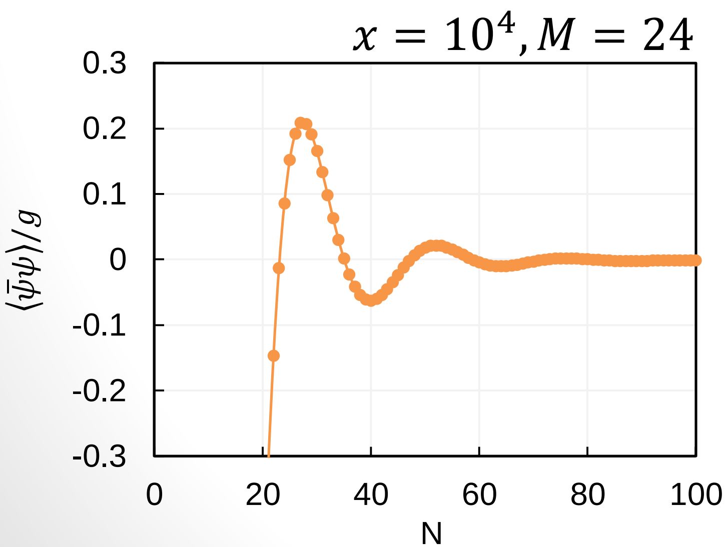
x	SCE+ED	MPS [5]
cont.	-0.091(5)	-0.092023(4)

[4] Bañuls, et al., JHEP 11 (2013) 158..
[5] Bañuls, et al., PoS(Lattice2013)332.
[6] de Forcrand, Nucl.Phys.Proc.Suppl. 63 (1998) 679-681.

Oscillations of chiral condensate while changing the SCE order N







Oscillations: fitting ansatz

- We have chosen the following fitting function:

$$\Sigma(N) = \Sigma(N \to \infty) + a\left(\frac{b}{N^3} + e^{-\alpha N}\right)\sin\left(\frac{2\pi}{T}N + \varphi\right)$$

for huge x

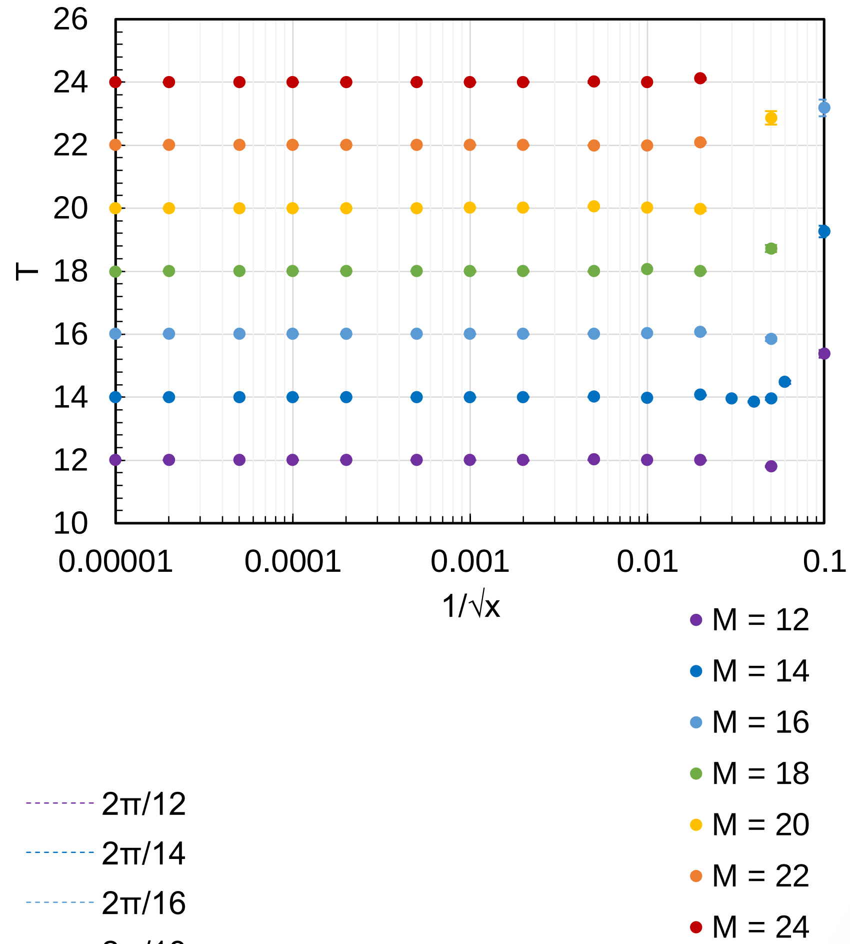
for small x

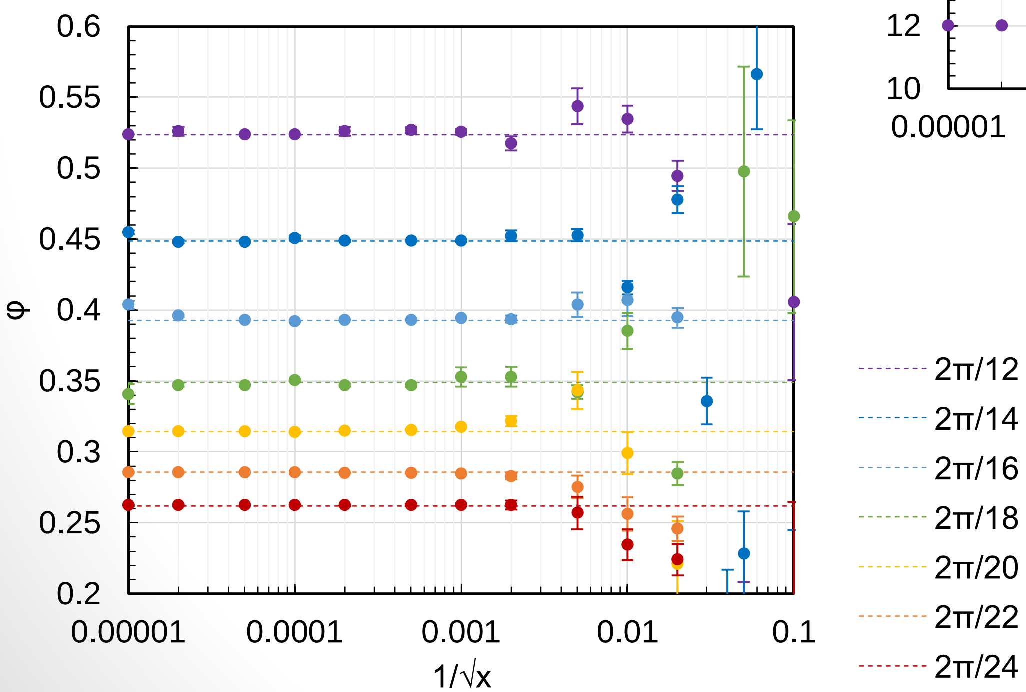
- If we can guess the fitting ansatz correctly, we can use small number of points to approximate saturated values, $\Sigma(N \to \infty)$.

Oscillations: fitting parameters

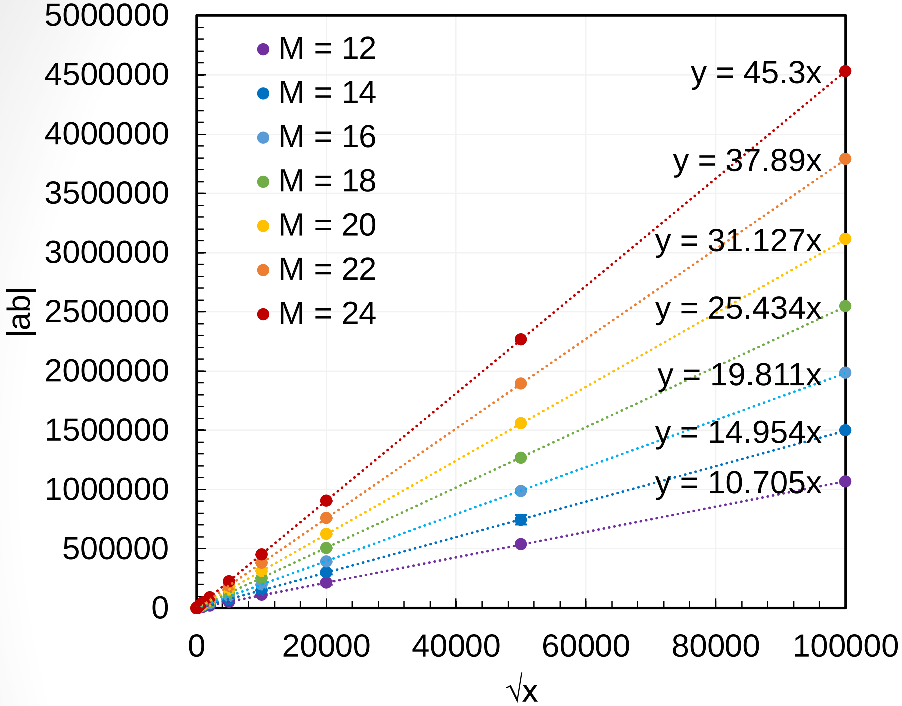
Phase and period:

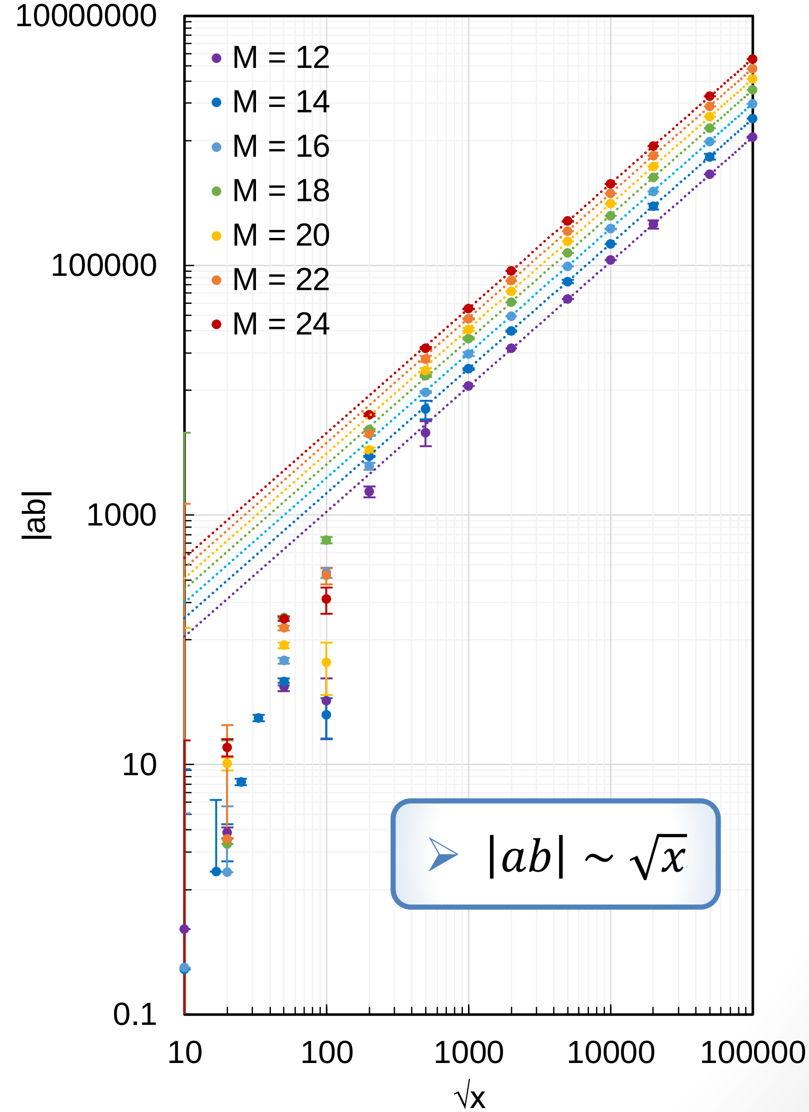
- $T = M$
- $\phi = \frac{2\pi}{M}$



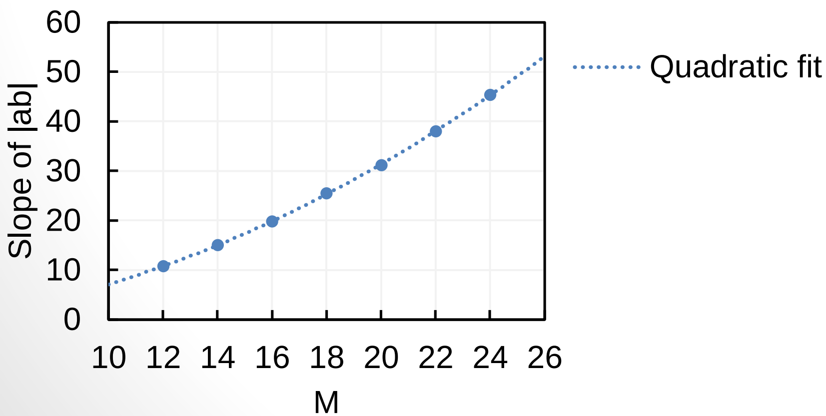


Fitting function: $\Sigma(N) = \Sigma(N \to \infty) + a\left(\frac{b}{N^3} + e^{-\alpha N}\right)\sin\left(\frac{2\pi}{T}N + \varphi\right)$



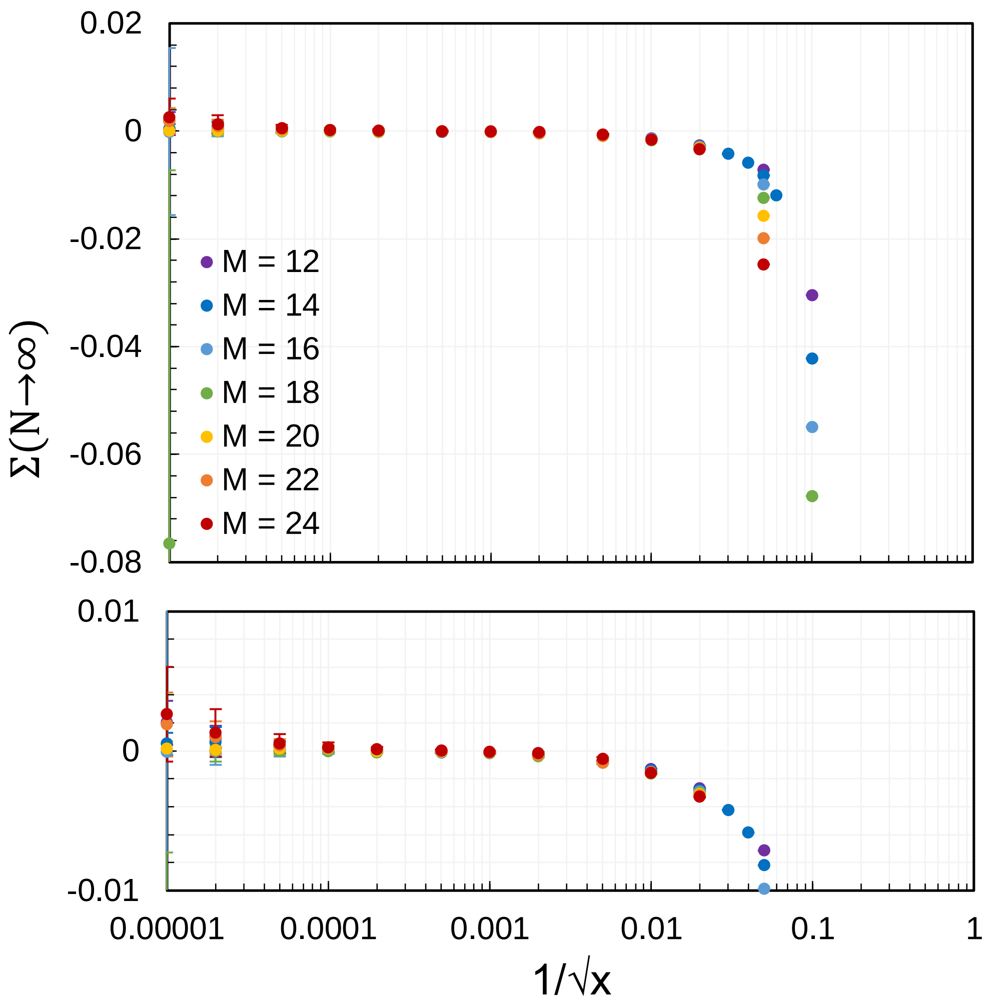


$\blacktriangleright\ |ab| \sim \sqrt{x}$



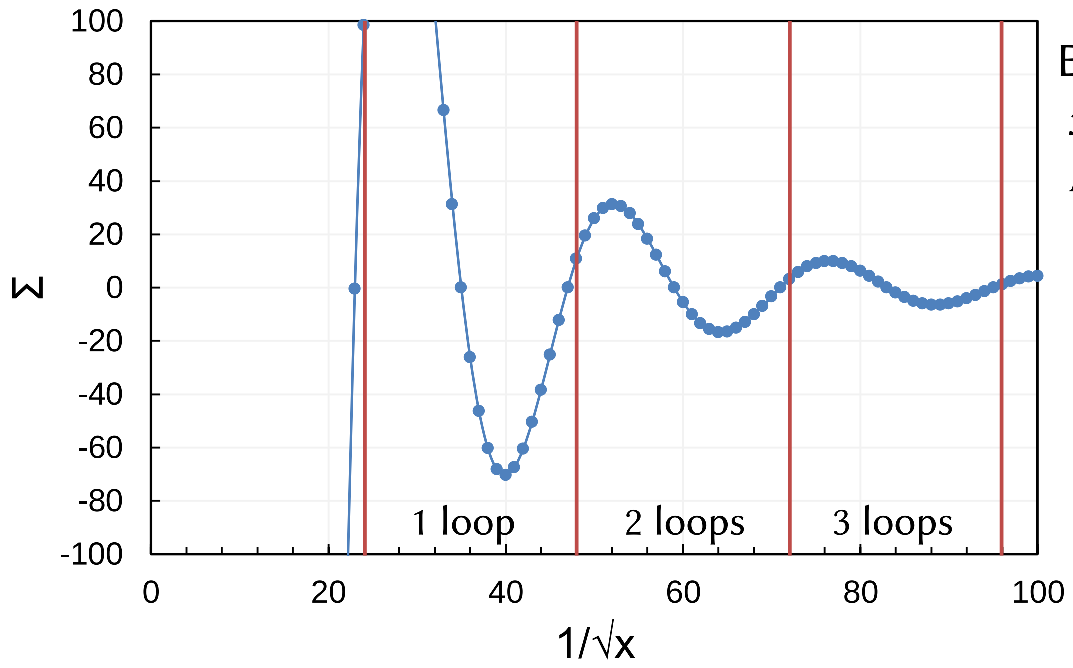
Fitting function: $\Sigma(N) = \Sigma(N \to \infty) + a\left(\frac{b}{N^3} + e^{-\alpha N}\right)\sin\left(\frac{2\pi}{T}N + \varphi\right)$

- High x: errors due to huge oscillations
- $\Sigma(N \to \infty)$ seems to go to zero – because of huge finite volume effects.



Oscillations of the chiral condensate and flux loops

- Every time $N = k\,M$, we reach the next flux loop in the system
- Period must reflect presence of the flux loops



Example:
$x = 10^{10}$
$M = 24$

- Final fitting ansatz:

$$\Sigma(N, M, x) = \Sigma(N \to \infty, M, x) + \left(A(M)\frac{\sqrt{x}}{N^3} + B(M, x)e^{-\alpha(M,x)N} \right) \sin\frac{2\pi}{M}(N + 1)$$

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Summary and outlook

Lattice Hamiltonian results for massless Schwinger model:

- Almost machine precision for GS energy and mass gaps
- Chiral condensate, but we have a problem with FVE

Future:

How to deal with finite volume effects in the chiral condensate?

Fitting functions?

Chiral condensate oscillations – further investigation

Fitting function?

Damped harmonic oscillator for small x?

Thanks for listening!

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