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Abstract. Classical linear and nonlinear control techniques applied to piecewise-smooth (PWS) systems can be ineffective if an additional bifurcation analysis is not made. Due to the presence of discontinuities, PWS systems present a wide variety of standard and non-standard bifurcations. Safe ranges of system and controller parameters can be established lest these bifurcations appear. This is studied by means of a simplified torsional model of an oilwell drillstring of three degrees of freedom (DOF). A PID-type controller is applied. The bifurcation analysis of the open-loop and the closed-loop systems is used to choose the controller parameters for which non-desired bit sticking situations are avoided. The control goal of driving the rotary velocities to a constant positive value is achieved despite the existence of a sliding motion.

Keywords: Discontinuous systems, sliding motions, bifurcation analysis, oilwell drillstrings, dry friction, stick-slip oscillations.

1 Introduction

Hybrid systems are dynamical systems consisting of continuous-time and discrete-event dynamics. They are characterized by discontinuous changes in system properties. PWS systems are an interesting subclass of hybrid systems. An example of PWS system is the model of a conventional vertical oilwell drillstring.

Oilwell drillstrings are systems exhibiting a wide variety of complex phenomena and non-desired oscillations. Stick-slip friction-induced oscillations and the permanent stuck-bit situation are particulary harmful [1]. Several approaches have appeared in order to model and control drillstring stick-slip oscillations. Most of them use lumped-parameter models of one DOF and two DOF (see [2] and references therein). In addition, in most of these works, no bifurcation

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analysis of system and controller parameters is made, and the influence of the weight on the bit (WOB) (a key drilling parameter) is not analysed.

Recently in [2], an analysis of bifurcations and transitions between several bit dynamics is reported for an n-DOF drillstring. Changes in drillstring dynamics are studied through variations in three parameters: 1) the WOB, 2) the rotary speed at the top-rotary system, 3) the torque given by the surface motor.

This paper follows [2] and sums up new results. On the one hand, the bit sticking problems are related to the existence of a sliding motion when the bit velocity is zero. On the other hand, the presence of multiple Hopf bifurcations in the vicinity of the standard equilibrium point when velocities are greater than zero is studied. A PID-type control which overcomes the existing sliding motion and drives the rotary velocities to a desired value is proposed. Controller parameters are chosen lest non-desired system transitions appear. These non-desired situations are permanent stuck bit, bit stick-slip oscillations and other periodic motions. A lumped-parameter torsional PWS model with three DOF including the bit-rock interaction is considered. This model is more general than the torsional lumped-parameter models of one and two DOF previously proposed.

2 A Torsional Discontinuous Model of the Drillstring

Three main parts of a conventional vertical oilwell drillstring are considered: 1) the rotating mechanism in the surface (inertia J_r), 2) the set of drill pipes which form a long pipeline (inertia J_p), 3) the bottom-hole assembly (BHA), which consists of the drill collars, the stabilizers, a heavy-weight drill pipe and the bit (inertia J_b). Hereinafter, the BHA will be also referred to as bit. A simplified torsional model is proposed in which the inertias are connected to each other by linear springs with torsional stiffness (k_t, k_tb) and torsional damping (c_t, c_tb).

The following system state vector is defined:

$$\mathbf{x} = (\dot{\varphi}_\mathrm{r},\, \varphi_\mathrm{r} - \varphi_\mathrm{p},\, \dot{\varphi}_\mathrm{p},\, \varphi_\mathrm{p} - \varphi_\mathrm{b},\, \dot{\varphi}_\mathrm{b})^\mathrm{T} = (x_1,\, x_2,\, x_3,\, x_4,\, x_5)^\mathrm{T}, \tag{1}$$

with φ_i, $\dot{\varphi}_\mathrm{i}$ ($i \in \{r, p, b\}$) the angular displacements and angular velocities of drillstring elements, respectively. At the top-drive system, a viscous damping torque is considered ($T_{\mathrm{a}_\mathrm{r}} = c_\mathrm{r} x_1$). $T_\mathrm{m} = u$ is the torque coming from the electrical motor at the surface, with u the control input. $T_\mathrm{b}(x_5) = T_{\mathrm{a}_\mathrm{b}}(x_5) + T_{\mathrm{f}_\mathrm{b}}(x_5)$ is the torque on the bit with $T_{\mathrm{a}_\mathrm{b}} = c_\mathrm{b} x_5$ approximating the influence of the mud drilling on the bit behaviour. $T_{\mathrm{f}_\mathrm{b}}(x_5)$ is the friction modelling the bit-rock contact, and

$$T_{\mathrm{f}_\mathrm{b}}(x_5) = W_\mathrm{ob} R_\mathrm{b} \left[\mu_{\mathrm{c}_\mathrm{b}} + (\mu_{\mathrm{s}_\mathrm{b}} - \mu_{\mathrm{c}_\mathrm{b}}) \exp^{-\frac{\gamma_\mathrm{b}}{v_\mathrm{f}}|x_5|}\right] \mathrm{sgn}(x_5), \tag{2}$$

with $W_\mathrm{ob} > 0$ the WOB, $R_\mathrm{b} > 0$ the bit radius; $\mu_{\mathrm{s}_\mathrm{b}}$, $\mu_{\mathrm{c}_\mathrm{b}} \in (0, 1)$ the static and Coulomb friction coefficients associated with J_b, $0 < \gamma_\mathrm{b} < 1$ and $v_\mathrm{f} > 0$. Finally, the drillstring behaviour is described by the following equations:

$$\dot{x}_1 = \frac{1}{J_\mathrm{r}}\left[-(c_\mathrm{t} + c_\mathrm{r})x_1 - k_\mathrm{t} x_2 + c_\mathrm{t} x_3 + u\right], \qquad \dot{x}_2 = x_1 - x_3,$$

$$\dot{x}_3 = \frac{1}{J_\mathrm{p}}\left[c_\mathrm{t} x_1 + k_\mathrm{t} x_2 - (c_\mathrm{t} + c_\mathrm{tb}) x_3 - k_\mathrm{tb}\, x_4 + c_\mathrm{tb}\, x_5\right], \quad \dot{x}_4 = x_3 - x_5, \quad (3)$$
$$\dot{x}_5 = \frac{1}{J_\mathrm{b}}\left[c_\mathrm{tb}\, x_3 + k_\mathrm{tb}\, x_4 - (c_\mathrm{tb} + c_\mathrm{b}) x_5 - T_{\mathrm{f_b}}(x_5)\right],$$

or in a compact form: $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u + \mathbf{T}_\mathrm{f}(\mathbf{x}(t))$, where $\mathbf{A}$, $\mathbf{B}$ are constant matrices depending on system parameters and $\mathbf{T_f}$ is the torque on the bit.

3 Transitions and Bifurcations in the Open-Loop System

Two key dynamical characteristics of system (3) determine its behaviour: 1) the existence of a sliding regime which implies the existence of a sliding surface locally attractive; 2) the existence of a unique standard equilibrium point when velocities are greater than zero, which is locally asymptotically stable depending on the values of W_ob, the torque u and the rotary velocity at the equilibrium.

The existence of a sliding motion (see [3]) in system (3) is directly related to different bit sticking phenomena, mainly, stick-slip motion and the permanent stuck-bit situation. System (3) is a PWS system of the form,

$$\dot{\mathbf{x}} = \begin{cases} \mathbf{f}^+(\mathbf{x}, W_\mathrm{ob}, u) = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{T}_\mathrm{f}(\mathbf{x})|_{T_{\mathrm{f_b}} = T_{\mathrm{f_b}}^+} & \text{if } x_5 > 0, \\ \mathbf{f}^-(\mathbf{x}, W_\mathrm{ob}, u) = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{T}_\mathrm{f}(\mathbf{x})|_{T_{\mathrm{f_b}} = T_{\mathrm{f_b}}^-} & \text{if } x_5 < 0, \end{cases} \quad (4)$$

where $T_{\mathrm{f_b}}^+$ and $T_{\mathrm{f_b}}^-$ are $T_{\mathrm{f_b}}(x_5)$ for $x_5 > 0$ and $x_5 < 0$, respectively. Let $\Sigma := \{\mathbf{x} \in \mathbb{R}^5 : x_5 = 0\}$ be the switching manifold, $\tilde{\Sigma} \subset \Sigma$ the sliding region with $\tilde{\Sigma} = \{\mathbf{x} \in \Sigma : |k_\mathrm{tb}\, x_4 + c_\mathrm{tb}\, x_3| < W_\mathrm{ob} R_\mathrm{b} \mu_{\mathrm{s_b}}\}$ and $\tilde{\mathbf{x}}$ the quasiequilibrium point existing on Σ, which can be shown to be asymptotically stable. If $x_5 > 0$ then the system is described by $\dot{\mathbf{x}} = \mathbf{f}^+(\mathbf{x}, W_\mathrm{ob}, u)$ and has a unique standard equilibrium point $\overline{\mathbf{x}}$ such that $\mathbf{f}^+(\overline{\mathbf{x}}, W_\mathrm{ob}, u) = 0$, which is the solution of the set of equations:

$$\overline{x}_1 = \overline{x}_3 = \overline{x}_5 > 0,\ u - (c_\mathrm{r} + c_\mathrm{b})\overline{x}_5 - T_{\mathrm{f_b}}^+(\overline{x}_5) = 0,\ \overline{x}_2 = \frac{h(\overline{x}_5, u)}{k_\mathrm{t}},\ \overline{x}_4 = \frac{h(\overline{x}_5, u)}{k_\mathrm{tb}},$$

with $h(\overline{x}_5, u) = \left[c_\mathrm{r} T_{\mathrm{f_b}}^+(\overline{x}_5) + c_\mathrm{b} u\right]/(c_\mathrm{r} + c_\mathrm{b})$ and $u > W_\mathrm{ob} R_\mathrm{b} \mu_{\mathrm{s_b}} > 0$.

Three main kinds of bit dynamical behaviours can be identified: 1) stick-slip at x_5, that is, the trajectory enters and leaves repeatedly the sliding mode; 2) permanent stuck bit, i.e., $\mathbf{x}(t) \in \tilde{\Sigma}$, $\forall t$; 3) the trajectory converges to the standard equilibrium $\overline{\mathbf{x}}$. The bit moves with a constant positive velocity (the third situation) when $\tilde{\mathbf{x}}$ is far away enough from the boundaries of $\tilde{\Sigma}$, which is accomplished when u is greater enough than $W_\mathrm{ob} R_\mathrm{b} \mu_{\mathrm{s_b}}$.

The local stability of $\overline{\mathbf{x}}$ must be also assured. Periodic orbits around it due to the presence of Hopf bifurcations (HB) can be avoided. Ranges of the WOB and velocities at equilibrium for which a HB may appear can be identified, they intersect the values for which stick-slip oscillations are present. The stability region of $\overline{\mathbf{x}}$ corresponds to low W_{ob} and high enough values of the rotary velocities.

4 Linear Control to Eliminate Non-desired Transitions

The control goal is to eliminate bit sticking problems and establish a stable standard equilibrium point in which the angular velocities are all equal to a desired positive velocity. This is accomplished by means of the following control:

$$u = K_1\, x_6 + K_2(\varOmega - x_1) + K_3(x_1 - x_5) + u^*, \tag{5}$$

with $x_6 = \int_0^t [\varOmega - x_1(\tau)]\, d\tau$, $\dot{x}_6 = \varOmega - x_1$, $\varOmega > 0$ the desired rotary velocity, K_i positive constants and $u^* = W_\mathrm{ob} R_\mathrm{b} \mu_{\mathrm{s_b}} > 0$ the minimum value of u for the system trajectory to cross the boundary of $\tilde{\varSigma}$. This value of u^* prevents the bit from sticking when control (5) is initially switched on. The closed-loop system has a unique standard equilibrium point $\overline{\mathbf{x}}_\mathbf{c}$ with the form,

$$\overline{x}_{\mathrm{c},1} = \overline{x}_{\mathrm{c},3} = \overline{x}_{\mathrm{c},5} = \varOmega,\ \overline{x}_{\mathrm{c},2} = \frac{h(\varOmega)}{k_\mathrm{t}},\ \overline{x}_{\mathrm{c},4} = \frac{h(\varOmega)}{k_\mathrm{tb}},\ \overline{x}_{\mathrm{c},6} = \frac{1}{K_1}\left[c_\mathrm{r}\varOmega + h(\varOmega) - u^*\right],$$

where $h(\varOmega) = \left[c_\mathrm{b}\varOmega + T^+_{\mathrm{f_b}}(\varOmega)\right]$. The dynamical changes introduced by control (5) in the open-loop system (3) are mainly: 1) the standard equilibrium point has the angular velocities equal to $\varOmega$; 2) the sliding motion is maintained, however, there is no quasiequilibrium point on the switching manifold, and the permanent stuck-bit situtation is eliminated; 3) periodic orbits (including stick-slip oscillations) may still arise in the system.

Stick-slip oscillations appear in the closed-loop system due to the existence of a sliding region $\tilde{\varSigma}$ locally attractive and a standard equilibrium which can become unstable or whose domain of attraction can be reduced due to the variation of the controller parameters, $\varOmega$ and W_ob. The analysis of the zero crossings of the real parts of the pairs of the complex eigenvalues of the Jacobian at $\overline{\mathbf{x}}_\mathrm{c}$ of the closed-loop system (referred to as ZC) is a good approach for studying the changing stability properties of $\overline{\mathbf{x}}_\mathrm{c}$. The ZC points might imply a Hopf bifurcation and be the origin of periodic orbits. The extension of the parameters region where no ZC point is present, is indicative of the domain of attraction of $\overline{\mathbf{x}}_\mathrm{c}$. When the domain of attraction of $\overline{\mathbf{x}}_\mathrm{c}$ increases, stick-slip motion may disappear.

Extensive simulations of the closed-loop system lead to establish safe ranges of controller parameters K_i in order to avoid bit sticking problems and the unstability of the equilibrium $\overline{\mathbf{x}}_\mathrm{c}$. The main conclusion is that for typical operation values of $\varOmega$ (between 10 and 14 rad/s), stick-slip oscillations disappear in the closed-loop system and trajectories converge to $\overline{\mathbf{x}}_\mathrm{c}$ with low enough values of K_3 ($K_3 \leqq K_3^\mathrm{a}$) despite variations in W_ob. For $K_3 > K_3^\mathrm{a}$, stick-slip oscillations will disappear if $K_2 > K_3$ and $K_2 - K_3$ is high enough. In this case, the higher K_2 is, the bigger the $(\varOmega, W_\mathrm{ob})$-region without ZC points is. K_1 does not imply significant changes in the curves $(\varOmega, W_\mathrm{ob})$ corresponding to ZC points which delimit safe parameters regions. The value of K_1 influences the transient system response: the higher K_1 is, the higher the overshooting in the velocities is.

Other non-standard bifurcations typical in PWS systems with sliding motion should be studied. The analysis shown could be successfully applied to other discontinuous mechanical systems exhibiting stick-slip oscillations and dry friction.

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