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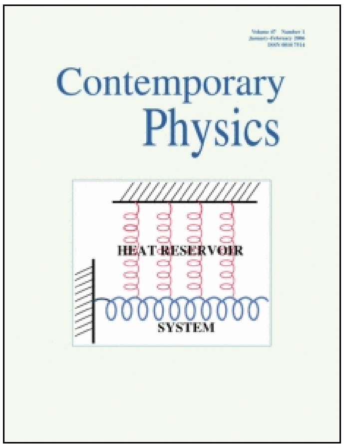

**Contemporary Physics**

Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713394025

# Quantum many-particle systems

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Online Publication Date: 01 March 1990

**To cite this Article** Bishop, R. F.(1990)'Quantum many-particle systems',Contemporary Physics,31:2,131 — 134
**To link to this Article: DOI:** 10.1080/00107519008222009
**URL:** http://dx.doi.org/10.1080/00107519008222009

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*Contemporary Physics*, 1990, volume 31, number 2, pages 131–134

# Essay review
# Quantum many-particle systems

R. F. BISHOP

A review of *Quantum Many-Particle Systems.* By JOHN W. NEGELE and HENRI ORLAND. (Addison Wesley, 1988.) [Pp. xviii + 459] £38·65 hbk. ISBN 0 201 12593 5. Scope: Textbook. Level: Postgraduate.

The subject of many-body physics or quantum many-body theory is *not* the rather specialised subfield of physics that it has often in the past been believed to be. On the contrary, at least at the most fundamental or microscopic level in each energy range that is typically used to characterise a particular branch of physics, we are nearly always faced with a system involving either many particles or many degrees of freedom. Furthermore, the underlying reason for this is so deeply ingrained in our present theoretical understanding of physical phenomena, that, with tongue only lightly in cheek, one might provocatively insist that the quantum many-body problem is almost synonymous with that problem that we otherwise call fundamental theoretical physics!

On the one hand there are fields like nuclear, atomic, molecular, and solid state physics where the fundamentally many-particle aspects of the subject and of its basic objects of interest, are immediately apparent. Thus, it is clear that atomic nuclei, atoms, molecules, solids and fluids are all interacting many-body systems. On the other hand, there are other fields such as elementary particle physics where it is perhaps less immediately obvious that one is seldom dealing with single or even few particles, rather than with intrinsically many-particle systems. For example, we know that at some deeper level of understanding even a 'fundamental' particle like a neutron or proton may be viewed as a cloud of mesons surrounding and compressing a smaller bag at its core in which the quarks are confined, or as three quarks interacting via gluons. Thus, our single nucleon has rapidly become a multi-particle problem. However, at least technically, and much more deeply and subtly too, it is indeed a truly infinite many-body problem since the underlying quantum field theory, in this case quantum chromodynamics, intrinsically allows the possibility of the virtual excitation of many particles out of the vacuum. In this sense even the vacuum, the ultimate zero-body problem, becomes endowed with an enormously complex structure due to this mechanism. Indeed it is probably no great exaggeration to say that in terms of quantum field theory, the structure of the physical vacuum is perhaps the most important many-body problem of all in this field of high-energy particle physics.

From this last example it is clear that quantum field theory may itself be viewed as a branch of quantum many-body theory. It is more usual however to draw the lines of demarcation between the two subjects by restricting the quantum many-body problem to systems which obey the non-relativistic (Schrödinger) quantum mechanics. Nevertheless, the overlaps between the two fields are strong. In particular, many of the original techniques of quantum many-body theory were adapted from their counterparts in relativistic quantum field theory. One may especially mention in this context such important tools as relativistic perturbation theory and the associated machinery of Feynman diagrams, which were themselves developed first in the arena of quantum electrodynamics.

Although the early work of Dirac and Heisenberg on the symmetry properties of the wavefunction of a system of many identical particles may be regarded as laying the original foundations of quantum many-body theory very soon after the birth of quantum mechanics itself, the modern era of the subject began about thirty years ago in the aftermath of the successful formulation of quantum electrodynamics by Feynman, Schwinger, Tomonaga, Dyson and others. The fruits of this first great blossoming of quantum many-body theory as a subject in its own right have been collected and well described in several excellent textbooks that had appeared by the early 1970s. A well known example from this period is the highly respected book of Fetter and Walecka (1971).

More recently, the subject of quantum many-body theory has undergone a second period of rapid expansion, during approximately the last decade or so. It has

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0010-7514/90 $3.00 © 1990 Taylor & Francis Ltd

also become more and more apparent from a number of disparate fields of application, that the quest for a fully microscopic description of physical systems that contain a large or infinite number of particles or degrees of freedom, and whose static or dynamical behaviour is governed by the laws of quantum mechanics, is one of enormous fundamental and practical importance. Indeed, as we have already seen, such systems and their properties pervade virtually the entire domain of theoretical physics. The time is now therefore particularly ripe for the consolidation and codification of the advances that this most recent period has seen. The present book is one of several recent additions to the textbook literature that hence ought rightly to be measured against the yardstick of the best of the older generation of books, most of which appeared approximately fifteen or more years ago.

Since the appearance of these older standard textbooks, the field of many-body physics has advanced significantly on two parallel fronts. In the first place, this period has witnessed either the advent or the further development and coming to maturity of several very powerful and rather universal methodologies for the fundamental description of many-body systems. These fully microscopic formalisms provide us with both a framework in which to imbed the many-body problem in all its generality, and a means with which to shed light on a large cross-section of its enormous diversity of features. Secondly, both these various general techniques and other more specialised methods that have been tailored to specific needs, have been more and more widely applied to the large variety of systems that exhibit many-body phenomena. Such phenomena range from the superfluidity of liquid helium to the fission of certain atomic nuclei; from the quantum Hall effects occurring in a two-dimensional electron gas with a strong magnetic field imposed in the perpendicular direction, to muon-catalysed fusion; from the high-temperature superconductivity of the recently discovered copper-oxide class of ceramic materials, to the confinement problem in quantum chromodynamics.

It is typical of most such observed interesting many-body behaviour that it cannot be simply and immediately understood in terms of a knowledge of the constituent particles and the interactions among them, even though we believe that all such properties of the system must ultimately be derivable from this information alone. Instead, the concepts and techniques of many-body theory must be brought to bear, starting from just such a microscopic (Hamiltonian) description. The many-body systems that have been studied in this context include both rather idealised models for which some exact results are often known, on the one hand, and very realistic models of actual physical systems on the other. The former are of particular use as yardsticks against which to measure, and hence compare, the various theoretical approaches. For the more specialised or phenomenological approaches they can often serve to indicate how accurate the basic starting-point may be for more realistic applications. For the formal microscopic techniques that are in principle exact when fully implemented, the model results are extremely useful in checking the validity, accuracy, and convergence properties of the various approximation schemes that are always necessary to utilise them in practice.

Although the techniques to deal with many-body systems have often arisen separately and independently in a wide range of different physical contexts, they have usually turned out to be of much more general interest. It is not surprising therefore that the modern arsenal of theoretical techniques which has been painstakingly assembled to attack the many-body problem, has had a rather chequered history of development. As we have attempted to indicate, these various theoretical tools and the associated physical insight that has come from them, have by now largely been incorporated into the unified field of study that we call quantum many-body theory. It is this body of common knowledge and techniques that this book aims to present for use as an integral and basic part of the education for students in disciplines as varied as quantum chemistry, atomic and molecular physics, solid state and condensed matter physics, nuclear physics, and particle physics and field theory.

Judged against the standards of its predecessors of 15 and more years ago, one might expect that a modern high-level textbook on many-body theory should have two main aims. The first and most important priority should be to convey the essential ideas of the more up-to-date developments not covered in the earlier texts. Secondly, we should also expect to see a new critique and emphasis placed on that portion of the older material that has retained its central importance as the basis for the later developments, or, alternatively, as either providing significant qualitative insight or quantitative accuracy in its own right. It will clearly also aid an understanding of the subject as a whole to see such of these earlier topics that have stood the test of time explained and illuminated by the use of the more modern and more powerful techniques, and from the vantage that they have provided.

On the basis of these criteria, Negele and Orland achieve a rather mixed pattern of success with this book. Doubtless, this is partly a reflection of the sheer scale of the recent developments and of the corpus of fundamental new techniques and applications. In many ways the present book is a curious mix of both the old and the new, fitting neatly into neither camp. While this could undoubtedly have been a strength of a book this size, and, perhaps, in some small measure may still be adjudged to be so, the total lack of coverage of the larger part of the most important new material in the field, is disappointing. However, the book does contain much of value that is difficult to find elsewhere in the textbook literature.

In spirit and in general approach, the present book is perhaps most like the recent work of Blaizot and Ripka (1986). As in this earlier work, Negele and Orland focus much of their presentation through the functional or Feynman path integral approach. There is no doubt that the intuitive appeal and powerfulness of these methods can lead to an economic and rather physical presentation of such important parts of the older material as many-body perturbation theory. It is also the case that such techniques can lend themselves to new approximation schemes or, equivalently, to rearrangements of the perturbation series that are not easily seen by focussing attention solely on the diagrams, as in the older presentations. The authors certainly exploit these advantages rather well. Nevertheless the path integral techniques should be kept in proper perspective, and it should be admitted that while they have undisputed pedagogical usefulness, their practical applicability as an analytic tool has been rather restricted. By concentrating so single-mindedly on this approach, the authors have denied themselves the possibility of discussing the numerous other alternative techniques that are also essentially non-perturbative, and which have, in numerous numerical applications to a wide variety of model and physical problems, been of much greater practical versatility and usefulness. Furthermore, by discussing only very few real applications of the techniques that are presented, the danger is exacerbated that the reader will acquire a very distorted and restricted knowledge of, and feel for, both the modern techniques of many-body theory, and their enormous range of successful applications to a diverse array of physical systems.

Among the most fundamental and more universal tools used nowadays in many-body theory, one numbers time-independent perturbation theory, Green function techniques, the configuration-interaction method, the coupled cluster method, variational techniques, and the method of correlated basis functions. Each of these techniques employs essentially analytic methods, and is based on a completely microscopic starting-point which is usually the many-body Hamiltonian. In each case a great deal of effort has been expended on the investigation of possible hierarchies of approximations, with the particular aim of trying to formulate them in such a way that the results at each order are guaranteed to improve systematically upon those obtained in the preceding order. It is clearly only within the confines of such a formalism that one can be said to possess a rigorous theoretical microscopic understanding of the system. Among the most important of the more heuristic or less theoretically fundamental treatments in the above sense, are the familiar Landau theory of phase transitions and the more recent polarisation potential methods of Pines and co-workers. Finally, the very important class of Monte Carlo or other similar stochastic simulation techniques has grown in importance in recent years with the advent of more and more powerful computers.

Of the fully microscopic analytical techniques, only the older perturbative and associated Green function methods are discussed by Negele and Orland. Even here, although their coverage of these topics is presented from a rather more modern point of view in comparison with the older textbooks, it is surprising to find much of the more recent interesting material omitted. For example, after the extensive coverage of both Green functions and the perturbative formalism in general, and the effective potential and irreducible diagrams in particular, it would have been natural to include a discussion of the parquet approach to many-body theory. This approach focuses very much on the effective interaction, and formulates it in terms of an integral equation, which thereby sums in a self-consistent fashion a very large and physically interesting set of Feynman diagrams. It is a method that derives the full two-body vertex function from some set of irreducible terms. As such, it illustrates in a concrete and physically appealing way much of the rather abstract material covered in this book, which is never explicitly applied therein to any actual many-body system. Since the parquet approach has also achieved some notable numerical success, especially for such bosonic systems as liquid helium-four, for which its explicit formulation is especially simple, an opportunity is lost to develop in the reader an intuitive feel for how the formalism can be utilised in practice. In a similar vein, the entire perturbative approach is based around the normal, non-interacting, ground state as the model unperturbed wavefunction. The possibility of generalisation is not discussed. Superfluid or similar 'abnormal' systems receive similar scant treatment. The reader is thereby denied the opportunity to consider applying the formal techniques of perturbation theory about the alternative well known Bardeen–Cooper–Schrieffer (BCS) state, for example.

The remaining analytic, but non-perturbative, methods are not even hinted at by Negele and Orland, despite the fact that, for example, the coupled cluster method, and variational techniques augmented by the method of correlated basis functions, undoubtedly represent in practice the two most powerful formulations of the general quantum many-body problem that are currently available. The extremely impressive and accurate array of their applications to such diverse systems of physical interest as complex atoms and molecules, the electron gas, finite atomic nuclei and infinite nuclear matter, and liquid helium, are likewise totally neglected in this book.

These omissions would not in themselves necessarily be important. They could perhaps be excused on considerations of space alone. Nevertheless the present treatment is so imbalanced that the impression that is left by this book is that, with the exception of the computationally-intensive stochastic techniques, the remaining methods of quantum many-body theory to calculate physical observables still basically reduce in practice to perturbative techniques for summing bigger and bigger classes of diagrams. From the discussion included, the further inescapable conclusion seems to be that either the convergence properties of these methods are essentially unknown or the necessary approximations are uncontrollable. While such a pessimistic statement might not have been entirely out of place in the older generation of textbooks, it is clearly not valid now. It would be particularly regrettable if the reader were to come away with quite the wrong understanding of the present status of many-body theory in both its formal developments and its applications. Since this book so obviously invites comparison with the earlier texts, and since it is actually advertised in the preface as including by comparison a number of the recent developments, there is a manifest danger of this occurring.

Despite these shortcomings there is still much of interest in the book. Worthy of special mention is the coverage of stochastic methods. This material is not easily found elsewhere. Indeed the description here is so good that one particularly regrets that similar compact coverage of the omitted alternative analytic techniques could not also have been included. Although the lack of applications has already been mentioned, a counterbalancing very positive feature is the use of the simple Lieb model of a one-dimensional many-particle system interacting via pairwise forces described by a delta-function potential, as a leitmotif throughout the book with which to illustrate the various techniques. The problems at the end of each chapter are also especially well chosen.

In conclusion, although the book contains much to commend it, it is probably best used as a teaching device either in conjunction with such more balanced and more carefully constructed modern textbooks as that of Blaizot and Ripka (1986), or to shed further light from a more modern viewpoint on the traditional material covered by such older texts as that of Fetter and Walecka (1971). Although this work does contain some good illustrative material, these nuggets are best sampled judiciously as supplementary material. Taken in its entirety the book is not sufficiently representative of the modern field of quantum many-body theory to be able to recommend it wholeheartedly.

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