Reviews, prices and endogenous information transmission

Reviews, Prices and Endogenous Information Transmission

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Very Preliminary and Incomplete. Comments Welcome

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Abstract

Empirical evidence suggests that online reviews are an important source of consumers information and a relevant determinant of the firms revenues. Little is known, however, about how prices and reviews affect each other. This paper proposes a dynamic game to investigate this relationship. A long-lived monopoly faces a sequence of short-lived consumers whose only information about the value of an experience good is the one contained in the reviews completed by previous buyers. Neither the monopoly nor the consumers have private information about the value of the good. After buying the good, the consumers observe a quality realisation that is correlated with the actual value of the good and decide whether to complete reviews. The consumers complete reviews according to a social rule that maximises the present value of current and future consumers utility. It is shown that a necessary condition for the existence of reviews is that the firm cannot fully appropriate the surplus generated by this increased information. Furthermore, the reviews induce a mean preserving spread on the posterior beliefs about the value of the good which, combined with the convexity with respect to the prior of the indirect utility and profit functions, implies that reviews are valuable for both the consumers and the firm. Hence, both parties are willing to face some cost in order to increase the information available in the market. The main result of the paper is that, from the firm's perspective, this cost takes the form of a discount in the price offered to current consumers.

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1 Introduction

Online reviews of products, services or business are an increasingly important source of consumers information about experience goods, i.e., goods whose quality is learned only after consumption. Recent empirical evidence suggests that reviews are also becoming a more relevant determinant of the firm's revenues, either because of their impact in the quantity demanded or because consumers are willing to pay a sort of "reputation premium" for products or services that have good reviews.[1]

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[1] See, for example Luca (2011), Chevalier and Mayzlin (2006). As shown by Doyle and Waterson (2012) and Resnick, Zeckhauser, Swanson, and Lockwood (2006), among others, the effect of reviews on revenues also seems to be present in the case of online auctions.

Closely related, though less documented, is the practice of offering important online discounts and then ask the buyers to complete reviews.[2] This practice seems to be increasingly used by recently established (or recently refurbished) small firms, like restaurants or pubs.[3] It also seems to be an increasingly common practice among restaurants or hotels in touristic areas. A common denominator to both situations is that the firm does not have complete information about the demand function it faces and so it values the information consumers can provide it. At the same time, being experience goods provided by new small firms or firms located far away from the consumer, it is unlikely that the consumer has ex ante information about the value of the firm's product or service.

Despite the growing importance of customers' reviews, their role in the firm's pricing decisions has not been studied. This paper proposes a dynamic model to investigate how prices and reviews affect each other. It considers a situation in which a long lived firm faces a sequence of short lived consumers whose only information about the value of the product is the one contained in the reviews completed by previous consumers. As in the examples above, it is further assumed that the firm does not know the actual value of the product either. After buying the product, the consumer observes a quality realisation and decides which review to complete (if any).

The results in this paper offer an explanation for those price discounts based on the value of the information contained in the reviews. It is shown that the information generated by the reviews is valuable for both, the consumers and the monopoly. As a result, the consumers and the firm "share" the cost of generating information. It is further shown that consumers are willing to complete reviews only if it is not too costly and the firm cannot appropriate all the surplus generated by the increased information. In this way, the incentives of the firm are "align" to those of the second period consumers.

Before analysing the results in more detail, it is important to note that when the consumer completes a review he is taking a costly action, the benefits of which he cannot (fully) appropriate.[4] Hence, the reviewing decision has some similarities with an agent's decision to contribute to the provision of a public good.[5] The free riding incentives in this context are analogous to the ones that originate the "paradox of no voting". Thus, to tackle this difficulty I borrow from the voting literature and I assume that consumers are *group-utilitarians*, i.e., they receive a positive payoff for acting according to a strategy that maximises consumers' aggregate utility.

The existence of the reviews induces a mean preserving spread on the agents' beliefs about the value of the good. As the posterior beliefs form a martingale with respect to the reviews

[2]Price discounts are offered through a variety of web pages, like groupon.com or vouchercodes.com, for example.

[3]The list of business that resort to this type of practices is considerably long. Apart from restaurants and pubs, it seems to be a common practice among recently established hairdressers, beauty saloons and various entertainment-related firms.

[4]In a way analogous to the situations I study, empirical studies on eBay show that most of the times the customer is not likely to buy again from the same seller, implying that he does not receive a direct benefit from completing a review. Yet, Resnick and Zeckhauser (2002) report that 52.1% of the buyers on eBay actually provide voluntary feedback about their sellers.

[5]Since I make the simplifying assumption that there is a continuum of consumers, the model in this paper suffers from an extreme version of free riding. Therefore, the standard result of suboptimal provision obtained in public good games apply to the games analysed in this thesis in a very extreme way, resulting in no complaints/reviews in equilibrium. See Osborne (2004).

completed by previous consumers, reviews do not affect the expected value of the posterior beliefs, but do increase their variability. Combined with the convexity of the indirect utility and the profit functions, the increased variability of the posterior beliefs results in the information contained in the reviews being valuable for both, the consumers and the firm. Hence, both parties are willing to face some cost in order to increase the information available in the market.

The paper shows that, from the firm's perspective, the cost of the information contained in the reviews takes the form of a "discount" in the price offered to current consumers. It is widely believed that the firm's decision to offer price discounts is due to an intention of "getting a good review". The result in this paper offers an alternative explanation. By reducing the current price, the firm increases current (expected) demand which in turn increases the probability with which the current consumer completes reviews. As this discount has the additional effect of compensating consumers for the cost of completing reviews, it also induces a reviewing rule that is more favourable to the firm (in the sense that it increases the expected future profits in the scenario with reviews).

From the perspective of the consumers, the price discount behaves as a "subsidy" to the reviewing activity and thus it has an effect similar to a reduction in the cost of completing reviews. It is further shown that the reviewing rule chosen by consumers is "softer" the lower is the cost of completing reviews. In this paper, a reviewing rule is softer than another rule if the posterior belief resulting from a bad review is higher and the one following a good review is lower. A softer reviewing rule has a higher positive impact on the firm's future profits because it induces a mean preserving spread relative to a tougher rule. As a result, the firm offers a higher price discount when it is easier for consumers to complete reviews (and thus, when it is more interested in increasing the probability with which the consumers complete reviews).

The paper shows that a necessary condition for the existence of reviews is that the firm cannot extract all the surplus generated by the increased information. Since the behaviour of the consumers and the firm changes according to the observed reviews, the informational content of the reviews has a positive value for both. As a result, the incentives of the firm are aligned with those of the consumers in the sense that both prefer the existence of a reviewing system over a situation with no information transmission. Consumers complete reviews in order to increase the sum of current and future consumers' expected utility. Hence, if the firm could appropriate all the surplus consumers would not complete reviews: completing reviews is a costly activity, then not even utilitarian consumers are willing to complete reviews if by doing so they do not improve the utility of those in their group (the consumers, in this case).

From a formal perspective, the utilitarian assumption implies that the game is strategically equivalent to a two persons game, in which both players are long lived. Therefore, the proposed reviewing game becomes analogous to a situation of a bilateral monopoly, in which the firm is the only potential "buyer" of information and the group of consumers are the only potential "suppliers". The equilibrium results suggest that the cost of completing reviews allocates the surplus created by that information between the firm and the consumers.

This paper is related with the large literature that studies how agents learn from the actions of others. See for example Bikhchandani, Hirshleifer, and Welch (1992), Smith and Sorensen (2000) and Bose, Orosel, Ottaviani, and Vesterlund (2006). In most part of that literature, the transmission of information is an externality: agents' actions carry information about their private signals, and so other agents can learn from those actions. The model in this paper adds to that literature because it endogenises consumers decision to transmit information. Knowing that the previous consumer bought is informative about his preferences over quality, but not about the actual value of the good. However, after observing a realisation of quality, the consumer may decide to complete a review, i.e., the consumer explicitly decides whether to transmit information and which information to transmit (which review to complete).

Kremer, Mansour, and Perry (2012) offers a normative analysis of a situation similar to the one considered here. They show that perfect information sharing through internet does not always support an optimal outcome.

This paper is also related to the literature on strategic information transmission and to the literature on public tests. Similar to the result in Crawford and Sobel (1982), the optimal set of messages in my model is maximum because the preferences of the "sender" and the "receiver" are aligned. However, the reviewing model proposed in this paper differs from the standard model of strategic transmission of information in that there is more than one "receiver", namely the second period consumers and the firm.

The model is also related to the literature on public tests.[6] Gill and Sgroi (2012) study a framework in which a firm can have its product publicly tested before launch and tests vary in their toughness. They show that the firm always prefers to have its product tested and that it will choose a test that is either very tough or very soft. From the firm's perspective, consumers' reviews also constitute a "public test" about its product, and I get a similar result to Gill and Sgroi's (2012) in the sense that the firm always prefer the existence of reviews. However, the characteristics of the test in the model presented in this paper are chosen by the consumers (and only indirectly affected by the firm).[7]

The rest of the paper is organised as follows. The next section describes the model, defines the reviewing rule and discusses its role on the public updating of beliefs. Sections 3 and 4 develop the building blocks for the equilibrium analysis of Section 5. Section 3 studies the optimal reviewing rule would information transmission be free, while Section 4 looks at how those result change when there exists a positive cost of completing reviews. Section 6 concludes. Appendix A contains all the proofs that are not in the text.

2 Basic Setup

A risk-neutral monopolist sells a good of unknown value to a sequence of consumers. The value of the good, v, can be high (H) or low (L), with $H > L \geq 0$. Nature selects v once and forever at the beginning of the game. Neither the firm nor the consumers observe it, but they have a prior belief $\lambda \in (0, 1)$ about the good being high value.

[6]See Gill and Sgroi (2008, 2012) and Lerner and Tirole (2006).

[7]Comparing the results of the reviewing model of this paper and the public test model as regards the toughness of the test is not as clear cut. However, the results suggest that in the model presented here the firm's preferred test is neither the softest possible nor the toughest.

There is a finite sequence of risk neutral buyers, each of which has a (potential) unit demand and lives for one period.[8] Consumers' preferences over quality are random and change every period. Consumer t's valuation of quality is γ_t, where each γ_t is independently distributed $U[0,1]$, and it is independent of any other random variable in the model. At the beginning of the period, the consumer learns his valuation for quality, which is not observed by the firm. The assumption that consumers are short lived implies that an individual consumer cannot learn the value of the good from his personal experience. Furthermore, the assumptions about γ_t together with the fact that the consumer has no private information about the value of the good before buying, imply that his buying decision contains no information about v either. As a result, the only information a consumer has about the value of the product before buying is the one contained in the reviews completed by previous buyers.

The sequence of events within any period t is as follows. Given the history of the game, a new generation of consumers choose the reviewing rule that will be followed if they buy the good. Then, the consumer observes γ_t and p_t and decides whether to buy or not. If he buys, he observes a realisation of quality, q_t, that is correlated with the real value of the good. Finally, the consumer decides whether to complete a review and which review to complete according to the previously chosen rule.

At the beginning of the period, the consumer observes the *reviews* completed by previous buyers, their preferences over quality and the prices offered by the monopolist in previous periods. The consumers in the previous period may have completed a review $i \in \{G, N, B\}$; where G means he completed a "Good" review, N means he did not complete a review ("remain silent") and B that he completed a "Bad" review. The consumer in period t uses this information together with the knowledge of the reviewing rule used by the previous consumers to update his beliefs about the probability of the good being high value (λ_t).[9] Given his beliefs and the price offered by the firm (p_t), the consumer decides whether to buy the good or not. If he buys, the consumer observes a realisation of quality, $q_t \in [q^0, q^K] \in \mathbb{R}_+$. This quality realisation is distributed conditional on the actual value of the good, $q_t \sim F_j(q)$ with $j \in \{L, H\}$ and $\mathbb{E}(q; H) = H$ and $\mathbb{E}(q; L) = L$.[10] It is assumed that no quality realisation is fully revealing of the product's value and that monotone likelihood ratio property holds, so $f_H(q)/f_L(q)$ is increasing in q.

The consumer's payoff from buying is $\gamma_t q_t - p_t$, while his payoff from not buying is zero. Thus, he buys if and only if the expected payoff from buying is positive: $\gamma_t \mathbb{E}_{\lambda_t}(q_t) \geq p_t$.[11] Without loss of generality, we can assume $H - L = 1$, and so $\mathbb{E}(q_t; \lambda_t) = \lambda_t H + (1-\lambda_t)L = L + \lambda_t$. Given λ_t and the price, the consumer buys if $\gamma_t \geq \frac{p_t}{\lambda_t + L}$.

From the firm's perspective, $P(\gamma_t \geq \frac{p_t}{\lambda_t + L})$ plays the role of the demand function: given γ_t, a higher price decreases expected demand, while a higher belief about the good being of high value increases demand. The firm does not observe γ_t but it knows the distribution from which it is drawn. Given consumer's prior belief and the price, expected profits in period t are:

[8] Alternatively, it can be considered that every period there is a continuum of identical consumers normalised to size one, who live during one period.

[9] As reviews are public, consumers' beliefs about the value of the good are "public beliefs".

[10] This assumption simplifies the notation and the algebra, but does not affect the results. All that is needed for the results is that $\mathbb{E}(q; H) > \mathbb{E}(q; L)$, which is implied by increasing monotone likelihood ratio.

[11] The weak inequality implies that if indifferent, the consumer buys.

$$\pi(p_t;\lambda_t)=(p_t-c)P\left(\gamma_t\geq\frac{p_t}{\lambda_t+L}\right)=(p_t-c)\left(\frac{\lambda_t+L-p_t}{\lambda_t+L}\right) \tag{1}$$

where $c<L$ is the constant marginal cost of production.[12]

2.1 Benchmark case: No Reviews

If there are no reviews, the possibility of transmitting information does not exist and so there is no updating of beliefs and $\lambda_t=\lambda\ \forall t$. The firm's optimal pricing strategy consists in offering in every period the price that maximises static profits, i.e., the price that solves:

$$\underset{p}{\text{Max}}\ \ \pi(p;\lambda)=(p-c)\left(\frac{\lambda+L-p}{\lambda+L}\right)$$

which implies:

$$\hat{p}(\lambda)=\frac{\lambda+L+c}{2} \tag{2}$$

Given λ, L and c, the maximum is unique because the profit function is strictly concave with respect to the price.[13] In this case, the firm's maximum expected profits in any period t are: $\hat{\pi}(\lambda)=\pi(\hat{p};\lambda)=\frac{(\lambda+L-c)^2}{4(\lambda+L)}$.

The consumer buys the good if $\gamma_t\mathbb{E}(q_t;\lambda)\geq p_t$. Given λ and $\hat{p}$, the probability that consumer t buys is the probability that $\gamma_t\geq\frac{\hat{p}(\lambda)}{\lambda+L}$, and his expected utility is:

$$\begin{aligned}\hat{u}(\lambda) &= u(\lambda,\hat{p}) \qquad (3)\\ &= P\left(\gamma_t\geq\frac{\hat{p}(\lambda)}{\lambda+L}\right)\left[\mathbb{E}_\gamma\left(\gamma_t|\gamma_t\geq\frac{\hat{p}(\lambda)}{\lambda+L}\right)(\lambda+L)-\hat{p}(\lambda)\right]\\ &= \frac{[\lambda+L-c]^2}{8(\lambda+L)}\end{aligned}$$

2.2 Reviews

The consumers complete reviews in order to maximise the sum of current and future consumers' net (expected) utility. Consumers are utilitarians, and so they are willing to follow the social norm that maximises the group's expected utility, as long as it is not too costly.[14] Consider a rule that determines two thresholds of quality realisations, $\bar{q}(p_t,\lambda_t)$ and $\underline{q}(p_t,\lambda_t)$, such that if period-$t$'s consumer receives a quality draw greater than or equal to $\bar{q}(p_t,\lambda_t)$ he completes a good review, and if he receives $q_t\leq\underline{q}(p_t,\lambda_t)$ he completes a bad review. Finally, if he receives a quality in between the thresholds, he completes no review. Denote by $\mathcal{R}_t(p_t,\lambda_t)=\{\underline{q}_t(p_t,\lambda_t),\bar{q}_t(p_t,\lambda_t)\}$ the reviewing rule followed by consumers in period t. To simplify notation, I use $\underline{q}_t$, $\bar{q}_t$ and $\mathcal{R}_t$ as shorthand notation for $\underline{q}(p_t,\lambda_t)$, $\bar{q}(p_t,\lambda_t)$ and $\mathcal{R}_t(p_t,\lambda_t)$, respectively.

In any period t, the expected utility of a consumer with prior λ_t depends on whether he buys the good or not and, if he buys, on whether he completes a review. It also depends on

[12]This assumption implies that the monopolist is willing to sell for every $\lambda_t\in[0,1]$. If $c\in(L,H)$ the monopolist would prefer to stop selling for some $\lambda_t>0$.

[13] $\frac{\partial^2\pi}{\partial p^2}=-\frac{2}{\lambda+L}<0$.

[14]Feddersen and Sandroni (2006).

how his current decisions affect the utility of future consumers. If the consumer in period t buys the good, his net utility is the utility he derives from the quality he receives net of the price he paid and the cost of completing a review (in case he does complete one). The expected utility of period-t consumer is:

$$u(\mathcal{R}_t;\lambda_t,p_t) = P\left(\gamma_t \geq \frac{p_t}{\lambda_t+L}\right)\left[\mathbb{E}_\gamma\left(\gamma_t|\gamma_t > \frac{p_t}{\lambda_t+L}\right)(\lambda_t+L) - p_t - \Psi(\mathcal{R}_t;h,\lambda_t)\right] + U(\mathcal{R}_t;\lambda_t,p_t) \tag{4}$$

where $(\lambda_t + L)$ is the expected quality given λ_t and $\Psi(h,\mathcal{R}_t,\lambda_t)$ is the expected cost of completing a review.[15] $U(\mathcal{R}_t;\lambda_t,p_t)$ is the expected utility of future consumers when the consumer in period t has beliefs λ_t, follows the reviewing rule $\mathcal{R}_t$ and pays a price p_t:[16]

$$\begin{aligned} U(\mathcal{R}_k;\lambda_k,p_k) &= \sum_{k=t+1}^{T}\left[P\left(\gamma_{k-1} \geq \frac{p_{k-1}}{\lambda_{k-1}+L}\right)\sum_{i\in\{G,B,N\}} P(i;\mathcal{R}_{k-1},\lambda_{k-1})u(\mathcal{R}_k;\lambda_k^i,p_k) + \right. \\ &+ \left. P\left(\gamma_{k-1} < \frac{p_{k-1}}{\lambda_{k-1}+L}\right)u(\mathcal{R}_k;\lambda_{k-1},p_k)\right] \end{aligned} \tag{5}$$

The first term is the expected utility of a period $k > t$ consumer when the previous consumer bought the good and completed review $i \in \{G,N,B\}$ according to the rule $\mathcal{R}_{k-1}$.[17] The second term is the expected utility when the previous consumer did not buy. In this case, there is no updating of beliefs and $\lambda_k = \lambda_{k-1}$.

The existence of the review system induces a sequential game between the firm and the consumers. Neither the firm nor the consumers know the actual value of the product, but the firm chooses its price knowing the reviewing rule consumers are going to follow. Consumers' problem in period t is to choose the reviewing rule $\mathcal{R}_t$ that maximises the sum of current and future consumers' expected utility, given their prior beliefs, their understanding of how future consumers will interpret the reviews and a proper anticipation of the firm's pricing strategy. The firm, on the other hand, behaves as a "Stackelberg follower": given the reviewing rule followed by consumers, it chooses the pricing strategy that maximises the sum of current and future expected profits. Therefore, the price offered by the firm in period t is a best response to the consumers' reviewing rule. As a result, an equilibrium of the reviewing game is defined as a sequence of strategies $\{\mathcal{R}_t^*,p_t^*\}_{t=0}^T$ such that, for every period t, $\mathcal{R}_t^*$ maximises (4) and p_t^* maximises the present value of the firm's profits, given $\mathcal{R}_t^*$.

It is worth noting that at the moment in which the consumers and the firm choose their actions ($\mathcal{R}_t$ and p_t) they have no more information about the actual value of the good than the one that is publicly available. Therefore, neither the reviewing rule nor the price are informative about the probability of the good being high value.

It becomes apparent from expression (4) that the price offered by the firm in period t affects the probability that the current consumer buys the good and, as a consequence, it affects the probability that current consumers transmit information to future consumers (and to the firm itself) through the reviews. The consumers' reviewing rule determines not only what information is transmitted (in the sense of which review is observed by the consumer

[15] This cost function is studied in detail in Section 4, where I look deeply into the effects of the costs of completed reviews on the optimal reviewing rule.

[16] In equilibrium p_t is the firm's best reply to the reviewing rule, so p_t is a shortcut for $p_t(\lambda_t,\mathcal{R}_t)$.

[17] λ_k^i is a shortcut for $\lambda_k^i(\lambda_{k-1};\mathcal{R}_{k-1})$

in period $t+1$), but also which inferences future consumers (and the firm) draw form the observed reviews. Both elements affect future consumers' willingness to pay for the good.

The problems of how much information is transmitted and which information is transmitted induce different tradeoffs for the agents. Therefore, I analyse the two problems separately before solving for the equilibrium strategies of the firm and the consumers. Furthermore, as two periods are enough to present the main results, I'll consider only the case with $T = 2$. To simplify the notation, time subscripts are omitted and λ and λ'_i denote the prior and the posterior beliefs after review i, respectively.[18]

The paper proceeds as follows. The remainder of this section analyses the updating of beliefs after each possible review and the role of the rule in that updating. The next section studies the impact of reviews in the firm's profits and the consumers' utility without taking into account the costs of transmitting information. Section 4 analyses how those results change when the cost of completing reviews is taken into account. Finally, section 5 studies the equilibrium reviewing rule and pricing strategy.

2.3 Updating: Public Beliefs

At the beginning of the second period the consumers (and the firm) use the reviews completed by past consumers to update their beliefs about the good being high value. The reviewing rule divides the space of quality realisations into three intervals, determining which realisations induce which reviews. Therefore, the reading the agents do into the reviews is a function of $\mathcal{R}$.[19]

After observing a review, and given the history up to that point, the consumer and the firm use Bayes' Rule to update their beliefs about the good being high value. This beliefs are "public" in the sense that they are entirely based on public information. As a result, after observing a review i both, the firm and the consumers assign the same probability to $v = H$. When observing a good review, consumers know the realisation of quality received by the previous consumer was higher than or equal to $\bar{q}$. As a result, their updated belief is:

$$\lambda'_G(\lambda; \mathcal{R}) = \frac{\lambda \int_{\bar{q}}^{q^K} f_H(q)dq}{\lambda \int_{\bar{q}}^{q^K} f_H(q)dq + (1-\lambda) \int_{\bar{q}}^{q^K} f_L(q)dq} \tag{6}$$

Analogously, after observing a bad review the consumer knows that the previous consumer received a quality realisation equal to or below the threshold $\underline{q}$; his beliefs after a bad review are:

$$\lambda'_B(\lambda; \mathcal{R}) = \frac{\lambda \int_{q^0}^{\underline{q}} f_H(q)dq}{\lambda \int_{q^0}^{\underline{q}} f_H(q)dq + (1-\lambda) \int_{q^0}^{\underline{q}} f_L(q)dq} \tag{7}$$

Finally, if the first period consumer does not complete a review, it might be because he did not buy the good or because he bought and received a quality realisation within the no reviewing region. In the first case, the consumer in the second period has nothing to learn from the absence of review, so $\lambda' = \lambda$. In the second case, the absence of review is informative about the quality realisation being somewhere "in the middle". The updating of beliefs in

[18] λ'_i is a shortcut for $\lambda'_i(\lambda, \mathcal{R})$.

[19] When the consumer in the second period updates his beliefs about the value of the good, γ_1 and p_1 are already known and so the reviewing rule can be considered as given when analysing the posterior beliefs.

the latter case is:

$$\lambda'_N(\lambda;\mathcal{R}) = \frac{\lambda \int_{\underline{q}}^{\bar{q}} f_H(q)dq}{\lambda \int_{\underline{q}}^{\bar{q}} f_H(q)dq + (1-\lambda)\int_{\underline{q}}^{\bar{q}} f_L(q)dq} \tag{8}$$

A good review increases the probability the agents assign to the good being high value, and so it constitutes "good news" in the sense that $\lambda'_G \in (\lambda, 1)$.[20] A bad review has the opposite effect: as it reflects a low quality realisation, it reduces the agents beliefs; thus, a bad review is "bad news" and $\lambda'_B \in (0, \lambda)$. Finally, when there are no reviews but the previous consumer bought the good, beliefs about the good being high value may increase or decrease depending on the conditional distributions of quality. However, λ'_N is always higher than the beliefs after observing a bad review, because it is an indication of a quality realisation above $\underline{q}$, and it is always smaller than their beliefs after observing a good review. The next two claims summarise these effects.

Claim 1. *Given $\mathcal{R}$ and $\lambda \in (0,1)$, good reviews are always good news about the value of the good being high, while bad reviews are always bad news. No reviews may be either good or bad news.*

Claim 2. *For every rule $\mathcal{R}$, no reviews is better than a bad review but worst than a good review: $\lambda'_G \geq \lambda'_N \geq \lambda'_B$.*

2.4 Role of $\mathcal{R} = \{\underline{q}, \bar{q}\}$

The reviewing rule is chosen by consumers before deciding whether to buy the good or not and so, it is chosen without having more information about the product's value than the one that is publicly available. As consumers are not better informed than future consumers or the firm when choosing $\mathcal{R}$, the reviewing rule itself contains no information about v.

However, the rule does affect the beliefs of an agent that observes the reviews. A higher $\bar{q}$ means that it requires a higher quality realisation to get a good review. As getting a good review is more difficult, consumers assign a higher probability to the good being of high value the higher is $\bar{q}$. Analogously, the higher is $\underline{q}$ the more likely it is that the firm gets a bad review, so a bad review is less damaging for higher values of $\underline{q}$. The thresholds of the reviewing rule also affect the inferences made after observing no reviews: observing that the previous consumer completed no reviews (given that he bought the good) is better news about the quality realisation he received the higher are $\underline{q}$ and $\bar{q}$, because they imply that the consumer remained silent for higher quality realisations. These intuitions are summarised in Claim 3.

Claim 3. *For any $\lambda \in (0,1)$, λ'_G is increasing in $\bar{q}$, and λ'_B is increasing in $\underline{q}$. Given that the previous consumer bought the good, the beliefs after observing that he completed no reviews is an increasing function of both, $\bar{q}$ and $\underline{q}$.*

[20]See Milgrom (1981).

3 Information Transmission

This section studies the effect of reviews on the expected payoffs of the firm and the consumers when the costs of transmitting information are not taken into account -i.e., when there is no cost of completing a review and the price of the first period is fixed. Isolating the effects of information transmission from its costs is useful in that it highlights the strategic considerations that will shape the equilibrium of the game.

Consider the last period of the game. The optimal reviewing rule has been determined at the beginning of the previous period, and it is thus given by the time the consumer observes a review. Furthermore, the consumer and the firm know whether the previous consumer bought the good or not, so the analysis can be conditional on the previous consumer having bought. After observing a review $i \in \{G, N, B\}$, the firm's optimal action is to offer the price that maximises its static profits given the observed review, $\hat{p}(\lambda'_i)$, and hence the expected utility of the second period consumer is $\hat{u}(\lambda'_i)$. In this context, the problem faced by the first period consumer is to choose the reviewing rule that maximises the expected utility of the next consumer. Denote $V(\mathcal{R}; \lambda) = \sum_i P(i; \mathcal{R}, \lambda)\hat{u}(\lambda'_i)$. Then, the consumers' problem is:

$$\max_{\{\underline{q}, \bar{q}\}} V(\mathcal{R}; \lambda) \tag{9}$$

A necessary condition for the existence of such a rule is that the information transmitted through the reviews increases the expected utility of second period consumers.[21] Whether this is the case or not depends on the curvature of the utility function because, as shown by the next claim, beliefs form a martingale.

Claim 4. *For every reviewing rule $\mathcal{R}$, and for every $\lambda \in (0, 1)$, beliefs form a martingale, i.e. $\mathbb{E}(\lambda'; \lambda, \mathcal{R}) = \lambda$.*

Proof. Conditional on the previous consumer having bought the good:[22]

$$\begin{aligned}
\mathbb{E}(\lambda'; \lambda, \mathcal{R}) &= \sum_{i \in \{G,N,B\}} P(i; \mathcal{R}, \lambda)\lambda'_i \\
&= \sum_{i \in \{G,N,B\}} P(i; \mathcal{R}, \lambda_t)\frac{\lambda \int_i f_H(q)dq}{P(i; \mathcal{R}, \lambda)} \\
&= \lambda \left[\int_{\bar{q}_t}^{q^K} f_H(q)dq + \int_{\underline{q}_t}^{\bar{q}_t} f_H(q)dq + \int_{q^0}^{\underline{q}_t} f_H(q)dq \right] \\
&= \lambda
\end{aligned}$$

□

The next Proposition shows that both the consumers and the firm prefer a rule in which reviews are completed with positive probability.

Proposition 1. *The information contained in the reviews increases the expected payoff of both the firm and the consumers.*

[21] Otherwise, consumers would receive no benefit from the reviews and so not even utilitarian consumers would be willing to complete reviews.

[22] It is shown in the Appendix that the martingale property also holds if the expectation is not conditional in the previous consumer having bought the good, for every $p \in (0, \lambda + L)$.

Proof. The consumers' (expected) utility is a convex function of λ'; thus, by Jensen inequality and the martingale property of the beliefs, it is higher when there is some information transmission:

$$\begin{aligned}\mathbb{E}_i(u;\mathcal{R},\lambda) &= \sum_{i\in\{G,N,B\}} P(i;\mathcal{R},\lambda)\hat{u}(\lambda_i') \\ &\geq \hat{u}\Big(\sum_{i\in\{G,N,B\}} P(i;\mathcal{R},\lambda)\lambda_i'\Big) \\ &= \hat{u}(\lambda)\end{aligned}$$

where $\hat{u}(\lambda)$ is the expected utility of a second period consumer when there there is no updating of beliefs.

A similar analysis holds for the firm's profits. Given that the previous consumer bought the good, the firm's expected profits when consumers can submit reviews are:

$$\mathbb{E}_i(\pi;\mathcal{R},\lambda) = \sum_{i\in\{G,N,B\}} P(i;\mathcal{R},\lambda)\hat{\pi}(\lambda_i')$$

Since the profit function is convex, the martingale property of the beliefs and Jensen's inequality imply:[23]

$$\begin{aligned}\mathbb{E}_i(\pi;\mathcal{R},\lambda) &= \sum_{i\in\{G,N,B\}} P(i;\mathcal{R},\lambda)\hat{\pi}(\lambda_i') \\ &\geq \hat{\pi}\Big(\sum_{i\in\{G,N,B\}} P(i;\mathcal{R},\lambda)\lambda_i'\Big) \\ &= \hat{\pi}(\lambda)\end{aligned}$$

□

Proposition 1 shows that, when its costs are not considered, the possibility of transmitting information through the reviews increases the (expected) payoff of both, the consumers and the firm. The reviews completed by the consumers are a function of the quality realisations they observed, which in turn are correlated with the actual v. Therefore, reviews are informative about the value of the good. The informativeness of the reviews increases expected profits and utility because it allows the firm and the consumers to adjust the price and the willingness to pay to a better approximation of v. It is important to note that this *"alignment"* of the incentives of the firm and the consumers holds because the monopolist cannot fully appropriate the additional surplus generated by the transmission of information. In the context of this paper, if the firm could appropriate all the surplus, leaving second period consumers indifferent between buying and not buying for every observed review, second period consumers' would be indifferent between receiving or not the information contained in the reviews. As a result, first period consumers would be indifferent between completing reviews or not when it is costless, but they would not complete reviews when there is a positive cost of doing it.

In order to determine the existence of an optimal reviewing rule that first period consumers are willing to follow, it is useful to look at the optimal amount of messages they would chose to

[23] Convexity of the expected profits is shown in Appendix A.

use when completing reviews is free.[24] The next Lemma shows that, when the first consumer buys the good, the expected utility of second period consumers is maximised by using all the available messages.[25] Proposition 2 uses the result in the Lemma to show the existence of a reviewing rule that maximises the expected utility of second period consumers.

Lemma 1. *Assume that there is no cost of transmitting information. Then, consumers' optimal reviewing rule assigns positive probability to all the available messages. The firm's (expected) profits are also higher with such a rule.*

The proof of the Lemma is in Appendix A. It shows that the addition of a third message induces a mean preserving spread with respect to the case in which there are only two messages. Given that the utility function is convex in the prior, this means that consumers always prefer using three messages instead of two. The introduction of a third message induces a finer partition of the set of quality realisations. As a result, the level of information received by the consumers in the second period is higher and so they can adjust their behaviour to a better approximation of the actual v. The firm's expected profits are convex in the belief too; hence, its payoff is also higher when consumers use all the available messages.

When there are no costs of completing reviews, the preferences of first and second period consumers are perfectly aligned. Therefore, the result in Lemma 1 is analogous to the one in Crawford and Sobel (1982). They show that the more similar are the preferences of the sender and the receiver, the larger is the maximal number of reports in equilibrium. The model in this paper differs from the standard model of strategic information transmission in that it has two "receivers" of the information, the future consumers and the firm. It is worth noting, however, that the firm is only an "indirect" receiver, because consumers aim when completing reviews is to transmit information to future consumers. The price offered by the firm in the first period may affect consumers' choice of the optimal amount of messages. I explore this possibility in Section 5.1.

An immediate implication of Lemma 1, is that there exists a rule, characterised by $\bar{q} < q^K$, $\underline{q} > q^0$ and $\bar{q} > \underline{q}$ that maximises the expected payoff of second-period consumers.

Proposition 2. *Expected second period utility is maximised for some rule* $\mathcal{R}_u = \{\underline{q}_u, \bar{q}_u\}$ *such that* $\underline{q}_u \in (0, \bar{q}_u)$ *and* $\bar{q}_u \in (\underline{q}_u, q^K)$.

Proof. The Proposition results from the fact that the rule maximises a continuous function over a non-empty and compact set. The fact that the consumers and the firm prefer a rule that uses three messages over a rule that uses only two means that their expected utility increases as $\underline{q}$ moves away from q^0 and $\bar{q}$ moves away from q^K. □

Furthermore, as shown in the next Lemma, this rule also maximises the firm's expected profits. As a result, the existence of the reviews aligns the incentives of the firm and the consumers.

[24] When there is no cost of completing a review, the three available messages (G, N and B) have the same unit cost and, given that the previous consumer bought, the three are informative about v.

[25] The implication of the Lemma is that, if we consider a system with M available messages and the cost of completing any two messages is the same, consumers will always choose a reviewing rule that assigns positive probability to all the M messages as this increases the precision of the information received by second period consumers. As discuss later in this section, this result is similar to the one in Crawford and Sobel (1982), and it is related to the fact that the preferences of the "sender" and the "receiver" are aligned.

Lemma 2. *The reviewing rule that maximises second period consumers' expected utility, $\mathcal{R}_u$, also maximises the firm's expected second period profits.*

Proof. After observing a review $i \in \{G, N, B\}$ the firm optimally sets the second period price at $\hat{p}_2(\lambda_i')$. At this price, the expected utility of a second period consumer is $\hat{u}_2(\lambda_i') = \frac{(\lambda_i'+L-c)^2}{8(\lambda_i'+L)}$ and the firm's (maximum) expected profit is $\hat{\pi}_2(\lambda_i') = \frac{(\lambda_i'+L-c)^2}{4(\lambda_i'+L)}$. As defined before, $V(\mathcal{R};\lambda) = \sum_{i\in\{G,N,B\}} P(i;\mathcal{R})\hat{u}_2(\lambda_i')$ and so $\mathbb{E}_i(\pi_2;\mathcal{R},\lambda) = 2V(\mathcal{R};\lambda)$. Then, if $\{\underline{q}_u, \bar{q}_u\}$ maximises $V(\cdot)$, it also maximises $\mathbb{E}_i(\pi_2;\mathcal{R},\lambda)$.

□

The more accurate the information the firm has about the value of v, the better it can adjust its second period price and thus, the higher are its expected second period profits. At the beginning of the second period both the firm and the consumers have the same information about value of the good. Hence, a rule that maximises the information available to second period consumers also maximises the one available to the firm.

Corollary 1. *The existence of information transmission through reviews aligns the incentives of the consumers and the firm.*

4 Cost of completing a review

The previous section showed that there exists a reviewing rule $\mathcal{R}_u = \{\underline{q}_u, \bar{q}_u\}$ that consumers are willing to follow when completing reviews is costless and the first period price is given. When the cost of completing a review is taken into account, the three messages available to the consumers (G, N and B) are not equivalent anymore. Conditional on the previous consumer having bought the good, the three available messages are informative about the quality realisation observed by the previous consumer, but while a good or a bad review have a positive cost, not completing a review is costless. When the firm's response in terms of first period price is not considered, the rule is not affected by (and does not affect) the probability that the first consumer buys. This section considers the effect of the cost of completing reviews when the first consumer bought the good. The main result is that, as long as the unit cost of completing a review is not very high, a rule that uses the three available messages is still optimal, but the set of quality realisations for which consumers do not complete reviews increases with the cost.

The cost of completing one review is $h > 0$; the total expected cost given a reviewing rule $\mathcal{R}$ is h times the probability of completing either a good or a bad review. Then, the total expected cost, $\Psi(\mathcal{R}; h, \lambda)$, is given by the following expression:

$$\begin{aligned}\Psi(\mathcal{R};h,\lambda) &= h\,[P(B;\mathcal{R},\lambda) + P(G;\mathcal{R},\lambda)] \qquad (10)\\ &= h\left[\lambda\left(\int_{q_0}^{\underline{q}} f_H(q)dq + \int_{\bar{q}}^{q_K} f_H(q)dq\right) + (1-\lambda)\left(\int_{q_0}^{\underline{q}} f_L(q)dq + \int_{\bar{q}}^{q_K} f_L(q)dq\right)\right]\end{aligned}$$

The first two terms in the second line define the probability of completing a good or a bad review conditional on the true value being H, while the last two terms measure the expected cost of completing a review conditional on the true value of the good being L. Consumers face the cost of completing a review only if they buy the good.

The expected cost of completing a review is an increasing function of h and $\underline{q}$, and a decreasing function of $\bar{q}$. A higher value of $\bar{q}$ reduces the probability that the consumer completes a good review and so it reduces the expected cost:

$$\frac{\partial \Psi(\mathcal{R}; h, \lambda)}{\partial \bar{q}} = h\frac{\partial P(G; \mathcal{R}, \lambda)}{\partial \bar{q}} = -h[\lambda f_H(\bar{q}) + (1-\lambda) f_L(\bar{q})] < 0 \tag{11}$$

Analogously, a higher $\underline{q}$ increases the probability of getting a quality realisation low enough so as to complete a bad review, which increases the expected cost:

$$\frac{\partial \Psi(\mathcal{R}; h, \lambda)}{\partial \underline{q}} = h\frac{\partial P(B; \mathcal{R}, \lambda)}{\partial \underline{q}} = h[\lambda f_H(\underline{q}) + (1-\lambda) f_L(\underline{q})] > 0 \tag{12}$$

When completing reviews is costless, first period consumers would be indifferent between any two rules that induce the same expected utility for the consumers in the second period. This is not true anymore when the cost the of completing reviews is taken into account: would there exist two rules that deliver the same value of $V(\cdot)$, first period consumers would now prefer the one that induces the smaller probability of completing either a good or a bad review. As a result, the optimal rule when completing reviews is not free makes a more extensive use of the "cheap message", N.

Proposition 3. *When there exists a positive cost h of completing a review, there exists an optimal rule $\mathcal{R}_c = \{\underline{q}^c, \bar{q}^c\}$ such that $\underline{q}^c < \underline{q}_u$ and $\bar{q}^c > \bar{q}_u$.*

Proof. From Proposition 1, there exists a reviewing rule $\mathcal{R}_u = \{\underline{q}_u, \bar{q}_u\}$ that maximises consumers' (expected) payoff when completing a review is costless. Now, consider how those thresholds change when the cost of completing reviews, $\Psi(\mathcal{R}; h, \lambda)$, is taken into account. Given p_1, the reviewing rule $\mathcal{R}$ affects the expected utility of the first period consumer through the cost of completing reviews and through its impact on future consumers' expected utility. The consumer's problem in this case is:

$$\max_{\{\underline{q}, \bar{q}\}} V(\mathcal{R}; \lambda) - \Psi(\mathcal{R}; h, \lambda)$$

Taken derivatives with respect to $\bar{q}$ and $\underline{q}$, the first order conditions are:

$$\frac{\partial V(\mathcal{R}; \lambda)}{\partial \bar{q}} - \frac{\partial \Psi(\mathcal{R}; h, \lambda)}{\partial \bar{q}} = 0 \tag{13}$$

$$\frac{\partial V(\mathcal{R}; \lambda)}{\partial \underline{q}} - \frac{\partial \Psi(\mathcal{R}; h, \lambda)}{\partial \underline{q}} = 0 \tag{14}$$

When $\mathcal{R} = \mathcal{R}_u = \{\underline{q}_u, \bar{q}_u\}$, the expression in (13) is positive because $\frac{\partial V(\mathcal{R}_u;\lambda)}{\partial \bar{q}} = 0$ as $\mathcal{R}_u$ maximises the second period's expected utility, but the derivative of $\Psi(\cdot)$ with respect to $\bar{q}$ is negative for every $\bar{q} \in [q_0, q_K]$. Therefore, when the cost of completing reviews is considered, the optimal cut-off quality for good reviews, $\bar{q}^c$, must be higher than $\bar{q}_u$. Analogously, the optimal cut-off quality for bad reviews, $\underline{q}^c$, is below $\underline{q}_u$ when the cost of completing reviews is taken into account. In this case, (14) evaluated at $\{\underline{q}, \bar{q}\} = \{\underline{q}_u, \bar{q}_u\}$ is negative: $\frac{\partial V(\mathcal{R}_u;\lambda)}{\partial \underline{q}} = 0$ and the cost of completing reviews is an increasing function of $\underline{q}$. As a result, $\underline{q}^c < \underline{q}_u$. □

When completing a review is costly, consumers face a trade off because transmitting more accurate information increases future consumers' expected payoff, but it reduces the payoff of current consumers. As the intermediate message is informative but costless, they solve the trade off by making a more extensive use of this "free message". As a result, $\bar{q}^c - \underline{q}^c > \bar{q}_u - \underline{q}_u$.

The reviewing rule used by consumers when h is positive may be considered "more tough" than $\mathcal{R}_u$. When consumers rely more extensively on the cheapest message (N), the set of quality realisations after which the first consumer completes a good review becomes smaller and so does the set after which he completes a bad review. As extreme reviews become less likely, the updating they induce becomes more extreme: a good review constitutes better news (has a higher positive impact on beliefs and profits) and a bad review is more damaging in the sense that it induces a higher reduction of the beliefs -i.e., $\lambda'_B(\lambda;\mathcal{R}_c) < \lambda'_B(\lambda;\mathcal{R}_u)$ and $\lambda'_G(\lambda;\mathcal{R}_c) > \lambda'_G(\lambda;\mathcal{R}_u)$.[26]

Whether it is still optimal for consumers to use the three available messages depends on the unit cost of completing reviews, h. As shown above, when the cost of completing a review, h is taken into account, the thresholds in consumers' optimal reviewing rule become closer to the extremes. However, more accurate information is still better for second period consumers. Therefore, as shown in Lemma 3, as long as h is not very high, consumers still prefer a rule such that $\bar{q}_c < q_K$ and $\underline{q}_c > q_0$.

Lemma 3. *There exists $\underline{h}(\lambda, \bar{q}^c)$ and $\bar{h}(\lambda, \underline{q}_c)$ such that for every $\lambda \in (0,1)$ and for every $h < min\{\underline{h}(\lambda, \bar{q}^c), \bar{h}(\lambda, \underline{q}_c)\}$, the optimal reviewing rule implies $\underline{q}^c > q^0$ and $\bar{q}^c < q^K$.*[27]

5 Equilibrium: Reviews and Price Discounts

This section solves for the equilibrium reviewing rule and price when $T = 2$. Given that neither the firm nor the consumers know the actual value of v, the equilibrium concept is similar to a subgame perfect equilibrium. The consumers move first and so their strategy is a reviewing rule $\mathcal{R}$. The firm chooses the first period price after consumers have chosen the rule and thus, its strategy assigns a price to every possible reviewing rule chosen by the consumers. The game is solved backwards: subsection 5.1 looks at the optimal first period price and subsection 5.2 presents the optimal reviewing rule and the equilibrium of the game, using the results from previous sections as building blocks.

5.1 Firm's pricing strategy

The firm's strategy in the two-period game consists of a first period price that maximises the sum of current and future profits, given a reviewing rule $\mathcal{R}$ and its own optimal pricing behaviour in the last period.[28] In the second period the firm will charge the optimal static price, given the public beliefs about the value of v -i.e., $p_2^* = \hat{p}_2(\lambda'_i)$ if the first period consumer bought the good and completed review i, and $p_2^* = \hat{p}_2(\lambda)$ if he did not buy. From Proposition 1, the information generated by the reviews increases the monopolist's expected second period profits. Therefore, the firm has incentives to reduce the first period price if by doing so it increases the probability that the first period consumer buys the good and completes reviews. As a price discount reduces first period's profits, the firm faces a standard trade off between

[26] The distribution of posterior beliefs induced by the rule $\mathcal{R}_c$ dominates stochastically of second order the one induced by $\mathcal{R}_u$. As both distributions have the same mean (by the martingale property of the beliefs) this means that the distribution induced by $\mathcal{R}_u$ is a mean preserving spread of the one induced by $\mathcal{R}_c$. Combined with the convexity of the utility function with respect to the beliefs, this implies that $V(\mathcal{R}_c, \lambda) < V(\mathcal{R}_u, \lambda)$.

[27] A formal proof of this result in Appendix A.

[28] The same is true when $T > 2$. I look at the two periods case because it is enough to derive the main intuitions.

current and future profits. The firm's optimisation problem, given a reviewing rule $\mathcal{R}$ is:[29]

$$\max_{p_1} \; \Pi(p_1;\lambda,\mathcal{R}) = \pi_1(p_1;\lambda) + P(\gamma_1 \geq \frac{p_1}{\lambda+L})\mathbb{E}_i(\hat{\pi}_2(\lambda_i')) + P(\gamma_1 < \frac{p_1}{\lambda+L})\hat{\pi}_2(\lambda) \tag{15}$$

The first term is the first period expected profit, $\pi_1(p_1;\lambda) = P(\gamma_1 \geq \frac{p_1}{\lambda+L})(p-c)$. The second term is the expected second period profit if the first consumer buys the good and so there is some transmission of information between periods -$\hat{\pi}_2(\lambda_i') = \pi_2(\hat{p}_2;\lambda_i')$. The last term is the firm's expected second period profit when the first period consumer does not buy (and so $\lambda' = \lambda$). From Proposition 1, $\mathbb{E}_i(\hat{\pi}_2(\lambda_i')) \geq \hat{\pi}_2(\lambda)$. This maximisation problem results in an optimal first period price:

$$\begin{aligned} p_1^*(\mathcal{R};\lambda) &= \frac{\lambda + L + c - [\mathbb{E}_i(\hat{\pi}_2(\lambda_i')) - \hat{\pi}_2(\lambda)]}{2} \\ &= \hat{p}_1(\lambda) - \frac{[\mathbb{E}_i(\hat{\pi}_2(\lambda_i')) - \hat{\pi}_2(\lambda)]}{2} \end{aligned} \tag{16}$$

which is smaller than the static optimal price by an amount that depends on the increase in the (expected) future profits induced by the reviews.[30] Denote the first period profits induced by price $p_1^*(\mathcal{R};\lambda)$ by $\pi_1^*(\lambda;\mathcal{R})$.

The price function in (16) is the firm's best reply to consumers' reviewing rule. The difference in expected second period profits, $[\mathbb{E}_i(\hat{\pi}_2(\lambda_i')) - \hat{\pi}_2(\lambda)]$, is the value for the firm of the information contained in the reviews. The firm is no better informed than consumers are about v and so the price it chooses in the first period is not a signal about the actual value of the good. From the firm's perspective, the role of $(\hat{p}_1 - p_1^*)$ is to assign probabilities between two possible states of the world: one in which the first period consumer buys the good, and so he completes reviews, and another state in which the first period consumer does not buy (and so $\lambda' = \lambda$). The firm's expected profits are higher in the first case, but increasing the probability of that state requires a reduction of p_1 that reduces (expected) first period profits. Therefore, $[\hat{\pi}_1(\lambda) - \pi_1^*(\lambda;\mathcal{R})] > 0$ constitutes a measure of the "price" paid by the firm in order to get information about v.[31] It is apparent from expression (16) that the price discount the firm is willing to offer depends on the benefit it expects to receive from the information contained in the reviews, which in turns depends on the rule chosen by consumers ($\mathcal{R}$).

5.2 Optimal Reviewing Rule and Equilibrium

The consumers' problem is to choose a reviewing rule that, given a proper anticipation of the firm's pricing strategy, maximises the sum of the utilities of first and second periods' consumers. Therefore, their strategy is a mapping from their prior beliefs (λ) and cost of completing reviews (h) into a pair of thresholds $\{\underline{q},\bar{q}\} \in [q_0,q_K]^2$. Their maximisation problem is as follows:

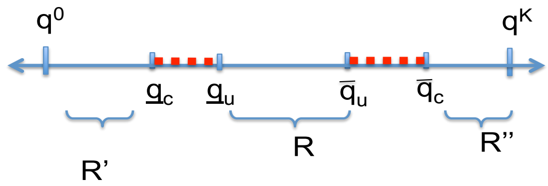
$$\max_{\{\underline{q},\bar{q}\}} P\left(\gamma_1 \geq \frac{p_1(\mathcal{R};\lambda)}{\lambda+L}\right)[u_1(p_1;\lambda) - \Psi(\mathcal{R};h,\lambda) + \mathbb{E}_i(\hat{u}_2(\lambda_i'))] + P\left(\gamma_1 < \frac{p_1(\mathcal{R};\lambda)}{\lambda+L}\right)\hat{u}_2(\lambda) \tag{17}$$

[29] Recall that λ_i' is a shortcut for $\lambda_i'(\lambda;\mathcal{R})$.

[30] The firm's maximisation problem considers the case in which the monopolist's discount factor is equal to one. Considering a discount factor $\delta \in (0,1)$ would result in a first period price closer to the static optimal price but would not change the main intuitions.

[31] It can also be considered as the price paid by the firm in order to have its product "tested" by consumers.

Figure 1: Reviewing Rule



where $p_1(\mathcal{R};\lambda)$ is the price function in (16). The expression above shows that the reviewing rule chosen by the consumers affects, through its effect on p_1, the probability that the first period consumers buys and, if he buys, his expected utility. The rule also affects the expected utility of the second period consumer because the reading the consumers do into the reviews affects their valuation for the good, which in turn determines the equilibrium values of $p_2(\cdot)$ and $u_2(\cdot)$.

From the consumers' perspective, the smaller first period price compensates part of the cost of completing reviews and so they optimally move the reviewing rule closer to the one that maximises $V(\cdot)$.[32] As shown in Lemma 2, such a rule also increases the firm's expected second period profits, which compensates the profits lost because of the discount. These intuitions are formalised in the following Equilibrium Proposition.

Proposition 4. Equilibrium. *Given $\lambda \in (0,1)$ and $h < \min\{\underline{h}, \bar{h}\}$, there exists a reviewing rule $\mathcal{R}^* = \{\underline{q}^*, \bar{q}^*\}$ and a first period price $p_1^*(\mathcal{R}^*,\lambda) < \hat{p}_1(\lambda)$, such that $\mathcal{R}^*$ is the consumers' best response to $p_1^*(\mathcal{R}^*,\lambda)$ and $p_1^*(\mathcal{R}^*,\lambda)$ is the best reply of the firm to $\mathcal{R}^*(\cdot)$. Furthermore, the equilibrium reviewing rule is such that $\underline{q}^* \in (\underline{q}^c, \underline{q}_u]$ and $\bar{q}^* \in [\bar{q}_u, \bar{q}^c)$.*

Proof. The conditions about λ and h guarantee that the consumers' problem has an interior solution. The result in the Proposition can be proved by contradiction. Considered the graph in Figure 1. The figure presents the thresholds of the rules $\mathcal{R}_c$ and $\mathcal{R}_u$; as shown in Proposition 3, $\bar{q}_c > \bar{q}_u$ and $\underline{q}_c < \underline{q}_u$. Consider a rule such that $\mathcal{R}$ in the Figure. This rule's thresholds are $\bar{q} < \bar{q}_u$ and $\underline{q} > \underline{q}_u$ (i.e., it is a "softer" rule than $\mathcal{R}_u$). Starting from rule $\mathcal{R}_u$, moving the thresholds according to the rule $\mathcal{R}$ reduces the expected utility of second period consumers, $V(\cdot)$. Furthermore, as this rule implies a higher probability of both, good and bad reviews relative to $\mathcal{R}_u$ and $\mathcal{R}_c$, it also increases the cost of completing reviews. As a result, $V(\mathcal{R},\lambda) - \Psi(\mathcal{R};h,\lambda) < V(\mathcal{R}_u,\lambda) - \Psi(\mathcal{R}_u;h,\lambda) < V(\mathcal{R}_c,\lambda) - \Psi(\mathcal{R}_c;h,\lambda)$. $\mathcal{R}$ would also imply a smaller expected utility for first period consumers: from Lemma 2, $\mathbb{E}(\pi_2;\hat{p}_2(\lambda_i'))$ is maximised when the reviewing rule is $\mathcal{R}_u$. Then, for any other rule the firm offers a smaller price discount, which reduces the expected utility of first period consumers. Then, starting from $\mathcal{R}_u$ consumers have no incentives to move to a "softer" rule like $\mathcal{R}$.

On the other hand, consider a rule like $\mathcal{R}'$ in the figure, with $\underline{q}' < \underline{q}_c$ and $\bar{q}' = \bar{q}_c$. Recall that $\mathcal{R}_c$ maximises the expected utility of second period consumers net of the cost of completing reviews. Therefore, $V(\mathcal{R}',\lambda) - \Psi(\mathcal{R}';h,\lambda) < V(\mathcal{R}_c,\lambda) - \Psi(\mathcal{R}_c;h,\lambda)$. Starting from $\mathcal{R}_c$, this new rule reduces the firm's expected second period profits when there are reviews, and thus it implies a smaller discount in the price of the first period. Therefore, the expected utility of first and second period consumers is smaller with a rule like $\mathcal{R}'$ than

[32] As discussed below, the fact that the price discount has a similar effect (from consumers' point of view) to a reduction in h does not mean that reducing p_1 and reducing h are substitutes from the firm's perspective.

with $\mathcal{R}_c$ and so consumers have no incentives to move to a rule tougher than $\mathcal{R}_c$. A similar analysis applies in the case of a rule like $\mathcal{R}''$, for which $\underline{q}' = \underline{q}_c$ but $\bar{q}' \geq \bar{q}_c$. Thus, consumers best response to the firm's pricing strategy must be a rule $\mathcal{R}^*$ such that $\underline{q}^* \in (\underline{q}^c, \underline{q}_u]$ and $\bar{q}^* \in [\bar{q}_u, \bar{q}^c)$ -i.e., a rule whose thresholds lie within the dotted part of the quality line of Figure 1.

□

Proposition 2 showed that there exists a reviewing rule $\mathcal{R}_u$ that maximises consumers expected second period utility when the cost of completing reviews is not taken into account. The firm's expected profits in the second period with reviews are also maximised by that rule. However, when consumers take into account the cost of competing reviews they move to a rule that induces smaller probabilities of completing good and bad reviews. As a result, a bad review has a more damaging impact on the firm's profits and a good review has a higher impact on the posterior (though it has a smaller probability). The firm is willing to pay a higher "price" for the first rule than for the second one and, as the price discount "subsidises" part of the cost of completing reviews, it induces first period consumers to choose a reviewing rule that is closer to the one that increases the expected payoff of both, the firm and the second period consumers.

Discussion

Role of h. The discount the firm is willing to offer to first period consumers increases as h decreases. The smaller is the cost of completing reviews the closer is the rule chosen by consumers to the one preferred by the firm ($\mathcal{R}_u$). As shown before, this implies that, the value of the information contained in the reviews increases and so the firm's best response it to reduce p_1 in order to increase the probability that the first consumer buys. In the limit in which $h = 0$, the price discount $(\hat{p}_1 - p_1^*)$ is maximum. This result implies that, contrary to some widespread beliefs, the price discount is not a substitute of a smaller cost of completing reviews, but it is instead its *complement*. It also suggests that as h decreases the burden of the "cost" of transmitting information moves from the consumers towards the firm. It is worth noting that a smaller h increases the expected payoff of both, the consumers and the firm.

A Bilateral Monopoly. The equilibrium results presented above can be easily associated with a situation of bilateral monopoly. Considering current and future consumers as a "group", it could be think that consumers are the only "suppliers" of the information contained in the reviews and the firm is the unique potential "buyer". Then, the "price" of that information is the price discount offered by the firm and the cost of completing reviews allocates the surplus between the two sides of the market. When the cost of completing a review increases, the expected cost of a given rule increases too. Given the discount offered by the firm, they the information offered by the firm, consumers move to a rule that assigns a smaller probability to either a good or a bad review. On the other extreme, when completing reviews is very cheap, the rule chosen by consumers is closer to the one preferred by the firm, and so it offers a higher discount.

6 Conclusions and Further Research

This paper proposes a dynamic game to explain how the reviews completed by consumers about the quality of an experience good and the pricing strategy of a monopoly firm affects each other. The results suggest that information is valuable for both, the firm and the consumers and so they are willing to "share" the costs of generating that information.

An important result of the paper is that a necessary condition for the existence of reviews in equilibrium is that the firm cannot appropriate all the surplus generated by the information in the reviews. As a consequence, second period consumers are not indifferent between observing the reviews or not and so first period consumers are willing to complete reviews (and the firm is willing to "pay" for them).

It is worth noting that the linearity of the demand function used in the model does not allow for the possibility that second period consumers do not get part of the surplus. However, the results may be affected by a change in the demand function. Under the assumptions made in this paper, would the firm be able to leave second period consumers indifferent between buy and not buying for every observed review, first period consumers would have less (or none) incentives to complete reviews. The firm would like to reduce the price of the second period, but it would face a commitment problem: in any finite game, after a review is observed the firm would have incentives to extract all the surplus. Solving backwards, consumers would not complete reviews if they complete reviews in order to increase the expected utility of future consumers.

The implications of this possibility are matter of future research. One possible explanation is that the reason we observe reviews is not related with consumers' intentions to maximise current and future consumers' (expected) payoff, as was assumed in this paper. Therefore, the implications of alternative assumptions about why consumers complete reviews (like anger, punishment or reciprocity, for example) should be considered.

A Proofs

Claim 1

For any rule $\mathcal{R}$, good reviews increase the probability consumers assign to the value of the good being high: $\lambda'_G(\lambda, \mathcal{R}) \geq \lambda \iff \int_{\bar{q}}^{q^K} f_H(q)dq \geq \int_{\bar{q}}^{q^K} f_L(q)dq$. This is true because increasing likelihood ratio implies first order stochastic dominance.[33]

Analogously, $\lambda'_B(\lambda; \mathcal{R}) \leq \lambda \iff \int_0^{\underline{q}} f_H(q)dq \leq \int_0^{\underline{q}} f_L(q)dq$, which is also implied by first order stochastic dominance.

Finally, observing that the previous consumer bought the good but he did not complete a review is bad news about v if $\lambda'_N(\lambda; \mathcal{R}) \leq \lambda$, or $\int_{\underline{q}}^{\bar{q}} f_H(q)dq \leq \int_{\underline{q}}^{\bar{q}} f_L(q)dq$. Whether this inequality holds or not depends on the quality realisation at which $f_H(q) = f_L(q)$.[34] To see this, denote by $\hat{q}$ the crossing point of the distributions and consider two extreme cases:

- If $\hat{q} \leq \underline{q}$, then $f_H(\underline{q}) \geq f_L(\underline{q})$. Increasing MLRP implies that the ratio $f_H(q)/f_L(q)$ is increasing in q, then $f_H(\underline{q}) \geq f_L(\underline{q})$ implies $f_H(q) \geq f_L(q)$ for every $q \in (\underline{q}, \bar{q})$ and so $\int_{\underline{q}}^{\bar{q}} f_H(q)dq \geq \int_{\underline{q}}^{\bar{q}} f_L(q)dq$ and $\lambda'_N(\lambda; \mathcal{R}) \geq \lambda$.
- On the other extreme, consider the case in which $\hat{q} \geq \bar{q}$. In this case, $f_H(\bar{q}) \leq f_L(\bar{q})$ and by increasing MLRP, this implies $f_H(q) \leq f_L(q)$ for every $q \in (\underline{q}, \bar{q})$. As a result, $\lambda'_N(\lambda; \mathcal{R}) \leq \lambda$.

As a result, no reviews are bad news when the crossing of the quality distributions is very high relative to the upper bound of the social norm, but they become more and more good news the closer is the crossing point to the lower bound of the social norm.

Claim 2

$\lambda'_G(\lambda; \mathcal{R}) \geq \lambda'_N(\lambda; \mathcal{R})$ for every social norm if and only if $\frac{\int_{\underline{q}}^{\bar{q}} f_L(q)dq}{\int_{\underline{q}}^{\bar{q}} f_H(q)dq} \geq \frac{\int_{\bar{q}}^{q^K} f_L(q)dq}{\int_{\bar{q}}^{q^K} f_H(q)dq}$ for every $\mathcal{R}$. This expression can be written as:

$$\frac{1 - F_H(\bar{q})}{F_H(\bar{q}) - F_H(\underline{q})} \geq \frac{1 - F_L(\bar{q})}{F_L(\bar{q}) - F_L(\underline{q})} \tag{18}$$

The inequality holds because first order stochastic dominance implies that the numerator of the left hand side is greater than that of the right hand side, while $F_H(\bar{q}) - F_H(\underline{q}) \leq F_L(\bar{q}) - F_L(\underline{q})$.

A similar argument can be used to show that $\lambda'_N(\lambda; \mathcal{R}) \geq \lambda'_B(\lambda; \mathcal{R})$. When the previous consumer bough the good, observing no reviews results in a higher posterior than observing a bad review if and only if:

$$\frac{F_H(\bar{q}) - F_H(\underline{q})}{F_H(\underline{q})} \geq \frac{F_L(\bar{q}) - F_L(\underline{q})}{F_L(\underline{q})} \tag{19}$$

for every rule $\{\bar{q}, \underline{q}\}$. To see that this inequality holds, note that both numerators are increasing functions of $\bar{q}$, but the left hand side increases at a rate $f_H(\bar{q})$ while the right hand

[33] First order stochastic dominance implies $F_H(q) \leq F_L(q)$ for every $q \in [q_0, q_K]$.

[34] As $F_H(q)$ dominates $F_L(q)$ in terms of the likelihood ratio, $f_H(q)$ and $f_L(q)$ cross only once.

side increases at a lower rate $f_L(\bar{q})$.[35] When $\bar{q} \to \underline{q}$, condition (19) becomes $\frac{F_L(\underline{q})}{f_L(\underline{q})} \geq \frac{F_H(\underline{q})}{f_H(\underline{q})}$, which holds because increasing monotone likelihood property implies reverse hazard rate dominance.[36] As $\bar{q}$ increases, the denominator of the left hand side of (19) increases faster than that of the right hand side and so, given any $\underline{q} > q^0$, condition (19) holds for any $\bar{q} \in (\underline{q}, q^K]$.

Claim 3

$$\frac{\partial \lambda'_G(\lambda; \underline{q}, \bar{q})}{\partial \bar{q}} = \frac{\lambda(1-\lambda)[f_L(\bar{q}) \int_{\bar{q}}^{q_K} f_H(q)dq - f_H(\bar{q}) \int_{\bar{q}}^{q_K} f_L(q)dq]}{[\lambda \int_{\bar{q}}^{q_K} f_H(q)dq + (1-\lambda \int_{\bar{q}}^{q_K} f_L(q)dq]^2} \geq 0 \tag{20}$$

The denominator is the probability of observing a good review squared, so it is positive. The sing of the numerator depends on the sign of $[f_L(\bar{q} \int_{\bar{q}}^{K} f_H(q)dq - f_H(\bar{q} \int_{\bar{q}}^{K} f_L(q)dq]$, which is positive as long as the distribution of quality conditional on H dominates in hazard rate sense the one conditional on L.[37] As hazard rate dominance is implied by increasing monotone likelihood ratio, the numerator is positive and the result in the Claim holds.

Consumers' beliefs after observing a bad review are increasing in $\underline{q}$:

$$\frac{\partial \lambda'_B(\lambda; \underline{q}, \bar{q})}{\partial \underline{q}} = \frac{\lambda(1-\lambda)[f_H(\underline{q}) \int_{q_0}^{\underline{q}} f_L(q)dq - f_L(\underline{q}) \int_{q_0}^{\underline{q}} f_H(q)dq]}{[\lambda \int_{q_0}^{\underline{q}} f_H(q)dq + (1-\lambda) \int_{q_0}^{\underline{q}} f_L(q)dq]^2} \geq 0 \tag{21}$$

The denominator is the probability of observing a bad review squared, so it is positive. The numerator is positive as long as $f_H(\underline{q}) \int_{q_0}^{\underline{q}} f_L(q)dq \geq f_L(\underline{q}) \int_{q_0}^{\underline{q}} f_H(q)dq$, which holds by inverse hazard rate dominance.[38]

A similar analysis shows that the updating after observing no reviews (when the previous consumer bought the good) is also an increasing function of the thresholds of the social rule:

$$\frac{\partial \lambda'_N(\lambda; \underline{q}, \bar{q})}{\partial \bar{q}} = \frac{\lambda(1-\lambda)[f_H(\bar{q}) \int_{\underline{q}}^{\bar{q}} f_L(q)dq - f_L(\bar{q}) \int_{\underline{q}}^{\bar{q}} f_H(q)dq]}{[\lambda \int_{\underline{q}}^{\bar{q}} f_H(q)dq + (1-\lambda) \int_{\underline{q}}^{\bar{q}} f_L(q)dq]^2} \geq 0 \tag{22}$$

$$\frac{\partial \lambda'_N(\lambda; \underline{q}, \bar{q})}{\partial \underline{q}} = \frac{\lambda(1-\lambda)[f_L(\underline{q}) \int_{\underline{q}}^{\bar{q}} f_H(q)dq - f_H(\underline{q}) \int_{\underline{q}}^{\bar{q}} f_L(q)dq]}{[\lambda \int_{\underline{q}}^{\bar{q}} f_H(q)dq + (1-\lambda) \int_{\underline{q}}^{\bar{q}} f_L(q)dq]^2} \geq 0 \tag{23}$$

Proposition 1

Convexity of the profit function

If the firm offers the optimal static price, $\hat{p}(\lambda'_i)$, its expected profits are $\pi(\lambda'_i) = \frac{(\lambda'_i+L-c)^2}{4(\lambda'_i+L)}$. Taking derivatives with respect to λ'_i:

$$\frac{\partial \pi(\lambda'_i)}{\partial \lambda'_i} = \frac{(\lambda'_i + L - c)(\lambda'_i + L + c)}{4(\lambda'_i + L)^2} \quad > 0 \text{ for every } L > c$$

[35]Increasing monotone likelihood implies that $f_H(q)/f_L(q)$ is an increasing function of q.

[36]Using L'Hopital's rule:

$$\lim_{\bar{q} \to \underline{q}} \frac{F_L(\bar{q}) - F_L(\underline{q})}{F_H(\bar{q}) - F_H(\underline{q})} = \frac{f_L(\underline{q})}{f_H(\underline{q})}$$

[37]Hazard rate dominance implies $f_L(\bar{q})[1 - F_H(\bar{q})] \geq f_H(\bar{q})[1 - F_L(\bar{q})]$. See Krishna (2002).

[38]Reverse hazard rate dominance implies $f_L(\underline{q})F_H(\underline{q}) \leq f_H(\underline{q})F_L(\underline{q})$. See Krishna (2002)

$$\frac{\partial^2 \pi(\lambda_i'; \mathcal{R})}{\partial {\lambda_i'}^2} = \frac{c^2}{4(\lambda_i' + L)^3} \quad > 0 \text{ for every } c > 0$$

Convexity of the utility function

The expected utility in any period t, given the consumer's prior λ_i' and a price p is:

$$\begin{aligned} u(\lambda_i'; \mathcal{R}) &= P(\gamma > \frac{p}{\lambda_i' + L}) \left[\mathbb{E}(\gamma | \gamma > \frac{p}{\lambda_i' + L})(\lambda_i' + L) - p \right] \\ &= \left[\frac{\lambda_i' + L - p}{\lambda_i' + L} \right] \left[\frac{\lambda_i' + L + p}{2} - p \right] \\ &= \frac{(\lambda_i' + L - p)^2}{2(\lambda_i' + L)} \end{aligned}$$

where I used the conditional expectation of γ:

$$\mathbb{E}\big(\gamma | \gamma \geq \frac{p}{\lambda + L}\big) = \frac{1}{1 - \frac{p}{\lambda + L}} \left[\int_{\frac{p}{\lambda + L}}^{1} x dx \right] = \frac{\lambda + L + p}{2(\lambda + L)}$$

Taking partial derivatives with respect to λ_i':

$$\frac{\partial u(\lambda_i'; \mathcal{R})}{\partial \lambda^i} = \frac{(\lambda_i' + L + p)(\lambda_i' + L - p)}{2(\lambda_i' + L)^2}$$

$$\frac{\partial^2 u(\lambda_i'; \mathcal{R})}{\partial {\lambda_i'}^2} = \frac{p^2}{(\lambda_i' + L)^3}$$

The second expression is positive for every $p > 0$, while the first one is positive as long as $p < L + c$. Then , consumers' utility function is increasing and convex with respect to λ_i' for every positive price at which some consumer is willing to buy. In particular, it is increasing and convex in the prior when $p = \hat{p}(\lambda_i') = \frac{\lambda_i' + L + c}{2}$. The expected utility of a consumer who observed review i, when the second period price is $\hat{p}(\lambda_i')$ is:[39]

$$u(\lambda_i'; \mathcal{R}) = \frac{[\lambda_i' + L - c]^2}{8(\lambda_i' + L)}$$

Taking derivatives with respect to λ_i':

$$\frac{\partial u(\lambda_i'; \mathcal{R})}{\partial \lambda^i} = \frac{(\lambda_i' + L - c)(\lambda_i' + L + c)}{8(\lambda_i' + L)^2} \quad > 0 \text{ for every } i \in \{G, N, B\} \text{ and } L > c$$

$$\frac{\partial^2 u(\lambda_i'; \mathcal{R})}{\partial {\lambda_i'}^2} = \frac{c^2}{4(\lambda_i' + L)^3} \quad > 0 \text{ for every } i \in \{G, N, B\} \text{ and } c > 0$$

Then, whichever the review completed by the previous consumer and the price set by the monopolist, the expected utility of the second consumer is increasing and convex in λ_i'.

[39] Given $p = \hat{p}(\lambda_i')$, the probability that the consumer buys the good is:

$$P(\gamma \geq \frac{\hat{p}}{\lambda_i' + L}) = \frac{\lambda_i' + L - p^*}{\lambda_i' + L} = \frac{\lambda_i' + L - c}{2(\lambda_i' + L)}$$

and the expected value of γ conditional on the consumer buying is:

$$\mathbb{E}(\gamma | \gamma \geq \frac{\hat{p}}{\lambda_i' + L}) = \frac{1}{1 - \frac{\hat{p}}{\lambda_i' + L}} [\int_{\frac{\hat{p}}{\lambda_i' + L}}^{1} x dx] = \frac{3\lambda_i' + 3L + c}{4(\lambda_i' + L)}$$

Claim 4

The martingale property of the beliefs also holds when the previous consumer bought the good with some probability in $(0,1)$:

$$\begin{aligned}
\mathbb{E}(\lambda_{t+1}|\lambda,\underline{q},\bar{q}) &= P\Big(\gamma \geq \frac{p}{\lambda+L}\Big)\sum_{i\in\{G,N,B\}} P(i)\lambda^i_{t+1} + P\Big(\gamma < \frac{p}{\lambda+L}\Big)\lambda \\
&= P\Big(\gamma \geq \frac{p}{\lambda+L}\Big)\lambda\left[\int_{\bar{q}}^{q^K} f_H(q)dq + \int_{\underline{q}}^{\bar{q}} f_H(q)dq + \int_{q^0}^{\underline{q}} f_H(q)dq\right] + P\Big(\gamma < \frac{p}{\lambda+L}\Big)\lambda \\
&= P\Big(\gamma \geq \frac{p}{\lambda+L}\Big)\lambda + P\Big(\gamma < \frac{p}{\lambda+L}\Big)\lambda \\
&= \lambda
\end{aligned}$$

Lemma 1

Assume that the first period consumer bought the good and that completing reviews is not costly. Consider two alternative social rules: one that uses two messages and another one that uses three messages. Each rule determines a distribution of posterior beliefs with mean λ (because of the martingale property of beliefs). Given that consumers' payoff is convex in λ', they prefers the rule with three messages over the one with two messages if and only if the second distribution of posterior beliefs is a mean preserving spread of the first one.[40]

Consider first a norm such that consumers complete a bad review if $q_1 \leq \hat{q}$ and a good review otherwise. Denote by λ^- the beliefs of second period consumers after observing a bad review and by λ^+ their beliefs after observing a good review. Denoting by $F(\cdot)$ the cumulative distribution of λ' induced by this norm, then: $F(\lambda^-) = P(\lambda' \leq \lambda^-) = P(q \leq \hat{q})$, $F(\lambda^+) = P(\lambda' \leq \lambda^+) = 1$. The expected value of λ' under this rule is:

$$\begin{aligned}
\mathbb{E}(\lambda'|\lambda,\hat{q}) &= P(\lambda^-)\lambda^- + P(\lambda^+)\lambda^+ \qquad (24)\\
&= [\lambda\int_{q_0}^{\hat{q}} f_H(q)dq + (1-\lambda)\int_{q_0}^{\hat{q}} f_L(q)dq]\frac{\lambda\int_{q_0}^{\hat{q}} f_H(q)dq}{\lambda\int_{q_0}^{\hat{q}} f_H(q)dq + (1-\lambda)\int_{q_0}^{\hat{q}} f_L(q)dq} + \\
&+ [\lambda\int_{\hat{q}}^{q^K} f_H(q)dq + (1-\lambda)\int_{\hat{q}}^{q^K} f_L(q)dq]\frac{\lambda\int_{\hat{q}}^{q^K} f_H(q)dq}{\lambda\int_{\hat{q}}^{q^K} f_H(q)dq + (1-\lambda)\int_{\hat{q}}^{q^K} f_L(q)dq} \\
&= \lambda[\int_{q_0}^{\hat{q}} f_H(q)dq + \int_{\hat{q}}^{q^K} f_H(q)dq] = \lambda
\end{aligned}$$

Consider an alternative rule in which first period consumers can send three different messages, G, N and B. They complete a bad review if the quality realisation was below a threshold $\underline{q}$, complete no reviews if $q_1 \in (\underline{q},\bar{q})$ and they complete a good review if $q_1 \geq \bar{q}$. Denote by λ^B, λ^N and λ^G consumers beliefs after observing a bad review, no review or a good review, respectively. Denote by $H(\cdot)$ the distribution of second period beliefs induced by this rule. Then, $H(\lambda^B) = P(\lambda' \leq \lambda^B) = P(q_1 \leq \underline{q})$, $H(\lambda^N) = P(\lambda_2 \leq \lambda^N) = P(q_1 \leq \bar{q})$ and $H(\lambda^G) = P(\lambda' \leq \lambda^G) = 1$. As shown in Claim 4, $\mathbb{E}(\lambda'|\lambda,\underline{q},\bar{q}) = \lambda$.

As both distributions of beliefs have the same mean and consumers' payoff function is convex in the beliefs, they will prefer the distribution induced by the second rule over the one induced by the first rule if the second one is a mean preserving spread of the first one or, equivalently, if $F(\cdot)$ dominates stochastically of second order $H(\cdot)$. A sufficient condition for this to be true is:

[40] Rothschild and Stiglitz (1970).

$$\int_0^{\lambda'} H(t)dt \geq \int_0^{\lambda'} F(t)dt \text{ for every } \lambda' \in [0,1] \tag{25}$$

Consider the case in which $\hat{q} = \bar{q}$.[41] In this case, $\lambda^N > \lambda^- > \lambda^B$ (see below) and $\lambda^+ = \lambda^G$. To show second order stochastic dominance it is necessary to show that condition (25) holds for every possible value of λ'. To start with, note that for $\lambda' < \lambda^-$ the distribution of beliefs under $H(\cdot)$ accumulates more mass than under $F(\cdot)$ because $F(\lambda' < \lambda^-) = 0$ while $H(\lambda' < \lambda^-) = H(\lambda^B) > 0$. The probability of observing a value of $\lambda' \in (\lambda^-, \lambda^N)$, on the other hand, is greater under $F(\cdot)$. However, as $P(\lambda^B)[\lambda^- - \lambda^B] = [P(\lambda^-) - P(\lambda^B)](\lambda^N - \lambda^-)$, where $P(\lambda^-) - P(\lambda^B) = P(\lambda^N)$,[42] both distributions accumulate the same mass for every $\lambda' > \lambda^N$. Then:

- For $\lambda' < \lambda^-$: $\int_0^{\lambda'} H(t)dt > \int_0^{\lambda'} F(t)dt$,
- For $\lambda' < \lambda^N$, $\int_0^{\lambda'} H(t)dt > \int_0^{\lambda'} F(t)dt$.
- For $\lambda' \geq \lambda^N$: $\int_0^{\lambda'} H(t)dt = \int_0^{\lambda'} F(t)dt$.

Then, the distribution of beliefs induced by the two-messages rule second order stochastically dominates the one induced by the three-messages rule. Together with the fact that both distributions have the same mean, this implies that $H(\cdot)$ is a mean preserving spread of $F(\cdot)$ and so consumers expected payoff is greater under the last rule. A similar result can be obtained for $\hat{q} = \underline{q}$ or for any other $\hat{q} \in (0,1)$.

The results above assume that $\lambda^N \leq \lambda^- \leq \lambda^B$. Now I show that those assumptions are correct.

$\lambda^- \leq \lambda^B \iff \frac{\int_{q^0}^{\underline{q}} f_L(q)dq}{\int_{q^0}^{\underline{q}} f_H(q)dq} \geq \frac{\int_{q^0}^{\hat{q}} f_L(q)dq}{\int_{q^0}^{\hat{q}} f_H(q)dq}$, which can be written as $\frac{F_H(\hat{q})}{F_H(\underline{q})} \geq \frac{F_L(\hat{q})}{F_L(\underline{q})}$. As we are assuming $\hat{q} = \bar{q}$, this is the same as $\frac{F_H(\bar{q})}{F_H(\underline{q})} \geq \frac{F_L(\bar{q})}{F_L(\underline{q})}$ which implies $\lambda^G(\bar{q}, \underline{q}) \geq \lambda^B(\bar{q}, \underline{q})$.

$\lambda^- \geq \lambda^N \iff \frac{F_L(\bar{q}) - F_L(\underline{q})}{F_H(\bar{q}) - F_H(\underline{q})} \geq \frac{F_L(\hat{q})}{F_H(\hat{q})}$. Using the fact that $\hat{q} = \bar{q}$, the previous condition becomes $\frac{F_L(\bar{q}) - F_L(\underline{q})}{F_H(\bar{q}) - F_H(\underline{q})} \geq \frac{F_L(\bar{q})}{F_H(\bar{q})}$, which holds because it implies $\lambda^G(\bar{q}, \underline{q}) \geq \lambda^N(\bar{q}, \underline{q})$.

Lemma 3

The result in Lemma 3 is an immediate implication of the results in the next two claims:

Claim 5. *For every $\lambda \in (0,1)$, $\bar{q} \in (q^0, q^K]$ there exists $\underline{h}(\lambda, \bar{q})$ such that $\frac{\partial V(\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0} - h\frac{\partial P(B;\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0} > 0$ for all $0 < h \leq \underline{h}(\lambda, \bar{q})$)*

Claim 6. *For every $\lambda \in (0,1)$, $\underline{q} \in [q^0, q^K)$ there exists $\bar{h}(\lambda, \underline{q})$ such that $\frac{\partial V(\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K} - h\frac{\partial P(G;\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K} < 0$ for all $0 < h \leq \bar{h}(\lambda, \underline{q})$).*

[41]A similar analysis can be done by assuming any other value of $\hat{q} \in [q^0, q^K]$ and the conclusions would not change.

[42]Given $\hat{q} = \bar{q}$, $P(\lambda^-) = P(\lambda^G)$, and $P(\lambda^G) - P(\lambda^B) = P(\lambda^N)$ by construction.

To see that the result in Claim 5 holds, consider what happens when $\underline{q} \to q^0$. From Lemma 1, $\frac{\partial V(\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0} > 0$ for every $\lambda \in (0,1)$ because when the cost of completing reviews is not taken into account, consumers' expected utility is greater when the reviewing rule uses the three available messages. On the other hand, $\frac{\partial P(B;\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0} = \lambda f_H(q^0) + (1-\lambda) f_L(q^0)$. If the distribution of quality realisations is such that $f_H(q^0) = f_L(q^0) = 0$, then $\frac{\partial P(B;\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0} = 0$ and the result in the Claim holds. If the distribution of quality realisations has fat tails and $f_H(q^0) > 0$ and $f_L(q^0) > 0$, then the result in the Claim holds as long as there exists $h > 0$ such that

$$\underline{h}(\lambda, \bar{q}) \leq \frac{\frac{\partial V(\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0}}{\frac{\partial P(B;\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0}} = \frac{\frac{\partial V(\mathcal{R},\lambda)}{\partial \underline{q}}|_{\underline{q}=q^0}}{\lambda f_H(q^0) + (1-\lambda) f_L(q^0)} \tag{26}$$

which holds because both, the numerator and the denominator are positive for every $\lambda \in (0,1)$ and for every $\bar{q} \in (q^0, q^K]$.

An analogous argument can be used to prove Claim 6. In this case the condition for consumers to prefer $\bar{q} < q^K$ is that $\frac{\partial V(\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K} - h\frac{\partial P(G;\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q_K} < 0$. From Lemma 1, as $\bar{q}$ moves away from q^K, consumers' expected utility $V(\cdot)$ increases; as a result, $\frac{\partial V(\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K} < 0$. Furthermore, $\frac{\partial P(G;\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K} = -[\lambda f_H(q^K) + (1-\lambda) f_L(q^K)] \leq 0$, with strict inequality if the distribution of quality realisations has "fat tails". If $f_H(q^K) = f_L(q^K) = 0$, the condition in the claim holds for every $\lambda \in (0,1)$ and for every $h > 0$. Otherwise, if the distribution of quality realisations assigns positive probability to the tails, the condition in the claim becomes:

$$\bar{h}(\lambda, \underline{q}) \leq -\frac{\frac{\partial V(\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K}}{\frac{\partial P(G;\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K}} = -\frac{\frac{\partial V(\mathcal{R},\lambda)}{\partial \bar{q}}|_{\bar{q}=q^K}}{\lambda f_H(q^K) + (1-\lambda) f_L(q^K)} \tag{27}$$

which is positive because, as mentioned above, the numerator is negative and the denominator is positive.

B Monotone Likelihood Ratio Property

For any $\theta_0, \theta_1 \in \Theta$ such that $\theta_1 \geq \theta_0$, MLRP implies $\frac{1-F(\bar{q}|\theta_1)}{F(\bar{q}|\theta_1)} \geq \frac{1-F(\bar{q}|\theta_0)}{F(\bar{q}|\theta_0)}$. The claim below shows why this is the case.

Claim 7. *$f(\cdot|\theta)$ satisfies MLRP, then for any $\theta_0, \theta_1 \in \Theta$ such that $\theta_1 \geq \theta_0$ we have $\frac{1-F(\bar{q}|\theta_1)}{F(\bar{q}|\theta_1)} \geq \frac{1-F(\bar{q}|\theta_0)}{F(\bar{q}|\theta_0)}$ and $\frac{1-F(\underline{q}|\theta_1)}{F(\underline{q}|\theta_1)} \geq \frac{F(\underline{q}|\theta_0)}{F(\underline{q}|\theta_1)}$.*

Proof. The family of densities $f(\cdot|\theta)$ satisfies the monotone likelihood ratio property if for all $q_1 \geq q_0$ and $\theta_1 \geq \theta_0$ we have:

$$f(q_1|\theta_1) f(q_0|\theta_0) \geq f(q_0|\theta_1) f(q_1|\theta_0) \tag{28}$$

Integrating both sides of this expression over q_0 from 0 (lower bound of q) to q_1:

$$\int_0^{q_1} f(q_1|\theta_1) f(q_0|\theta_0) dq_0 \geq \int_0^{q_1} f(q_0|\theta_1) f(q_1|\theta_0) dq_0$$

$$f(q_1|\theta_1)F(q_1|\theta_0) \geq F(q_1|\theta_1)f(q_1|\theta_0)$$

Let $a = q_1$, we have:

$$\frac{f(a|\theta_1)}{f(a|\theta_0)} \geq \frac{F(a|\theta_1)}{F(a|\theta_0)} \tag{29}$$

Integrating both sides of (28) with respect to q_1, from q_0 to 1 (upper bound of q):

$$\int_{q_0}^{1} f(q_1|\theta_1)f(q_0|\theta_0)dq_1 \geq \int_{q_0}^{1} f(q_0|\theta_1)f(q_1|\theta_0)dq_1$$

Let $a = q_0$, we have:

$$\frac{1 - F(a|\theta_1)}{1 - F(a|\theta_0)} \geq \frac{f(a|\theta_1)}{f(a|\theta_0)} \tag{30}$$

Combining (29) and (30), we have:

$$\frac{1 - F(a|\theta_1)}{1 - F(a|\theta_0)} \geq \frac{F(a|\theta_1)}{F(a|\theta_0)}$$

This result holds for any $a \in [0, 1]$. In particular: $\frac{1-F(\bar{q}|\theta_1)}{1-F(\bar{q}|\theta_0)} \geq \frac{F(\bar{q}|\theta_1)}{F(\bar{q}|\theta_0)}$ and $\frac{1-F(\underline{q}|\theta_1)}{1-F(\underline{q}|\theta_0)} \geq \frac{F(\underline{q}|\theta_1)}{F(\underline{q}|\theta_0)}$.

□

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