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Nuclear Physics B (Proc. Suppl.) 83–84 (2000) 956–958

NUCLEAR PHYSICS B
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A Hamiltonian Many-Body Approach to SU(N) Lattice Gauge Theory

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We study the low-lying energy spectrum for the SU(N) one-plaquette problem by use of a maximal-tree gauge and angular variables. We show that the one-plaquette problem reduces to a problem of N fermions on a circle, which is solved numerically.

1. INTRODUCTION

In order to solve a field theory from first principles, one must deal with the large number of degrees of freedom involved. Monte Carlo techniques are the preferred approach, and state-of-the-art numerical results for masses and other static ground-state properties for QCD all originate from Euclidean field theory treated using Monte Carlo methods. However, some questions cannot readily be answered in Euclidean field theory, and sometimes the relation between the real-time (Hamiltonian) results and their Euclidean counterpart is not clear. The vacuum wave functional, for example, can only be defined in a Hamiltonian approach.

Although the QCD vacuum is rich with non-perturbative phenomena, which leave their imprint on the particles which interact strongly, it is invariant under space-group transformations. Therefore, the many degrees of freedom of the system are restricted to a subset invariant under these space symmetries. The coupled cluster method [1] naturally uses theses symmetries, as one can restrict the wave functional to contain correlation operators that depend not on the absolute position but only on the relative orientation of the correlated partners (i.e., plaquettes). Similarly, the vacuum is invariant under gauge transformations which can be implemented from the start.

However, we must construct a Hamiltonian which requires that the gauge is fixed so the canonical variables can be determined, and the theory quantised. In the continuum theory this problem can only be dealt with perturbatively. On the lattice we can use the maximal-tree gauge fixing [2], which results in a proper set of conjugate variables.

Eventually, we can determine the ground-state energy for different trial wave functionals that can be improved systematically within the coupled cluster method. In order to filter out the special properties of SU(3), we study SU(N) gauge theories for several values of N. For present purposes we restrict ourselves to a relatively simple wave functional, in order to investigate useful representations of the problem.

2. THEORY

In the maximal-tree gauge the links [3] on a maximal tree, which connects all lattice sites in an unique way (Fig. 1), are associated with the irrelevant gauge degrees of freedom. The remaining links, $\{m\}$, when connected with a path from and to the origin (Fig. 1), are the canonical variables, X_m, of the Hamiltonian theory in the colourless sector [2]. The electric operator E_l^a associated with the link l on the maximal tree can be transformed by a body-frame rotation, such that when acting on the X-variables one of the following the relations, depending on the position of the link,

$$E_l^a X_m = -\frac{1}{2}\lambda^a X_m \quad , \quad E_l^a X_m = \frac{1}{2} X_m \lambda^a \quad ,$$

$$E_l^a X_m = -\frac{1}{2}\lambda^a X_m + \frac{1}{2} X_m \lambda^a \quad , \tag{1}$$

where l is part of the path leading up to the the link m in the first case, part of the path from the link m back to the origin in the second case, and part of both paths in the third case. Therefore the electric operator generate long-distance

PII S0920-5632(00)00373-X

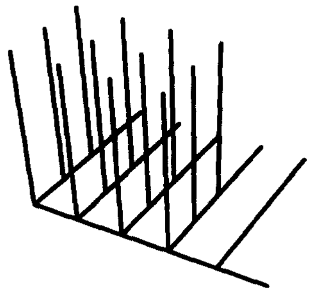

Figure 1. A particular choice of maximal tree.

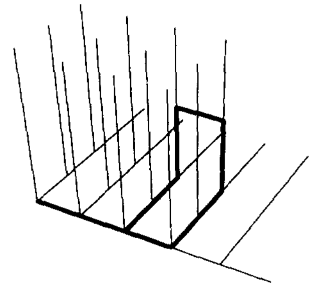

Figure 2. A contour, from and to the origin, associated with a canonical variable.

interactions between two variables X_m and $X_{m'}$ associated with two links m and m' on the lattice.

After establishing the independent variables, X_m, and the canonically conjugate variables, E_l^a, we can investigate some simple trial wave functionals. Two wave functionals spring immediately to mind; the sum and the product of functions of one-plaquette [3] variables:

$$\langle\{U\}|\Sigma\rangle = \sum_{\text{plaquettes } \alpha} f(\mathrm{Tr}U_\alpha) \ , \tag{2}$$

$$\langle\{U\}|\Pi\rangle = \prod_{\text{plaquettes } \alpha} f(\mathrm{Tr}U_\alpha) \ , \tag{3}$$

where the products of links, U_α, around a plaquette can be expressed as products of X's and $X^\dagger$'s. Both wave functionals lead to the direct sum of one-plaquette problems, as the cross term in the electric operator vanishes under integration ($\alpha \neq \beta$):

$$\langle\Pi|[E_l^a \ln f(\mathrm{Tr}U_\alpha)][E_l^a \ln f(\mathrm{Tr}U_\beta)]|\Pi\rangle = 0 \quad . \tag{4}$$

For the one-plaquette problem the functions depend on the eigenvalues x_i of U only [4]. It can be expressed in a basis of group characters [5], labelled by the partition $\lambda_1 > \cdots > \lambda_N = 0$, which are the eigenstates of the electric operator. However, for the weak-coupling limit it is useful to notice that the group characters are ratios of determinants $\det[x_i^{\lambda_j}]/\det[x_i^{j-1}]$, where x_i^{j-1} and $x_i^{\lambda_j}$ denote the entries of the matrix, and the one-plaquette problem can be mapped onto an N-fermion problem on a circle if we introduce angular variables $\exp\{i\phi_i\} = x_i$, satisfying the constraint $\sum_{i=1}^N \phi_i = 0$. We multiply the original, symmetric wave function by $\det[x_i^{j-1}]$ to generate a completely antisymmetric wave function. This simplifies the Hamiltonian considerably. The electric operator yields the kinetic energy of N particles on a circle minus their centre-of-mass energy.

3. RESULTS AND CONCLUSIONS

For the low-lying spectra shown in Fig. 2 we solve the linear eigenvalue equations numerically. For an accuracy of six decimal places we require a basis with approximately 100-200 eigenstates of the electric operator. For SU(2), the spectrum is given by the odd characteristic values of the Mathieu equation. For $N = 3, 4$, and 5, the spectra are much richer. They include both crossings, indicating the presence of a symmetry, and avoided crossings. These features are being investigat-

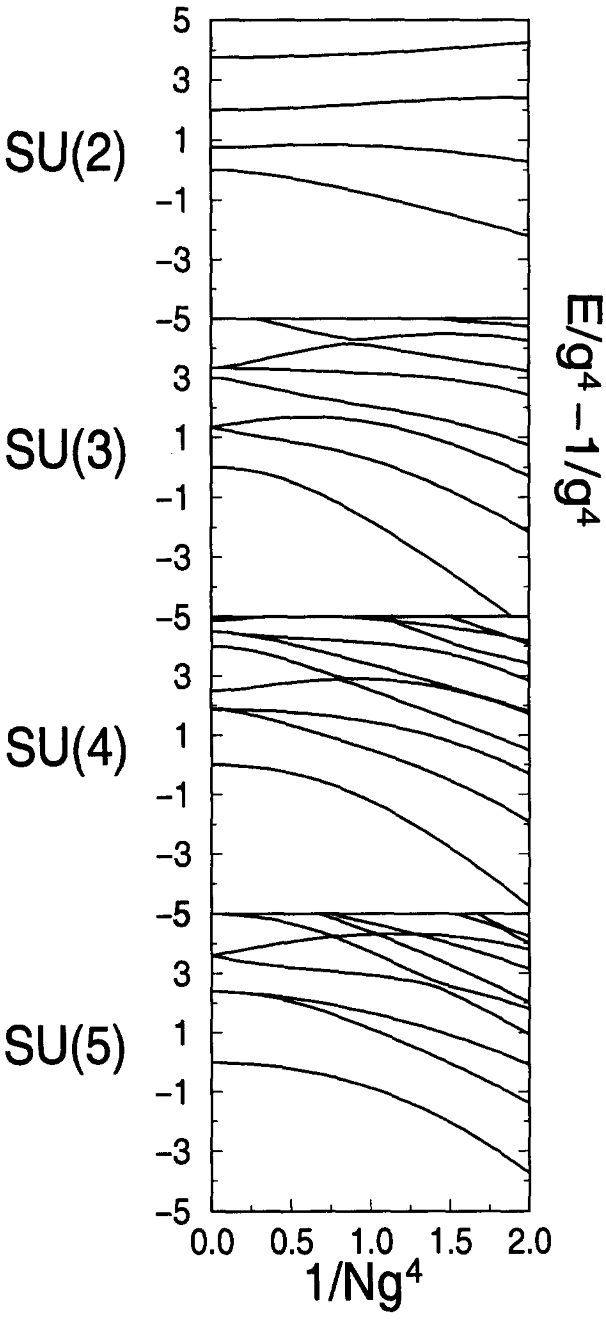

Figure 3. The spectra for the SU(2), SU(3), SU(4), and SU(5) one-plaquette problem, after subtraction of the linear term g^{-4}.

ed.

The extension of this approach to include spatial correlations is also under investigation. The simplicity of the angular variables formulation is promising for more elaborate wave functionals [1]. We hope that the one-plaquette wave functional can serve as a first step in describing a spatially correlated wave functional.

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