



The University of Manchester Research

# Analysis and structural behavior of cylindrical steel tanks under seismic effects

## Link to publication record in Manchester Research Explorer

## Citation for published version (APA):

Maraveas, C. (2011). Analysis and structural behavior of cylindrical steel tanks under seismic effects. In *The 12th International Conference on Metal Structures – ICMS, Wrocaw 2011* (pp. 476-485)

## **Published in:**

The 12th International Conference on Metal Structures - ICMS, Wrocaw 2011

## Citing this paper

Please note that where the full-text provided on Manchester Research Explorer is the Author Accepted Manuscript or Proof version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version.

## **General rights**

Copyright and moral rights for the publications made accessible in the Research Explorer are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

## Takedown policy

If you believe that this document breaches copyright please refer to the University of Manchester's Takedown Procedures [http://man.ac.uk/04Y6Bo] or contact uml.scholarlycommunications@manchester.ac.uk providing relevant details, so we can investigate your claim.



## Analysis and structural behavior of cylindrical steel tanks under seismic effects

C. Maraveas\*

C. Maraveas Partnership, Athens, Greece

*ABSTRACT:* The use of large steel oil storage tanks that provide adequate safety and reasonable economy has always been an issue of major concern for the petroleum industry. To this end, efficient design and comprehensive analysis of such structures is important. This paper addresses specific issues regarding the analysis and behavior of cylindrical self supported tanks under earth-quake loading and more importantly their seismic design, by comparing current codes used in practice with the finite element method (FEM). The results refer to two large cylindrical tanks and they show that even though the current design methods fail to describe their exact behavior, they do not violate fundamental safety considerations.

Keywords: tanks, steel, seismic loading

## 1. INTRODUCTION

## 1.1. Scope of work

The response of tanks to seismic excitation has always been a major concern for their safety and efficient design. When imposed on tanks, earthquake action can cause various types of failure, namely: i) detachment of the base plate from the circumferential shell ii) "elephant's foot" failure type iii) detachment of the piping connected to the tank iv) damage to the shell resulting from hydro-dynamic pressure v) sinking of the floating roof (if any). This paper attempts to investigate the resourcefulness of current codes regarding the seismic response of steel tanks by comparison with the Finite Element Method, in order for such failures to be prevented. For this purpose, two large cylindrical steel tanks designed according to the American Standard API 650 [1] were analyzed following the principals of the Finite Element Method. Furthermore, analysis of the two tanks based on the specifications of Eurocode 8 [2] and the Greek seismic code EAK 2000 [3] was performed in order to examine their applicability on the topic.

## 1.2. Theoretical background of analysis

The determination of the seismic response of tanks requires extensive and rigorous analysis, which, in most cases, demands long computational times. Despite the existing codes, the simplifying assumptions and analyses that have been performed since the early seventies, the major issue of understanding their exact seismic behavior is still incomplete and under investigation. Until recently, publications of relative research work focused on approximate procedures and simulation models in an attempt to approach the subject. However, in any case, specialized knowledge and

<sup>\*</sup> corresponding author: c.maraveas@maraveas.gr

reliable assumptions regarding the interaction between tank and contained liquid are necessary in order to simulate the complex effects of hydrodynamic loading (such as the impulsive pressure, the convective pressure component etc.) in conjunction with generating a compatible finite element mesh for the shell of the steel tank.

#### 1.3. Comments on current design philosophy

Based on the general provisions of the American Standard API 650 [1] and Eurocode 8 [2], the analysis and behavior of tanks subjected to earthquake loading focuses on the following: tanks of cylindrical shape; rigid or elastic foundation; tanks based fully or partially on their foundation. According to the current design philosophy, two distinct modes of vibration for the contained liquid of rigid vertical tanks subjected to horizontal ground motion are defined: the rigid impulsive mode, in which the contained liquid follows the rigid motion of the tank and the convective mode, in which the contained liquid moves vertically due to the incompatibility with the shell of the tank. This vertical motion of the liquid surface is called sloshing. Greiner and Kettler [4] also mention a third mode for flexible tanks, the "flexible impulsive mode" which accounts for the relative displacement of liquid and flexible wall with respect to the bottom plate of the tank.

The American Standard API 650 [1] provides – via the definition of certain parameters and specifications – guidelines and an analytical procedure by which to design the aforementioned tanks. Based on the geometric characteristics of the tank, the respective maximum hydrostatic pressure as well as its seismic response is determined. A unique element of the standard is its applicability on all continents, because it takes into account the seismicity and ground acceleration of the region in which the tank will be constructed.

On the other hand, Eurocode 8 [2] is constrained to more simplified methods and applications which follow the international state-of-the-art trends on the topic. The background of the code is more theoretical and its major application field is cylindrical tanks anchored on rigid foundation. An important feature relevant the design of tanks is that it includes an analytical relationship for the calculation of the maximum wave height due to seismic action. It should be mentioned, however, that the code is still being developed and additions are to be made. Therefore, in certain cases, bibliographical references are given in order to provide insight for issues not completely covered by the specific code.

#### 1.4. Description of the steel tanks

Two large cylindrical flat-bottom steel tanks (T-776 and T-761) were designed for the oil refinery of Motor Oil Hellas S.A., in Korinthos, Greece. The tanks are self-supported, unanchored and tank T-776 supports a cone roof at its top. Their geometric characteristics as well as steel grade are given in Table 1. For the purpose of investigating their behavior to seismic excitation, several cases in which the tanks were filled at different liquid levels were studied. The six cases in total (four for tank T-776 and two for tank T-761) are also listed in Table 1.

Tank ID Case ID Liquid Tank Tank Steel Grade Steel F   level (m) Height (mm) Diameter (mm) Terre Diameter (mm) Terre							
level (m) Height (mm) Diameter (mm)   T-776 I 18.83 20032 46950.13 BS4360 GR50C Yes   T-776 II 14.12 20032 46950.13 BS4360 GR50C Yes   T-776 III 9.42 20032 46950.13 BS4360 GR50C Yes   T-776 IV 4.70 20032 46950.13 BS4360 GR50C Yes   T-776 IV 4.70 20032 46950.13 BS4360 GR50C Yes   T-761 I 16.83 20350 88556.10 ASTM A36 M No   T-761 II 11.40 20350 88556.10 ASTM A36 M No	Tank ID	Case ID	Liquid	Tank	Tank	Steel Grade	Steel Roof
T-776 I 18.83 20032 46950.13 BS4360 GR50C Yes   T-776 II 14.12 20032 46950.13 BS4360 GR50C Yes   T-776 III 9.42 20032 46950.13 BS4360 GR50C Yes   T-776 III 9.42 20032 46950.13 BS4360 GR50C Yes   T-776 IV 4.70 20032 46950.13 BS4360 GR50C Yes   T-761 I 16.83 20350 88556.10 ASTM A36 M No   T-761 II 11.40 20350 88556.10 ASTM A36 M No			level (m)	Height (mm)	Diameter (mm)		
T-776 II 14.12 20032 46950.13 BS4360 GR50C Yes   T-776 III 9.42 20032 46950.13 BS4360 GR50C Yes   T-776 IV 4.70 20032 46950.13 BS4360 GR50C Yes   T-761 I 16.83 20350 88556.10 ASTM A36 M No   T-761 II 11.40 20350 88556.10 ASTM A36 M No	T-776	Ι	18.83	20032	46950.13	BS4360 GR50C	Yes
T-776 III 9.42 20032 46950.13 BS4360 GR50C Yes   T-776 IV 4.70 20032 46950.13 BS4360 GR50C Yes   T-761 I 16.83 20350 88556.10 ASTM A36 M No   T-761 II 11.40 20350 88556.10 ASTM A36 M No	T-776	II	14.12	20032	46950.13	BS4360 GR50C	Yes
T-776 IV 4.70 20032 46950.13 BS4360 GR50C Yes   T-761 I 16.83 20350 88556.10 ASTM A36 M No   T-761 II 11.40 20350 88556.10 ASTM A36 M No	T-776	III	9.42	20032	46950.13	BS4360 GR50C	Yes
T-761 I 16.83 20350 88556.10 ASTM A36 M No   T-761 II 11.40 20350 88556.10 ASTM A36 M No	T-776	IV	4.70	20032	46950.13	BS4360 GR50C	Yes
T-761 II 11.40 20350 88556.10 ASTM A36 M No	T-761	Ι	16.83	20350	88556.10	ASTM A36 M	No
	T-761	II	11.40	20350	88556.10	ASTM A36 M	No

Table 1. Geometric characteristics of tanks.

## 2. API 650 SEISMIC PROVISIONS FOR LARGE STORAGE TANKS

#### 2.1. General comments and procedure

The current provisions of API 650 [1] specify a design procedure against seismic action (Appendix E – seismic design of storage tanks), during which seismic lateral forces and the respective overturning moment are calculated. This is achieved by defining two "effective" masses for the contained liquid, in order to account for the two pressure components (rigid impulsive and convective corresponding to the fundamental sloshing mode) resulting from the imposed earthquake excitation. The code requirements sufficiently deal with the overturning stability of the tank and the buckling of its circumferential shell due to compressive stresses. However, the code does not provide a method for calculating the tensile stresses developed under seismic loading, but simply states that they will remain at acceptable levels when the calculated lateral forces are applied. Despite providing a formula for the period of the first sloshing mode, the code does not include one for the maximum wave height.

#### 2.2. Calculations regarding the tanks

Following the above design procedure for horizontal acceleration 0.24g (Korinthos, Greece) and a soil parameter  $S_3 = 1.5$  (per API 650 code [1]), the effective masses  $W_1$  (effective mass content that moves in unison with the tank shell) and  $W_2$  (effective mass content that moves in the first sloshing mode), the overturning moment and the maximum longitudinal compressive stress for the two tanks are calculated and summarized in Table 2.

Tank ID	Liquid level (m)	Effective mass W1 (kN)	Effective mass W <sub>2</sub> (kN)	Overturning Moment (kNm)	Max longitudinal comprsssive stress (MPa)
	10.02	11//01	11//01	0.10.6.10	0.00
T-776	18.83	146624	146624	243649	0.60
T-776	14.12	87959	146598	117182	0.33
1 //0	14.12	01757	140570	11/102	0.55
T-776	9.42	34230	114101	34759	0.15
T-776	4.70	14639	65062	7481	0.11
T-761	16.83	201735	726249	299731	0.24
T-761	11.40	136850	547402	137996	0.18

Table 2. Calculations according to API 650.

## 3. SEISMIC DESIGN OF TANKS ACCORDING TO EUROCODE 8 AND EAK 2000

## 3.1. Code provisions

Regarding the response of tanks to seismic action, Eurocode 8 [2] (Annex A, EN 1998-4) focuses on rigid vertical cylindrical tanks. More specifically, it provides analytical expressions for calculating the rigid impulsive pressure and its resultants (the impulsive base shear and moment), as well as the convective pressure and its resultants (the convective base shear and moment) that originate from lateral ground motion. The maximum wave height is also given analytically, based on the first sloshing mode. Furthermore, the Eurocode [2] also permits the omission of the tank's shell mass contribution to the sum of the dynamic and inertia forces, given the fact that it is much smaller than the hydrodynamic forces.

On the contrary, the Greek seismic code EAK 2000 [3] does not provide specific expressions or a design procedure for the seismic response of tanks. Nevertheless, it defines a response spectrum that is better adjusted to the Greek seismicity and can be used in conjunction with the formulas of Eurocode 8 [2]. Figure 1 presents graphically the response spectra defined in EAK 2000 [3] and Eurocode 8 (EN 1998-1) [5] for various soil conditions.



Figure 1: Response spectra per EAK 2000 (left side) and Eurocode 8 (right side).

#### 3.2. Calculations regarding the tanks

Taking advantage of the fact that analytical expressions are defined in Eurocode 8 [2], the rigid impulsive pressure, the convective pressure and maximum wave height of the tanks were calculated and are summarized in Table 3. Moreover, because the Greek seismic code EAK 2000 [3] does not provide such expressions, the respective formulas of Eurocode 8 [2] were used in combination with the EAK 2000 [3] response spectrum and the results are summarized in Table 4.

Tank ID	Liquid level (m)	Maximum rigid impulsive pres- sure (kPa)	Maximum convective pres- sure (kPa)	Maximum wave height (m)
T-776	18.83	55.201	11.875	1.13
T-776	14.12	47.315	6.683	0.64
T-776	9.42	30.755	2.084	0.20
T-776	4.70	14.195	0.997	0.10
T-761	16.83	41.656	2.138	0.18
T-761	11.40	17.852	1.882	0.18

Table 3. Seismic response calculations according to the Eurocode 8.

Table 4. Seismic response	calculations acco	rding to the	Greek	seismic	code	EAK	2000.
---------------------------	-------------------	--------------	-------	---------	------	-----	-------

Tank ID	Liquid level (m)	Maximum rigid impulsive pres- sure (kPa)	Maximum convective pres- sure (kPa)	Maximum wave height (m)
T-776	18.83	51.26	33.93	3.24
T-776	14.12	43.94	28.61	2.73
T-776	9.42	28.56	25.33	2.42
T-776	4.70	13.18	21.83	2.08
T-761	16.83	38.68	46.84	3.93
T-761	11.40	16.58	36.06	3.39

## 4. FINITE ELEMENT SIMULATIONS OF THE TANKS

The investigation of the behavior of the tanks under seismic action was conducted with the application of the Finite Element Method (FEM). For each tank and level of contained fluid, a separate 3D finite element model was created. More specifically, all the relevant analyses were performed using the commercial program ANSYS [6], which is widely used for simulating similar systems that involve solid and liquid mass interaction. Four finite element models were created for tank T-776 and two for tank T-761. A detailed description regarding the generation and meshing of the finite element models for each tank follows.

4.1. Description and generation of the 3D finite element models

For the geometrical shape of the tank and more specifically for the simulation of the circumferential shell, the bottom plate and the steel roof (Tank T-776 only), isoparametric 3-node (triangular) and 4-node shell finite elements (ANSYS "shell 63" elements [6]) were used. These elements combine the membrane action and bending and have six degrees of freedom per node.

The simulation of the contained liquid was accomplished by the use solid elements (ANSYS "solid 45" elements [6]). These elements have three translational degrees of freedom per node and were given uncompressible fluid properties in order to simulate the behavior of the contained liquid mass effectively. This type of simulation has been widely used by many researchers [4], [7], [8] to reliably simulate the translational motion of liquids under static and dynamic loading.

The meshing and the generation of the finite elements for the circumferential shell of the tanks were based on the range of the width of the plates that were used to construct them. This mesh simulates the real situation effectively and gives the opportunity to obtain reliable results, especially regarding stresses at sensitive parts of the shell, such as the welding connecting the plates. Furthermore, every shell finite element was given the appropriate thickness and material properties based on the actual geometric characteristics and steel grade mentioned. Damping for both tanks was assumed to be 2%. Figures 2 and 3 show the shell and solid elements used in the simulation models for tanks T-776 and T-761 respectively.



Figure 2: Shell finite elements for the simulation of the circumferential wall, the base plate and roof (left side) and solid finite elements for the simulation of the contained liquid (right side) for Tank T-776.



Figure 3: Shell finite elements for the simulation of the circumferential wall and the base plate (left side) and solid finite elements for the simulation of the contained liquid (right side) for Tank T-761.

## 4.2. Boundary conditions

The reliability of the simulations and consequently the legitimacy of the results depend on the application of the correct boundary conditions between the two subsystems (liquid-steel tank). More specifically, boundary conditions were applied: i) on the top liquid surface incorporating the linear wave theory for the simulation of the maximum wave height due to seismic loading ii) between the circumferential shell of the tank and the contained liquid by proper kinematic coupling among the pertinent solid-shell nodes iii) between the base plate and the and the contained liquid by application of proper kinematic constraints on the nodes of the solid elements iv) among the shell elements of the roof and the circumferential shell to simulate their connection v) between the overall tankliquid system and the soil.

## 4.3. Imposed Loading

The loads that were taken into account for the analyses are the following:

- Dead load from the self-weight of the steel tanks
- Dead load from the weight of the contained liquid (conservatively assumed to be water)
- Uniform live load at the roof of tank T-776, with a characteristic value of q<sub>k</sub> = 1.20kPa (according to API 650 [1])
- Seismic loading (inputted as dynamic loading) according to Eurocode 8 [5]

## 4.4. Analysis Procedure

Initially, linear static analysis of the tanks and the contained liquid for each case previously explained was performed. In this type of analysis, only gravity loads were applied and the maximum hydrostatic pressure on the shell, as well as the reactions on the base plate were calculated.

Following that, a modal analysis for each model was performed, according to which the modal frequencies, periods and for the system tank-liquid mass were calculated. The total vibrated mass was calculated based on the defined specific weight of the shell elements (simulating the steel tank) and the solid elements (simulating contained liquid mass).

Even though it might be expected that the first modal shape would be adequate for drawing necessary conclusions regarding the seismic response characteristics (such as the maximum wave height) of each tank, a large number of modal shapes (over 100) was extracted in order to achieve a better simulation of its behavior. Next, spectral analyses of the tanks were performed according to the pertaining response spectrum (specified in Eurocode 8 [5]) and the maximum response (deflection, stress) for each mode was calculated. The final stresses and deflections due to seismic were derived by combining the respective modal response values according to the Complete Quadratic Combination (CQC) rule for 2% of the critical damping. It should be noted that due to the symmetrical geometry of the tanks only one spatial direction was examined.

## 4.5. Analysis Results

The FEM analysis results focus on the generated principal stresses and the maximum wave height resulting from earthquake loading. Figures 4 to 9 show the two principal stresses ( $\sigma_1$  and  $\sigma_2$ ) and Figures 10 and 11 the maximum wave height of the tanks. Table 5 summarizes the maximum values of the stresses (tensile and compressive) and the maximum wave height as well as the first natural vibration period of the studied tanks.



Figure 4: 1st Principal Stress (left) and 2nd Principal Stress (right) due to seismic excitation for Tank T-776 / Case I (max liquid level: 18.83m).



Figure 5: 1st Principal Stress (left) and 2nd Principal Stress (right) due to seismic excitation for Tank T-776 / Case II (max liquid level: 14.12m).



Figure 6: 1st Principal Stress (left) and 2nd Principal Stress (right) due to seismic excitation for Tank T-776 / Case III (max liquid level: 9.42m).



Figure 7: 1st Principal Stress (left) and 2nd Principal Stress (right) due to seismic excitation for Tank T-776 / Case IV (max liquid level: 4.70m).



Figure 8: 1st Principal Stress (left) and 2nd Principal Stress (right) due to seismic excitation for Tank T-761 / Case I (max liquid level: 16.38m).



Figure 9: 1st Principal Stress (left) and 2nd Principal Stress (right) due to seismic excitation for Tank T-761 / Case II (max liquid level: 11.40m).



Figure 10: Maximum wave height for Tank T-776: Case I (bottom right), Case II (bottom left), Case III (top right) and Case IV (top left).



Figure 11: Maximum wave height for Tank T-761: Case I (left) and Case II (right).

Tank ID	Liquid level (m)	Maximum Tensi- le stress (MPa)	Maximum compressive stress (MPa)	Maximum wave height (m)	First natural vibration pe- riod (sec)
T-776	18.83	262.77	4.80	0.96	6.50
T-776	14.12	211.74	11.59	0.81	8.11
T-776	9.42	166.93	11.20	0.64	9.41
T-776	4.70	97.25	17.26	0.28	12.03
T-761	16.83	184.02	16.01	0.32	12.21
T-761	11.40	134.92	10.90	0.23	14.80

Table 5. Maximum tensile stresses, compressive stresses and wave height due to earthquake loading presented together with the first natural vibration period of the analysed tanks (FEM analysis).

As expected, the tensile stresses increase with the increase of liquid level, whereas more mass is being vibrated and the hydrodynamic forces are greater. The location of maximum tensile stress is close to the liquid surface, because it is influenced by the pressure resulting from the generated wave. The compressive stresses on the other hand generally reduce with increasing liquid level, with the geometry of the tank (diameter, height and steel roof) playing a significant role. The maximum wave height increases with the liquid level, as a result of the reduction in the sloshing period. It should also be noted that the first natural vibration modal shape for each analysed case is very similar to the figures depicting the maximum wave height (Figure 10 and Figure 11).

#### 5. COMPARISON AMONG CODES AND FEM ANALYSIS

#### 5.1. Stresses

Taking into account the fact that API 650 [1] is design-oriented and that Eurocode 8 [2] does not provide formulas for directly calculating stresses due to seismic loading, a direct comparison with the FE Method is not possible. It can be stated, however, that because the tanks were designed according to API 650 [1] and the maximum tensile stresses are below the yield stress of the material, the FEM analysis produces results in agreement with the current design philosophy. It should also be noted that the actual stresses are less than those produced by the FEM analysis, based on the fact that the contained liquid was assumed to be water. A direct comparison (Table 6) of the compressive stresses resulting from the FEM analysis with the values calculated according to API 650 [1] shows that the latter are significantly lower. These stresses, however, are still well below the design limit of the API 650 code [1].

Tank ID	Liquid level (m)	API 650 (MPa)	FEM Analysis (MPa)
T-776	18.83	0.60	4.80
T-776	14.12	0.33	11.59
T-776	9.42	0.15	11.20
T-776	4.70	0.11	17.26
T-761	16.83	0.24	16.01
T-761	11.40	0.18	10.90

Table 6. Comparison of maximum compressive stresses.

## 5.2. Maximum wave height

Table 7 summarizes the maximum wave height comparison among the two codes (Eurocode 8 [2] and EAK 2000 [3]) and the FEM analysis. It can be observed that the values obtained from Eurocode 8 [2] and FEM analysis are comparable, while the Greek seismic code EAK 2000 [3] yields much higher results (as high as approximately 20 for low liquid levels). This large discrepancy can be attributed to the fact that the response spectrum of EAK 2000 [3] has a lower bound of 0.25 (Figure 1), hence even very high sloshing periods have a non-negligible contribution to the maximum wave height. On the contrary, the Eurocode [5] calculates the same values with a response spectrum that converges to zero for higher values and thus produces similar results with the FEM analysis.

Tank ID	Case ID	Liquid level (m)	EAK 2000 (m) (Analytical Calculations)	Eurocode 8 (m) (Analytical Cal- culations)	FEM Analysis (m)
T-776	Ι	18.83	3.24	1.13	0.96
T-776	II	14.12	2.73	0.64	0.81
T-776	III	9.42	2.42	0.20	0.64
T-776	IV	4.70	2.08	0.10	0.28
T-761	Ι	16.83	3.93	0.18	0.32
T-761	II	11.40	3.39	0.18	0.23

Table 7.	Com	parison	of ma	aximum	wave	height	for the	containe	d liqu	uid o	during	g earthc	Juake	excitatio	n.
						<i>u</i>						-			

## 6. CONCLUSIONS

- 1. Compressive stresses calculated according to both API 650 [1] and FEM analysis (despite the discrepancies in their magnitude) are very low (below 35% of the acceptable limit determined in API 650 [1]) and in general are not expected to cause local buckling of the tank's shell during earthquake excitation.
- The maximum wave height can be calculated with satisfactory accuracy according to Eurocode 8 [2]. On the other hand, the results obtained from the response spectrum of the Greek seismic code EAK 2000 [3] are unrealistic due to the lower bound specified for its high-period range.
- 3. Despite being conservative, the FEM analysis tensile stresses do not exceed the yield stress of the material at any point, strengthening the argument that current design methods (even though approximate) produce results on the safe side.

## ACKNOWLEDGEMENTS

I would like to thank Motor Oil Hellas, Korinthos Refineries S.A. and the supervisor of the project Mrs Sofia Dimitrakaki for their contribution.

## REFERENCES

- [1] API Standard 650: 1998. *Welded Steel Tanks for Oil Storage*. Tenth Edition, Addendum 1 (2000), Addendum 2 (2001), Addendum 3 2003.
- [2] EN 1998-4:2006. Design of structures for earthquake resistance Part 4: Silos, tanks and pipelines.
- [3] Greek code for Seismic Resistant Structures EAK 2000:2000.
- [4] Greiner R., Kettler M.: Seismic design calculations of large cylindrical flat-bottom storage tanks. National Steel Conference, Xanthi, Greece 2005, 1, 228-235.
- [5] EN 1998-1:2004. Design of structures for earthquake resistance Part 1: General rules, seismic actions and rules for buildings.
- [6] Swanson Analysis Systems, Inc.: ANSYS Software (1992).
- [7] Cho J.R., Song J.M., Lee J.K.: Finite element techniques for the free-vibration and seismic analysis of liquid-storage tanks. Finite Elements in Analysis and Design 37, (2001), 467-483.
- [8] Liu W.K.: Finite element procedures for fluid-structure interactions and application to liquid storage tanks. Nuclear Engineering and Design 65, (1981), 221-238.