"Efficient Estimation of the Probability of Small-disturbance Instability of Large Uncertain Power Systems"

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Efficient Estimation of the Probability of Small-disturbance Instability of Large Uncertain Power Systems

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***Abstract*—This paper proposes a methodology to efficiently estimate the probability of small-disturbance rotor angle instability of uncertain power systems. Traditional Monte Carlo (MC) approaches are computationally intensive and inefficient, particularly when used to study low probability conditions which result in small disturbance instabilities and develop into serious outage events high impact. The proposed methodology uses importance sampling to focus on conditions which contain the high information content required to make relevant decisions about low probability events. Latin Hypercube Sampling (LHS) is used to efficiently bound the search space and identify operating conditions leading to a marginally stable or unstable system response. The proposed approach is demonstrated on a model of a multi-area transmission network with a significant capacity of intermittent generation connected through a multi-terminal Voltage Source Converter-based High Voltage Direct Current (VSC-HVDC) grid. It is demonstrated that the methodology yields accurate results with just a small fraction of the sample points required using a conventional numerical MC approach.**

***Index Terms*—Eigenvalues, electromechanical oscillations, importance sampling, instability, Monte Carlo, probability, small disturbance stability, uncertainty.**

I. Introduction

POWER systems are operated under increasingly variable conditions due to the proliferation of intermittent renewable energy sources and new types of system loads. There is also a drive for greater asset utilization to improve the efficiency and economics of power networks which is resulting in operation closer to system capacity limits. With systems more exposed to uncertain and variable conditions, standard deterministic approaches towards stability and security analysis are inadequate. Probabilistic studies are required to correctly represent the variability inherent in power system operation and to provide statistical results that incorporate uncertainty.

Small-disturbance stability relates to the ability of a power system to maintain stability following the small variations that naturally and continuously occur during daily operation. As synchronous generators regain stability following disturbances, oscillations are seen in their rotor speeds, resulting in power oscillations throughout the system. Low frequency inter-area electromechanical oscillations (or modes) are inherent in all large power systems [1], and can be exacerbated by increased power transfer across key transmission corridors and the use of high gain excitation systems to enhance the transient stability of power systems. Furthermore, future power systems may be characterized by reduced capacity margins, fewer conventionally regulated generation units, and lower system inertia (caused by converter-connected renewable energy sources). In such systems, there is an increased likelihood that troublesome inter-area modes develop into potentially unstable conditions requiring increased monitoring and control.

The importance and benefits of probabilistic approaches for power system stability analysis has been highlighted in previous research [2]–[4]. These studies demonstrated accurate depictions of the modal variation in the case of uncertain operating conditions. They, however, are dependent on numerical Monte Carlo (MC) sampling and the large number of required samples (typically thousands) can limit their potential use for practical size systems. The standard MC-based approach applied to realistic size power systems can be too computationally intensive for fast assessment applications or for repeated probabilistic studies, particularly when identifying low probability conditions which may develop into serious outage events with high impact.

Different approaches have been proposed in the past to attempt to reduce the computational burden of these probabilistic stability studies whilst maintaining the accuracy and detail of the results. Point estimate methods [5], [6], analytical cumulant-based approaches [7], [8], and the probabilistic collocation method [9], [10] can all be applied to probabilistic small-disturbance stability studies – though much of the focus has been on probabilistic load flow to date. All of the above methods can be used to derive the probability density functions *(pdfs)* of a variable system output based on known input uncertainty using significantly fewer sample points than required for traditional numerical approaches.

The efficient sampling approaches detailed in [5]–[10] can provide very accurate results when the output *pdfs* are symmetrical (or nearly symmetrical with low skewness). However, although they do not impose Gaussian representations, it has been observed that they become less efficient and more inaccurate when distributions are *long-tailed*. It has been observed in the past, and also shown within this study, that in the case

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of probabilistic small-disturbance stability studies, the resulting *pdfs* of modal damping factors can be extremely skewed. The methods proposed in [5]–[10] are therefore unable to provide sufficiently accurate results and so an alternative efficient estimation approach is required.

The methodology proposed in this paper to resolve the above issues is based on the concept of *importance sampling*. Importance sampling has been used in reliability studies [11], [12] and in machine learning applications [13], [14] in the past to focus sampling effort on the high information content areas of search space in order to make decisions about low probability events. The approach is analogous to the intensification rules utilized within *tabu search* optimization to bias effort towards promising search space regions [15]. For application with probabilistic small-disturbance studies, discussed in this paper, the importance sampling concept is combined with an efficient Latin Hypercube Sampling (LHS) based operational search space characterization to initially identify marginally stable or unstable conditions. Furthermore, the balance between efficiency and accuracy of the approach is discussed and appropriate threshold parameters introduced. The methodology is demonstrated using a multi-area transmission network with significant portion of intermittent generation connected through a multi-terminal VSC-HVDC network.

II. Methodology

This section will describe the benchmark approach – the traditional Monte Carlo approach – before describing the steps of the proposed novel efficient method.

A. Monte Carlo Approach

The traditional numerical Monte Carlo (MC) approach, which relies on extensive and repeated random sampling of system uncertainties [16], provides the benchmark which the proposed methodology is (and other similar approaches have previously been) assessed against in terms of both accuracy and computational burden.

For each input set, randomly generated using the MC approach, a deterministic study is performed (consisting of load flow, system linearization, eigenvalue analysis, and modal identification) in order to calculate the details of critical system mode. It is necessary to run very large numbers of full deterministic studies to ensure that the distribution of output variables accurately represents the true system behavior and that all relevant combinations of input variables are taken into account. This is a significant limitation with respect to the application of the MC method for probabilistic studies of large power systems (using standard computational hardware).

In this work, the stopping rule (1) is used to limit the number of simulations.

$$\varepsilon_{\bar{X}_N} = \frac{\Phi^{-1}\left(1-\frac{\delta}{2}\right)\sqrt{\frac{\sigma^2\left(X_N\right)}{N}}}{\bar{X}_N} \tag{1}$$

In (1), $\Phi^{-1}(\bullet)$ is the inverse Gaussian conditional probability distribution *(cdf)* with a mean of zero and standard deviation of one, $\sigma^2(\bullet)$ represents the variance of a sample, δ represents the desired confidence level, and X_N is a sample of measured outputs consisting of N samples. This rule, from [2], is based upon calculating the error of the *sample mean* $\bar{X}_N$ (compared to the unknown population mean). Simulations are stopped if the *sample mean error* $\varepsilon_{\bar{X}_N}$ falls below a specified threshold. In this work, $\delta = 0.01$ and MC simulations are stopped when $\varepsilon_{\bar{X}_N} \le 0.01$ to ensure 99% confidence that the difference between the true and sampled mean values is less than 1% of the true mean value.

B. Overview of the Proposed Methodology

Whilst the traditional numerical MC approach could provide sufficiently accurate results, the number of simulations required may be extremely high. The aim of this research is to focus the analysis on conditions which lead to marginally stable or unstable system behavior and avoid wasting computational effort on large numbers of stable conditions.

This approach is loosely termed *importance sampling* – an MC variance reduction technique in which the sampling process is re-oriented towards a desired region of the search space [11]–[14]. The proposed methodology can be summarized as follows:

1. Identify and bound conditions leading to unstable or marginally stable system oscillations.
2. Simulate conditions within these bounds only to collect samples of unstable and marginally stable scenarios.
3. Fit a probability distribution to these sampled points.
4. Determine the probability of instability from the gathered data.

Each step of this methodology is subsequently described fully using theoretical examples for clarity.

C. Establishing Conditions Resulting in Poor Performance

Each uncertain system parameter γ represents an additional dimension in the operational search space. Initial deterministic simulations are required to characterize this operational search space and establish which conditions result in poor system performance. The simplest approach is to divide this n-dimensional search space into a number of smaller hypercubes and test the performance in each one [11], [14]. This approach is easily implemented and results in a very thorough characterization of the search space. However, for m subdivisions of each dimension, a total of m^n simulations are required, and the technique quickly becomes unsuitable as n rises. The novel methodology proposed in this paper makes use of efficient search space sampling techniques to quickly identify the regions of poor performance.

1) Search Space Characterization

It is proposed that search space characterization is performed using only a small number of samples. This limited number of samples is selected across the entire search space (with variations in all dimensions) and the system performance is assessed at each point. This results in a set of system operational scenarios and their corresponding measures of system performance. For this work concerned with small-disturbance stability, the critical mode damping factor ζ serves as an ideal measure of system performance – describing the dominant post-disturbance oscillatory nature of the power system. Full deterministic studies are performed for each operational scenario to determine ζ.

2) Linear Regression Model

A simple linear regression model is produced from the set of operational scenarios and observed ζ values [17]. This model can subsequently be used to provide an estimate of the critical mode damping $\hat{\zeta}$ for any set of uncertain system parameters $\Gamma = [\gamma_1 \cdots \gamma_n]$. This linear regression model is determined by finding the set of coefficients β that satisfy the equation $\zeta = \beta\mathbf{\Gamma}$, in which $\mathbf{\Gamma}$ is a matrix whose rows contain the uncertain parameters Γ corresponding to the observed values in ζ.

It should be noted that it is not being suggested that $\hat{\zeta}$ will be an extremely accurate estimation of ζ, this is not the purpose of the model, nor can the system complexities and nonlinearities be captured using such a simple model. This linear regression model is used to *point towards* the regions of the search space which are likely to result in poor performance. These regions will then become the focus of more sampling, ensuring the computational effort is directed at the areas of the search space with high information content.

3) Latin Hypercube Sampling

The generation of the sampling points is completed using Latin Hypercube Sampling (LHS) to ensure that the whole search space is evenly sampled. For LHS, each dimension is subdivided and sample points are selected so that each subdivision of each parameter is selected once. Furthermore, samples are selected which maximize the minimum distance between sampled points in the multidimensional hyperplane – further improving the search space coverage.

The difference between random sampling and LHS within the same intervals is shown in Fig. 1. It is clear that random sampling may result in large areas of the search space not being sampled. It is highly important that all areas are evenly sampled to capture the low probability unstable conditions. In this research, equidistant (as opposed to equiprobable) points are used to ensure sampling of the *tail* regions of distributions. It should be noted that other methods such as low discrepancy sequences or $\Lambda\Pi_\tau$ sequence-based sampling which also ensure even search space sampling could be used if desired [18], [19].

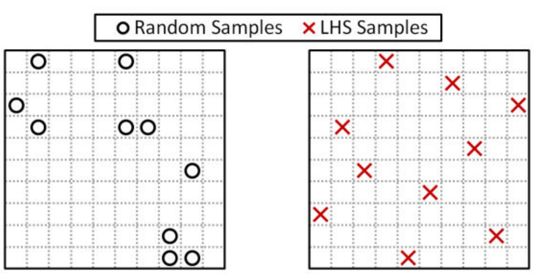

Fig. 1: Comparison of random and Latin hypercube sampling examples in two dimensions (every row and column is sampled with LHS).

Care must be taken when scaling this approach to applications with very large numbers of uncertainties as, like many sampling processes, LHS suffers from the curse of dimensionality. Optimizing designs to ensure good space filling properties as n increases becomes increasingly difficult. One possible solution is to identify and focus on the uncertainties which are most critical in determining system performance and therefore reducing n. However, this is a non-trivial task, requiring a global sensitivity analysis across all uncertainties and the entire search space. In many instances, a brute force approach to such a study would be far more computationally expensive than the probabilistic study itself.

D. Importance Sampling

The linear regression model can be used to identify the regions of the operational search space that should be sampled. *Importance sampling* describes a variety of techniques which aim to redirect the sampling process to regions of distributions that are of greater importance [11]–[14] . In this work, this is achieved by scaling the multivariate *pdf*.

The degree of *pdf* region scaling can be variable and is used at its extreme in this work so that only samples which fall inside the identified boundaries are sampled. An example of this is illustrated (in two dimensions) in Fig. 2 in which it is assumed that the boundaries of the generic uncertainties γ_1 and γ_2 have been determined (as described in the following section). It is important to note that the shape of the probability distribution within the boundary region is retained.

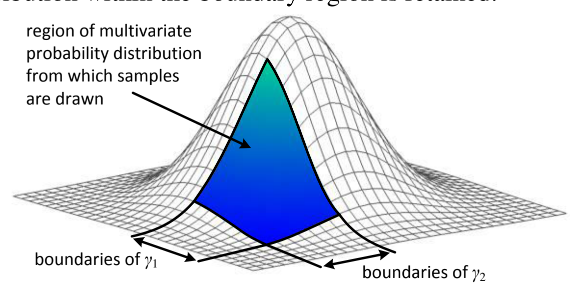

Fig. 2: Example of the extreme importance sampling used (in two dimensions) – only the highlighted region is sampled.

The linear regression model can be used to map the estimated multi-dimensional boundaries that enclose a given level of system performance. The system performance is bounded using a damping estimate threshold $\hat{\zeta}^{thresh}$, below which performance is deemed poor (and sampling is required). The multivariate *pdf* describing the entire system operational search space can be subsequently scaled so that regions outside these boundaries have a probability density of zero, and so that the multidimensional integral of the retained *pdf* equals one. The resulting *pdf* will have discontinuities at boundaries and will require a complex mathematical description.

For practical implementation, it is far simpler to generate samples from the whole distribution (i.e. the wireframe *pdf* in Fig. 2), determine $\hat{\zeta}$ using the linear regression model, and discard sample points which result in modal damping estimates above $\hat{\zeta}^{thresh}$ (i.e. the samples that are outside the highlighted region in Fig. 2). The computational burden of generating points from the whole *pdf* and testing them with the regression model is negligible compared to the full deterministic studies. This practical implementation, therefore, does not reduce the improved efficiency of the proposed methodology.

For each retained sample (from the shaded area in Fig. 2), a full deterministic simulation is performed and the critical mode damping recorded. The aim is to collect samples which result in system performance below the damping exceedance threshold ζ_{exc}^{thresh}. In this work where performance degrades with decreasing damping, the term *exceedance* is used to describe values *below* the threshold. Only the *exceedance samples* below this threshold will be used when fitting the final distribution and determining the probability of instability. This threshold value is set close to the stability boundary. The *exceedance* region of the search space must be enclosed by the *importance sampling* region – therefore $\hat{\zeta}^{thresh}$ should be set greater than ζ_{exc}^{thresh} to allow for the errors introduced by the linear regression model. In this work, $\hat{\zeta}^{thresh}$ is set as three times the standard deviation of the regression model residuals. Assuming an approximately Gaussian residual error distribution, this 3σ value will ensure that the vast majority of measured ζ values below zero are captured. This sampling must continue until a sufficient number of exceedance samples n_{exc}^{req} are collected to accurately fit a distribution.

E. Fitting a Distribution

Once the required number n_{exc}^{req} is obtained, distribution fitting can be completed. The ultimate purpose of the previous steps (identifying regions of poor performance, and importance sampling) is to generate samples which populate the tail region of the whole *pdf* of critical mode damping. By using only the exceedance values it is possible to fit a Generalized Pareto (GP) tail distribution from which the final probability of instability can be determined. A GP distribution is described by (2) for exceedance values ζ_{exc}.

$$y = \left(\frac{1}{\beta}\right)\left(1+\frac{k\zeta_{exc}}{\beta}\right)^{-1-\frac{1}{k}} \tag{2}$$

In (2), k represents the *shape* parameter and β is the *scale* parameter (commonly σ, but called β here to avoid confusion with standard deviation). Distribution fitting can be completed using a wide variety of software programs. The GP distribution is often used to represent the tail regions of *pdfs* [20].

F. Determining the Probability of Instability

The final probability of instability $P(\zeta < 0\%)$ can be approximated using (3). Each term in (3) is determined during a different stage of this proposed methodology and all are required to produce an accurate estimation of the probability of instability. Note that this is only an approximation (as are all sample-based methodologies including the full MC approach). In the following sections, the terms *sampling* and *sampled points* refer to the *importance sampling* stage and not the initial LHS used to generate the linear regression model.

$$\begin{aligned} P(\zeta < 0\%) \approx\, & P\left(\hat{\zeta} < \hat{\zeta}^{thresh}\right) \\ & \times P\left(\zeta < \zeta_{exc}^{thresh} \mid \hat{\zeta} < \hat{\zeta}^{thresh}\right) \\ & \times P\left(\zeta_{exc} < 0\% \mid \zeta < \zeta_{exc}^{thresh}, \hat{\zeta} < \hat{\zeta}^{thresh}\right) \end{aligned} \tag{3}$$

1) $P\left(\hat{\zeta} < \hat{\zeta}^{thresh}\right)$

The first term in (3) is the probability that the estimated damping is less than $\hat{\zeta}^{thresh}$. This represents the proportion of the whole search space which is sampled. It can be determined empirically from the number of sampled points n_s and the number of discarded points n_d as $n_s/(n_s+n_d)$. However, this may be subject to errors caused by the relatively small values of n_s and n_d. Instead, it is advised that an additional analysis is completed using a large number of generated candidate input sets Γ to determine $P\left(\hat{\zeta} < \hat{\zeta}^{thresh}\right)$ more accurately, as the computational burden is negligible.

2) $P\left(\zeta < \zeta_{exc}^{thresh} \mid \hat{\zeta} < \hat{\zeta}^{thresh}\right)$

The second term in (3) is the probability of sampling a point with a modal damping value below the exceedance threshold ζ_{exc}^{thresh} (conditional on the point having been sampled). As sampling is completed until n_{exc}^{req} exceedance points are obtained, this probability can be calculated empirically as n_{exc}^{req}/n_s. However, this is again likely to be subject to errors caused by the small sample sizes. It is not possible to run an additional large sample size as the process requires full deterministic studies which are the source of the computational burden. Rather, a kernel smoothing density estimate can be used to produce a *pdf* for all sampled points [21]. This can subsequently be used to determine the probability of exceeding the threshold. The use of a smoothing density estimate reduces the error introduced by the small sample size.

3) $P\left(\zeta_{exc} < 0\% \mid \zeta < \zeta_{exc}^{thresh}, \hat{\zeta} < \hat{\zeta}^{thresh}\right)$

The final term in (3) is the probability that an exceedance sample will be unstable. This probability is conditional on the fact that the point must be sampled, and the measured damping must be less than ζ_{exc}^{thresh}. This probability is calculated from the exceedance value GP distribution. It is also possible to calculate confidence intervals on this probability (based on the confidence intervals of the GP fit). A summary of the full proposed methodology to efficiently estimate the probability of small-disturbance instability in uncertain power systems is presented in Fig. 3.

G. Methodology Generalization

It should be noted that determining the probability of instability is simply a special case of determining the probability of damping to be less than a given boundary (in this case zero). It is also possible to use the same methodology to determine the probability of poorly damped oscillations by simply selecting boundary greater than damping equal to zero. The methodology will be valid provided the *tail region* of the true damping *pdf* is being sampled. Provided that the probability of poorly damped oscillations is low, we can be confident that sampling is occurring from the distribution tails. Therefore the applicability of the methodology extends beyond instability determination, improving its practicality further.

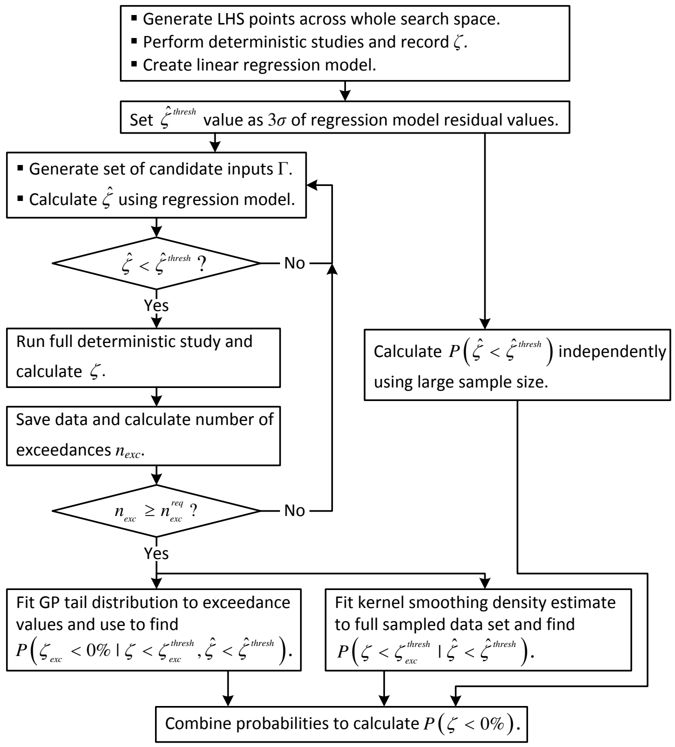

Fig. 3: Flow chart summary of the proposed efficient methodology.

III. Test Network & System Uncertainties

The methods described within this paper are illustrated using a heavily modified version of the 16 machine, 68 bus reduced order representation of the New England Test System and the New York Power System (NETS & NYPS). The network (including modifications) is shown in Fig. 4. System analysis and simulations are all performed within the MATLAB/Simulink environment making use of modified MATPOWER [22] functions to perform optimal power flows.

A. System Details

Generators G1–13 are modeled and controlled as detailed in [23] with G1–8 under slow DC excitation (IEEE-DC1A), G9 equipped with a fast acting static exciter (IEEE-ST1A) and a Power System Stabilizer (PSS), and G10–13 under constant manual excitation. All generators are represented by full sixth order models. System loads are modeled as constant impedance. Full system details, generator and exciter parameters are given in [23] with PSS settings for G9 taken from [24].

The standard test network in [23] also contains three large generators representing external networks which import approximately 2.1 GW into the NYPS area under nominal loading. In the modified network, the generators, buses, and lines composing these external networks have been replaced with a five line, six terminal VSC-based multi-terminal HVDC (VSC-MTDC) network connected to five 450 MW wind farms (GWF-A–E).

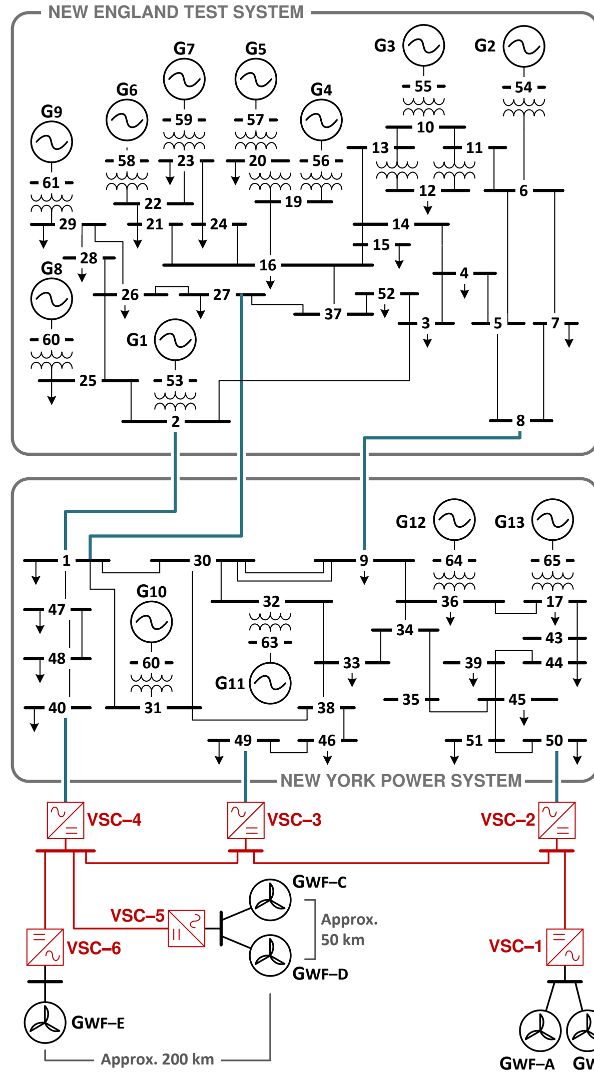

Fig. 4: 16 machine, 68 bus reduced order model of NETS & NYPS.

1) VSC-MTDC System Details

The VSC-MTDC system enables power transfer from the wind farms into the AC network. Each converter station is modeled as an injection of active and reactive power neglecting device switching operations [25]. Converter station controllers and DC lines are modeled as presented in [26].

All converter stations connected to the NYPS region (*VSC-2–4*) regulate active power injection into the AC system using a *DC voltage droop* characteristic [27]. Additionally, *VSC-2–4* support the local AC voltage at the connection bus through reactive power injection. These converters, therefore, operate in *Active Power Voltage Droop–AC Voltage* control mode.

The active power injected into the MTDC system at each converter station connected to a wind farm (*VSC-1,5,6*) is determined by the output of the renewable energy sources. Reactive power is supplied as required to support the renewable generation. These converters operate in *AC Frequency–AC Voltage* control mode.

2) Wind Farm System Details

Five 450 MW wind farms (GWF-A–E) are connected to the test network through the VSC-MTDC system. As the converters connected to the wind farms operate in *AC Frequency–AC Voltage* control, all power produced by the wind farm is transferred to the VSC-MTDC system.

3) Operational Constraints

An optimal power flow solution is used within this work to more accurately generate representative system operating points. All voltages are constrained within the range 0.9–1.1 pu. Generator cost data, and active and reactive power limits can be found in [28].

B. System Uncertainties

A primary aim of having an efficient accurate method to determine $P(\zeta < 0\%)$ is to incorporate it into probabilistic studies incorporating uncertainty. A wide variety of uncertainties are present in power systems, including (but not limited to) electricity demand forecasts, generation output from renewable energy resources, failures and faults on the system, the material properties of equipment, and the composition of system loads connected at any time. In this study the uncertainty surrounding operational forecasts (particularly for intermittent generation) has been considered as an illustrative example in order to study the resulting variations in the damping of critical system oscillations. Appropriate modelling of all uncertainties in the system would yield the most accurate representation of true system performance; however this will also increase the problem complexity and initial data requirements.

1) Modeling Loading Uncertainty

Variations in the loading values at each AC system bus represent uncertainty in loading forecasts. The correlation of different load types is also considered with loads categorized as *residential* or *industrial* based on their nominal power factor (values over 0.9 are classed as *residential*). Correlation coefficients ρ between different loads are $\rho = 0.8$ between *residential* loads, $\rho = 0.4$ between *industrial* loads, and $\rho = 0.2$ between *residential and industrial* loads [29]. All loads follow a Gaussian distribution with mean values equal to nominal loading scaled by the forecast loading factor, and standard deviation derived from the forecast uncertainty.

2) Modeling Wind Generation Uncertainty

Variations in wind power generation around the forecast value are also modeled as following a Gaussian distribution (though any distribution can be used if required). Correlation is modeled between the power generated at the wind farms. It is assumed that wind farms connected to the same converter station are 50 km apart, and wind farms connected to different converter stations are 200 km apart. The correlation coefficients used are sourced from [30] resulting in $\rho = 0.73$ and $\rho = 0.58$ for 50 km and 200 km separation respectively. Mean values are selected as the forecast generation value, with standard deviation based on the level of forecast uncertainty.

3) Forecast Uncertainty

The uncertainty associated with forecasts (of loading and of wind generation) can be modeled based on typical forecast errors. Fig. 5 displays the relationship between the forecast error and the forecast horizon for system loading and for offshore wind generation, sourced from [30] and [31] respectively. *MAPE* refers to the *mean absolute percentage error* relative to the forecast value. *NAME* refers to the *normalized absolute mean error* relative to the maximum rated generation capacity of the wind farm. These are both derivatives of the *mean absolute deviation* (MAD). It is clear that there is far greater uncertainty surrounding the forecast of wind generation.

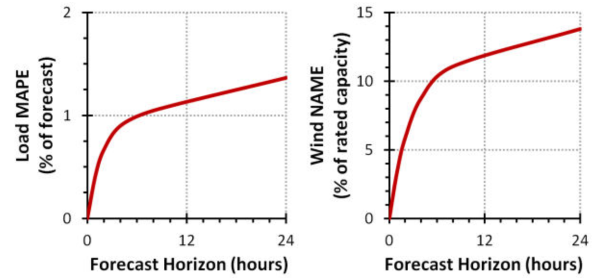

Fig. 5: Typical forecast errors for system loading and wind generation, adopted from [30] and [31].

For any forecast point, the error can be translated from *MAD* to a standard deviation σ using the scaling relationship (4) for Gaussian distributions.

$$MAD = \left(\sqrt{2/\pi}\right)\sigma \tag{4}$$

It should be noted that the long-term uncertainty of wind speed is, in this illustrative example, negated by the fact that a forecast value exists. Variation around the forecast value is modelled but the longer-term distribution is not. Over long time scales (when the full range of possible operating scenarios may occur), a Weibull distribution may more accurately represents the variation in wind speed at locations. However, this is not a valid representation for short time scales. Since the focus of this study is on a short time scale, it is not necessary to include the uncertainty of fluctuations over longer periods of time and to describe it by appropriate distributions. Instead the uncertainty in the forecasted value can be modeled by Gaussian distribution in piece-wise manner for each short period of time of interest. If the focus involves longer periods of time then variation in forecasted values can be additionally described using appropriate probability distributions.

4) VSC-MTDC Operational Uncertainty

The VSC-MTDC grid operates to deliver all power from the intermittent wind resources to the AC network and therefore the uncertainty surrounding its operation is tied to the wind farm power output. A power sharing strategy is utilized within this study wherein *VSC-2* delivers 40% of the total wind power production, *VSC-3* delivers 20%, and *VSC-4* acts as a *slack* to account for the losses within the VSC-MTDC system (approximately 2.5% of total power transmission). This *slack* behavior only occurs during the DC load flow solution. For dynamic studies the *droop* characteristic ensures that active power variations are shared by all converters.

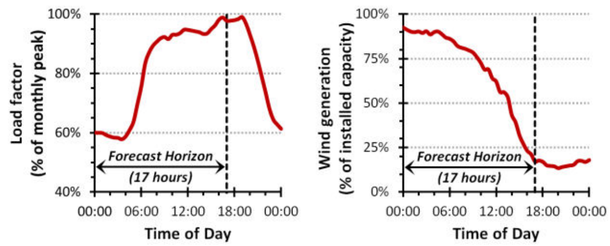

Fig. 6: Load and wind generation data for the UK network (17-Sep-13).

C. Operating Scenario

An operating scenario is selected to represent forecast conditions and illustrate the efficiency of the proposed method. As the test system being used is a modified version of a reduced order equivalent, no true forecast data is available for such a study. To overcome this limitation in this study, historical data for the loading level and wind generation in the UK network [32] is used to represent forecasts which are scaled to the operating point of the test system. This data is representative of expected loading and wind generation patterns seen in other similar transmission systems. Data for the 17 September 2013 (an arbitrary selection) is used, shown in Fig. 6. It is assumed that this small-disturbance stability assessment is made at a time of 00:00 for the forecast conditions at 17:00 (an arbitrary forecast horizon of 17 hours). Forecast values are 97.74% for the system load factor, with wind generation of 17.29% (of installed capacity). Based on this forecast horizon, standard deviations σ for the modeled uncertainty of loading and wind generation are equal to 1.53% and 16.00% respectively – scaled from Fig. 5 using (4).

It should be noted that a shorter time horizon would improve the accuracy of the forecast, reduce output (i.e. modal damping) variation, and therefore reduce the number of MC samples required if using a full MC-based methodology. Consequently, this would potentially reduce the efficiency gains of the proposed methodology. However, reduced output variation would at the same time improve the accuracy of the regression model, and improve the success rate when sampling exceedance values, therefore improving the efficiency of this proposed methodology. Establishing the appropriate balance between these two factors is an interesting area for further investigation in particular in the case of large system studies.

IV. Application & Results

A. Traditional Monte Carlo Approach

A traditional MC approach is completed first to establish the benchmark for comparison with the proposed methodology. The simulations are terminated using the stopping rule (1), resulting in a total of 11,586 simulations. The probability of instability clearly could be calculated empirically from this data set. However, errors will be introduced when calculating this probability due to the low number of occurrences of instability sampled. $P(\zeta < 0\%)$ is therefore determined by fitting a GP distribution to the exceedance samples (as for the proposed efficient methodology).

An exceedance threshold of $\zeta_{exc}^{thresh} = 0.5$ % has been selected for this study and is shown to give good results. Other values near to the stability boundary could also be selected. Note that lower values will increase the total number of deterministic studies required when completing the efficient methodology in order to obtain sufficient number of *exceedance samples* and will reduce the efficiency of this proposed technique. Higher values of ζ_{exc}^{thresh} will increase the range of the determined confidence interval and will therefore reduce the accuracy of the methodology. These two aspects must be balanced according to the requirements of the analysis.

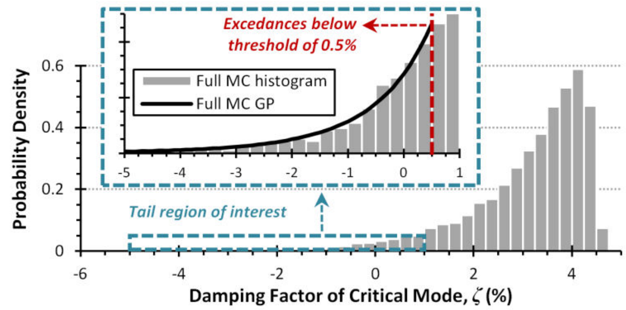

Fig. 7: Results from full MC simulation and GP distribution fit.

The results from the MC study are presented in Fig. 7 including the GP distribution. An additional benefit of using the GP fit is that it enables the derivation of a confidence interval for $P(\zeta < 0\%)$. The obtained results are $P(\zeta < 0\%) = 3.34\%$ with a 95% confidence range of 3.12–3.54%. It is also very evident from Fig. 7 that the GP distribution is a very good fit to the tail of the damping factor histogram. The GP distribution has been tested against a number of alternative distributions (including Weibull, exponential, lognormal, and generalized extreme value) and it was found to provide *the most likely* (i.e. the *lowest log-likelihood*) distribution fit.

B. Proposed Efficient Methodology

In total, the sources of uncertainties within the test system consist of the 32 load buses and the wind farms. As the wind power is transmitted through the VSC-MTDC grid and shared amongst the AC grid injection points, it is possible to consider the wind generation as a single uncertain parameter. The operational search space is therefore described in 33 dimensions. These 33 uncertainties considered are the loading values for each bus and the total power generated from the wind farms, all of which are variable due to the uncertainty in the forecast values.

For this illustrative example, an OPF solution is used with non-varying cost functions. Therefore, the same values of loading and wind generation will always result in the same generator dispatch. In practical situations it may be necessary to also model the variations in OPF cost function parameters to capture the variations in power flow direction and magnitude that may occur for identical loading scenarios. If modelled, such variations will require close scrutiny to ensure the modelled conditions are representative of the true system variation. This can be included in this methodology, though it will increase the number of dimensions in the operational search space.

1) Creating the Linear Regression Model

The linear regression model was created using 100 LHS points and the full set of 33 uncertainties. Each input parameter was sampled evenly in the range of $\pm 3\sigma$, encompassing 99.7% of the distribution of each parameter. It was found that trimming the obtained data set of samples to remove those resulting in high critical mode damping improved the accuracy of the regression model at predicting poor performance. A sensitivity analysis and discussion on the number of points required to create the linear regression model is can be found in Section IV.D.

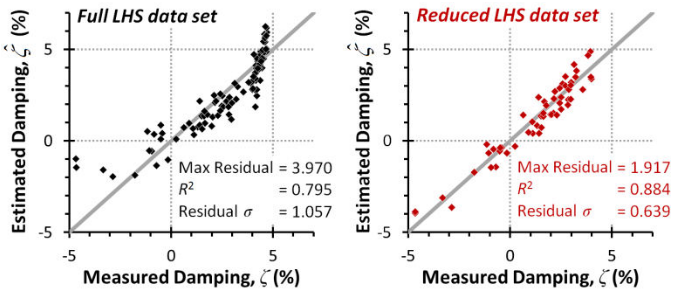

Fig. 8: Estimation accuracy of different linear regression models.

This is demonstrated in Fig. 8 where the differences can be seen in the estimation accuracy of a linear regression model created from the *full LHS data set* and a *reduced LHS data set*. A trimming threshold of $\zeta < 4\%$ was found to provide a good balance between reducing the residuals and containing sufficient data to characterize the search space. The included statistics in Fig. 8 reveal the reduction in the maximum residual value, the increase in the coefficient of determination (R^2), and the decrease in residual standard deviation when the reduced data set is used. The results are also revealed visually to be more *balanced*, particularly for low damping estimates. This trimming of the LHS data set was performed following the collection of all the sample data points.

The damping estimation threshold is set according to the stated methodology as 3σ of the regression model residuals. This results in a value of $\hat{\zeta}^{thresh} = 1.917\ \%$.

2) Establishing n_{exc}^{req}

The results from the full MC results are used here to establish a value for n_{exc}^{req} – the number of exceedance samples required. The full MC data set (11,586 points) includes 556 (4.8%) exceedance values. In order to assess the effect that the number of exceedance values has on the accuracy of the analysis, exceedance values were removed from the data set at random and the probability of instability (and confidence interval) were recalculated. The results are shown in Fig. 9.

It is evident that the estimated value for $P(\zeta < 0\%)$ remains approximately constant whilst the confidence interval steadily increases as exceedance samples are removed from the data set. The confidence interval range is plotted separately in Fig. 10 where it can be seen to display a very steady increase and can be used to establish the desired value for n_{exc}^{req}. A threshold of 1% (in absolute probability terms) is set on this range, suggesting that at least 90 exceedance samples are required. This value of n_{exc}^{req} is used when completing the efficient estimation of the probability of instability.

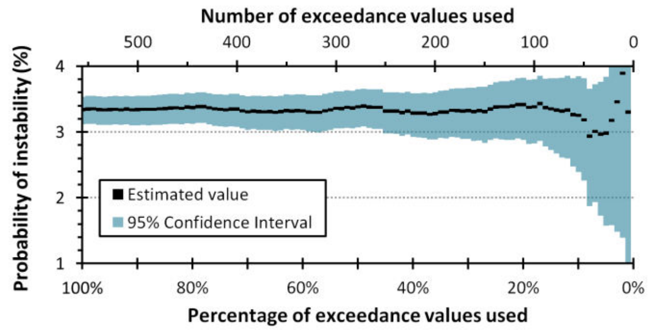

Fig. 9: Effect of the number of exceedance values on accuracy.

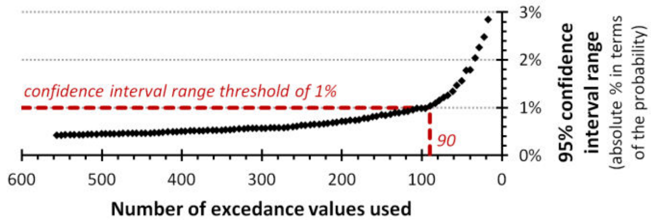

Fig. 10: Confidence interval range for the probability of instability.

C. Comparison of Results

The proposed efficient methodology was completed using the thresholds and values established in the previous sections ($\hat{\zeta}^{thresh} = 1.917\ \%$, $\zeta_{exc}^{thresh} = 0.5\ \%$, and $n_{exc}^{req} = 90$). The resulting methodology required 308 full deterministic studies in order to obtain the required 90 exceedance samples. Including the initial 100 LHS simulations performed to construct the linear regression model, a total of 408 deterministic studies were performed. This represents a 96.5% reduction in the number of samples used (or only 3.5% of the original number of samples) – a very significant improvement in the computational efficiency achieved by the proposed methodology.

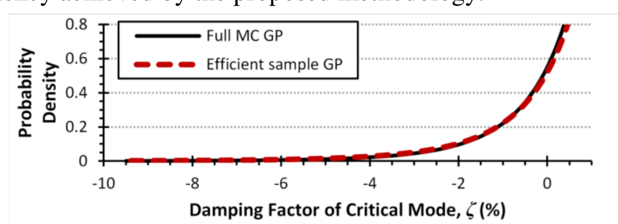

Fig. 11: Comparison of GP distributions.

TABLE 1
PROBABILITY OF INSTABILITY VALUES AND CONFIDENCE INTERVALS

Method	$P(\zeta < 0\%)$	*95% Confidence*	*Total Simulations*	*Total Time Taken*
Full MC	3.34%	3.12 – 3.54%	11,586	9 hr 39 m
Efficient	3.30%	2.75 – 3.72%	408	21 m

The accuracy of the proposed methodology must also be assessed. Fig. 11 shows the resulting GP distributions and Table 1 displays the results obtained from both the full MC-based approach and the efficient method. It is evident that the efficient method produces an accurate estimate of $P(\zeta < 0\%)$ which has a relative error of just 1.2% compared to full MC-based approach. However, the confidence interval is larger, covering 0.97% (compared to 0.42% for the full MC-based approach). This larger confidence interval is expected though, since n_{exc}^{req} was selected as 90 based on a chosen confidence interval threshold of 1% (in Fig. 10). It would be possible to set n_{exc}^{req} to a higher value to reduce the range of the 95% confidence interval and improve the accuracy of the method. The results from Fig. 10 suggest that more than 400 exceedance samples are required to reduce the confidence interval to less than 0.5%. This would, however, result in a decrease in the efficiency of the approach. This tradeoff between the accuracy and confidence level must be appraised in advance depending on the needs and scope of the study. The times taken, shown in Table 1 to the nearest minute, are based on simulation using a PC with 2.66 GHz quad-core CPU and 4 GB RAM.

Note that the setting of n_{exc}^{req} is critical and will require careful validation for practical applications. It is also possible that there could be particular operational scenarios where n_{exc}^{req} will exceed the number of exceedance samples (and the methodology would continue searching indefinitely). To avoid this, it is recommended that the search is limited to a maximum sample number. The system operator should be alerted if this is reached to highlight the need for more detailed studies, perhaps using a full MC approach.

D. LHS-based Regression Model Sensitivity Analysis

The linear regression model is critical to the success of this technique by *steering* the approach towards regions of the search space which contain high levels of information. In this example, this is poorly damped or unstable operating conditions. It is therefore required that a linear regression model that is suitably accurate can be ensured. To illustrate this part of the methodology more clearly, sensitivity analysis has been performed investigating the effectiveness of the LHS-based linear regression model against the number of sample points used to build the model.

It has been previously stated that guaranteeing a very accurate model is *not the objective*. What is desired is a model which can estimate damping (particularly low damping) with *reasonable accuracy*. The full simulations at these targeted points will then determine the true damping values. Fig. 12 shows the result of the sensitivity analysis, and the improvement made to the regression model by using more sample points.

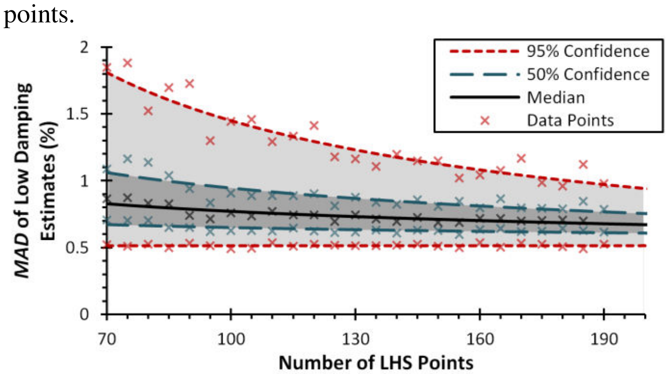

Fig. 12: Effect of increasing LHS samples on accuracy of linear regression model.

With respect to Fig. 12, *MAD* refers to the *mean absolute deviation* of damping estimates from new regression models (produced following the method in Section II.B.2) tested against true damping values from the data set gathered during the full MC-based approach. As the regression models are produced using points where damping factor is less than 4%, only these points are used to test the models, resulting in 8543 test points for each model. The MAD is calculated by comparing the true damping with the model estimates and is reported as an *absolute percentage* (in terms of the damping factor). A total of 100 regression models have been tested for each number of sample points shown and the trends for key percentiles (2.5%, 25%, 50%, 75%, and 97.5%) plotted on Fig. 12. In total over 2500 regression models were tested.

The lower bound for the number of points is dictated by the fact that there must be more sample points than uncertainties. Furthermore, as some points (with damping factor over 4%) are discarded (see Section IV.B.1), the number of points must be reasonably larger than the number of uncertainties. In this work, it was found that approximately 60 points were required with 33 uncertainties. It is therefore desirable to only model those uncertainties which have a significant effect on system performance.

It can be seen that using 100 LHS points in the regression model will provide a MAD in the range 0.51–1.45% (with 95% confidence). It should be noted the maximum value in this range is lower than $\hat{\zeta}^{thresh}$ used in this study (the threshold designed to account for regression model inaccuracies). If desired, the number of sample points can be increased. For example, using 200 points will provide a MAD in the range 0.51–0.94% (95% confidence). Whilst this will reduce the methodology efficiency, the overall computational savings compared with a full MC-based approach will still be significant.

V. Conclusions

The paper presented a novel methodology for the efficient determination of the probability of small-disturbance rotor angle instability in uncertain power systems. Traditional numerical approaches are too computationally intensive for repeated probabilistic studies and the efficient sampling techniques proposed to date for use with probabilistic power system stability analysis – such as point estimate methods, analytical cumulant approaches, and the probabilistic collocation method – are unable to accurately reproduce the *long-tailed* distributions which are often associated with modal distributions. The methodology established within this paper addresses this gap and has been shown to accurately determine the probability of instability using a small fraction (only 3.5% for this illustrative test system) of the samples required by the traditional MC-based approach. Comprehensive numerical justifications have been provided for the threshold limits used. Furthermore, the effect of altering these thresholds has been discussed, acknowledging the balance between efficiency and accuracy that this method permits.

The combination of efficient Latin hypercube sampling-based search space characterization and importance sampling could also be utilized in other aspects of power system analysis to improve the efficiency of probabilistic analysis. Logical extensions include applications in other areas of stability analysis – particularly transient stability analysis where a reduction in the computational burden associated with multiple time-based simulations would be welcome. It is hoped that this method may enable *online* probabilistic analysis, or optimization based upon repeated probabilistic studies, for example for stability-related damping controller designs.

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