

# Statistical top-down approach for energy loss estimation in distribution systems

# Statistical Top-Down Approach for Energy Loss Estimation in Distribution Systems

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***Abstract*— This work proposes a statistical top-down methodology for energy loss estimation in medium voltage (MV) distribution systems. A statistical model is used to adjust the load parameters (i.e., ZIP coefficients) of the aggregated load allocated to each secondary transformer along the MV feeder. This adjustment process also results in the estimation of the corresponding energy losses. The information required by the proposed methodology is limited to the feeder topology, conductors, rated capacity of the transformers, and the voltage and power measurements at the primary substation during the period of analysis. If available, additional information from meters installed along the feeder can be used to improve the estimation. To illustrate the approach, a real Brazilian 13.8kV feeder is used. The results, compared with other methodologies available in the literature, demonstrate the benefits of the proposed methodology.**

***Index Terms*—Energy losses, load allocation, distribution system, ZIP coefficients.**

## NOMENCLATURE

*Measurements*

| | |
|---|---|
| $P_1^{SE}(t)$ | Active power at the substation at the time $t$. |
| $Q_1^{SE}(t)$ | Reactive power at the substation at the time $t$. |
| $V_1^{SE}(t)$ | Voltage at the substation at the time $t$. |
| $P_j^{msr}(t)$ | Active power consumed by the load $j$ with meter. |
| $Q_j^{msr}(t)$ | Reactive power consumed by the load $j$ with meter. |
| $V_j^{msr}(t)$ | Voltage at the bus $j$ with meter. |
| $P_1^{SE\,(nom)}$ | Active power at the substation for nominal conditions on the system. |
| $Q_1^{SE\,(nom)}$ | Reactive power at the substation for nominal conditions on the system. |
| $V_1^{ref}$ | Reference voltage at the substation (nominal voltage of the system). |
| $P_i^{nom}$ | Rated active power of the load $i$. |
| $Q_i^{nom}$ | Rated reactive power of the load $i$ |

*Parameters*

| | |
|---|---|
| $x_i$ | Fraction of the active power at the substation allocated to the load $i$. |
| $y_i$ | Fraction of the reactive power at the substation allocated to the load $i$. |
| $\boldsymbol{\alpha_P},\boldsymbol{\beta_P},\boldsymbol{\gamma_P}$ | Vector of load parameters for the active power. Each element of the vector is related to each load of the system. |
| $\boldsymbol{\alpha_Q},\boldsymbol{\beta_Q},\boldsymbol{\gamma_Q}$ | Vector of load parameters for the reactive power. Each element of the vector is related to each load of the system. |
| $u_i$ | Parameter of correlation between the voltage substation and voltage at bus $i$. |

*Variables*

| | |
|---|---|
| $P_i(t)$ | Active power allocated to the load $i$ in each time interval $t$. |
| $Q_i(t)$ | Reactive power allocated to the load $i$ in each time interval $t$. |
| $L_P^k(t)$ | Active power loss of the system at iteration $k$ in each time interval $t$. |
| $L_Q^k(t)$ | Reactive power loss of the system at iteration $k$ in each time interval $t$. |

*Sets*

| | |
|---|---|
| $\Omega_L$ | Set of loads at the system. |
| $\Omega_M$ | Set of loads with meters. |
| $\Omega_{NM}$ | Set of loads without meters. |
| $\Omega_T$ | Set of $T$ time intervals of the period of analysis. |

## I. INTRODUCTION

In electrical distribution systems, one of the greatest challenges for utilities is the estimation of the technical energy losses on the feeders. In [1] the authors estimate that the energy losses throughout the world's electric distribution networks vary from country to country between 3.7% and 26.7% of the electricity use, which implies that there is a large potential for improvement. Specifically in Brazil, the correct evaluation of the energy losses provides valuable information for the regulator to establish the energy distribution tariffs.

There are different ways for estimating energy losses, but due to the difficulty for modeling precisely the equipment of the system, as well as the energy consumed of each load, the energy losses estimation can lead to huge errors. In addition, the difficulty to split technical energy losses and non-technical energy losses, which is usually caused by metering errors, unmetered company or customer use and billing cycle errors [2], aggravates the problem.

In the literature, several works can be found that face this issue. In [3], the average demand is used for estimating the energy losses. Artificial intelligence techniques, like fuzzy logic [4] and decision trees based algorithms [1] are also applied to solve the problem. In Brazil, the methodology established for the energy losses estimation on medium voltage for utilities is based on the average power loss during a period, computed by a multiple linear regression model provided by the National Agency of Electric Energy (ANEEL in Portuguese) [5]. None of those methods considers the voltage effect to estimate the energy losses, which can vary at the substation and along the feeder and, therefore, influence the energy losses estimation.

In this paper, it is proposed a new methodology based on a statistical model and a Top-Down approach for energy loss estimation. Hereinafter the proposed method is called Statistic Top-Down Approach (STDA). To be more specific, the methodology attempts to estimate technical energy losses along a period by allocating parameters of the load model applied, taking into account the measurements of voltages and power at the substation and, when available, the measurements of voltages and power demanded by loads with meters installed at the transformers. The main contribution of the proposed method is the application of a statistical model for energy losses estimation using network information and the correlation between the power consumed and the voltage, which is usually neglected by other methods.

To describe the proposed methodology in detail and its features, this paper is organized as follows: Section II describes the proposed methodology for energy loss estimation; Section III presents a case of study using a real feeder from Brazil. Additionally, in this section, a comparison with other methods takes place; finally, Section IV presents the conclusions of the work.

## II. Proposed Methodology

The application of the proposed methodology requires information about the feeder: topology, line impedances, nominal power of the transformers, a database containing voltages and power measured at the substation and, when it is available, the voltage and power measured at the transformers along the feeder. In order to improve the model, the data can be organized by clusters, according to its level of load. The clusters are desired for the statistical models because it uses the similarities of the load pattern during the period of time considered.

To estimate the energy losses, firstly, the load parameters should be adjusted to allocate loads properly. Then, in order to apply the proposed methodology, a modified ZIP model is established. Considering that the power supplied by the substation is distributed to every load on the feeder plus the power losses, the power reference of the modified ZIP model for each load may be expressed as a percentage of the power at the substation. In addition, since the voltages at the nodes are not available, in this proposed model, they are substituted by a percentage of the voltage at the substation. Thus, the modified ZIP model can be written for each load *i*, for active and reactive power, as follows:

$$P_i(t) = x_i P_1^{SE}(t) \left\{ \alpha_{p_i} \left( u_i \cdot \frac{V_1^{SE}(t)}{V_1^{ref}} \right)^2 + \beta_{p_i} \left( u_i \cdot \frac{V_1^{SE}(t)}{V_1^{ref}} \right) + \gamma_{p_i} \right\} \quad (1)$$

$$Q_i(t) = y_i Q_1^{SE}(t) \left\{ \alpha_{Q_i} \left( u_i \cdot \frac{V_1^{SE}(t)}{V_1^{ref}} \right)^2 + \beta_{Q_i} \left( u_i \cdot \frac{V_1^{SE}(t)}{V_1^{ref}} \right) + \gamma_{Q_i} \right\} \quad (2)$$

The proposed methodology uses an optimization model to adjust the parameters of the load model, minimizing, in an iterative process, the square difference between power measured at the substation and the sum of the power allocated for each load plus the power losses during the period of analysis. The convergence is achieved when no significant change is observed in the power losses computed in each iteration for each time interval. As a result, the energy losses are a by-product of the proposed method for the corresponding period. Fig. 1 shows the flowchart of the proposed methodology.

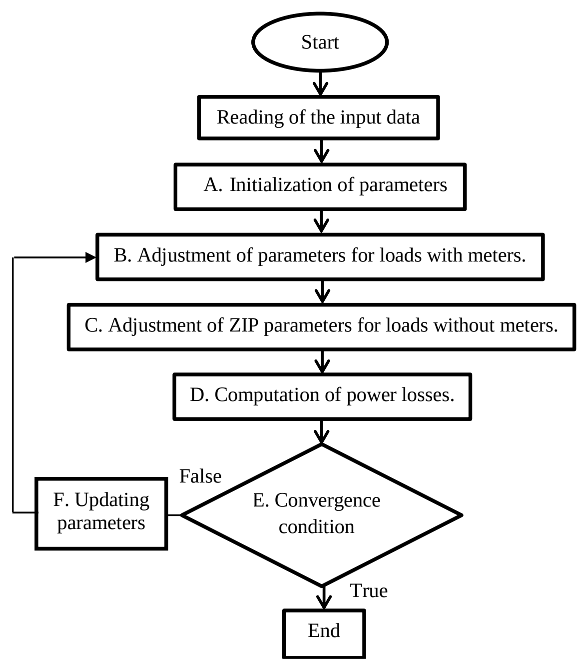


Figure 1. Flowchart of the methodology proposed for the estimation of power losses.

Each stage of the process is explained as follows.

### A. Initialization of parameters

Consider an iteration counter $k$ set to zero. The active and reactive power losses at iteration $k$ must be set to zero for each time interval of the period of analysis. The parameter $u_i$ must be set to one for each load $i$ without meter installed.

### B. Adjustment of parameters of the loads with meters

Before the adjustment process, some constraints for the load parameters, with or without meters, must be described:

*1) Constraints for **x** and **y**:* The parameters $x_i$ and $y_i$ are used to allocate the active and reactive power at the substation to each load $i$. In order to reduce the search space, the $x_i$ and $y_i$ are bounded around the relation between the power of the load and the power at the substation, both in nominal conditions as shown in (3) and (4).

$$LB_i \cdot \frac{P_i^{nom}}{P_1^{SE\,(nom)}} \leq x_i \leq UB_i \cdot \frac{P_i^{nom}}{P_1^{SE\,(nom)}},\ \forall i \in \Omega_L \tag{3}$$

$$LB_i \cdot \frac{Q_i^{nom}}{Q_1^{SE\,(nom)}} \leq y_i \leq UB_i \cdot \frac{Q_i^{nom}}{Q_1^{SE\,(nom)}},\ \forall i \in \Omega_L \tag{4}$$

The $LB_i$ and $UB_i$ are boundary factors to define the lower and upper bound of the search space $x_i$ and $y_i$.

*2) Constraints for the ZIP parameters:* As usually done for traditional ZIP model, for each load $i$ the sum of the parameters $\alpha_i$ , $\beta_i$ and $\gamma_i$ is equal to one for the active and reactive component.

Before the parameters adjustment in the proposed methodology, an estimation of $u_j$ in for each load $j$ with meter installed should take place. This adjustment is done by using the Least Square method, in which $u_j$ is calculated by the relation between the voltage measured at load $j$ and the voltage at the substation, as shown in (5):

$$\min_{\boldsymbol{u}} \left\{ \sum_{t \in \Omega_T} \left[ \frac{V_j^{msr}(t) - u_j \cdot V_1^{SE}(t)}{V_j^{msr}(t)} \right]^2 \right\}, \forall j \in \Omega_M \tag{5}$$

In addition, the parameters of the loads with meters are adjusted using the Least Square method to minimize the error between the power measured of the loads with meters and their allocated power computed by the proposed model as follows:

$$\min_{\substack{x,\alpha_P,\beta_P,\gamma_P,\\ y,\alpha_Q,\beta_Q,\gamma_Q}} \left\{ \sum_{t \in \Omega_T} \left( \left[ \frac{P_j^{msr}(t) - P_j(t)}{P_j^{msr}(t)} \right]^2 + \left[ \frac{Q_j^{msr}(t) - Q_j(t)}{Q_j^{msr}(t)} \right]^2 \right) \right\}, \forall j \in \Omega_M \tag{6}$$

### C. Adjustment of the loads parameters without meters

The Least Squares method is applied to minimize the error between the power measured at the substation and the sum of power in each load and the power losses calculated in the previous iteration for all time intervals of the period of analysis. Then, for $k = k + 1$:

$$\min_{x,\alpha_P,\beta_P,\gamma_P} \left\{ \sum_{t \in \Omega_T} \left[ \frac{P_1^{SE}(t) - \sum_{i \in \Omega_M} P_i(t) - \sum_{j \in \Omega_{NM}} P_j(t) - L_P^{k-1}(t)}{P_1^{SE}(t)} \right]^2 \right\} \tag{7}$$

$$\min_{y,\alpha_Q,\beta_Q,\gamma_Q} \left\{ \sum_{t \in \Omega_T} \left[ \frac{Q_1^{SE}(t) - \sum_{i \in \Omega_M} Q_i(t) - \sum_{j \in \Omega_{NM}} Q_j(t) - L_Q^{k-1}(t)}{Q_1^{SE}(t)} \right]^2 \right\} \tag{8}$$

### D. Computation of power losses

After the steps presented, the power losses are recalculated through a power flow method using the power allocated to the loads in step C and the measurements of the voltage at the substation.

### E. Verification of the convergence condition

In this step, the absolute comparison between the power losses computed in the current iteration and the previous interaction is verified according to the following expressions:

$$\left| L_P^k(t) - L_P^{k-1}(t) \right| < Tolerance,\ \forall t \in \Omega_T \tag{9}$$

$$\left| L_Q^k(t) - L_Q^{k-1}(t) \right| < Tolerance,\ \forall t \in \Omega_T \tag{10}$$

If the both conditions are satisfied, then the convergence is reached. Otherwise, the process must continue, readjusting the vectors ($\boldsymbol{x}, \boldsymbol{\alpha_P}, \boldsymbol{\beta_P}, \boldsymbol{\gamma_P}, \boldsymbol{y}, \boldsymbol{\alpha_Q}, \boldsymbol{\beta_Q}$ and $\boldsymbol{\gamma_Q}$) of parameters.

### F. Updating the parameters u

In each iteration, the vector of parameters $\boldsymbol{u}$ for loads without meter must be updated. The criterion adopted to obtain a representative value was the average of the set of relation values between the computed voltages of the load and the voltages at the substation for the corresponding time intervals as shown in (11)

$$u_j = \frac{1}{T} \sum_{t \in \Omega_T} \frac{V_j^{cal}(t)}{V_1^{SE}(t)}, \forall j \in \Omega_{NM} \tag{11}$$

Where T represents the number of time intervals.

## III. Case Study

To evaluate the performance of the proposed methodology, a real feeder from a utility company of State of São Paulo-Brazil was used. The nominal voltage and power of this system are 13.8kV and 4500kVA. Fig. 2 shows the 23 nodes feeder with a substation at the first node and loads at the remaining nodes. The buses and lines data for this system are presented in Table I and Table II respectively. More information of the feeder can be found in [6].

For this case study, a database was generated by computing a power flow solution for 50 days with a 15-minute time interval and considering different types of load models for each distribution transformer (node) on the feeder. The load factor profile during a day was obtained according to [7]. To highlight the features of the proposed methodology, the voltage and power load profile at the substation are strongly correlated, with a maximum variation of 5% around the nominal voltage. Fig. 3

shows the 5, 50 and 95% quantiles of the daily apparent power and voltages at the substation along the 50 days. Additionally, the voltage of each load are between 0.975% and 1.025% of its nominal value. With all information, the theoretical energy losses can be calculated through a power flow solution. This value is used in this section to validate the proposed methodology and to compare the results with other representative methods estimation.

TABLE I. BUSES DATA

| Buses | Active load (kW) | Reactive load (kVAr) |
|---|---|---|
| 2 | 1229 | 505 |
| 3 | 80 | 39 |
| 4 | 36 | 17 |
| 5 | 671 | 325 |
| 6 | 176 | 85 |
| 7 | 64 | 31 |
| 8 | 266 | 129 |
| 9 | 72 | 35 |
| 10 | 108 | 52 |
| 11 | 124 | 60 |
| 12 | 28 | 14 |
| 13 | 52 | 25 |
| 14 | 308 | 149 |
| 15 | 16 | 8 |
| 16 | 32 | 16 |
| 17 | 56 | 27 |
| 18 | 68 | 33 |
| 19 | 72 | 35 |
| 20 | 28 | 13 |
| 21 | 72 | 35 |
| 22 | 36 | 17 |
| 23 | 36 | 17 |

TABLE II. LINES DATA

| Initial node | Final node | Resistance (ohm) | Reactance (ohm) | Length (m) |
|---|---|---|---|---|
| 1 | 2 | 0.1104 | 0.1415 | 300 |
| 2 | 3 | 1.1773 | 1.5094 | 3200 |
| 3 | 4 | 1.2141 | 1.5566 | 3300 |
| 4 | 5 | 0.3532 | 0.4528 | 960 |
| 5 | 6 | 0.1112 | 0.1018 | 200 |
| 6 | 7 | 0.3893 | 0.3562 | 700 |
| 7 | 8 | 0.8343 | 0.7634 | 1500 |
| 5 | 9 | 0.5224 | 0.6698 | 1420 |
| 9 | 10 | 0.4783 | 0.6132 | 1300 |
| 9 | 11 | 2.8358 | 1.4145 | 2700 |
| 11 | 12 | 2.7308 | 1.3621 | 2600 |
| 2 | 13 | 1.2604 | 0.6287 | 1200 |
| 13 | 14 | 4.5163 | 2.2528 | 4300 |
| 14 | 15 | 2.3107 | 1.1526 | 2200 |
| 15 | 16 | 3.3610 | 1.6765 | 3200 |
| 16 | 17 | 4.8314 | 2.4099 | 4600 |
| 15 | 18 | 12.4976 | 4.1793 | 7300 |
| 18 | 19 | 8.3888 | 2.8053 | 4900 |
| 13 | 20 | 0.4831 | 0.2410 | 460 |
| 20 | 21 | 6.0917 | 3.0386 | 3480 |
| 21 | 22 | 2.7308 | 1.3621 | 2600 |
| 21 | 23 | 2.9408 | 1.4669 | 2800 |

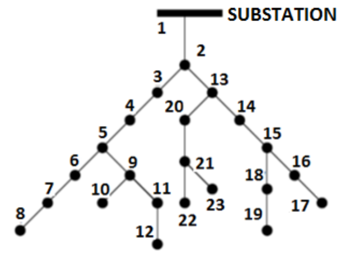


Figure 2. 23 nodes feeder of the case study.

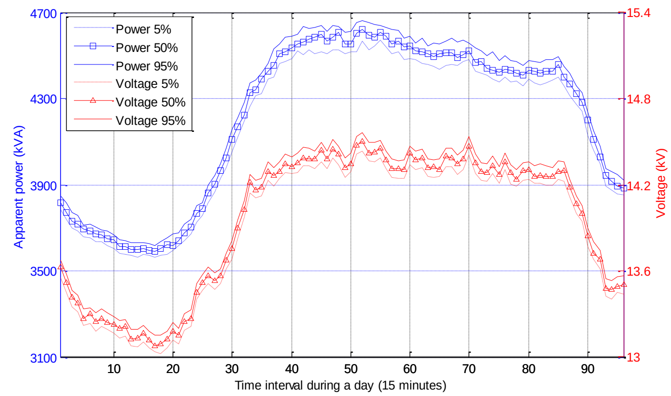


Figure 3. Quantiles for the apparent power and voltage at the substation.

In order to improve the approach of the estimations, the model was applied to three different level of loads during the day (light, medium and peak load), organized by clusters. Note that for each cluster, every load on the feeder has one load model for active and reactive power. Using the three clusters aforementioned, two tests were performed to estimate the energy losses using the Optimization Toolbox of MATLAB [8] to adjust the parameters. The first one considers that there is just one meter at the substation. Therefore, the input data only has values of voltage, active and reactive power at the substation. The second one considers meters at the substation and at the nodes 2 and 18, i.e., the input data contains also measurements of voltages and power consumed by loads connected at the nodes 2 and 18. These tests are called STDA 1 and STDA 2, respectively. In both, the following considerations were taken into account:

- For the constraints (2) and (3), a lower and upper bound factor are 0.8 and 1.2 for every load in the system.
- For the convergence conditions expressed in (8) and (9), it was used a tolerance of 0.45W and 0.45VAr for the active and reactive power, which represents $10^{-5}$% of the peak load measured at the substation.

After applied the proposed methodology for the STDA 1 and STDA 2, the methodology performance is represented by the information in Table I. From this table, it can be seen that the number of iterations for convergence of each cluster is relatively small, however the computation time is longer because the number of power flow computation and the minimization processes of the methodology. The computation time is proportional to the size of the system and the amount of input data.

TABLE III. METHODOLOGY PERFORMANCE

| | ***Iterations performed for each cluster*** | |
|---|---|---|
| | ***STDA 1*** | ***STDA 2*** |
| ***Light demand*** | 8 | 11 |
| ***Medium demand*** | 10 | 12 |
| ***Peak demand*** | 12 | 8 |
| ***Computation time (seconds)*** | 490.80 | 386.45 |

The optimization result of the parameter $x$ for each load adjusted for STDA 1 and STDA 2 is presented in Fig. 4 and Fig. 5. In these figures, it is appreciable that the corresponding values of the loads in the buses 2, 5, 8 and 14 indicate that the loads connected to those nodes demand more power from the system, which agrees with their nominal values presented in Table I. For the sake of space, the results are presented only for the active power component. However, it is important to highlight that a similar behavior can be observed for the vector of reactive power $\boldsymbol{y}$.

The results of the parameter $u$, adjusted for STDA 1 and STDA 2, are shown in Fig. 6 and Fig. 7. As seen in the figures, the values are less than 1 because they are limited to the voltage at the substation and the lowest values (e.g. nodes 12 and 19) indicate the more distance nodes from the substation.

Fig. 8 and Fig. 9 show the results of the active load parameters adjusted for the light demand for STDA 1 and STDA 2. From these figures it is observed that more weight is given to the power constant type of load, represented by the gamma parameter. This happens because the search space of the power is larger than the search space of the voltage, which leads the algorithm to focus in this parameter in most nodes. A similar behavior was observed for medium and peak load tests.

After the parameters adjustment, the power losses, which are a by-product of the model, can be compared with to the real power losses from the database used. Table IV shows the mean absolute percentage error (MAPE) obtained of the active power losses in each test performed between the estimated model and the theoretical value.

Additionally, the results obtained with the proposed methodology are compared to those estimated using the New Top-Down (NTD) methodology [3] and the current methodology applied in Brazil and established by the ANNEL [5]. The NTD methodology estimates the energy losses by calculating a product of a loss factor (based on the power supplied by the substation), the power losses for maximum demand conditions through a power flow solution and the number of time intervals along the period of analysis. The ANNEL methodology estimates the energy losses by calculating the product of a loss coefficient (based on the power supplied by the substation), the average power loss computed by a multiple linear regression equation (established by the ANNEL) and the number of time intervals along the period of analysis.

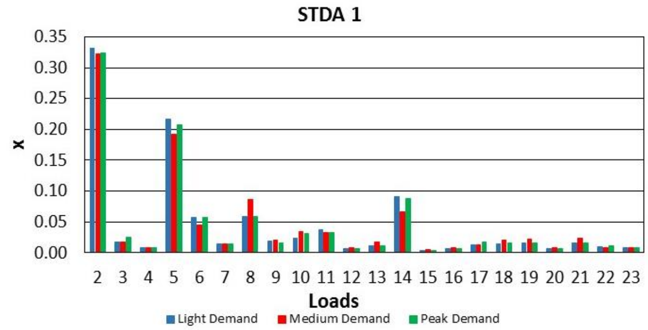


Figure 4. Results of the x parameter for the test STDA 1.

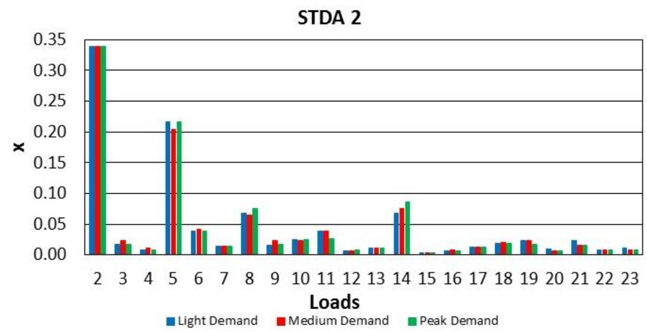


Figure 5. Results of the x parameter for the test STDA 2.

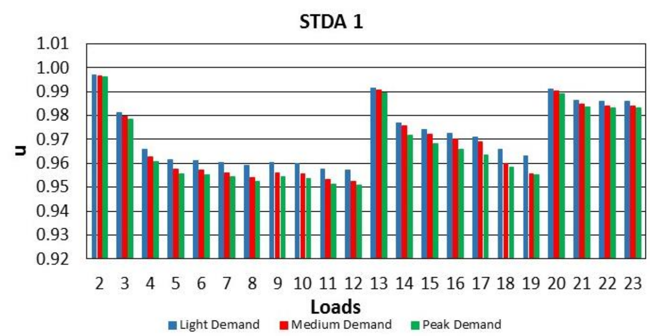


Figure 6. Results of the voltage correlation parameters for the test STDA 1.

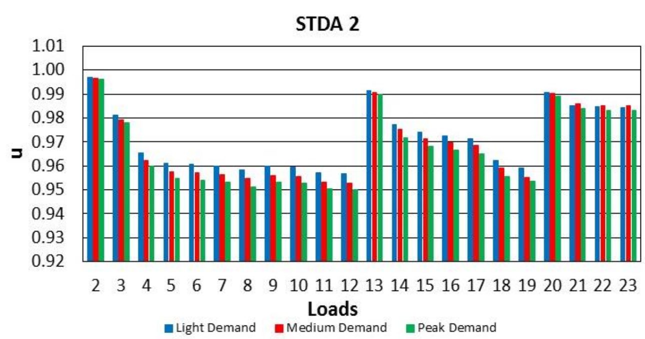


Figure 7. Results of the voltage correlation parameters for the test STDA 2.

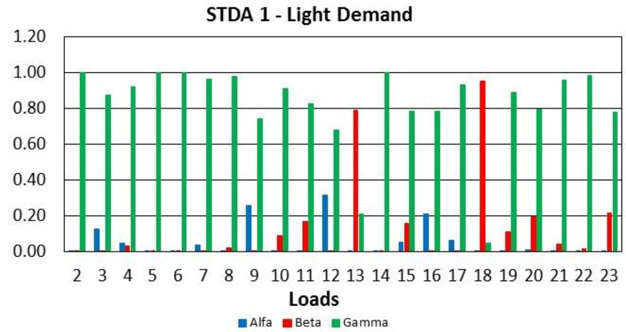


Figure 8. Results of the load parameters for the test STDA 1.

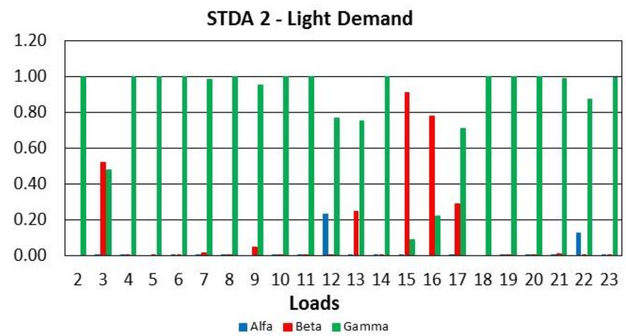


Figure 9. Results of the load parameters for the test STDA 2.

TABLE IV. MAPE OF THE POWER LOSSES ESTIMATION

| *MAPE (%)* | |
|---|---|
| ***STDA 1*** | ***STDA 2*** |
| 5.08 | 1.31 |

TABLE V. ESTIMATION OF ENERGY LOSSES

| **Energy losses during 50 days (kWh)** | | | | |
|---|---|---|---|---|
| ***Real*** | ***STDA 1*** | ***STDA 2*** | ***NTD*** | ***ANEEL*** |
| 95703 | 87494 | 98249 | 105126 | 142357 |

TABLE VI. PERCENTAGE OF ENERGY LOSS

| **Percentage of Energy loss on the system (%)** | | | | |
|---|---|---|---|---|
| ***Real*** | ***STDA 1*** | ***STDA 2*** | ***NTD*** | ***ANEEL*** |
| 2.01 | 1.91 | 2.15 | 2.30 | 3.11 |

TABLE VII. ABSOLUTE PERCENTAGE ERROR

| **Absolute percentage errors (%)** | | | |
|---|---|---|---|
| ***STDA 1*** | ***STDA 2*** | ***NTD*** | ***ANEEL*** |
| 8.60 | 2.70 | 9.84 | 48.70 |

Table V shows the real energy losses of the system and the values estimated by the aforementioned methodologies. Table VI shows the percentage of the energy loss to the total distributed energy. Table VII shows the Absolute Percentage Error of the losses between the different methodologies analyzed and the theoretical value.

As it can be seen in Table VII, the best result was reached by applying the STDA method. The results indicated that the more information available from meters allocated in the network, the more accurate the results will be.

## IV. Conclusions

In this paper, a new method, called Statistical Top-Down Approach (STDA), for energy loss estimation in distribution systems was presented. The novelty of the proposed method is the application of a model that considers the voltage drop of the system to estimate the power in each load, taking into account the power flow results to estimate the energy losses. The case study demonstrates that the proposed methodology estimates energy losses more accurately than other methodologies such as the New Top-Down (NTD) approach and the one produced by ANNEL (the Brazilian regulator). The results indicate that the proposed method is promising, particularly considering that the number of meters to be installed in medium voltage distribution networks is likely to rise in the next few years.

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