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Strut-Braced Wing Modelling with a Reduced Order Beam Model

C.P. Szczyglowski[1], C. Howcroft[1], S.A. Neild[2], B. Titurus[1], J.Z. Jiang[2]
J.E. Cooper[1], E. Coetzee[3]

[1]Dept. of Aerospace Engineering, University of Bristol
[2]Dept. of Mechanical Engineering, University of Bristol
[3]Airbus Operations Ltd.

Abstract

In recent years there has been much interest in the study of strut-braced wings, as they potentially offer the opportunity to design lightweight wings with increased wingspan. This work includes NASA's development of a strut-based configuration as part of the Subsonic Ultra Green Aircraft Research (SUGAR) project. Optimisation strategies, based on linear structural models have been proposed to size such a wing. Here, a simple sizing study is conducted on the SUGAR planform using empirical formula and a linear Nastran structural model. The resulting wing is then analysed using a novel nonlinear structural solver to assess the effect of including geometric nonlinearities on the predicted wing response. It is shown that for the maximum stress levels considered geometric nonlinearity has only a slight effect on the deflections of the sized wing model.

I. Introduction

Significant reductions in aircraft fuel burn are required if the aviation industry is to meet the stringent environmental challenges that have been posed by the civil aviation authorities. One solution is the implementation of a new aircraft concept that can provide a step change in performance over the current commercial airliner design. One such concept is the strut-braced wing (SBW) aircraft; originally championed by Maurice Hurel during the 1950s[1] it has since become the subject of a major study conducted by NASA as part of the Subsonic Ultra Green Aircraft Research (SUGAR) project[2].

A strut-braced wing provides many benefits over a traditional cantilever design. Firstly, the loads alleviation provided by the strut means that the inboard wing section can have a reduced chord and thickness compared to an equivalent cantilever wing. The strut also allows a larger wingspan to be achieved which, when combined with the reduced chord, results in an increased aspect ratio providing an overall aerodynamic benefit. Further aerodynamic benefits are also possible as the decreased wing profile inboard of the strut enables a reduction in both wave and parasitic drag, however the interference drag associated with the truss structure can have a detrimental effect on performance if it is not properly designed. Despite this, numerous studies have shown that a truss-braced wing can provide a significant reduction in take-off weight and fuel burn compared to a cantilever wing aircraft [3, 4, 5, 6].

The increased aspect ratio and reduced thickness of strut-braced wing designs leads to a slender, more flexible structure that is capable of undergoing large deformations[7]. These large deformations cannot be accurately captured using linear analysis and so nonlinear methods must be adopted if the structural deformations and internal loads are to be modelled correctly. Several nonlinear methods are available for the aeroelastic modelling of highly flexible structures including multi-body methods[8], intrinsic beam formulations[9], and various NASTRAN algorithms. However, the application of these methods to SBWs has been limited.

Historically most of the research dedicated to strut-braced wings has been concerned with the overall design and optimisation of the aircraft. Early studies focussed on establishing the performance benefits of a SBW compared to a traditional cantilever design. Various aircraft were studied including, a regional turboprop[3], business jets[4], high-altitude research aircraft[5] and long-range military transports[6]. In each case it was found that the strut-braced wing would provide a benefit in terms of aircraft weight and fuel-burn, however, some aspects of the aircraft such as manufacturing cost and productivity were questioned[4].

During the 1990s a major study was undertaken at Virginia Tech University with the aim of applying multi-disciplinary optimisation (MDO) techniques to the design of a SBW. This study established some of the key challenges facing SBW design, including: strut buckling[10], aeroelastic effects[11] and reliable prediction of strut-wing interference drag[12]. Several researchers have continued to investigate the multi-disciplinary optimisation and design of SBW aircraft, such as teams at ONERA[13], Virginia Tech[14] and Stanford[15].

Arguably the biggest research effort into the strut-braced wing concept is NASA's Subsonic Ultra Green Aircraft Research project. The SUGAR project is tasked with exploring novel aircraft concepts and developing innovative technologies that will improve aircraft performance in-line with the goals set out by the global aviation community[16]. The SBW aircraft was one of the main concepts carried forward from the preliminary exploratory studies and is now in the advance concept development stage. This involves various studies into the aerodynamic, structural and weight characteristics as well as the aeroelastic behaviour of the SBW. Several technical reports have been generated[2, 7, 17, 18] which provide extensive data on the SBW variants currently under investigation, including details on aircraft geometry as well as mass and stiffness properties.

In addition to the research dedicated to the detailed design of strut-braced wing aircraft there has been a recent surge in publications concerned with developing initial sizing and wing weight estimation tools. Locatelli *et al.*[19] developed a physics based method for sizing cantilevered and strut-braced wings which uses an iterative process to calculate the strut reaction force and a lumped boom area model to size the wing cross-section. Chiozzotto[20] presented a wing-weight estimation tool for strut-braced wing aircraft that incorporates some aeroelastic effects, including flexible effects on wing lift distribution and calculation of aileron reversal and divergence speeds, as well as including composite materials in the initial wing sizing process. Aspects of both of these papers have been used in the development of the SBW sizing tool presented in Section 3 of the present work.

This paper will consider the influence of nonlinear geometric effects on the load and deflection predictions of an example SBW. Firstly, a novel, low-order method for modelling the geometric nonlinearity in strut-braced wings using a piecewise shape function method is introduced in Section 2. Some verification results are shown as well as a discussion on the adaptation of this method to SBWs. Section 3 starts by providing an overview of a NASTRAN-based sizing and optimisation process based on the work by [19] and [20] which has been used to perform some initial design studies. Then the static aeroelastic response of a SBW is investigated using the nonlinear method presented in Section 2 and comparisons are made with the linear NASTRAN results. Conclusions are drawn in Section 4.

II. Nonlinear Beam Modelling

The nonlinear beam model used in this research is based on an intrinsic, Euler-Ritz beam formulation developed by Howcroft *et al.* [8, 21]. This method characterises the orientation of a local beam axis using a set of Euler angles (θ,ϕ,ψ) which in turn uniquely define the shape of the beam in 3D space. This is depicted in Figure 1 where e_x, e_y and e_z are the local orientation vectors at a point (s) along the beam and Γ is the position of the beam line in the XYZ coordinate system; determined by integrating the spanwise variation of the local

e_y vector and axial strain (ϵ) along the beam.

$$\Gamma(s) = \int_0^s (1 + \epsilon(s))e_y(s)d\tilde{s} \quad (1)$$

Following the theory presented in [21] it is possible to develop an expression for the energy functional of a beam subject to external forces and moments using the principal of virtual work

$$\Pi = W_K + W_F + W_M \quad (2)$$

where Π is the energy functional, W_K is the work done by the beam strain, W_F is the work done by external forces and W_M is the work done by external moments. The complete expressions for these terms can be found in [21].

The equation of motion governing the static behaviour of the beam can be found via application of Hamilton's Principle

$$\frac{\partial \Pi}{\partial q} = \frac{\partial W_K}{\partial q} + \frac{\partial W_F}{\partial q} + \frac{\partial W_M}{\partial q} \quad (3)$$

where q is the generalised coordinate. In all subsequent equations the ∂ symbol denotes partial differentiation with respect to q.

As described in [21], each degree of freedom (DOF), which for this system are the Euler angles and the axial strain, is defined as a linear combination of shape functions which are prescribed along the spanwise coordinate s. The specific weighting of each shape function is determined by the values of the state vector q, therefore, the solution of (3) can be obtained via the variation of the shape function weights until an equilibrium position is found. Any number or type of shape functions can be chosen however the accuracy of the solution is dependent on this choice. It has been shown that the use of orthogonal shapes gives the fastest and most robust performance in terms of solving the static system [21]. The equilibrium solution of this system in terms of shape function weights may be found using one of MATLAB's built-in nonlinear equation solvers such as *fsolve* although the number of function evaluations can be reduced with the application of a more sophisticated algorithm.

When looking to apply this beam modelling method to SBWs two approaches are available:

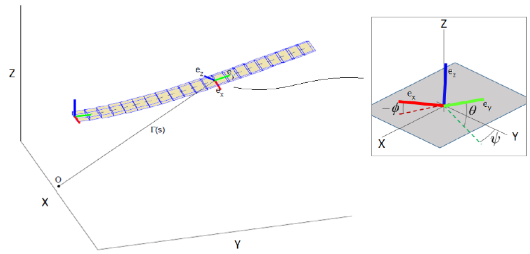


Figure 1: *Example beam deflection showing the local beam axis and Euler angle set [21]*

global shape functions, or a piecewise description which joins multiple sets of shape functions each one capturing the orientations of an individual wing segment. Using global shape functions would enable the largest reduction in the order of the model as the entire wing structure is captured with each shape function, however, generating such shape functions is not a trivial exercise and new shape functions would need to be generated each time that the structure was modified.

An approach built on piecewise shape functions is more promising as it allows extra structural elements to be added without needing to generate new shape functions and it means discontinuities in the wing structure, such as kink points, can be added with relative ease. The approach detailed in this paper uses the latter method.

i. Application of the piecewise shape function method

We will first consider the case of a cantilevered beam modelled using piecewise shape functions. By splitting the beam into several segments new internal boundary points are introduced which were not present in the previous beam modelling formulation. In order to satisfy the equilibrium of work over the entire beam additional energy terms must be incorporated into (2) that ensure that the orientation of the local axis (see Figure 1) is matched at each internal boundary.

$$\Delta\vartheta = \vartheta_i - \vartheta_{k_i} \quad (4)$$

where $\Delta\vartheta$ is the generic orientation constraint, ϑ is the rotation vector and ϑ_k prescribes the rotation vector relating to the optional desired kink angles.

This constraint is then introduced into the energy functional by applying a Lagrange multiplier.

$$\Pi^* = \Pi + \sum_{i=1}^{nB} \lambda_{\vartheta_i} \cdot \Delta\vartheta \tag{5}$$

where Π^* is the modified energy functional with rotation constraints, nB is the number of internal boundaries in the beam element and λ_ϑ is the Lagrange multiplier relating to the orientation constraint.

Applying Hamilton's principle yields the equations of motion for a multi-segmented beam subject to externally applied static forces and moments.

$$\partial\Pi^* = \partial\Pi + \sum_{i=1}^{nB} \partial\lambda_{\vartheta_i} \cdot \Delta\vartheta_i + \lambda_{\vartheta_i} \cdot \partial\Delta\vartheta_i \tag{6}$$

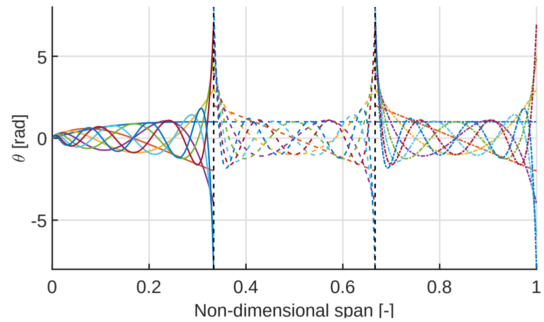
For the case of a single beam modelled with piecewise shape functions this is the only condition that is required to ensure that the segmented and non-segmented models match. This approach was verified by comparing against the classical test case of a fixed-free cantilevered beam undergoing pure bending [22]. The segmented beam model consisted of three segments with eight shape functions in each degree of freedom (DOF). The model has a length of 1m and a bending stiffness of 2Nm^2. The applied tip moment is determined by

$$M_x = \beta \frac{EI\pi}{L} \tag{7}$$

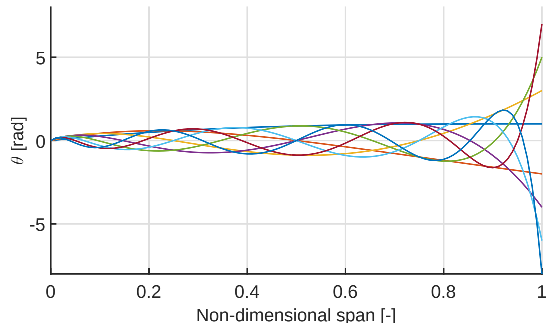
where EI is the beam bending stiffness and L is the beam length; β is a load factor and when $\beta = 2$ the beam deflects into a full circle. Analytic expressions for the beam deflections are available in [23].

Figure 2 shows shape functions that contribute to the overall orientation angle θ for the cases where the beam is split into three segments (Figure 2a) and where the beam is treated globally (Figure 2b). For each set the kinematic root condition $\theta = \phi = \psi = 0$ is satisfied (see [21] for further discussion on boundary conditions).

Considering the deflection of the three-segment piecewise shape function model, Figure 3 shows the prediction for a variety of loading conditions. Excellent agreement between the analytical and numerical results were obtained in all cases.



(a) *Example shape functions for a three segment beam*



(b) *Example shape functions for a one segment beam*

Figure 2: *Example shape functions for a segmented and non-segmented beam element*

ii. Modelling multiple beam elements

Expanding the formulation to include multiple inter-connected beam elements requires a further term to be added to (5). In this instance it is required that the displacement in the global reference frame (Γ) at every inter-element boundary must be the same.

$$\Delta\Gamma = \Gamma_1 - \Gamma_2 \tag{8}$$

The implementation of this constraint takes a similar form to (6), including the use of a Lagrange multiplier to enforce the constraint. Adding the displacement constraint to the modified energy function (5) and taking the partial derivative with respect to q provides the equation of motion for a mutli-element beam system modelled using piecewise shape functions.

$$\partial\Pi^+ = \partial\Pi^* + \sum_{j=1}^{nE} \partial\lambda_{\Gamma_j} \cdot \Delta\Gamma_j + \lambda_{\Gamma_j} \cdot \partial\Delta\Gamma_j \tag{9}$$

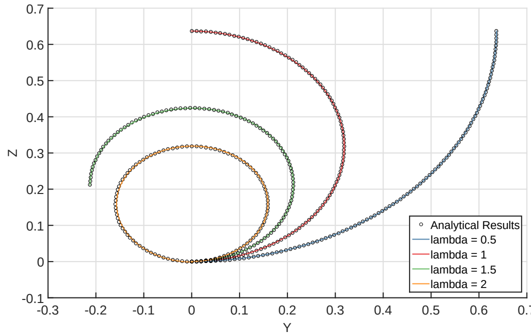


Figure 3: *Comparison of the analytical and numerical results for a cantilevered beam subject to pure bending*

where Π^{+} is the modified energy functional with displacement and rotation constraints, nE is the number of inter-element boundaries in the beam model and λ_{Γ} is the Lagrange multiplier relating to the displacement constraint. Some preliminary results have been generated using this method and are shown in Figure 5. In this study the effect of increasing the number of shape functions in each segment is investigated. The wing model for this case is the classic HALE wing [23] with a strut attached at the half-span and a single jury-strut located at the midpoint of the strut and connecting vertically to the undeformed wing, as in Figure 7. The beam properties for the strut

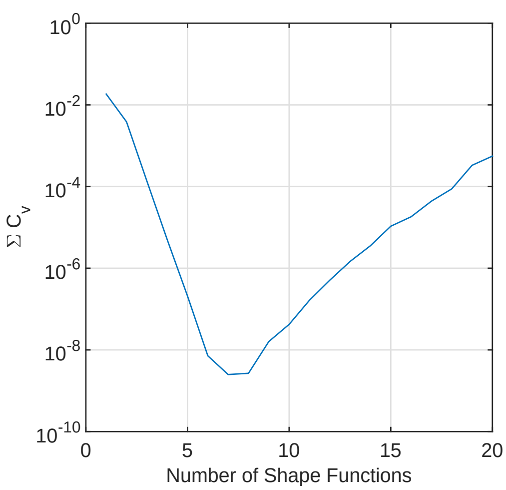


Figure 4: *Variation of total convergence criteria with respect to the number of shape functions in each beam element*

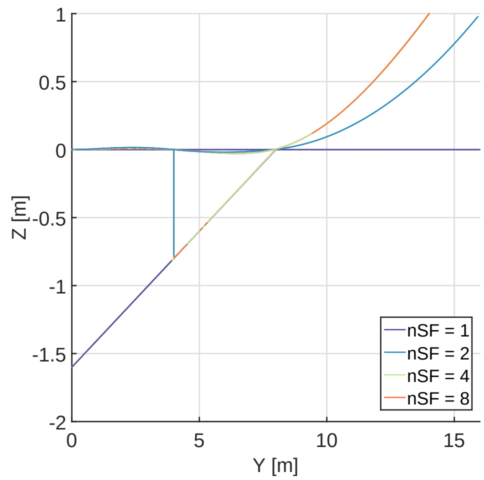


Figure 5: *SBW deflected shape for a varying number of shapes in each segment*

and the jury-strut are the same as the HALE wing and an axial stiffness of 2×10^{6} has been assumed for all beam elements.

The HALE wing has been used as it enables a better understanding of the shape functions required to capture large deflections. The load case is a 500N follower tip load in local y axes and a 150N follower tip load in the local z axes as this will induce reasonably large deflections, thus making it suitable as a test case.

In order to provide a quantitive measure of the accuracy of the nonlinear solution the convergence criteria from [21] has been modified to include the forces and moments from the displacement and orientation constraints. Figure 4 shows that as the number of shape functions in each DOF increases the solver achieves an improved convergence up to the point where eight shape functions are used in each DOF for each beam element. After this point the convergence value increases indicating a decrease in accuracy. This is because even though a higher order shape function may have very little participation in the solution it can still introduce noise, which for this simple load case has a significant effect. Therefore, there is a strong argument for using a decreased number of shape functions in the degrees of freedom where a low order solution is expected, such as the axial degree of freedom.

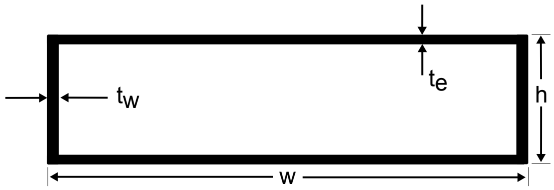


Figure 6: *Schematic of the torque box section used in the sizing process*

iii. Aerodynamic Modelling

The nonlinear beam model can be coupled to any arbitrary aerodynamic formulation. For this research aerodynamic forces are generated using a vortex lattice method (VLM). The implementation of the VLM aerodynamics matches the method described in [8].

III. Strut-Braced Wing Sizing

A strut-braced wing sizing tool has been developed in order to facilitate further design studies and research into the strut-braced wing concept. Initially the conceptual method introduced in [20] was used to generate a beam model based on the SUGAR VOLT planform [7]. This method assumed that the loads alleviation provided by the strut resulted in constant internal loads inboard of the strut. As noted by Chiozzotto[20] this was an overly-conservative assumption and as a result the total wing weight was almost double that of the SUGAR VOLT aircraft.

Several modifications have been made to the previous method so as to achieve a closer match with the SUGAR aircraft. First amongst these is the utilisation of NASTRAN's aeroelastic solution sequence (SOL 144) to generate the aerodynamic loads and subsequent structural loads which are used to size the wing. Secondly, the Euler buckling check has been expanded to consider buckling of the inboard wing section and the jury-strut as these have been shown to be critical design cases[7]. In addition to this the axial load in the wing due to the strut reaction force is now considered when sizing the wing box components[19]. Finally, the aircraft vertical trim is handled by SOL 144, which gives a better estimate of the wing loading when compared to [20].

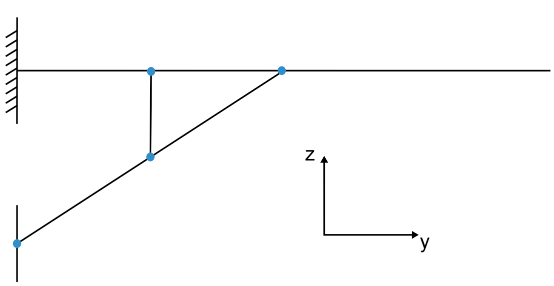


Figure 7: *Schematic of SBW showing joint types at inter-element boundaries*

The wing geometry and load cases used in the sizing process have been extracted from the SUGAR project technical reports[7]. However, in contrast to the SUGAR study, here the wing is sized for static aeroelastic load cases only; the capability to model gust responses will be added at a later stage.

The torque box of the wing, strut and jury-strut is modelled as a simple rectangular box (see Figure 6). Stringers are accounted for by considering a smeared skin approach as in [20], although, a boom area method[19] is also perfectly acceptable. At this stage the section is assumed to be symmetric, with the top and bottom covers (t_e) and front and rear spars (t_w) having equal thicknesses. This is a simple approach that is not representative of a real aircraft wing box; however, it is suitable for initial design studies.

The covers are assumed to carry direct stresses due to bending and shear stresses due to torques whereas the web components carry shear loads only. The equations for sizing the wing box and rib components can be found in [20]. In the current study the sizing equations have been modified to include the contributions from the axial loading[19].

Figure 7 shows the joint types at each of the inter-element boundary positions. For this study the strut-root, strut-wing, strut-jury and jury-wing connections are all assumed to be pinned about the local x axis. This means that out-of-plane bending moments are not transferred across these joints, however, all forces as well as in-plane and torque moments are transmitted.

The NASTRAN structural model uses linear beam elements (CBAR) and the aerodynamic forces are generated using the doublet-lattice method (DLM) with 20 spanwise and 4 chordwise panels. A beam spline is used to interpolate the aerodynamic forces to the structural nodes. Figure 8 shows an overview of the sizing and optimisation process. In the present study the optimisation loop is not used, instead the output from the sizing process represents a minimum weight configuration as it is likely that the flutter and divergence constraints would be violated and the wing stiffness increased.

For the initial SBW configuration the strut and jury strut were attached to the wing at the wing-box shear centre. This meant that the

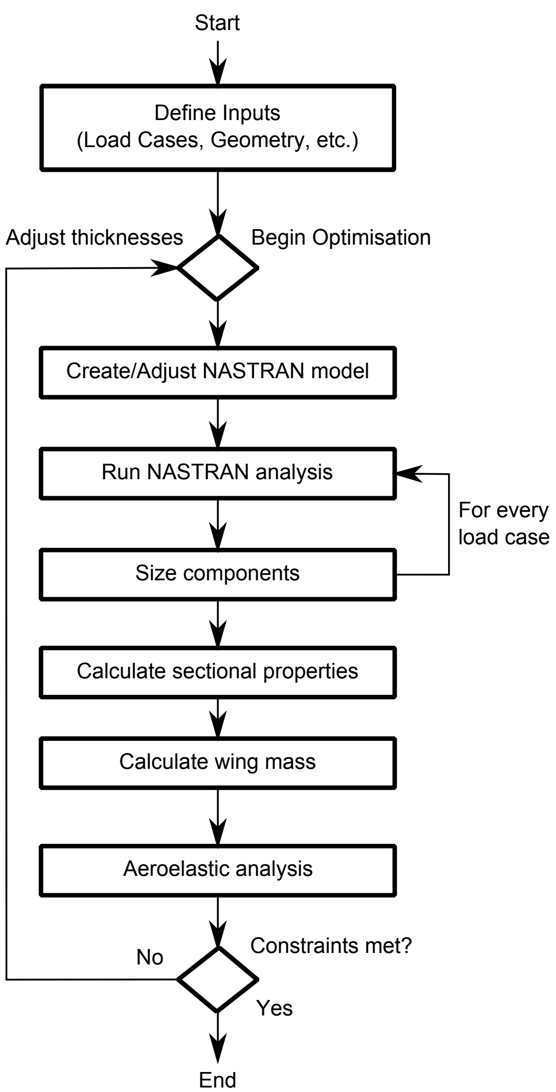


Figure 8: *SBW sizing and optimisation process*

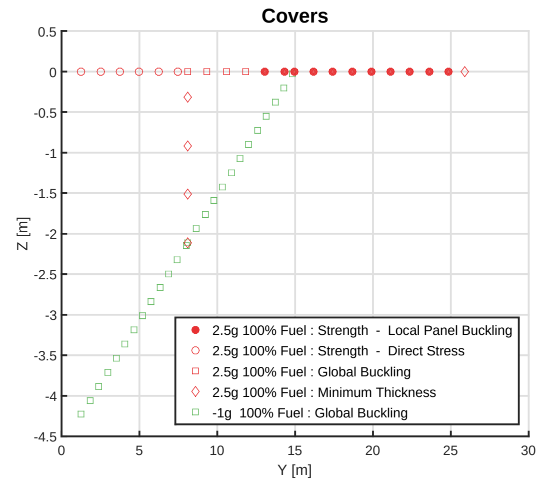
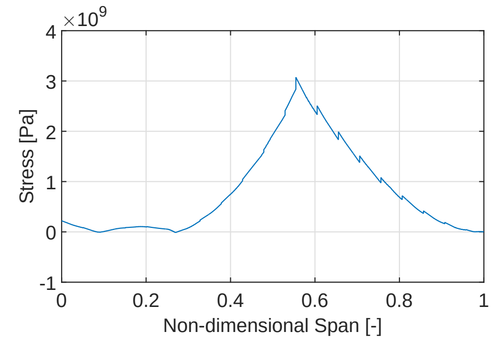


Figure 9: *Component sizing diagram for wing, strut and jury-strut box covers*

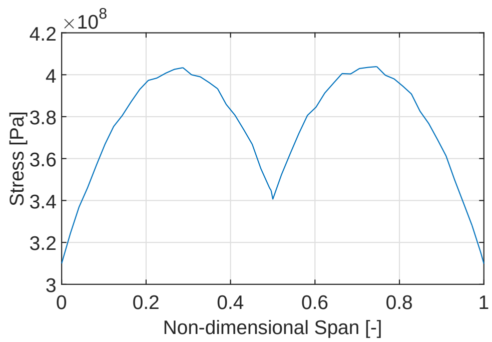
truss-structure provided no loads alleviation to the twist degree-of-freedom which led to excessive torque loading inboard of the strut attachment point. The resulting wing structure was dominated by torque loading and was significantly over-sized.

It was clear that the assumption of a fully-symmetric section was overly simplistic as it prevented the full benefits of the strut passive loads alleviation from being exploited. As a first step towards rectifying this the beam eccentricity was reduced by shifting the aerodynamic mesh rearwards by a small fraction of the tip chord; which, for the aerodynamically loaded wing, is analogous to moving the shear centre slightly towards the front spar. As it is typical to have a thicker front spar this was deemed to be an acceptable change. Running the sizing again for this new model showed a significant reduction in wing weight. Further studies will investigate the passive load alleviation effects of moving the strut attachment point along the chordwise direction.

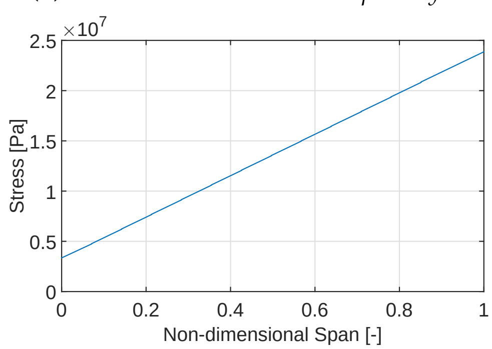
Figure 9 shows the component sizing diagram for the wing, strut and jury-strut covers. As expected, the primary strut is sized for global buckling during the -1g load case. The global buckling constraint has also been activated for the inboard wing section between the jury and primary strut attachment points, indicating that the axial force component cannot be neglected during the initial sizing process.



(a) *Maximum direct stress in the wing*



(b) *Maximum direct stress in the primary strut*



(c) *Maximum direct stress in the jury strut*

Figure 10: *Maximum direct stress distribution in the wing (a), strut (b) and jury- strut (c) for the 2.5g, 100% fuel case*

Most of the outer wing section has been sized by inter-panel buckling requirements as a result of the large curvatures that are present in this part of the structure during the 2.5g load case. Perhaps most interesting is that the jury-strut components have been sized by minimum thickness requirements. This would suggest that trim load cases alone are not sufficient for sizing the jury-strut and other load cases, such as gust loading, may be necessary.

The maximum direct stress distribution in the wing, strut and jury-strut is shown in Figure 10 for the 2.5g, maximum fuel load case. The maximum stress in the wing occurs at the attachment point of the primary strut which is to be expected due to the combined effects of large bending curvatures and the strut reaction force. The 'bow-tie' stress distribution in the primary strut is a result of the pinned connections between the wing, fuselage and jury strut, causing the bending stress to go to zero at these points. The resulting stress value at the strut root and tip is due to axial loading only.

It is interesting to note that the distinctive bow-tie shape of the direct stress distribution in the primary strut matches exactly the primary strut chord shape on the SUGAR VOLT aircraft. If the strut chord was defined as a design variable in the optimisation problem then it is possible that the strut geometry would converge to the SUGAR VOLT case for some combination of direct stress and buckling loads. This capability could be added in future iterations of the sizing tool.

A comparison of the final wing stiffness distribution for the three wing models is shown in Figure 11. The out-of-plane bending stiffness matches the SUGAR VOLT data reasonably well, which is surprising considering the simplicity of the box model in this analysis, however, the conceptual method has over-sized the cover components which has led to an increased in-plane and torsional stiffness distribution. The NASTRAN sizing method has an increased axial stiffness as it has accounted for the axial wing loading inboard of the strut attachment point although, it is impossible to compare this to the SUGAR VOLT as no information is available for the axial stiffness distribution.

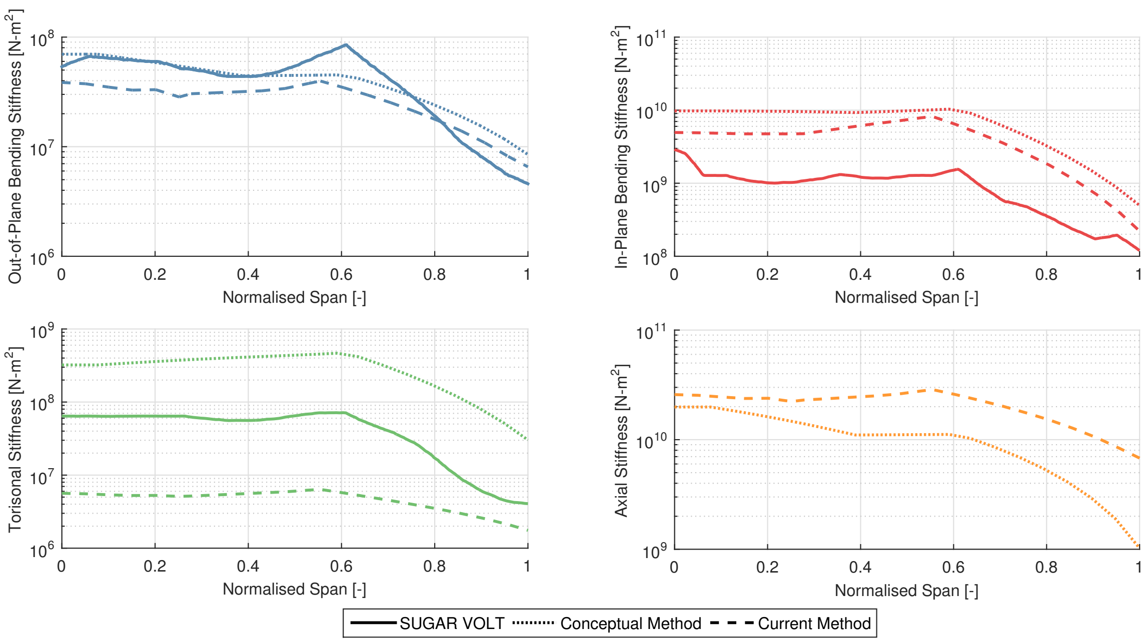


Figure 11: *Comparison of wing stiffness for the three sizing methods*

The final wing weights for the two sizing methods and the SUGAR VOLT aircraft are shown in Table 1. The current sizing process overestimates the wing weight by around 40% although this is not surprising considering the simplicity of the wing box model. It is anticipated that refinement of the box geometry and adjusting the strut-wing attachment point towards the front spar will cause a significant reduction in wing weight and lead to better convergence between the two models.

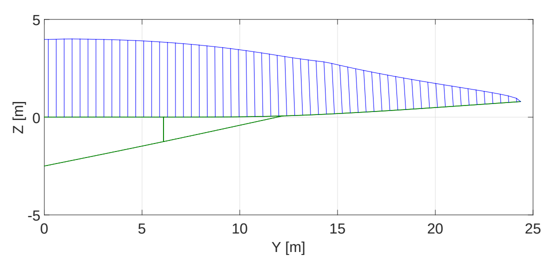


Figure 12: *Nonlinear deflected shape and aerodynamic force distribution for the 2.5g 100% fuel load case*

IV. Nonlinear Modelling of a SBW

The nonlinear beam model from Section 2 is used to analyse the sized wing model from the NASTRAN sizing process. Aerodynamic loads are calculated using VLM and the aircraft is trimmed for the 2.5g, max fuel load case. Figure 12 shows that for this wing model the deflections for the 2.5g load case are not significant and can be considered to be linear, therefore, from inspecting the deflections, geometric nonlinearity has an almost negligible effect. Further analysis will continue to focus on the effect that geometric nonlinearity has on a SBW configuration as it is anticipated that as the sizing and optimisation process is refined the model will become more flexible and therefore nonlinear effects may need to be included.

Table 1: *Wing Weight comparison for the three methods*

Sizing Method	**Wing Weight**
Conceptual Method	13,940kg
Modified Conceptual + NASTRAN	10,400kg
SUGAR VOLT [7]	7561kg

V. Conclusion

This paper has presented a novel method for nonlinear beam modelling of strut-braced wings based on a piecewise weighted shape function method. Verification of the method has been carried out for the case of a cantilevered beam and excellent agreement has been achieved with analytical results. The importance of choosing the correct number of shape functions has also been highlighted and some sample results for a SBW have been shown.

Additionally, a strut-braced wing sizing process using NASTRAN has been detailed and some of the key design considerations have been highlighted. Wing weights and stiffness distributions have been compared against the SUGAR VOLT aircraft and results from a preliminary sizing process. The NASTRAN-based method has provided an improved estimate of the wing weight compared to the conceptual method however further weight refinements are required in order to match the SUGAR VOLT data.

Future work will focus on expanding the capability of the NASTRAN sizing process with the aim to add in divergence and flutter constraints to enable full optimisation of the wing as well as including a more accurate torque box model. In addition, it is planned to incorporate the nonlinear modelling tool within the optimiser to assess whether the maximum loads which govern the sizing are effected by nonlinear effects, such as large deformations or nonlinear stiffening.

VI. Acknowledgements

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References

[1] M Hurel. The advantages of high aspect ratios. *Interavia*, 7(12):695–699, 1952.

[2] Marty K Bradley and Christopher K Droney. Subsonic ultra green aircraft research phase ii: N+ 4 advanced concept development. 2012.

[3] Paul H Park. Fuel consumption of a strutted vs cantilever-winged short-haul transport with aeroelastic considerations. *Journal of Aircraft*, 17(12):856–860, 1980.

[4] RV Turriziani, WA Lovell, GL Martin, JE Price, EE Swanson, and GF Washburn. Preliminary design characteristics of a subsonic business jet concept employing an aspect ratio 25 strut braced wing. 1980.

[5] Paul M Smith, John DeYoung, William A Lovell, Jack E Price, and G Fred Washburn. A study of high-altitude manned research aircraft employing strut-braced wings of high-aspect-ratio. 1981.

[6] Robert M Kulfan and John D Vachal. Wing planform geometry effects on large subsonic military transport airplanes. Technical report, DTIC Document, 1978.

[7] Marty K Bradley, Christopher K Droney, and Timothy J Allen. Subsonic ultra green aircraft research. phase ii-volume i; truss braced wing design exploration. 2015.

[8] Chris Howcroft, Dario Calderon, Luke Lambert, Michele Castellani, Jonathan E Cooper, Mark H Lowenberg, and Simon Neild. Aeroelastic modelling of highly flexible wings. In *15th Dynamics Specialists Conference*, page 1798, 2016.

[9] Dewey H Hodges. A mixed variational formulation based on exact intrinsic equations for dynamics of moving beams. *International journal of solids and structures*, 26(11):1253–1273, 1990.

[10] Amir H Naghshineh-Pour. Structural optimization and design of a strut-braced wing aircraft. 1998.

[11] Erwin Sulaeman. *Effect of compressive force on aeroelastic stability of a strut-braced wing*. PhD thesis, Virginia Polytechnic Institute and State University, 2001.

[12] Philippe-André Tétrault, Joseph A Schetz, and Bernard Grossman. *Numerical prediction of the interference drag of a streamlined strut intersecting a surface in transonic flow*. University Microfilms, 2004.

[13] G Carrier, O Atinault, S Dequand, JL Hantrais-Gervois, C Liauzun, B Paluch, AM Rodde, and C Toussaint. Investigation of a strut-braced wing configuration for future commercial transport. In *28th Congress of the International Council of the Aeronautical Sciences*, pages 2012–1. ICAS Bonn, 2012.

[14] Ohad Gur, Manav Bhatia, William H Mason, Joseph A Schetz, Rakesh K Kapania, and Taewoo Nam. Development of a framework for truss-braced wing conceptual mdo. *Structural and Multidisciplinary Optimization*, 44(2):277–298, 2011.

[15] Anil Variyar, Thomas D Economon, and Juan J Alonso. Multifidelity conceptual design and optimization of strut-braced wing aircraft using physics based methods. In *54th AIAA Aerospace Sciences Meeting*, page 2000, 2016.

[16] 1European Commission et al. Flightpath 2050: EuropeŠs vision for aviation. *Report of the High Level Group on Aviation Research, Publications Office of the European Union, Luxembourg*, 2011.

[17] Marty K Bradley and Christopher K Droney. Subsonic ultra green aircraft research. 2011.

[18] Marty K Bradley, Timothy J Allen, and Christopher Droney. Subsonic ultra green aircraft research: Phase ii-volume iii-truss braced wing aeroelastic test report. 2014.

[19] Davide Locatelli, Benjamin Riggins, Rakesh K Kapania, Joseph A Schetz, and Thomas Poquet. A physics-based methodology for cantilever and strut-braced wing weight estimation. In *54th AIAA Aerospace Sciences Meeting*, page 0780, 2016.

[20] Gabriel P Chiozzotto. Wing weight estimation in conceptual design: a method for strut-braced wings considering static aeroelastic effects. *CEAS Aeronautical Journal*, 7(3):499–519, 2016.

[21] C. Howcroft, S.A. Neild, M.H. Lowenberg, and J.E. Cooper. Aeroelastic Modelling of Highly Flexible Wings, To appear in proceedings of USD2012 International Conference on Uncertainty in Structural Dynamics, Leuven, Belgium, 19-21 September. 2016.

[22] Juan C Simo and Loc Vu-Quoc. A three-dimensional finite-strain rod model. part ii: Computational aspects. *Computer methods in applied mechanics and engineering*, 58(1):79–116, 1986.

[23] Nicholas A Johnson. A verification and validation of the geometrically exact beam theory with legendre spectral finite elements for wind turbine blade analysis. 2007.