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Performance Analysis of Coordinated Transmission for Stochastic Cellular Network

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Abstract—**Inter-cell interference mitigation techniques are playing important roles to improve the system performance, especially for dense network. Among them, the coordinated transmission has been used to tackle the interference problem. In this paper, we investigated the performances of coordinated transmission with stochastic network modeling and derived its expression on coverage probability and average rate with different number of base station coordination. The numerical results have been presented to compare the performance with frequency reuse technique. The results suggest that the coordinated transmission can perform better than frequency reuse in terms of coverage probability and average rate depending on the scheduling of the transmission of coordinated base stations.**

Index Terms—**Frequency reuse, stochastic network modeling, coordinated transmission**

I. INTRODUCTION

Due to the explosive use of smart phones , computing devices and machine-type of sensors, the population of users connecting cellular networks has reached an unprecedented level and demands the network to deliver huge volume of data. It is expected that the cellular networks will have to increase its capacity by 1000 times within next decade and to take up this challenge, the networks recently have been overhauled within standard organizations , industry consortium and academia. There are a couple of approaches to fundamentally increase the network capacity, one of them is to shrink the cells into 'small cells' and deploy them everywhere and consequently improve the system capacity due to spatial reuse. Small cell means the network will become much denser and as a result, the locations of the base stations (BS) within cellular networks are more random, especially the BS (femto base station for example) nowadays can be deployed by subscribers.

Meanwhile, infrastructure cellular networks are normally interference-limited and the inter-cell interference is a big concern for network performance , especially the network coverage, which become particularly critical for highly dense network. Therefore, inter-cell interference mitigation techniques play an important role to improve the system performance. One simple way to mitigate the interference is the frequency reuse where there are a given number of channel bands and each BS is allocated one of them. Since the number of BSs using the same channel band is reduced , the interference will decrease hence the system coverage will be increased. There are variant frequency reuse methods depending on how to allocate the channels to BSs, for example, random allocation and coloring-based allocation. More recently, the coordinated transmission has been used to tackle the interference problem in the latest cellular systems, for example, the COMP (coordinated mulipoint) in LTE. In concept, there are two major approaches regarding to the coordinated transmission: **Joint transmission**: Coordinated transmission occurs where there is coordination between multiple entities - base stations - that are simultaneously transmitting to users, i.e a user will receive the same data from n BSs who provide n strongest received power for the user. **Coordinated scheduling**: This is a form of coordination where a user is communicating with a single transmission base station. However the communication is made with an exchange of control among several coordinated entities. i.e when a user is receiving the signal from its serving BSs, the $n-1$ strongest interferencing BSs will mute their transmission. Both the n BS coordinated transmission approaches have their con and pro. Instead of comparison, we will focus on the analytical performance on the latter in following sections

To properly model the random/stochastic cellular network has been challenging and the stochastic geometry based modeling seems to be promising and performs better than the conventional hexagonal modeling [1], [2], see Fig.1 for exemplary network layout. In stochastic modeling, the base stations(BSs) are placed randomly and the locations of the BSs are modeled normally as a homogeneous Poison point process (PPP), where the performance matrics like coverage and ergodic rate can be determined with a given radio channel, see [1]–[7] and the references therein. Regarding to interference mitigation for stochastic cellular network, the works in [1], [4], [7] have been done on the analytical performance for frequency reuse. The coordinated transmission, at the moment, haven't been fully analytically studied and there are very few works looking into it and this motivates us to investigate its analytical performance in terms of coverage and average rate. In this paper , the major contribution is to derive the analytical expression of the coverage probability and average rate for coordinated transmission and investigate its numerical results as well as compare it with frequency reuse.

The rest of the paper is organized as follows: Section II describes the system model, and Section III and Section IV analyze the coverage probability and average rate for
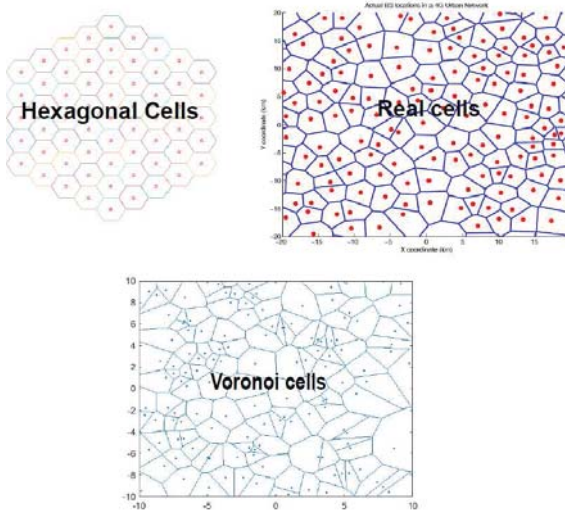
Fig. 1. A realization of Poisson point process (Voinoro cells), real netowrk deployment [1] and conventional hexagonal cells

coordinated transmission, respectively. In Section V, numerical results have been presented to evaluate the performance. Then Section VI concludes the paper.

II. SYSTEM MODEL

A. Poisson Point Process

For better illustration of the analysis in the remaining sections, here we briefly describe the PPP Ω which is characterized mainly by two fundamental properties:

1) The numbers of points falling within any disjoint regions are independent random variables ;

2) The random number of points of $\Omega = \{v_i\}$ falling within a region A has a Poisson distribution as

$$\mathbb{P}(\mathrm{N}(A) = k) = \frac{\Lambda(A)^k e^{-\Lambda(A)}}{k!}, \qquad (1)$$

where $\mathrm{N}(A)$ is the number of points in A and $\Lambda(A) = \int_A \Lambda(\mathrm{d}v) = \int_A \lambda(v)\mathrm{d}v$ is the intensity measure of the region A, and $\lambda(v)$ is the intensity function. For homogeneous PPP, $\lambda(v) = \lambda$ and $\Lambda(A) = \lambda|A|$, where $|A|$ is the area of the region A.

Lemma 1: A homogeneous PPP $\Omega = \{v_i\}$ in 2-dimensional planar space, where the v_i is the coordinates of point i to the origin, then the PPP $\Omega_1 = \{\|v_i\|\}$ is an inhomogeneous PPP with intensity function of $\lambda(x) = 2\pi\lambda x$

The proof of *Lemma 1* can be done by using the mapping theorem of PPP [8] [9]. In this paper, the *Lemma 1* will be applied to the random network we are working on where for a given location, the distances x from the BSs to the location form an inhomogeneous PPP with intensity function as $2\pi\lambda x$.

B. Network deployment

We consider a network having BSs spatially distributed as a homogeneous Poisson point process (PPP) Φ with intensity

λ in the Euclidean plane, where the probability to have k BSs in the region A follows $\mathbb{P}(k) = \frac{(\lambda|A|)^k}{k!}\exp(-\lambda|A|)$ and $|A|$ is the area of the region A. A user is assumed to connect a base station who is closest and is called the user's serving BS, namely the users will fall into the Voronoi cell of their serving BS , resulting in coverage area of a BS that comprises a Voronoi tessellation on the plane as shown in Fig. 1

C. Coverage and average rate

The path loss is given by $l(r) = r^{-\alpha}$, where $\alpha > 2$ and r is the distance from a transmitter to receiver, and All the BSs transmit with the same power. The fading channel between a BS and a user is assumed to be Rayleigh channel , hence the fading power h follows exponential distribution with mean μ, ie. $h \sim \exp(1/\mu)$. Without loss of generality, we assume unit transmit power , then we can have , for a typical location as the origin, the downlink signal-to-interference-plus-noise ratio (SINR) at the origin is given by

$$S_c = \frac{hr^{-\alpha}}{\sigma^2 + \sum_{i\in\Phi\backslash o} h_i r_i^{-\alpha}}$$
$$= \frac{hr^{-\alpha}}{\sigma^2 + I_r} \qquad (2)$$

where I_r is the aggregated interference and o is the location of the serving BS.

The coverage probability is defined as the probability of the SINR larger than a given threshold τ, equivalently, the complemnetary cumulative distribution function(CCDF) of the SINR,

$$p_c(\tau, \lambda, \alpha) = \mathbb{P}(S_c > \tau) \qquad (3)$$

The average rate of the typical location is defined as

$$q_c = \mathbb{E}[\ln(1 + S_c)] \qquad (4)$$

where the average is taken over both the fading distribution and the spatial PPP.

Our research problem here is to derive the coverage probability and the average rate for coordinated transmission in the following section.

III. COVERAGE PROBABILITY ANALYSIS

By conditioning on the distance from the origin to the serving BS, the coverage in (3) can be expanded as

$$\mathbb{P}(S_c > \tau) = \mathbb{E}_R\left[P(S_c > \tau|r)\right]$$
$$= \int_{r>0} \mathbb{P}(S_c > \tau|r) f_R(r)\mathrm{d}r \qquad (5)$$

where $f_R(r) = 2\pi\lambda r \exp(-\lambda\pi r^2)$ is the pdf of r to the closest BS. Then the coverage probability at a typical location (origin) is given as [1]

$$p_c(\tau, \lambda, \alpha) = \pi\lambda \int_0^\infty e^{-\pi\lambda(1+\rho(\tau,\alpha))-\mu\tau\sigma^2 v^{\alpha/2}}\mathrm{d}v \qquad (6)$$

where $\rho(\tau, \alpha) = \tau^{2/\alpha}\int_{\tau^{-2/\alpha}}^\infty \frac{1}{1+u^{\alpha/2}}\mathrm{d}u$

A. Coverage probability of frequency reuse

Assuming there are δ channel bands and we allocate one of them to each BS randomly, similar to (6), the coverage probability is given as [1]

$$p_c(\tau, \lambda, \alpha, \delta) = \pi\lambda \int_0^\infty e^{-\pi\lambda(1+\frac{1}{\delta}\rho(\tau,\alpha))-\mu\tau\sigma^2 v^{\alpha/2}}\mathrm{d}v \quad (7)$$

It should be noted that $\delta = 1$ corresponds to the case without frequency reuse.

B. Coverage probability of coordinated transmission

Now we are working on the coverage probablity for co-ordinated transmission. Re-order the points in PPP $\Phi = \{\phi_1, \phi_2, ...\}$ with ascending order in terms of distance to the origin, and define $\Phi' = \Phi\backslash\{\phi_1, ..., \phi_n\}$, then the coverage of coordinated transmission with n-BS coordination can be represented as

$$\begin{aligned}
p_t(\tau, \lambda, \alpha, n) &= \mathbb{P}(S_t > \tau)\\
&= \mathbb{P}\left(\frac{hr^{-\alpha}}{\sigma^2 + \sum_{i\in\Phi'} w_i h_i r_i^{-\alpha}} > \tau\right)\\
&= \mathbb{P}\left(\frac{hr^{-\alpha}}{\sigma^2 + I_{r_n}} > \tau\right)\\
&= \mathbb{E}_R\left[\mathbb{P}\left(h > \tau r^\alpha(\sigma^2 + I_{r_n})|r\right)\right] \quad (8)
\end{aligned}$$

where r_n is the distance from the nth closest BS , I_{r_n} is the aggregated interferences from the BSs whose distances to the origin are larger than r_n, $S_t = \frac{hr^{-\alpha}}{\sigma^2 + \sum_{i\in\Phi'} w_i h_i r_i^{-\alpha}}$ is the SINR with coordinated scheduling and w_i is the Bernoulli variable with probability p_w , i.e. $\mathbb{P}(w_i = 1) = p_w$. The w_i is used to model the coordinated transmissions among the interferencing BSs to the origin. How to choose the probability depends on the scheduling of the transmission of the BSs. Since the BSs are distributed uniformly for any realization of PPP and if we assume the users are also uniformly distributed in the network, it is reasonable to assume that each BS has equal transmission, then $p_w = \frac{1}{n}$. Then the conditional probability in the RHS of (8) can be expanded as

$$\begin{aligned}
&\mathbb{P}\left(h > \tau r^\alpha(\sigma^2 + I_{r_n})|r\right)\\
&= \int_r^\infty f_{R_n}(r_n|r)\mathbb{P}\left(h > \tau r^\alpha(\sigma^2 + I_{r_n})|r, r_n\right)\mathrm{d}r_n\\
&= \int_r^\infty f_{R_n}(r_n|r)\mathbb{E}_{I_{r_n}}\left[\mathbb{P}\left(h > \tau r^\alpha(\sigma^2 + I_{r_n})|r, r_n, I_{r_n}\right)\right]\mathrm{d}r_n\\
&\overset{(a)}{=} \int_r^\infty f_{R_n}(r_n|r)\mathbb{E}_{I_{r_n}}[\exp(-\mu\tau r^\alpha(\sigma^2 + I_{r_n})]\mathrm{d}r_n\\
&= \int_r^\infty f_{R_n}(r_n|r)\exp(-\mu\tau\ r^\alpha\sigma^2)\mathcal{L}_{I_{r_n}}(\mu\tau r^\alpha)\mathrm{d}r_n \quad (9)
\end{aligned}$$

where $f_{R_n}(r_n|r)$ is the pdf of r_n conditioning on r , $\mathcal{L}_{I_{r_n}}(\theta)$ is the Lapalace transform of random variable I_{r_n} at θ and

(a) follows the fact that the channel power is exponential distributed. Letting $\theta = \mu\tau r^\alpha$, we can have

$$\begin{aligned}
\mathcal{L}_{I_{r_n}}(\theta) &= \mathbb{E}_{I_{r_n}}[\exp(-\theta I_{r_n})]\\
&= \mathbb{E}_{\Phi,h}\left[e^{-\theta\sum_{i\in\Phi'} w_i h_i r_i^{-\alpha}}\right]\\
&= \mathbb{E}_{\Phi,h}\left[\prod_{\Phi'} e^{-\theta w_i h_i r_i^{-\alpha}}\right]\\
&\overset{(b)}{=} \mathbb{E}_\Phi\left[\prod_{\Phi'} \mathbb{E}_h\left[e^{-\theta h w_i r_i^{-\alpha}}\right]\right]\\
&\overset{(c)}{=} \exp\left\{-2\pi\lambda p_w\int_{r_n}^\infty(1 - \mathbb{E}_h[\exp(-\theta h v^{-\alpha})])v\mathrm{d}v\right\}\\
&\overset{(d)}{=} \exp\left\{-2\pi\lambda p_w\int_{r_n}^\infty\left(1 - \frac{\mu}{\mu + \theta v^{-\alpha}}\right)v\mathrm{d}v\right\}
\end{aligned}$$
$$(10)$$

where (b) follows that the channel is iid distributed and independence from the PPP Φ, (c) follows the Campbells theorem [8] on the expectation of summation of points over PPP Φ' which is thinned by probability p_w , and (d) follows the fact that the channel power is exponential distributed. Plugging (10),$\theta = \mu\tau r^\alpha$ and (9) into (8) gives

$$\begin{aligned}
&p_t(\tau, \lambda, \alpha, n)\\
&= \int_0^\infty\int_r^\infty f_{R_n}(r_n|r)f_R(r)\times\\
&e^{-\mu\tau\ r^\alpha\sigma^2 - 2\pi\lambda p_w\int_{r_n}^\infty(1-\frac{1}{1+\tau r^\alpha v^{-\alpha}})v\mathrm{d}v}\mathrm{d}r_n\mathrm{d}r \quad (11)
\end{aligned}$$

we slightly abuse the notation using $r_1 = r$, then the pdf of r_n for $n > 1$ conditioning on r_1 is given by

$$\begin{aligned}
f_{R_n}(r_n|r_1) &= \mathbb{P}(\texttt{exactly } n-2 \texttt{ nodes in } \mathbb{I}_{(r_1,r_n)})\times\lambda_{r_n}\\
&= \frac{\Lambda(\mathbb{I}_{(r_1,r_n)})^{n-2}}{(n-2)!}e^{\Lambda(\mathbb{I}_{(r_1,r_n)})}\times\lambda_{r_n}\\
&= \frac{\left(\int_{r_1}^{r_n}\lambda_y\mathrm{d}y\right)^{n-2}}{(n-2)!}e^{-\int_{r_1}^{r_n}\lambda_y\mathrm{d}y}\times\lambda_{r_n}\\
&= \frac{2\pi\lambda r_n\left(\pi\lambda(r_n^2 - r_1^2)\right)^{n-2}}{(n-2)!}e^{-\pi\lambda(r_n^2 - r_1^2)} \quad (12)
\end{aligned}$$

where $\Lambda(\mathbb{I}_{(r_1,r_n)})$ means intensity measurement in the interval (r_1, r_n) and $\lambda_y = 2\pi\lambda y$ is the intensity function according to *Lemma 1*. Plugging (12) and $f_R(r) = 2\pi\lambda r\exp(-\lambda\pi r^2)$ into (11) gives

$$\begin{aligned}
&p_t(\tau, \lambda, \alpha, n)\\
&= \int_0^\infty\int_{r_1}^\infty (2\pi\lambda)^2 r_1 r_n\frac{(\lambda\pi(r_n^2 - r_1^2))^{n-2}}{(n-2)!}\times\\
&e^{-\pi\lambda r_n^2 - \mu\tau\ r_1^\alpha\sigma^2 - 2\pi\lambda p_w\int_{r_n}^\infty(1-\frac{1}{1+\tau r_1^\alpha v^{-\alpha}})v\mathrm{d}v}\mathrm{d}r_n\mathrm{d}r_1 \quad (13)
\end{aligned}$$

IV. AVERAGE RATE ANALYSIS

A. Average rate of frequency reuse

The average rate for frequency reuse is given as [1]

$$q_c(\lambda, \alpha, \delta) = \frac{2\pi\lambda}{\delta} \int_0^\infty \int_0^\infty re^{-\pi\lambda r^2 - \mu(e^t-1)\sigma^2 r^\alpha} \times$$
$$\mathcal{L}_{I_r}(\mu(e^t-1)r^\alpha)\mathrm{d}t\mathrm{d}r \quad (14)$$

where $\mathcal{L}_{I_r}(\mu(e^t-1)r^\alpha)$ is

$$\exp\left(-\frac{\lambda r^2 (e^t-1)^{2/\alpha}}{\delta} \int_{(e^t-1)^{-2/\alpha}}^\infty \frac{1}{1+g^{\alpha/2}}\mathrm{d}g\right)$$

where $\delta = 1$ is the case without frequency reuse.

B. Average rate of coordinated transmission

we denote the average rate at the origin as $q_t = \mathbb{E}[\ln(1+S_t)]$, it can be expanded as

$$q_t(\lambda, \alpha, n)$$
$$\overset{(e)}{=} p_w \mathbb{E}[\ln(1+S_t)]$$
$$\overset{(f)}{=} \int_0^\infty p_w \mathbb{P}[\ln(1+S_t) > t]\mathrm{d}t$$
$$= \int_0^\infty p_w \mathbb{P}[S_t > e^t - 1]\mathrm{d}t$$
$$\overset{(g)}{=} \int_0^\infty \int_0^\infty \int_{r_1}^\infty (2\lambda\pi)^2 p_w r_1 r_n \frac{(\lambda\pi(r_n^2 - r_1^2))^{n-2}}{(n-2)!} e^{-\pi\lambda r_n^2} \times$$
$$e^{-\mu(e^t-1)\ r_1^\alpha \sigma^2 - 2\pi\lambda p_w \int_{r_n}^\infty (1 - \frac{1}{1+(e^t-1)r_1^\alpha v^{-\alpha}})v\mathrm{d}v}\mathrm{d}r_n\mathrm{d}r_1\mathrm{d}t$$
$$(15)$$

where (e) follows that the probability of each BS to transmit is p_w, (f)follows that for positive random variable X, $\mathbb{E}[X] = \int_0^\infty \mathbb{P}(X > t)(\mathrm{d})t$ and (g) follows the result in (13)

V. NUMERICAL ANALYSIS

In this section, we will investigate the numerical results to illustrate the coverage and average rate performance of coordinated transmission.

A. Coordinated transmission vs. Frequency reuse

In the Fig.2, we present the coverage probabilities of coordinated transmission (CT as shown in the figure) as well as frequency reuse(RF as shown in the figure) with $\alpha = 2.5$. As it can be seen that both coordination transmission and frequency reuse can significantly improve the coverage. When it comes to the comparison between the coordinated transmission and frequency reuse, it is fair to compare the case where the number of BS coordination in coordinated transmission is the same as the number of the channel bands in the frequency reuse, i.e. $\delta = n$. From the figure, in terms of coverage, the coordinate transmission performs better than frequency reuse.
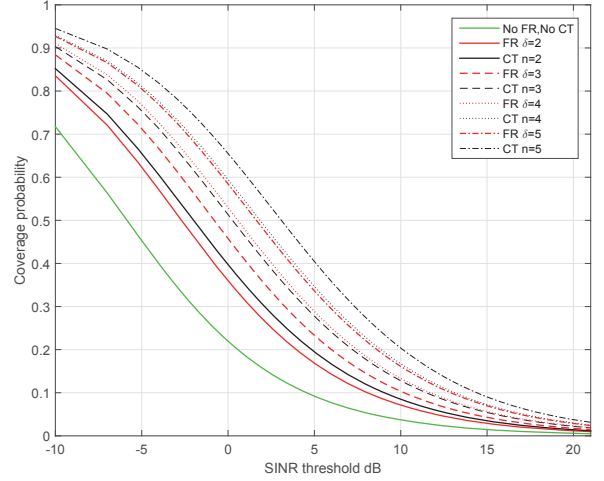

Fig. 2. A comparison of coverage probability of coordinated transmission and frequency reuse, with $\alpha = 2.5$, different number δ of channel bands for frequency reuse and different number n of BSs coordination with $p_w = 1/n$, $\lambda = 3000/\pi$ per square kilometer, no noise

B. Coordinated transmission with different α and p_w

Even though it has been pointed out that it is fairly reasonable to assume $p_w = 1/n$ for modeling the interferencing BSs within coordinated transmission, it is worth showing the coverage performance with different p_w to demonstrate the impact of the coordinated scheduling of BSs. In Fig.3 , 3 cases i.e. $p_w = 1$, $p_w = \frac{1}{n}$ and $p_w = \frac{1}{1.5\times n}$ have been checked for the coverage probability where $p_w = 1$ represents the case all the interferencing are transmitting which is impractical but can be a benchmark or kind of lower bound, and $p_w = \frac{1}{1.5\times n}$ represents the case better than equal BS coordinated scheduling transmission. From the figure, it is expected that the performances indeed vary with different scheduling cases: $p_w = 1$ has worst performance while $p_w = \frac{1}{1.5\times n}$ achieves the best among the three.

C. Average rate

The Fig.4 illustrates the average rate performance for both coordinated transmission and frequency reuse with different different α and the number of channel bands or coordinated BSs. It can be seen from the figure that the coordinated transmission performs better than frequency reuse, and it is also interesting to see that both frequency reuse and coordinated transmission actually degrade the average rate comparing with the case without interference mitigation. This is primarily due to the fact that even though the coordinated transmission and frequency reuse increase the SINR ,hence the coverage , at the same time the bandwidth in frequency reuse and the transmission probability (p_w) in coordinated transmission for each BS have been reduced, hence degrading the average rate. The results suggest that more sophisticated interference mitigation techniques, for example, fractional frequency reuse, are needed to improve the coverage as well as average rate.
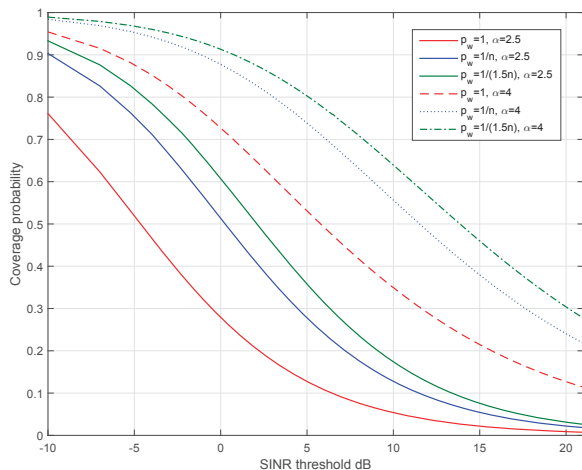
Fig. 3. Coverage probabilities of coordinated transmission with $n = 3$ different α and p_w, $\lambda = 3000/\pi$ per square kilometer , no noise

Fig. 4. A comparison of average rate of coordinated transmission and frequency reuse, with $\alpha = 2.5$ and $\alpha = 4$, $p_w = 1/n$, $\lambda = 3000/\pi$ per square kilometer, no noise

VI. CONCLUSION

In this paper we have studied the coverage probability and the average rate of coordinated transmission for stochastic cellular network which is modeled by a homogeneous Poisson point process. The analytical expressions for both coverage probability and average rate have been derived and numerical results were presented to illustrate the analytical performance of coordinated transmission where the results show that coordinated transmission can significantly improve the system coverage and perform better than frequency reuse in terms of coverage and average rate. Similar to the frequency reuse, the coordinated transmission degrade the average rate comparing with the case without interference mitigation which suggests more sophisticated interference mitigation techniques are need if both coverage and average rate are needed to be improved. Meanwhile, the performances vary depending on how to schedule the BSs to coordinate transmission and investigating the scheduling is our future study.

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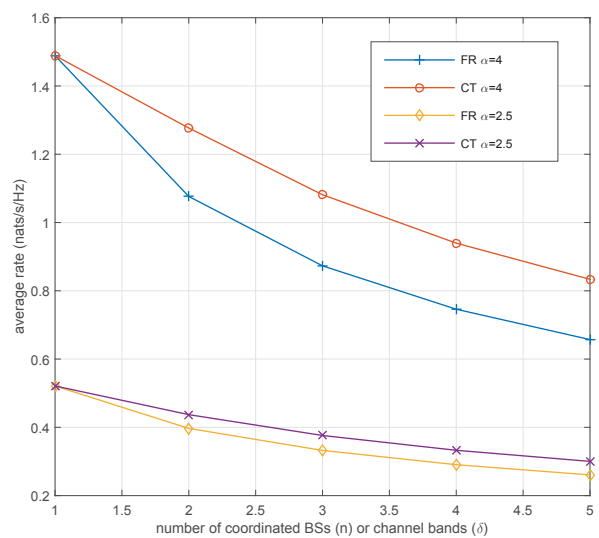[5] H. Dhillon, R. Ganti, F. Baccelli, and J. Andrews, "Modeling and analysis of k-tier downlink heterogeneous cellular networks," *Selected Areas in Communications, IEEE Journal on*, vol. 30, no. 3, pp. 550–560, April 2012.
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