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On the distribution and mean of received power in stochastic cellular network

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Abstract

This paper exploits the distribution and mean of received power for cellular network with stochastic network modeling to study the difference between the two cell association criteria, i.e. the strongest received power based cell association and the closest distance based cell association. Consequently we derive the analytical expression of the distribution and the mean of the nth strongest received power and the received power from the nth nearest base station and the derivations have been confirmed by simulation results. From both the simulation results and analytical results, we can see that the distributions of received power with both association criteria vary with different path loss exponents α: there is a clear gap between the two when the exponent is small and the two are almost the same with larger exponent. Meanwhile, the analysis on the mean of the received power suggests that under a certain converged condition,i.e. $n > \alpha/2$, the average of the received power from the nth closest BS is actually larger than the nth strongest one.

1. Introduction

The wireless cellular systems are getting increasingly dense in order to deliver high volume of data due to the huge demand of explosive smart phones and computing devices. As a result, the topologies of recent cellular networks are not regular-shaped any more, especially the base station (femto base station for example) nowadays can be deployed by subscribers, and it has been challenging to properly model the new networks. Modeling the networks with stochastic geometry has been well studied in recent years and seems to be promising, see [1,2] and the references therein. Indeed,

even though the stochastic network modeling does not totally match the real deployment, it does look more similar to the real deployment and performs better than the conventional hexagonal grid modeling [3] [4] . In stochastic modeling, the base stations(BSs) are placed randomly and the locations of the BSs are modeled normally as a homogeneous Poison point process (PPP), where the performance metrics like coverage and ergodic rate can be determined if the radio channel and cell association criterion are given. Regarding to the cell association, a user will be connected to a BS either with distance-based association where the connected is closest to the user or receive signal strength(RSS) based association where the connected BS provides the strongest received power to the user. While the vast majority of the existing stochastic modeling work assumed the distance-based cell association [2–6], the RSS-based cell association has been used in the practical system [7] and is worth being exploited within stochastic modeling. One basic research issue regarding to the two cell associations is that what is the difference between the two association in terms of distribution and moment of the strongest and the closest received powers. More recently, cooperative transmission has been used in the practical system where a user need to connect multiple BSs simultaneously or with time-division manner, not just with only one strongest or closest BS [8]. Therefore, it will be more interesting to derive the distribution and moment for the received power from an arbitrary BS which provide insightful results not only for cell association, but also for other topics like cooperative transmission. At the moment , only few relevant results are available in the literature. In [9] and [10], a tier is chosen for a user within the multi-tier network based on the strongest received signal, but the user is connected to a BS within the chosen tier based on the nearest distance. In [11], the distribution of distances in Poisson point processes has been studied and the results was applied for bounding the interference and ad hoc routing. In [12], the CDF of signal to noise ratio (SIR) with RSS based association was presented to model lattice cellular networks

Different from previous work, we particularly focus on the distribution and mean of the nth strongest re-

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ceived power and the one from the nth closest BS, aiming to illustrate the difference between the two cell associations. In this paper, we model the received power as a marked PPP by attaching the marks (fading channel) to the ground PPP which is an inhomogeneous PPP in line-space transformed from a planar homogeneous PPP, and use the marked PPP to analyze the distribution and moment of the received power. The main contributions are as follows

- We derive analytical expressions for the distributions of the strongest received power and the closest received power which is invariant to fading channels, and generalize them for the nth strongest and the nth closest received power.

- Close-forms of the average of the above-mentioned received power are also obtained and their comparison suggests the average of the received power from the nth closest BS is larger than the nth strongest received power under a certain converged condition

- Simulation results are presented to show that out analytical results closely match the performance observed from the simulations.

For convenience, the nth closest received power means the received power from the nth closest BS and they are interchangeable in the whole paper.

The rest of the paper is organized as follows: Section 2 introduces the system model and the derivations of the distribution and the mean of the received power have been elaborated in Section 3 and Section 4, respectively. In Section 5, numerical results and simulations have been presented to evaluate the derivation and illustrate comparison. Then Section 6 concludes the paper.

2. Poisson point process and system model

2.1. Poisson point process

Here we briefly describe the PPP for better illustration of the analysis in the remaining sections. A PPP Ω is characterized mainly by two fundamental properties:

1) The numbers of points falling within any disjoint regions are independent random variables ;

2) The random number of points of $\Omega = \{v_i\}$ falling within a region A has a Poisson distribution as

$$\mathbb{P}(\mathrm{N}(A) = k) = \frac{\Lambda(A)^k e^{-\Lambda(A)}}{k!}, \qquad (1)$$

where $\mathrm{N}(A)$ is the number of points in A and $\Lambda(A) = \int_A \Lambda(\mathrm{d}v) = \int_A \lambda_v \mathrm{d}v$ is the intensity measure of the re-

gion A, and λ_v is the intensity function. For homogeneous PPP, $\lambda_v = \lambda$ and $\Lambda(A) = \lambda|A|$, where $|A|$ is the area of the region A. A PPP $\Omega = \{v_i\}$ can be made into a marked PPP $\Theta = \{v_i, m_i\}$ by attaching a mark m_i to each point v_i of the process. [13] It can be shown that the intensity measure for the independent marked point process is $\Lambda(d(v,m)) = M_v(\mathrm{d}m)\Lambda(\mathrm{d}v)$ where M_v is the probability measure on dm, interpreted as the distribution of the mark of a point at v. Assuming $v \in B$ and $m \in L$, (1) will be slightly changed into (2)

$$\mathbb{P}(N(B \times L) = k)$$
$$= \frac{\left[\int_{B \times L} \Lambda(\mathrm{d}(v,m))\right]^k}{k!} \times \exp\left(-\int_{B \times L} \Lambda(\mathrm{d}(v,m))\right) \tag{2}$$

2.2. System model

We consider a network having BSs spatially distributed as a homogeneous PPP Φ with intensity λ in the Euclidean plane, where the probability to have k BSs in the region A follows $\mathbb{P}(N(A) = k) = \frac{(\lambda|A|)^k}{k!} \exp(-\lambda|A|)$ as mentioned in 2.1. In the network, a user is assumed to be associated with a BS according to either the closest-distance based or strongest received power based association criterion and the BS is called the user's serving BS and the received power from the user's serving BS is called desired power. The path loss is given by $l(r) = r^{-\alpha}$, where $\alpha(\alpha > 2)$ is the path loss exponent and r is the distance from a transmitter to a receiver, and it is assumed that all the BSs transmit with the same power. The fading power between a BS and a user is denoted by h , and its distribution $f_H(h)$ depends on the type of the channel. In this paper, we consider two typical types of channels:

1) Rayleigh channel which power follows $f_H(h) = \frac{1}{\mu} \exp\left(\frac{-h}{\mu}\right)$ with mean μ.

2) Lognormal channel which power follows $f_H(h) = \frac{1}{h\sigma\sqrt{2pi}} \exp\left\{-\frac{(\ln(h)-\mu)^2}{2\sigma^2}\right\}$ with mean $\exp(\mu + \sigma^2/2)$ and variance $(e^{\sigma^2} - 1)e^{2\mu+\sigma^2}$.

Both channels have been intensively used in scientific community.The Rayleigh channel is relatively simple to maintain mathematical tractability as well as provide insightful engineering results, whilst the Lognormal channel usually is used to capture the shadowing in practical systems.

For a given location as the origin, the received power from a BS is given by

$$Y(r,h) = P_t h r^{-\alpha} \tag{3}$$

where P_t is the transmit power at the BS. In the next section, we will study the distributions of the desired power

associated with cell associations, i.e. the strongest received power and the received power from the serving BS, and furthermore extend it to any n^{th} strongest received power and any received power from the nth nearest BS.

3. Distribution of received power

3.1. Distribution of the received power from the serving BS

We firstly converge the 2-dimension PPP in planar space into one dimension PPP of BSs in line space in which the 'point arrival times' are $\Phi = \{r_i\}$ with intensity $\lambda_r = 2\pi\lambda r$ [14]. Define a point process $\mathcal{X} = \{x_i = P_t r_i^{-\alpha}\}$ for $x = P_t r^{-\alpha} = f(r)$ and further define a marked point process $\mathcal{Y} = \{y_i : y_i = h_i x_i\}$ to represent the received power.

Lemma 1 The marked point process $\mathcal{Y}$ is an inhomogenoues PPP with intensity as

$$\lambda_y = \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} y^{-2/\alpha-1} \mathbb{E}_H(h^{2/\alpha}) \qquad (4)$$

Subject to

$$\mathbb{E}_H(h^{2/\alpha}) < \infty$$

Proof : According to the mapping theorem of PPP [14] [15], $\mathcal{X}$ is also a PPP and its intensity can be obtained as

$$\lambda_x = \lambda(f^{-1}(x)) \left| \frac{\partial f^{-1}(x)}{\partial x} \right|$$
$$= \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} x^{-2/\alpha-1} \qquad (5)$$

Since $\mathcal{Y}$ is the displacement of the point process $\mathcal{X}$, according to the displacement theorem of PPP [14], $\mathcal{Y}$ is a PPP and we can calculate its intensity as

$$\lambda_y = \int_0^\infty \lambda_x \rho(x,y) dx$$
$$= \int_0^\infty \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} x^{-2/\alpha-1} \rho(x,y) dx$$
$$= \int_0^\infty \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} x^{-2/\alpha-1} f_H(y/x) \frac{1}{x} dx$$
$$\overset{z=y/x}{=\!=\!=} \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} \int_0^\infty y^{-2/\alpha-1} z^{2/\alpha} f_H(z) dz$$
$$= \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} y^{-2/\alpha-1} \mathbb{E}_H(h^{2/\alpha}) \qquad (6)$$

where $\rho(x,y)$ is the probability of y at x and obviously $\mathbb{E}_H(h^{2/\alpha}) < \infty$ in (6) need to be satisfied to keep $\lambda_y < \infty$ to complete the proof.

It is interesting to see that the intensity of the $\mathcal{Y}$ can be applied to arbitrary fading channel so long as $\mathbb{E}_H(h^{2/\alpha})$ converges which is assumed in the following analysis. Now we will investigate the distribution of the received signal from the serving BS.

Theorem 1 (Distribution of the strongest received power) Let $S = \max\{Y(r,h)\}$ be the strongest received power, then its distribution is

$$\mathbb{P}(S \leq z) = \exp\left(-\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_H(h^{2/\alpha})\right) \qquad (7)$$

Proof: $\max(Y) \leq z$ means that there are no points of the marked point process $\mathcal{Y}$ falling into the interval $(z, +\infty)$, so

$$\mathbb{P}(S \leq z) = \mathbb{P}(\max(Y) \leq z)$$
$$\overset{(a)}{=} \exp\left(-\int_z^\infty \lambda_y dy\right)$$
$$= \exp\left(-\frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} \int_z^\infty y^{-2/\alpha-1} \mathbb{E}_H(h^{2/\alpha}) dy\right)$$
$$= \exp\left(-\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_H(h^{2/\alpha})\right) \qquad (8)$$

where (a) follows the null probability of the marked point process $\mathcal{Y}$ in the interval $(z, +\infty)$ and Q.E.D. Accordingly the pdf of S is given by

$$f_S(z) = \frac{\partial \mathbb{P}(S \leq z)}{\partial z}$$
$$= \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} z^{-2/\alpha-1} \mathbb{E}_H(h^{2/\alpha}) \times$$
$$\exp\left(-\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_H(h^{2/\alpha})\right) \qquad (9)$$

For Rayleigh channel with unit mean, (8) and (9) will become (10) and (11), respectively.

$$\mathbb{P}(S_{Rayleigh} \leq z) = \exp\left(-\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \Gamma(2/\alpha+1)\right) \qquad (10)$$

$$f_{S_{Rayleigh}}(z) = \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} z^{-2/\alpha-1} \Gamma(2/\alpha+1) \times$$
$$\exp\left(-\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \Gamma(2/\alpha+1)\right) \qquad (11)$$

where $\Gamma(t) = \int_0^\infty v^{t-1} e^{-v} dv$ is the Gamma function.

Theorem 2 (Distribution of the closest received power) Let $R_1 = \min(r)$ and Y_{R_1} be the distance from the origin to the closest BS and the received power from the closest BS, respectively, then the distribution of Y_{R_1} is given by

$$\mathbb{P}(Y_{R_1} \leq z) = \int_0^\infty \exp(-\pi\lambda(P_t h)^{2/\alpha} z^{-2/\alpha}) f_H(h) dh \qquad (12)$$

Proof : we firstly have

$$\mathbb{P}(Y_{R_1} \leq z) = \mathbb{P}(R_1 \geq (\frac{z}{P_t h})^{-1/\alpha})$$

Denote $B(o,t)$ as the circle region with radius t to the center i.e. the origin. $R_1 \geq (\frac{z}{P_t h})^{-1/\alpha}$ means there are no points of $\Phi = \{r_i\}$ falling into $B(o,(\frac{z}{P_t h})^{-1/\alpha})$, so

$$\mathbb{P}(R_1 \geq (\frac{z}{P_t h})^{-1/\alpha}) =$$
$$\overset{(b)}{=} \mathbb{E}_H \left[\mathbb{P}\left(R_1 \geq (\frac{z}{P_t h})^{-1/\alpha}|h \right) \right]$$
$$= \mathbb{E}_H \left\{ \mathbb{P}\left(N\left(B\left(o,(\frac{z}{P_t h})^{-1/\alpha}\right) \right) = 0|h \right) \right\}$$
$$\overset{(c)}{=} \mathbb{E}_H \left\{ \exp(-\pi\lambda(P_t h)^{2/\alpha}z^{-2/\alpha}) \right\}$$
$$= \int_0^\infty \exp(-\pi\lambda(P_t h)^{2/\alpha}z^{-2/\alpha})f_H(h)\mathrm{d}h \qquad (13)$$

where (b) means the average is taken over the fading distribution and (c) follows the null probability of the point process $\Phi = \{r_i\}$ within $B(o,(\frac{z}{P_t h})^{-1/\alpha})$,and Q.E.D. The pdf of Y_{R_1} is accordingly obtained as

$$f_{Y_{R_1}}(z) = \frac{\partial \mathbb{P}(Y_{R_1} \leq z)}{\partial z}$$
$$= \frac{2}{\alpha}\pi\lambda P_t^{2/\alpha}z^{-2/\alpha-1}\int_0^\infty h^{2/\alpha} \times \qquad (14)$$
$$\exp(-\pi\lambda(P_t h)^{2/\alpha}z^{-2/\alpha})f_H(h)\mathrm{d}h$$

3.2. Distribution of the received power from the nth connecting BS

Now we further extend the analysis to the distribution of the received power from the nth strongest BS, followed by the one from the nth closest BS. Since the received power is an inhomogeneous Poisson point process in line space with intensity $\lambda_y = \frac{2\pi\lambda}{\alpha}P_t^{2/\alpha}y^{-2/\alpha-1}\mathbb{E}_H(h^{2/\alpha})$ as in (6), we can calculate its intensity measure in interval $(z,+\infty)$ as follows

$$\Lambda_{(z,+\infty)} = \int_z^\infty \Lambda_y \mathrm{d}y$$
$$= \frac{2\pi\lambda}{\alpha}P_t^{2/\alpha}\int_z^\infty y^{-2/\alpha-1}\mathbb{E}_h(h^{2/\alpha})\mathrm{d}y$$
$$= \pi\lambda P_t^{2/\alpha}z^{-2/\alpha}\mathbb{E}_h(h^{2/\alpha}) \qquad (15)$$

Theorem 3 (Distribution of the nth strongest received power) Let Y_n be the nth strongest received power. Its distribution is given by

$$\mathbb{P}(Y_{R_n} \leq z) = \frac{\Gamma\left[\pi\lambda P_t^{2/\alpha}z^{-2/\alpha}\mathbb{E}_H(h^{2/\alpha}),n\right]}{(n-1)!} \qquad (16)$$

Proof: The complementary cumulative distribution function of Y_{R_n} is the probability that there are less than n points in $\mathscr{Y}$ within $(z,+\infty)$, so

$$\mathbb{P}(Y_n \leq z) = \mathbb{P}(0,...,n-1 \text{ points in}(z,+\infty))$$
$$= \sum_{k=0}^{n-1} \frac{\Lambda_{(z,+\infty)}^k}{k!}e^{-\Lambda_{(z,+\infty)}}$$
$$= \sum_{k=0}^{n-1} \frac{(\pi\lambda P_t^{2/\alpha}z^{-2/\alpha}\mathbb{E}_h(h^{2/\alpha}))^k}{k!} \times$$
$$\exp\left(-\pi\lambda P_t^{2/\alpha}z^{-2/\alpha}\mathbb{E}_H(h^{2/\alpha})\right) \qquad (17)$$

Using the incomplete Gamma function $\Gamma(t,k) = (k-1)!e^{-t}\sum_{i=0}^{k-1}\frac{t^i}{i!}$ for integer k to replace the RHS of (17) gives the final result of (16). Similar to (9), the pdf of Y_n can be calculated as in (18). **Theorem 4** (Distribution of the nth closest received power) Let R_n be the distance to the nth closest base station and Y_{R_n} be the received power of the nth closest base station, then the distribution of Y_{R_n} is given by

$$\mathbb{P}(Y_{R_n} \leq z) = \sum_{k=0}^{n-1} \int_0^\infty \frac{(\lambda\pi P_t^{2/\alpha}h^{2/\alpha}z^{-2/\alpha})^k}{k!} \times$$
$$\exp(-\lambda\pi P_t^{2/\alpha}h^{2/\alpha}z^{-2/\alpha})f_H(h)\mathrm{d}h \quad (19)$$

Proof : The proof is given in Appendix

Accordingly, its pdf is given in (20). Although the expression for both the nth strongest and closest received power have been derived as in Theorem 4 and Theorem 5, it is still hard to analytically compare the both in terms of distribution, having said that, we are able to evaluate them and illustrate their difference through numerical results which will be presented in section 5. Furthermore, we will study their average and compare them analytically in the next section.

4. The mean of the received power

In this section,we are now studying the average for both n^{th} strongest and closest received power and the comparison between them.

Theorem 5 (Mean of the received power) Let $\bar{Y}_n = \mathbb{E}_{Y_n}[z]$ be the mean of the nth strongest received power and $\bar{Y}_{R_n} = \mathbb{E}_{Y_{R_n}}[x]$ be the mean of received power from the nth nearest BS. For $n > \alpha/2$, we have

$$\bar{Y}_n = \left(\pi\lambda P_t^{2/\alpha}\mathbb{E}_h(h^{2/\alpha})\right)^{\alpha/2} \frac{\Gamma(n-\alpha/2)}{\Gamma(n)} \qquad (21)$$

and

$$\bar{Y}_{R_n} = \Gamma(n-\alpha/2)\frac{(\lambda\pi P_t^{2/\alpha})^{\alpha/2}}{\Gamma(n)}\mathbb{E}_h[h] \qquad (22)$$

$$f_{Y_n}(z) = \frac{\partial \mathbb{P}(Y_n \leq z)}{\partial z}$$

$$= \frac{2\pi\lambda}{\alpha} P_t^{2/\alpha} z^{-2/\alpha - 1} \mathbb{E}_h(h^{2/\alpha}) e^{-\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_h(h^{2/\alpha})} \times$$

$$\left(\sum_{k=0}^{n-1} \frac{(\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_h(h^{2/\alpha}))^k}{k!} - \sum_{k=1}^{n-1} \frac{(\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_h(h^{2/\alpha}))^{k-1}}{(k-1)!} \right)$$

$$= \frac{2}{z\alpha} \frac{(\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_h(h^{2/\alpha}))^n}{(n-1)!} e^{-\pi\lambda P_t^{2/\alpha} z^{-2/\alpha} \mathbb{E}_h(h^{2/\alpha})} \tag{18}$$

$$f_{Y_{R_n}}(z) = \frac{\partial \mathbb{P}(Y_{R_n} \leq z)}{\partial z}$$

$$= \int_0^\infty \frac{2}{\alpha} \lambda\pi P_t^{2/\alpha} h^{2/\alpha} z^{-2/\alpha - 1} \exp(-\lambda\pi P_t^{2/\alpha} h^{2/\alpha} z^{-2/\alpha}) \times$$

$$\left(\sum_{k=0}^{n-1} \frac{(\lambda\pi P_t^{2/\alpha} h^{2/\alpha} z^{-2/\alpha})^k}{k!} - \sum_{k=1}^{n-1} \frac{(\lambda\pi P_t^{2/\alpha} h^{2/\alpha} z^{-2/\alpha})^{k-1}}{(k-1)!} \right) f_H(h) \mathrm{d}h$$

$$= \int_0^\infty \frac{2}{\alpha} z^{-2n/\alpha - 1} \frac{(\lambda\pi P_t^{2/\alpha} h^{2/\alpha})^n}{(n-1)!} \exp(-\lambda\pi P_t^{2/\alpha} h^{2/\alpha} z^{-2/\alpha}) f_H(h) \mathrm{d}h \tag{20}$$

and their ratio is given by

$$\frac{\bar{Y}_n}{\bar{Y}_{R_n}} \leq 1 \tag{23}$$

Proof: the proof is given in Appendix

The condition $n > \alpha/2$ suggests that the mean of Y_n and Y_{R_n} does not always converge, for example , the $\bar{Y}_{R_1}$ and $\bar{Y}_{R_1}$ is not able to converge. This is due to the fact that with the PPP modeling it is likely a BS can be indefinitely close to the origin,hence causing the convergence problem. But this will not be an issue in practice since there will be a limited minimal distance from BS to users in practical networks. Taking the limited minimal distance into account is not within this paper and will be our future work. It is interesting to see that the ratio $\frac{\bar{Y}_n}{\bar{Y}_{R_n}}$ is only dependent of the channel and the path loss exponent , and is regardless of n. Furthermore $\bar{Y}_{R_n}$ is always larger than or equal to $\bar{Y}_n$. It should be noted that this only holds under the convergence condition and cannot be applied to non-convergence condition, for example,$\bar{Y}_1$ is obviously always larger than or equal to $\bar{Y}_{R_1}$. Intuitively, the ratio in (23) might provide a good implication to the case when a user need to connect multiple n BSs simultaneously, for example cooperative transmission, that connecting to n BSs with mixture of RSS-based and distance-based association is probably better than that with only RSS-based or distance-based association.
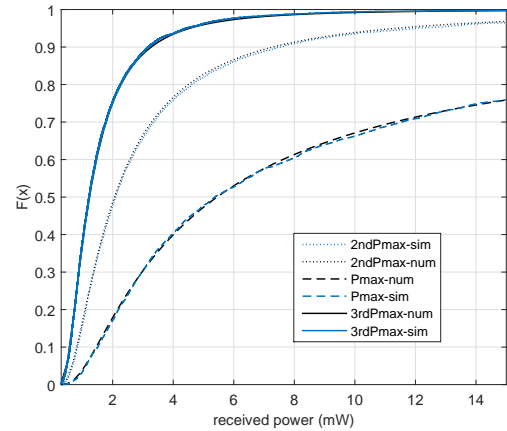

Figure 1. CDF of the $1^{st}, 2^{nd}$ and 3rd strongest received power, Rayleigh channel with power mean 1, $\lambda = 2000$ **BSs per square kilometer,** $\alpha = 2.2$**, transmit power of 1 Watt**
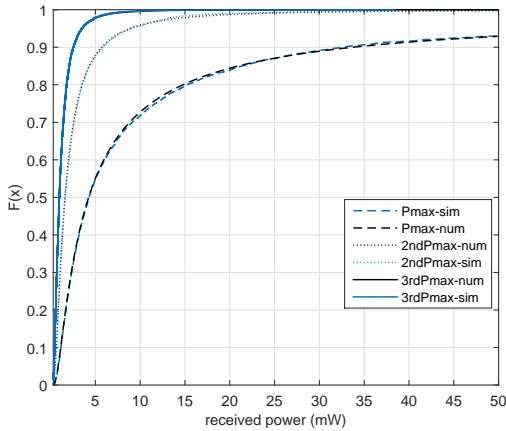
Figure 2. CDF of the $1^{st}, 2^{nd}$ **and** 3rd **strongest received power, lognormal channel with mean** 1 **and variance** 5**,** $\lambda = 2000$ **BSs per square kilometer,** $\alpha = 2.2$ **, transmit power of 1 Watt**
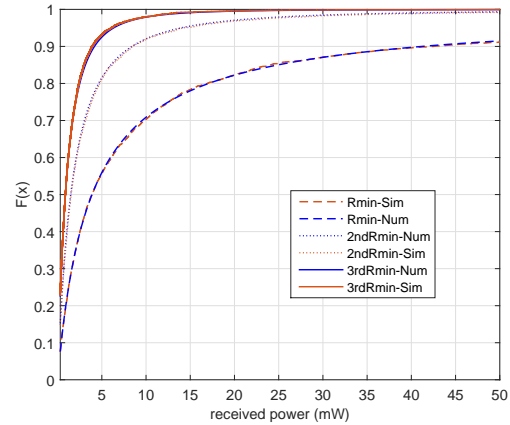

Figure 3. CDF of received power from the $1^{st}, 2^{nd}$ **and** 3rd **nearest BSs, Rayleigh channel with power mean** 1**,** $\lambda = 2000$ **BSs per square kilometer,** $\alpha = 2.2$**,transmit power of 1 Watt**

5. Numerical Results

In this section, we will evaluate the numerical analysis through Monte Carlo simulations. In the simulations, we randomly dropp BSs in a large area centering at the origin according to the Poisson distribution with the intensity of 2000 BSs per square kilometer. Then the received power from any BS to the origin can be calculated if the channel and path loss exponent are given and the simulation results on the distribution and mean of the received powers can be obtained. At the same time, the numerical results from the analytical expressions are also plotted to be checked with the simulation results. We tested the results with both Rayleigh channel and Lognormal channel. Figure1 and Figure2 show the CDF of the $1^{st}, 2^{nd}$ and 3rd strongest received power for both channels and it can be seen that the simulation results perfectly confirm the numerical results. Similarly, Figure3 illustrates both simulation and numerical results for the received powers from the $1^{st}, 2^{nd}$ and 3rd closest BSs, with Rayleigh channel. Again, the simulation results match the analytical performance well. In the figures, the 'Pmax' , '2ndPmax' and '3rdPmax' mean the strongest, second strongest and third strongest received power respectively, and 'Rmin' , '2ndRmin' and '3rdRmin' mean received power from the nearest, sencond, third nearest BSs repectively, while 'sim' and 'num' mean simulation result and numerical result respectively.

Figure4 shows simulation and numerical results of the CDF of Y_1 and Y_{R_1} with two different α values i.e.

$\alpha = 2.2$ and $\alpha = 3$ with Lognormal channel. We can see from the figure there is clearly a gap between the two when the α is small with $\alpha = 2.2$ here and the two are almost the same for $\alpha = 3$ which suggests that the path loss exponent has impact on the association strategies. Intuitively speaking, a larger α implies a heavier path loss, which makes it harder for a farther BS to compete with a nearer BS in terms of RSS. In more detail, although the radio channel gain of a farther BS might be higher than that of a nearer BS, the RSS-based associated with a farther BS is more likely to be smaller than that associated with a nearer BS, particularly when α is relatively large. Therefore, the performance of the two investigated user association strategies converges, e.g., when $\alpha = 3$ in our analysis

Figure5 illustrates the average of the 3rd strongest received power and the one from 3rd nearest BS with variant α values for Reyleigh channel. From the figure, the simulation results are consistent with the numerical results and the 3rd nearest averaged received power is always larger than the 3rd strongest received power.

6. Conclusion

In this paper we have studied the distribution and the average of the received power from the nth nearest BS and the nth strongest received power to the origin for stochastic network. The analytical performances have been derived and simulation results also were provided to confirm the analytical results. The study showed that
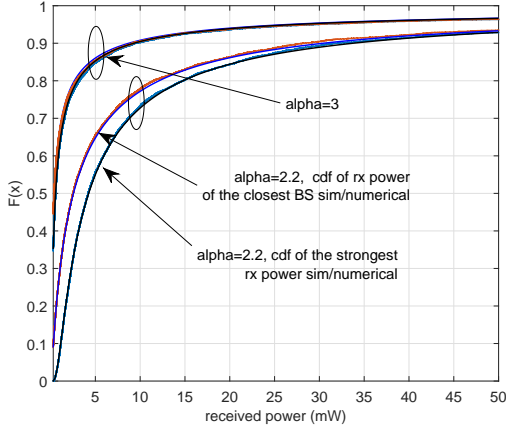
Figure 4. Comparison of CDF of the strongest rx power and rx power from the nearest BSs with $\alpha = 2.2$ and $\alpha = 3$, lognormal channel with mean 1 and variance 5, $\lambda = 2000$ BSs per square kilometer, transmit power of 1 Watt

Figure 5. Average received power of the 3rd strongest rx power and from the 3rd nearest BSs vs α, Rayleigh channel with unit mean, $\lambda = 2000$ BSs per square kilometer, transmit power of 1 Watt

it does have some difference between the two, especially when the path loss exponent is small, which imply that there might be some impacts on the other performance metrics such as coverage and system capacity using different cell associations, which is one of our future work. At the same time, analysis also suggests that for $n > \alpha/2$, the nth nearest received power is always larger than the nth strongest received power, which provides an good implication on the association to multiple BSs in the cases when a user need to communicate with multiple BSs simutaneously, for example, the cooperative transmission.

Appendix

Proof of theorem 4

Beginning with the definition of distribution , we have

$$
\begin{aligned}
&\mathbb{P}(Y_{R_n} \leq z) \\
&= \mathbb{P}(R_n \geq (\frac{z}{P_t h})^{-1/\alpha}) \\
&= \mathbb{E}_H\{\mathbb{P}(R_n \geq (\frac{z}{P_t h})^{-1/\alpha}|h)\}
\end{aligned} \tag{24}
$$

$\mathbb{P}(R_n \geq (\frac{z}{P_t h})^{-1/\alpha}$ is the probability that there are less than n points of $\Phi = \{r_i\}$ falling into $B\left(o, (\frac{z}{P_t h})^{-1/\alpha}\right)$,
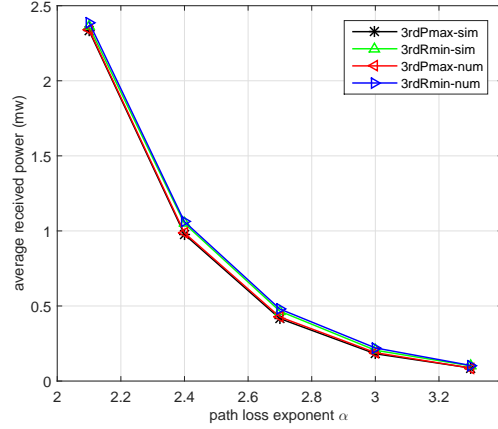
so

$$
\begin{aligned}
&\mathbb{P}(Y_{R_n} \leq z) \\
&= \mathbb{E}_H\left\{\sum_{k=0}^{n-1} \frac{(\lambda\pi(P_t h)^{2/\alpha}z^{-2/\alpha})^k}{k!}\exp(-\lambda\pi(P_t h)^{2/\alpha}z^{-2/\alpha})\right\} \\
&= \sum_{k=0}^{n-1}\int_0^\infty \frac{(\lambda\pi P_t^{2/\alpha}h^{2/\alpha}z^{-2/\alpha})^k}{k!} \times \\
&\quad \exp(-\lambda\pi P_t^{2/\alpha}h^{2/\alpha}z^{-2/\alpha})f_H(h)\mathrm{d}h
\end{aligned} \tag{25}
$$

which completes the proof.

Proof of theorem 5

By plugging (18) into $\bar{Y}_n$, we can have

$$
\begin{aligned}
\bar{Y}_n &= \mathbb{E}_{Y_n}[z] \\
&= \int_0^\infty \frac{2}{\alpha}\frac{(\pi\lambda P_t^{2/\alpha}z^{-2/\alpha}\mathbb{E}_h(h^{2/\alpha}))^n}{(n-1)!} \times \\
&\quad \exp\left\{-\pi\lambda P_t^{2/\alpha}z^{-2/\alpha}\mathbb{E}_h(h^{2/\alpha})\right\}\mathrm{d}z
\end{aligned} \tag{26}
$$

Employing a change of variables $x = z^{-2/\alpha}$ results in

$$
\begin{aligned}
\bar{Y}_n &= \int_0^\infty \frac{(\pi\lambda P_t^{2/\alpha}\mathbb{E}_h(h^{2/\alpha}))^n}{(n-1)!} \times \\
&\quad x^{n-\alpha/2-1}\exp\left\{-\pi\lambda P_t^{2/\alpha}x\mathbb{E}_h(h^{2/\alpha})\right\}\mathrm{d}z \\
&\stackrel{(d)}{=} \left(\pi\lambda P_t^{2/\alpha}\mathbb{E}_h(h^{2/\alpha})\right)^{\alpha/2}\frac{\Gamma(n-\alpha/2)}{\Gamma(n)}
\end{aligned} \tag{27}
$$

which completes the proof of (21).

By plugging (20) into $\bar{Y}_{R_n}$, we can have

$$
\begin{aligned}
\bar{Y}_{R_n} &= \mathbb{E}_{Y_{R_n}}[z] \\
&= \int_0^\infty \int_0^\infty \frac{2}{\alpha} z^{-2n/\alpha} \frac{(\lambda \pi P_t^{2/\alpha} h^{2/\alpha})^n}{(n-1)!} \times \\
&\quad \exp\left(-\lambda \pi P_t^{2/\alpha} h^{2/\alpha} z^{-2/\alpha}\right) \mathrm{d}z f_H(h)\mathrm{d}h \qquad (28)
\end{aligned}
$$

Similarly employing a change of variables $x = z^{-2/\alpha}$ results in

$$
\begin{aligned}
\bar{Y}_{R_n} &= \int_0^\infty \int_0^\infty \frac{(\lambda \pi P_t^{2/\alpha} h^{2/\alpha})^n}{(n-1)!} x^{n-\alpha/2-1} \times \\
&\quad \exp\left(-\lambda \pi P_t^{2/\alpha} h^{2/\alpha} x\right) \mathrm{d}x f_H(h)\mathrm{d}h \\
&\overset{(d)}{=} \int_0^\infty \Gamma(n-\alpha/2) \frac{(\lambda \pi P_t^{2/\alpha})^{\alpha/2}}{\Gamma(n)} h f_H(h)\mathrm{d}h \\
&= \Gamma(n-\alpha/2) \frac{(\lambda \pi P_t^{2/\alpha})^{\alpha/2}}{\Gamma(n)} \mathbb{E}_h[h] \qquad (29)
\end{aligned}
$$

which completes the proof of (22). The (d) in (26) and (28) follows the Gamma functions $\int_0^\infty x^b e^{-ax} dx = \frac{\Gamma(b+1)}{a^{b+1}}$.

From (21) and (22), we can see that the convergence condition for both Y_n and Y_{R_n} is $n > \alpha/2$, and under the convergence condition the ratio of Y_n over Y_{R_n} is given as

$$
\begin{aligned}
\frac{\bar{Y}_n}{\bar{Y}_{R_n}} &= \frac{\mathbb{E}_H[h^{2/\alpha}]^{\alpha/2}}{\mathbb{E}_H[h]} \\
&\overset{(e)}{\leq} 1 \qquad (30)
\end{aligned}
$$

where (e) follows the Lyapunov inequality: if $0 < s \leq t$, then $\mathbb{E}[x^s]^{1/s} \leq \mathbb{E}[x^t]^{1/t}$, and Q.E.D.

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