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USE OF ELASTIC FOLLOW-UP TO STUDY THE EFFECT OF DISPLACEMENT CONTROLLED LOADING ON PLASTIC COLLAPSE PRESSURES FOR CIRCUMFERENTIALLY CRACKED PIPES

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ABSTRACT

Structural integrity assessments of pressurised pipes consider plastic collapse as a potential failure mode. This paper uses finite element analysis to explore the effect of the pipe end boundary conditions on the collapse pressure. Two end conditions are considered: a fixed axial load and a fixed axial displacement. The fixed axial displacement condition represents a long-range axial residual stress. In the R6 structural integrity assessment procedure long-range residual stress is associated with elastic follow-up. However, no guidance is given on whether the level of elastic follow-up is sufficient to justify treating long-range residual stress as a primary stress.

In this paper, a method is proposed to estimate elastic follow-up of an internally pressurised pipe containing a fully circumferential crack. It is found that the elastic follow-up is related to the length of the pipe. A short pipe that contains a fully circumferential crack, subjected to a displacement induced axial stress, has a global collapse that is not modified by the fixed displacement condition. The short pipe corresponds to a small elastic follow-up factor, Z. However, as the elastic follow-up factor increases, the presence of long-range residual stress starts to make a contribution to global collapse. When elastic follow-up is significant, a long-range residual stress has the same effect on global collapse as does a mechanical stress.

NOMENCLATURE

R_i, R_o	Internal and external radius
R_m	Mean radius
L	Length of the pipe
t	Thickness of the pipe
a	Depth of the crack
θ	Half crack angle
E	Young's modulus
ν	Poisson's ratio
σ_y	Yield stress
P, N	Internal pressure, initially introduced end load
P_o, N_o	Plastic collapse pressure and collapse end load for an unflawed pipe
ε, ε_a, ε_h	Strain, axial strain, hoop strain
σ, σ_a, σ_h	Stress, axial stress, hoop stress
A	Area
δ	Displacement
K	Stiffness
α	Relative stiffness ratio
Z	Elastic follow-up factor

INTRODUCTION

Nuclear and conventional power plants contain lengths of pipe work carrying pressurised water, steam and gas. This pipework is fabricated from short lengths of pipe butt welded together. It is of importance to determine the plastic collapse loads for pipes subjected to internal pressure. Residual stress may exist during the process of welding and installation. Additionally, if the pipe is restrained during welding restraint stresses are distributed through the whole structure and the characteristic length of the distribution is related to the dimensions of the structure. Such restraint stresses are referred to as long-range residual stresses. The R6 structural integrity assessment code [1] classifies stresses contributing to plastic collapse of the structure as primary stresses and those that do not contribute to plastic collapse as secondary stresses. R6 notes that long-range residual stresses are associated with significant elastic follow-up, and therefore may be classified as primary stress. However, R6 has no detail about what level of elastic follow-up is deemed to be significant and therefore it is not clear whether a long-range residual stress should be considered to be primary, secondary or between the two.

Previous work [2-12] has evaluated the plastic collapse load for pipes but these solutions have not considered the influence of residual stress, particularly long-range residual stress. Miller [2] defined the global collapse pressure as the load resulting in unbounded plastic strain in the structure and local collapse as the load leading to yielding of the local ligament or local net section. He also gave analytical solutions for the plastic collapse pressure for closed-end pipes containing circumferential cracks. Kim and Shim et al. [3-5] improved the accuracy of the existing analytical solutions of global collapse loads of pipes containing defects under single and combined loadings. Staat et al. [7-8] used finite element methods to give solutions for local and global collapse loads of closed-end pipes with flaws. They also treated local net section yielding as local collapse, although Shen and Tyson [11] realised that the term *local collapse* is a misnomer since the yielded region is constrained and therefore the component can sustain further increases in pressure. However, neither elastic follow-up nor long-range residual stress was considered in terms of plastic collapse solutions.

This paper investigates the influence of long-range residual stress on local net section yield and global collapse pressures of uncracked and full circumferentially cracked open-ended pipes. The fixed displacement controlled end condition is taken to represent long-range residual stress. Elastic follow-up is considered by varying the length of the pipes. Detailed axisymmetric finite element (FE) models were used, together with an elastic-perfectly plastic material model. Results are presented for global collapse and local net section yield pressures of open-ended pipes subjected to pre-fixed end load or pre-fixed end displacement.

First, quantification of elastic follow-up for a simple structure is considered. This is followed by determining global collapse and local net section yield of unflawed and full circumferentially cracked pipes.

ELASTIC FOLLOW-UP

The concept of elastic follow-up was first used in creep stress relaxation by Robinson [13]. He introduced this concept to deal with the problem of creep concentrations in high temperature piping systems. He illustrated the creep behaviour of a bolt in a rigid flange and a bolt in an elastic flange to explain the problem. The bolt in the elastic flange exhibits more permanent creep deformation in a specific time period due to “follow-up elasticity”. Boyle and Nakamura [14] explained the definition of elastic follow-up by illustrating a two bar structure in series subjected to fixed displacement under creep. They noted that if there is no additional strain accumulated during elastic follow-up, the stress is relaxed during creep. However, if there is significantly additional strain accumulated during elastic follow-up, the stress is not relaxed during creep.

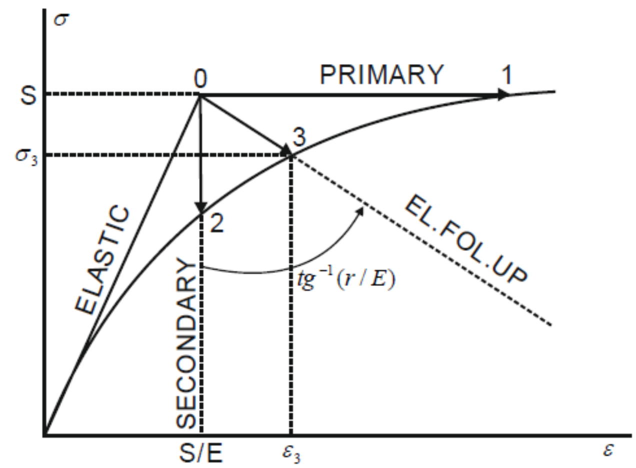


Fig. 1 Stress-strain curve showing elastic follow-up between the fixed load and fixed displacement boundary conditions [15]

This concept was extended by Roche [15] to be used for classifying stresses in integrity assessments. Roche [15] illustrated the difference between fixed displacement and fixed load conditions by using an example of a wire under tension. He presented an intermediate case which is loaded by means of a spring in series. Roche defined elastic follow-up using the plot of stress versus strain presented in Fig. 1. It is noted that the true stress point in this state is very different from the fictitious elastic state. He then associated the fixed-load condition with primary stress which contributes to plastic collapse and the fixed displacement with secondary stress which does not contribute to plastic collapse. For the intermediate case if the spring is very flexible, it is considered as the fixed-load condition but if the spring is very rigid, it is considered as the fixed-displacement condition. Elastic follow-up was defined as the inverse of the modulus of the gradient of line corresponding to the intermediate case, marked as “EL. FOL. UP” in Fig. 1.

Recently, Hadidi-Moud and Smith [16, 17] extended the definition of elastic follow-up in the high temperature integrity assessment procedure, R5 [18], to explore elastic follow-up due to plasticity. In a similar way to creep, a softening behaviour in a region of a structure may also be caused by plasticity or the presence of crack [16]. They used a series bar structure and a parallel bar structure to study the softening behaviour due to plasticity. Here, the definition of elastic follow-up in R5 is adopted to quantify elastic follow-up in a three series bar structure shown in Fig. 2 due to plasticity. It is assumed that the material exhibits elastic perfectly plastic behaviour in bar 1 while bar 2 is elastic during loading.

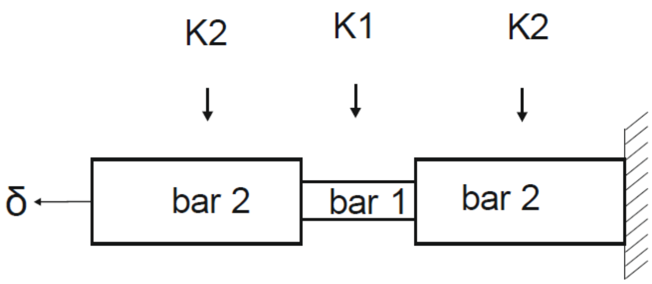


Fig.2 A three series bar structure subjected to fixed displacement, δ

The elastic follow-up factor, Z, in R5 [18] is defined by

$$Z = \frac{(\varepsilon)_{final} - \left(\varepsilon_{eq}^{el}\right)_{final}}{(\varepsilon)_{initial} - \left(\varepsilon_{eq}^{el}\right)_{final}} \tag{1}$$

where

$$\left(\varepsilon_{eq}^{el}\right)_{final} = (\sigma)_{final} / E \tag{2}$$

and $\varepsilon_{initial}$, ε_{final} and σ_{final} are the initial strain, final strain and final stress.

For the purely elastic response of the three series bar structure the strains in the bars are:

$$\varepsilon_2 = \frac{\delta}{L_2}\left(\frac{1}{2+\alpha}\right) \tag{3}$$

$$\varepsilon_1^e = \frac{\delta}{L_1}\left(\frac{\alpha}{2+\alpha}\right) \tag{4}$$

The stiffness ratio of bar 2 to bar 1 is

$$\alpha = \frac{K_2}{K_1} \tag{5}$$

where L_1 and L_2 are the lengths of bar 1 and bar 2; K_1 and K_2 are the stiffnesses for bar 1 and bar 2.

When the displacement increases, plastic strain will accumulate in bar 1 but bar 2 remains elastic. At this moment, the displacement must satisfy the following condition:

$$\delta \geq \left(\frac{\sigma_y^1}{E_1}\right) L_1 \left(\frac{2+\alpha}{\alpha}\right) \tag{6}$$

where σ_y^1 is the yield stress for bar 1 and E_1 is the Young's Modulus for bar 1.

As elastic perfectly plastic material is considered, bar 1 continues to extend with no additional load. The strains in bar 1 and bar 2 are given by:

$$\varepsilon_2 = \left(\frac{\sigma_y^1}{E_1}\right)\left(\frac{L_1}{L_2}\right)\left(\frac{1}{\alpha}\right) \tag{7}$$

$$\varepsilon_1^{ep} = \frac{\delta}{L_1} - \left(\frac{\sigma_y^1}{E_1}\right)\left(\frac{2}{\alpha}\right) \tag{8}$$

Incorporating Eq. 4 and Eq. 8 into Eq. 1 ($\varepsilon_1^e = (\varepsilon)_{initial}$, $\varepsilon_1^{ep} = (\varepsilon)_{final}$), we have elastic follow-up factor, Z, as

$$Z = \frac{\alpha + 2}{\alpha} \tag{9}$$

From this equation, it is found that when the stiffness ratio tends to zero, the elastic follow-up factor becomes infinite. In this case, applied displacements act in the same way as applied loads and will therefore contribute to plastic collapse. However, if the stiffness ratio tends to infinity, the elastic follow-up approaches 1 so that the applied displacement does not contribute to plastic collapse. For the cases that the elastic follow-up is in between the above two extreme cases, the fixed displacement condition can be partially considered as a load control. Therefore, depending on the value of elastic follow-up factor, Z, the applied displacement will contribute in different ways to plastic collapse.

In the following, two structures are considered. One is unflawed pipes, the other is circumferentially cracked pipes.

PLASTIC COLLAPSE OF UNFLAWED PIPES

For thin-walled, elastic-perfectly plastic pipes subjected to pre-tension controlled by mechanical load, the pipes are unable to sustain a pressure higher than that to cause first yield. However, if the pre-tension is controlled by fixed displacement, the unflawed pipes can sustain a pressure higher than that to cause first yield. Since there is no elastic follow-up in the unflawed pipes, the loading introduced by fixed displacement can be relaxed when the pressure is higher than that to cause first yield. Plastic collapse occurs when the displacement controlled stresses are removed and an asymptotic value can be found in the pressure-radial displacement curve. This asymptotic value is the collapse pressure. The pressure to cause first yield and the collapse pressure may therefore be calculated using a straightforward analytical solution by substituting the hoop and axial stress into a yield criterion. The results of this analytical solution are then compared with finite element (FE) solutions for both thin and thick-walled pipes.

When an axial load N is applied to a thin pipe the axial stress σ_a is given by

$$\sigma_a = \frac{N}{2\pi Rt} \tag{10}$$

where R is the radius of the pipe and t is the wall thickness. When the axial load is increased, plastic collapse will occur in the absence of internal pressure when axial stress reaches the yield stress σ_y. The end load N_o to cause plastic collapse is:

$$N_o = 2\pi R t \sigma_y \tag{11}$$

When an internal pressure is applied to the pipe the hoop stress is:

$$\sigma_h = \frac{PR}{t} \tag{12}$$

where P is the internal pressure. In the absence of axial loading, plastic collapse will occur when the hoop stress reaches the yield stress. The internal pressure to cause plastic collapse is then

$$P_o = \frac{\sigma_y t}{R} \tag{13}$$

For the case of a pipe with a fixed axial load subjected to additional internal pressure which is increased until plastic collapse occurs. The fixed tensile load is defined by

$$N = \beta N_o \tag{14}$$

where $0 \le \beta \le 1$. Using von Mises yield criterion, the pressure P_C to cause plastic collapse (i.e. the first yield) for the combined fixed axial load and internal pressure conditions is found to be

$$P_C = P_y = \frac{P_o}{2}\left(\beta + \sqrt{4 - 3\beta^2}\right) \tag{15}$$

Fig. 3 shows the normalised pressure (P_y / P_o) versus the normalised axial load (N / N_o) for the fixed tensile load condition. The maximum pressure that can be sustained by the pipe occurs when $N / N_o = \sqrt{1/3}$.

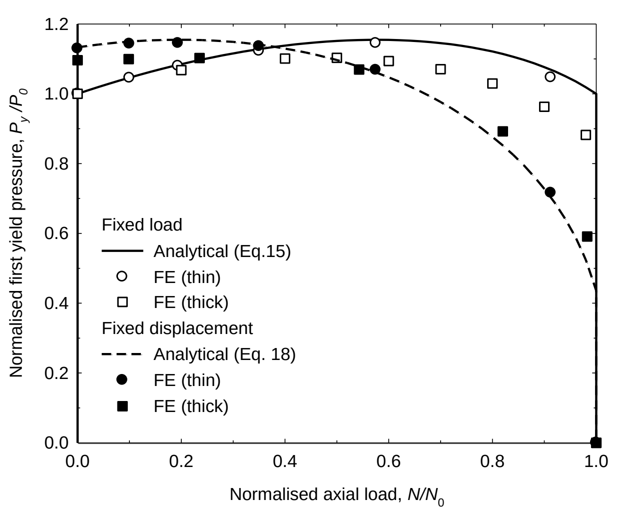


Fig. 3 Analytical and FE calculations for the first yield pressure versus initial end load for unflawed thick and thin-walled pipes under fixed tensile load and displacement conditions

For pressurised pipes with pre-tension controlled by fixed displacement conditions, the axial strain is given by

$$\varepsilon_a = \frac{1}{E}\left(\sigma_a - v\sigma_h\right) = \frac{\beta\sigma_y}{E} \tag{16}$$

where E is Young's Modulus and v is Poisson's ratio. For non-zero internal pressure, the axial stress is obtained from

$$\sigma_a = \beta\sigma_y + v\sigma_h \tag{17}$$

Using von Mises yield criterion, the pressure P_y to cause first yield for the combined fixed tensile displacement and internal pressure conditions can be shown to be

$$P_C = \frac{P_o}{2\left(v^2 - v + 1\right)}\left[\beta(1-2v) + \sqrt{\beta^2(2v-1)^2 - 4\left(v^2 - v + 1\right)\left(\beta^2 - 1\right)}\right] \tag{18}$$

Fig. 3 also shows the normalised pressure to cause first yield versus the normalised axial displacement for the fixed displacement condition with a Poisson's ratio of 0.33. The maximum pressure to reach the first yield is almost as that for the fixed load condition but occurs at a smaller value of N / N_o.

For displacement control, the real collapse pressure occurs when the initial stress induced by fixed displacement is removed. Therefore, the collapse pressure is independent on the initial stress induced by displacement and the axial stress at the collapse is found to be

$$\sigma_a = v\sigma_h \tag{19}$$

As this stress is due to Poisson's effect, it is called as Poisson's stress. Using von Mises yield criterion, the collapse pressure, P_C, can be shown to be

$$P_C = \frac{P_o}{\sqrt{v^2 - v + 1}} \tag{20}$$

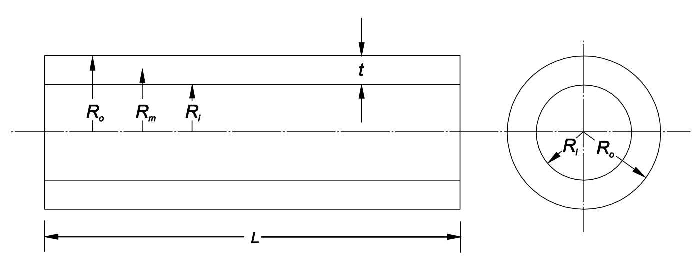


Fig. 4 Geometry of an unflawed pipe

For thick-walled pipes an FE approach using the ABAQUS FE code [19] has been used to determine the yield pressure and the collapse pressure. Fig. 4 shows a thick-walled unflawed pipe with inner radius R_i, outer radius R_o, mean radius R_m, length L and thickness t. An axisymmetric FE model was generated with a regular mesh of 10 elements through the thickness of the pipe and 40 elements along the length. Quadratic elements with reduced integration (type CAX8R) were adopted. A value of 10 mm for the wall thickness and the mean radius was given by $R_m / t = 5$. The length of the pipe was given by $L / 2\pi R_m = 0.64$. For comparison with the results of the analytical solutions, an FE model was also generated for a thin-walled, elastic perfectly plastic pipe by setting $R_m / t = 30$. The pressures to cause yielding were found and the collapse pressures were derived by increasing internal pressure until the pressure-radial displacement curve approaches to an asymptotic value. It was found that small strain FE analysis and large strain FE analysis produced the same results.

The yielding results of the FE approach are superimposed on those of the analytical solutions in Fig. 3. The normalised yield pressures for the thick-walled pipe are similar to those for the thin-walled pipe. The results of the analytical solutions and the FE for the thin-walled pipe are in good agreement. For the global collapse pressure, the FE results are shown in Fig. 5. It is noted that the global collapse pressure for combined axial load and internal pressure are the same as the yield pressure. This is because the axial load remains constant as the pressure is increased. However, this is not the case for combined fixed axial displacement and internal pressure. It is found in Fig. 5 that the fixed displacement controlled end conditions do not contribute to global collapse pressures. After the yield pressures are reached, the initial stresses are relaxed to allow higher pressure to be applied until global collapse occurs. It is also found that the normalised pressures for displacement control are larger than 1. This is due to the Poisson effect. The results of the analytical solutions and the FE for the thin-walled pipes are in good agreement.

In addition, there is no elastic follow-up for the unflawed pipe under fixed displacement end conditions. FE analyses were also carried out for longer pipes, and similar results shown in Fig. 5 were obtained. It is therefore concluded that long-range residual stress does not contribute to plastic collapse of unflawed pipes but mechanical axial loading does.

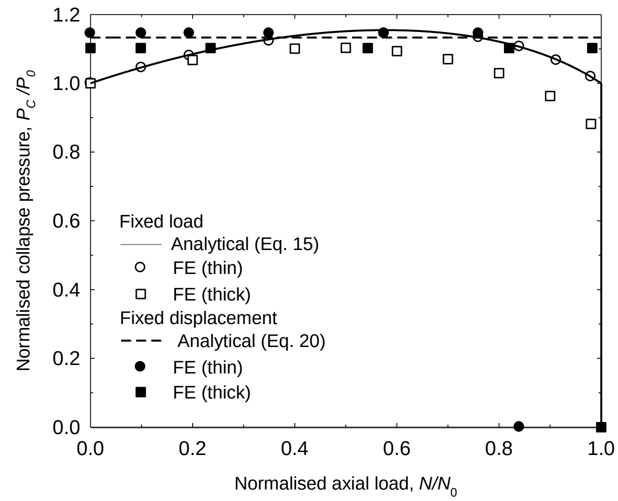


Fig. 5 Analytical and FE calculations for the collapse pressure versus initial end load for unflawed thick and thin-walled pipes under fixed tensile load and displacement conditions

PLASTIC COLLAPSE OF CIRCUMFERENTIALLY CRACKED PIPES

We now examine the plastic collapse of a thick pipe containing a fully circumferential crack as shown in Fig. 6. Only one value of the crack depth has been considered given by a / t =0.75. Other depths of crack have been investigated but the general form of the results is similar. Additionally, in this work only external cracks have been considered since internal cracks behave similarly.

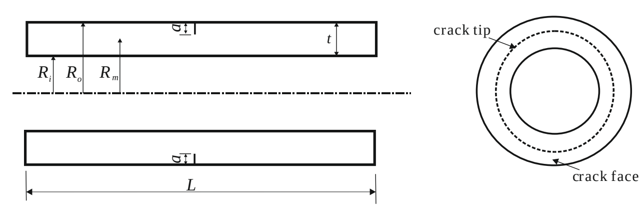


Fig. 6 Geometry of the pipe containing a fully circumferential crack

As for the unflawed pipe, only the open-ended condition has been addressed. For these analyses the same thickness and internal and external radius for the thick pipe without a flaw have been used, that is t =10 mm and R_m / t =5. The length of pipe is examined given by $L / 2\pi R_m$ =0.64. In addition, analyses have been carried out for longer pipes with $L / 2\pi R_m$ =12.73 to explore the influence of length on the results.

Fig. 7 shows the axisymmetric FE model to analyse the case of a pipe containing a fully circumferential crack. Quadratic axisymmetric elements with reduced integration were used (type CAX8R). A sensitivity study was performed to investigate the appropriateness of the meshes. Different levels of coarse and fine meshes were adopted to study the crack driving force and limit load of the cracked pipe under tension. This sensitivity study gave the confidence of the meshes used in this work. It can be seen from Fig. 7 that finer meshes were made around the crack section. In the crack tip region, a ring of wedge-shaped elements was adopted.

In addition, either fixed axial load or fixed axial displacement conditions were considered, combined with constraint equations to ensure that nodes at the end of the pipe have the same axial displacement. The material properties were the same as for the condition of unflawed pipes. Once the appropriate axial condition was applied, the internal pressure was increased until collapse occured.

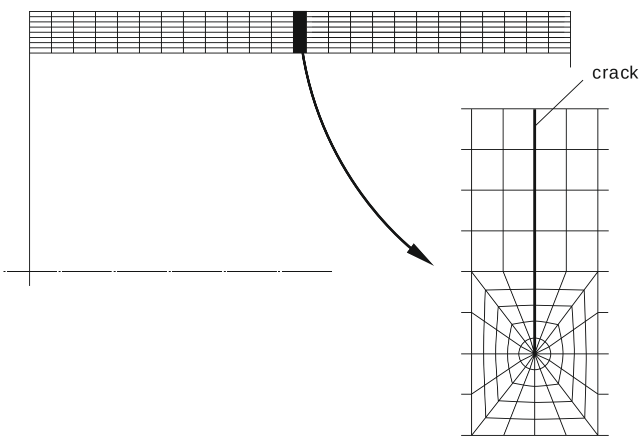


Fig. 7 FE meshes used for the analysis of a full circumferentially cracked pipe

Fig. 8 illustrates the relationship between the internal pressure, the radial displacement and the corresponding local net section yield and global collapse pressures subjected to internal pressure only. The pressure is normalised using the collapse pressure P_o for a pipe without a crack with the same wall thickness and mean radius in the absence of axial loading. For a normalised pressure of 0.92 the plastic zone has spread across the ligament. This pressure is termed here the local net section yield pressure. As the pressure is increased, the displacement increases rapidly with a small increase of internal pressure. Finally, the pressure is asymptotic to the global collapse pressure shown as GC in Fig. 8. Both small and large strains were carried out. However, for all subsequent results, the large strain global collapse pressures are used when they are different from the small strain pressures since the results from large strain analysis take into account geometrical nonlinear effects and large deformation.

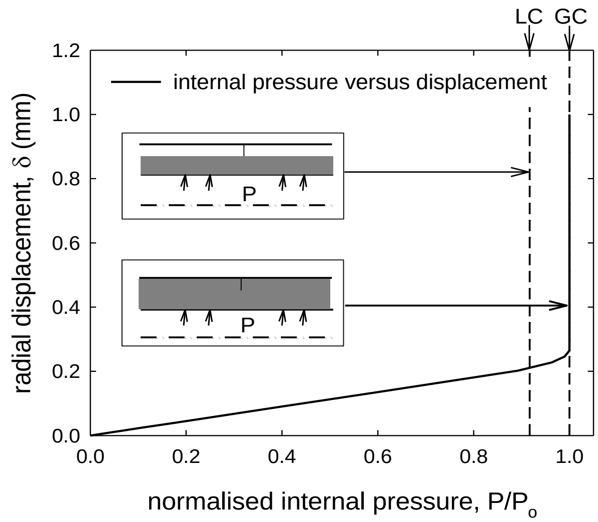
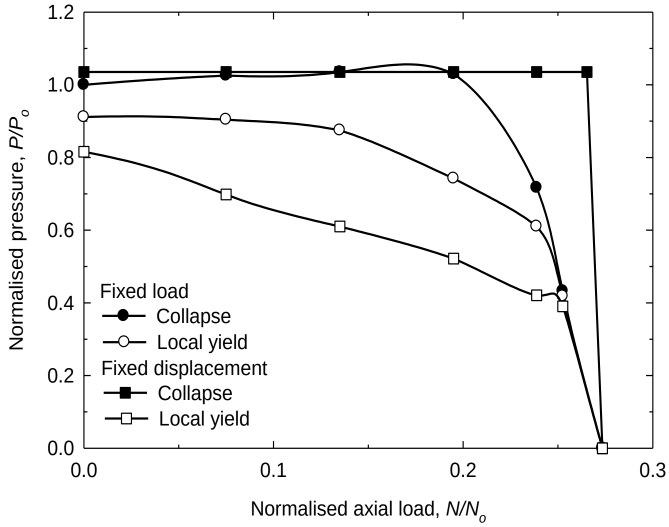


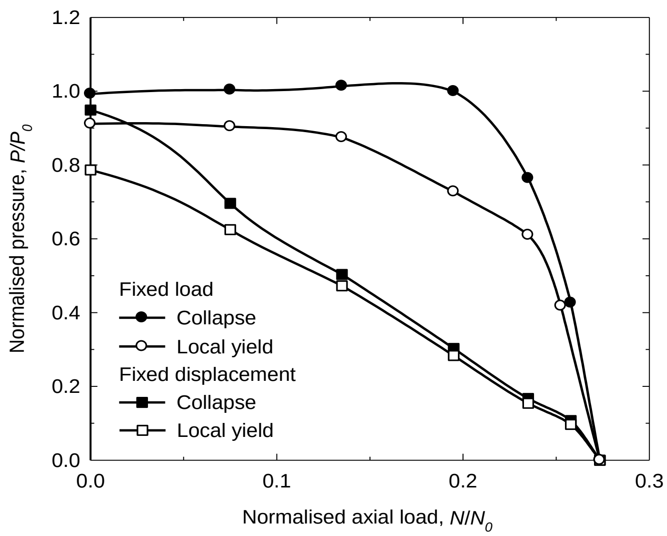
Fig. 8 global collapse (GC) and local net section yield (LC) pressure calculated for a pipe containing a fully circumferential crack subjected to internal pressure only using FE method

The techniques to determine the local yielding and collapse pressures, as shown in Fig. 8, were repeated for different fixed axial load and axial displacement conditions to give the results shown in Fig. 9 (a) and (b) for the cases of a short pipe with length of $L/2\pi R_m$ =0.64 and a long pipe with length of $L/2\pi R_m$ =12.73 containing a fully circumferential crack. Here the axial load is normalised using the axial load to cause collapse N_o for a pipe without a flaw with the same wall thickness and mean radius in the absence of pressure. If a tensile axial load or displacement is applied, collapse occurs without additional pressure when the normalised axial load is equal to 0.27 for both short and long pipes.

In terms of the short pipe, Fig. 9 (a) shows that fixed load controlled end conditions have influence on both local net section yield and global collapse pressures, especially when normalised axial load is larger than 0.2. The same influence of fixed loading controlled end conditions on local net section yield and global collapse pressures can be found for the case of long pipe, as shown in Fig. 9 (b). This demonstrates that the length of the pipe does not affect the contribution of load controlled end conditions on local net section yield and global collapse pressures.



(a) $L/2\pi R_m$ =0.64

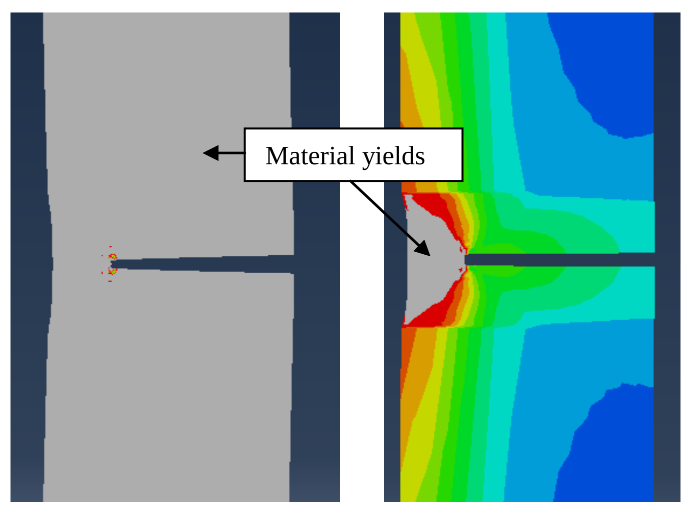


(b) $L/2\pi R_m$ =12.74

Fig. 9 Global collapse pressure and pressure to cause local net section yield versus initial end load for (a) a short pipe and (b) a long pipe containing a fully circumferential crack with crack depth of *a/t*=0.75 under fixed axial load and axial displacement conditions

However, the length of pipe does affect the contribution of displacement controlled loading on local net section yield and global collapse pressures. For a short pipe, Fig. 9 (a) shows that the local net section yield pressure is reduced as the axial tensile load induced by fixed displacement increases. However, the global collapse pressure remains constant with the change of displacement controlled loading. This is due to the relaxation of displacement controlled stress as plasticity accumulates when pressure approaches to global collapse. This can be seen from Fig. 10 (a) which shows the material is everywhere at yield when global collapse occurs. When local net section yield occurs, the displacement controlled stress is not removed and therefore contributes to local net section yield. However, for pressurised pipes containing a circumferential crack, global collapse is more relevant than local net section yield as the global collapse pressure is the limiting pressure while local net section yield is not a failure condition.

For the long pipe, Fig. 9 (b) shows that displacement controlled loading reduces both local net section yield and global collapse pressures. Local net section yield pressures are even lower than those obtained from the short pipe. Additionally, local net section yield pressures are close to global collapse pressures for the long pipe. This is because the long pipe containing a fully circumferential crack exhibits significant elastic follow-up and therefore displacement controlled stress does not relax before collapse. When collapse is about to occur, the plastic zone is limited to the ligament region, as shown in Fig. 10 (b). The displacement controlled stress together with an additional axial stress generated due to the Poisson's effect dominates the global collapse. If this additional axial stress is added to the fixed load controlled end conditions, the global collapse pressures obtained are the same as the cases for displacement controlled end conditions. Therefore, without this additional axial stress for fixed load conditions, both local yield and global collapse pressures are higher than the cases for fixed displacement conditions, as shown in Fig. 9 (b).



(a) $L/2\pi R_m$ =0.64 (b) $L/2\pi R_m$ =12.74

Fig. 10 Accumulation of plasticity obtained from FE analysis at the onset of the global collapse for (a) a short pipe and (b) a long pipe containing a fully circumferential crack with crack depth of *a/t*=0.75 when normalised axial load controlled by fixed displacement is 0.2

Since the contribution of long-range residual stress (displacement controlled loading conditions) to plastic collapse depends on the significance of elastic follow-up, the elastic follow-up for different lengths of pipe containing the same fully circumferential crack is discussed in the next section.

DISCUSSION

For an unflawed pipe, Fig. 5 shows the collapse pressure versus axial load for two cases, one where the axial load was fixed and one where the axial load was a result of a fixed axial displacement. For the first case the axial load remains constant with varying pressure while for the second case the axial load changes with pressure. The collapse pressure depends on the axial load for the first case but does not depend on the axial load for the second case. This is not affected by the length of the pipe. The R6 procedure classifies stresses contributing to plastic collapse as primary stresses and stresses which do not contribute to plastic collapse as secondary stresses. Therefore the stresses generated by the fixed axial load are classified as primary stresses while the stresses generated by the fixed displacement condition are classified as secondary stresses.

Conversely, Fig. 9 showed that the pressure to cause collapse of a pipe containing a fully circumferential crack may be affected by the magnitude of the fixed axial displacement depending on the degree of significance of elastic follow-up. This elastic follow-up is considered by varying the length of the pipe in this work. A method for estimating elastic follow-up in an open-ended pipe containing a fully circumferential crack is suggested.

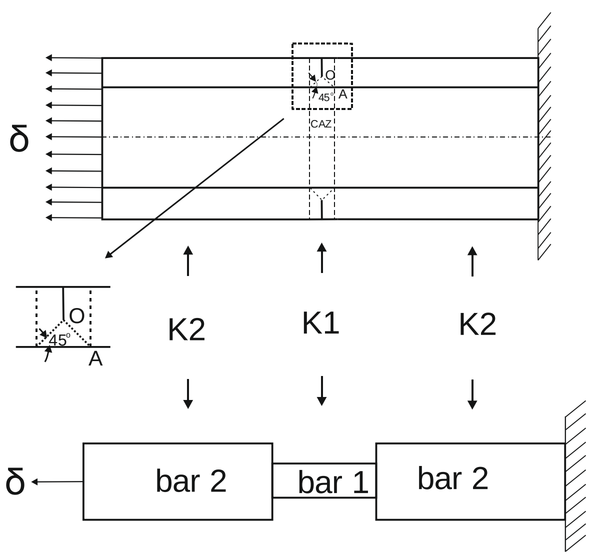


Fig. 11 Crack affected zone for a pressurised pipe with a fully circumferential crack

Hadidi-Moud and Smith [17] noted that for a thick pipe with external circumferential hoop crack in tension, a series bar formed by a central spring representing the crack affected zone (CAZ) and two linear hollow bars representing the remaining length of the pipe could be used to define the equivalent spring system. However, they did not provide specific rule to define the boundary of CAZ, but nevertheless suggested that the boundary of CAZ was a range of normal distances from the crack plane equal to 1, 2, 3 and 4 times the crack depth [17]. Here, an alternative method is used to determine the boundary of the CAZ for a fully circumferentially cracked pipe. This is proposed in Fig. 11 which illustrates a region defined by the extent of the plastic zone that penetrates the ligament along the line OA. The angle between the line OA and the inner surface is about 45 degrees which agrees with the plastic zone shown in Fig. 10 (b). Additionally, this is also in accordance with slip-line field theory. Based on the CAZ defined in Fig. 11, the pipe can be treated as a three-series bar structure. The CAZ is represented by bar 1 with a small area while the unflawed region is represented by bar 2. Therefore, the method used to estimate the elastic follow-up factor, *Z*, for the three-series bar structure is also used for pressurised pipes containing a fully circumferential crack under fixed tensile displacement controlled end conditions. Based on the CAZ defined in Fig. 11, the relative stiffness ratio of K2 to K1 can be calculated by FE elastic analysis, and therefore elastic follow-up can be estimated via Eq. 9.

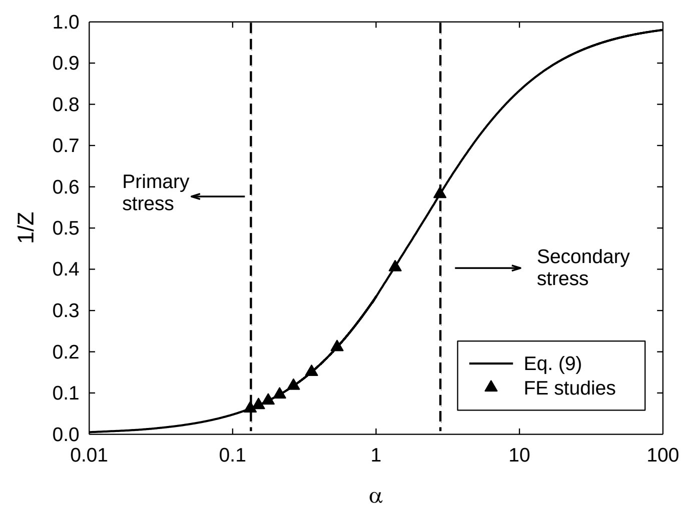


Fig. 12 Variation of elastic follow-up factor with stiffness ratio for a pressurised pipe containing a fully circumferential crack with crack depth a/t=0.75

Fig. 12 shows elastic follow-up factor against the relative stiffness ratio. By examining different lengths of pipes containing the same fully circumferential crack, different values of the relative stiffness ratio can be determined via FE analyses. As the length of the cracked pipe increases, the relative stiffness ratio decreases. This leads to the increase of elastic follow-up, *Z*, in the structure. When the elastic follow up is significant (Z>10 in Fig. 12), the displacement controlled stress does not relax and contribute to plastic collapse. Such displacement controlled stress can be classified as primary stress. However, if the relative stiffness between the unflawed region and the CAZ is high, the elastic follow-up is low (Z<2 in Fig. 12). At this situation, the displacement controlled stress is relaxed by the plasticity before collapse and therefore should be classified as secondary stress. Between these two extreme cases, displacement controlled stress is partially relaxed but the unrelaxed part will contribute to plastic collapse. This is also demonstrated by Fig. 13. For the short pipe ($L/2\pi R_m$ =0.64), the elastic follow-up factor is 1.72. Since the elastic follow-up is low, the global collapse pressure is not affected by the initial stress induced by fixed displacement. As the length of the pipe increases, the global collapse pressure is decreased. When the pipe is sufficiently long, the elastic follow-up is significant and the initial tensile stress induced by fixed displacement has the same effect as the stress controlled by a fixed load on global collapse.

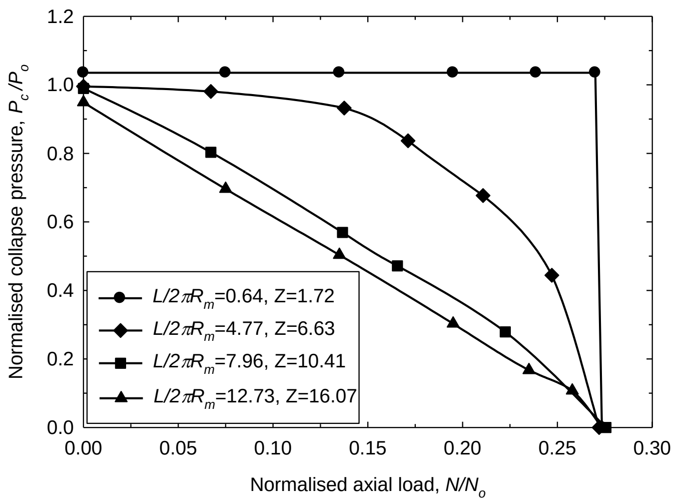


Fig. 13 Global collapse pressure versus initially axial load induced by fixed displacement for different lengths of open-ended pipes containing a fully circumferential crack with crack depth of *a/t*=0.75

CONCLUSION

This paper has presented results from a finite element based study of local net section yield and global collapse pressure for unflawed and circumferentially cracked pipes when pre-fixed load and displacement controlled conditions are present.

For unflawed pipes the pressure to cause collapse is not affected by the condition of fixed axial displacement. However, the fixed load controlled end conditions have an influence on collapse pressure. The length of the pipe does not affect the collapse pressure as no elastic follow-up exists in unflawed pipes.

For pipes containing a fully circumferential crack, the collapse pressure is affected by the load for fixed axial displacement depending on the degree of elastic follow-up. The CAZ approach was developed to evaluate elastic follow-up in the fully circumferentially cracked pipe. The elastic follow-up for a short pipe may be insignificant and long-range residual stress may not contribute to global collapse. However, as the length of pipe increases, the elastic follow-up becomes significant and therefore long-range residual stress will contribute to global collapse.

This paper demonstrates that depending on the degree of elastic follow-up in the structure, long-range residual stress can be classified into three categories: secondary stress (completely removed), primary stress (remain constant) and stress in between secondary and primary stress (partially relaxed).

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