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Systems with bilinear stiffness: extraction of backbone curves and identification

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ABSTRACT

With the need to improve system performance, aerospace and automotive structures are being designed with much lighter construction and also much less inherent damping. A consequence of this is that structural nonlinearities have a much greater effect on the static and dynamic performance. Although there has been significant effort recently towards the extension of modal analysis to identify structural nonlinearities, these techniques are still not at a stage where they can be used on industrial size structures. This paper describe an experimental investigations on a structure model containing two bilinear stiffness elements. A method for the estimation of the characteristic backbone curves of the nonlinear system is first discussed and then used to characterise the active nonlinear elements from experimental data. Conclusions are drawn as to the most effective way to extend the proposed method for future industrial applications.

Keywords: Bilinear stiffness, Experimental identification, Resonance decay method, identification of nonlinearities.

Introduction

The development of novel materials together with the increasing computational capabilities have brought to light a variety of novel and more efficient design solutions to diverse engineering problems. These solutions usually result in extremely flexible structures that are particularly susceptible to exhibiting nonlinear effects that occur at large structural deflections. Others structural systems can also combine nonlinear phenomena due to discontinuities (like mobile joints or connections) and nonlinear material properties. Therefore, the understanding of nonlinear dynamic systems and their performance in operational and under extreme loading conditions is an increasingly important research topic with potentially strong impact in many industrial sectors.

One useful tool capable of offering a better understanding of the behaviour of nonlinear systems is the backbone curve [11, 12]. This defines the natural frequency as a function of the amplitude of the system response when neither damping nor forcing are present.

This paper studies a scaled structural model that features two bilinear stiffness elements. The model is, at a first approximation, linear at low vibration levels but the contribution of the nonlinear elements becomes significant at larger excitation levels. Our interest is to investigate the free vibration response originated from initial conditions belonging to particular steady–state responses of the nonlinear system. The Resonance Decay Method (RDM) [6] is used here, as this enables the excitation of individual modes of the system independently. Once the structure is vibrating at the desired resonance condition, the forcing is removed and the resulting free vibration response can be analysed. This strategy has proven to be able to isolate distinct characteristics of the examined nonlinear system [1, 10].

One promising technique for nonlinear identification consists in curve–fitting the resulting decay response directly to identify active nonlinear elements [4, 6]. This requires prior knowledge of the nature of the nonlinearity to be curve–fitted. The problem of deciding which terms are required for an accurate identification has previously been addressed using some forms of the Forward Selection, Backward Elimination or other techniques based on soft computing [5, 9]. Notwithstanding, a suitable strategy to select proper candidates of the nonlinearities is still unclear.

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In this paper, the RDM method is used to excite the structure at a number of resonance frequencies that enables measurements to be made in regimes where the nonlinearities are more active. This facilitates both the estimation of backbone curves from the experimental decay records and more information to be contained in the backbone curves, thus enabling a appropriate identification of the nonlinearities.

Backbone curves from experimental data

Our interest is to examine free vibration responses originated from initial conditions that lay on one steady–state condition of the system. This strategy has proven to be able to isolate distinct characteristics of the examined dynamical system. Consequently, in this approach the signal used to estimate the backbone curves of the nonlinear structure is generated in accordance with the Resonance Decay Method (RDM) [6]. In this technique, individual modes of the system can be excited independently by applying an appropriated force pattern. Such a force pattern is determined by using the normal–force mode appropriation method, that enables for extracting the undamped natural frequency and normal–modes shapes of a linearised structural system [13]. After the appropriated force pattern is computed, this is applied to the system at the relevant frequency using harmonic excitation. The input is then removed and the model undergoes free vibration from the steady–state reached. As long as the level of vibration in the steady–state is large enough to activate the structural nonlinearities, the generated decaying response can be use to estimate the system's backbone curves that offer significant information about the system and its dynamic variables.

Two main features namely instantaneous frequency and amplitude envelope are estimated in the interest of revealing the active nonlinear elements.

While there are many procedures for calculating instantaneous frequency such as the Wigner–Ville distribution [7] and the Hilbert transform [3], the process used here is based on the detection of the zero–crossing points of the response signal. Once the sequence of the crossing times (t^o) is properly determined, the first estimation of the instantaneous frequency is computed as the inverse of the instantaneous period along one complete cycle. This frequency is assigned to the crossing time at the centre of this respective interval. Imperfects in the prediction arising from noise and other sampling issues are smoothed out by means of a moving average (MA) filter. In spite of its simplicity, the moving average filter exhibits excellent properties in reducing random noise while is able to retain a sharper step response. Hence, the final estimation of the instantness frequency derives from a processed data that retains only the dominant frequency variation along time. We note that this procedure assumes that the instantaneous frequency estimated is not significantly altered by the dissipative forces acting on the system.

To estimate the instantaneous amplitude envelope over time, the maximum absolute value of the decaying response and its corresponding occurring time are measured within each interval $\{t^o_i, t^o_{i+1}\}$. These sequence of numbers is used in conjunction with a standard polynomial interpolating function to determine the value of the envelope amplitude at the same times at which the instantaneous frequency has been evaluated.

As a final step, the backbone curve can be obtained as a function of above estimated instantaneous frequency and amplitude enveloped parametrised by time.

Experimental example

Results of a model structure containing two element with bilinear stiffness are used here to illustrate the applicability of backbone curves as a tool for characterising and modelling nonlinear systems. The test structure approximately represents the configuration of an aircraft wing having two underwing stores (e.g., engines) with nonlinear pylon connections. The model consists of a rectangular aluminium plate hung by bungee chords and two lumped masses suspended underneath via pylon plates, as shown in Figure 1.

The nonlinear connection is built by fixing the pylon plates onto the wing using two bespoke clamps. This connecting element features internal curve surfaces that produce a characteristic nonlinear behaviour (See details in Figure 1b). This wing structure was previously tested in [10] using the RDM approach. The model is excited by an electrodynamic shaker (LDS V201) and instrumented with 9 piezoelectric accelerometers (PCB 33M07) and one force sensors (PCB 208C03) to measure the shaker driving force. Additionally, a laser Doppler vibrometer (PDV-100) is used to measure the velocity in the horizontal direction of one of the stores. Laser readings are used only to verify the processed data after numerically integrating the acquired acceleration signals. The vibration tests were controlled and recorded by using the data acquisition systems LMS SCADAS Lab.

A number of tests were performed to gain insights into the overall structural behaviour of the wing model. To begin with, the pylon connection was tested statically to determine the actual nonlinearity acting on the system. Figure 2 (left) shows the resulting bilinear hardening stiffness exhibited by the clamped connection of one of the pylons when it is loaded in a pseudo–static test cyclically. Several dynamic tests were also conducted at different applied vibration levels. The driving force employed to vibrate the model was a burst random signal with a bandwidth of 256Hz. Figure 2 presents five FRFs for different increasing levels of excitation. In their estimation, 25 averages are considered. It can be seen that the FRFs do not overlay each other for several modes indicating the presence of nonlinearities. Interestingly, the zoomed plot within the figure revels a clear softening effect that is opposite to what might be expected from the preliminary pseudo–static tests. This fact will be further discuss later. We note that the frequency shift is mainly exhibited by the first and second resonance

Fig. 1 Test structure representing the configuration of an aircraft wing having two underwing stores supported by pylons. a) General view of the experiment setup. b) Detailed view of the nonlinear connection.

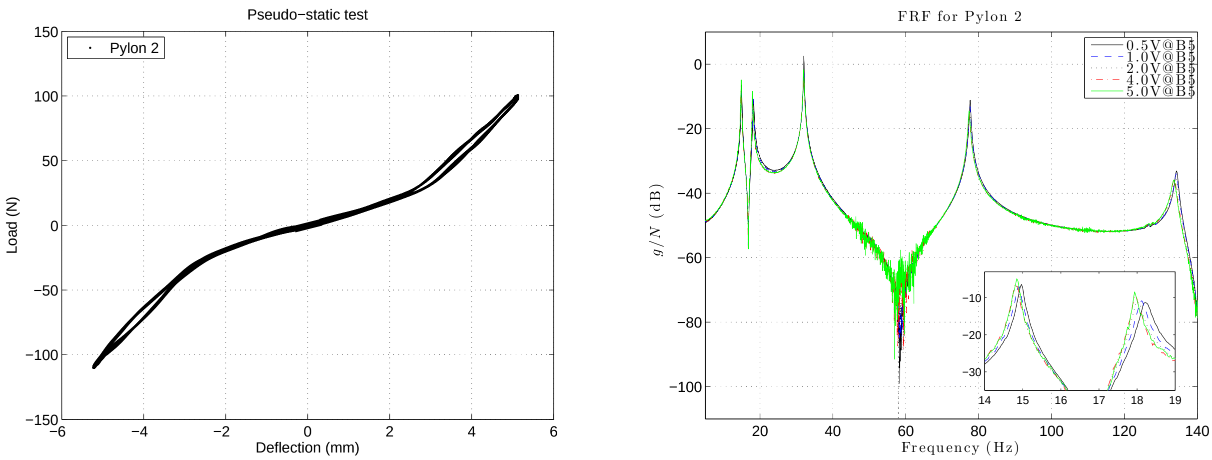

Fig. 2 Pseudo–static test on one isolated clamp connection showing a bilinear hardening stiffness [left]. Set of FRFs for different levels of vibration via random excitation. Note the softening effect exhibited by the resonance peaks in the zoomed box [right].

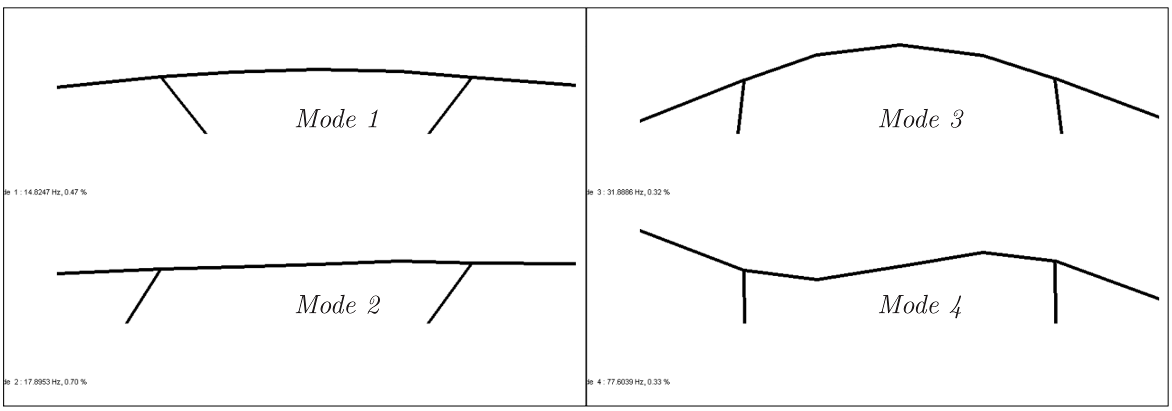

Fig. 3 First four mode shapes of the underlying linear system of the test model.

frequencies located around 16Hz and 18Hz respectively.

Classical modal testing [2] is used to obtain the shapes corresponding to the first four vibration modes, see Figure 3. The first and second modes describe the stores moving in anti–phase and in phase respectively. Whereas the third and fourth correspond to modes that are dominated by the first and second bending modes of the wing respectively, with minimal interaction of the stores. Therefore, the nonlinear behaviour is expected to mainly affect the structure when vibrating in one of the first two modes. In the present paper, we only discusse the results associated with to the first resonance frequency.

Following the procedure described above for the experimental estimation of backbone curves, the first tasks consists in applying the RDM method for the first resonance frequency in order to obtain suitable decay records. After applying the normal–force mode appropriation method [13], results showed that not more than one shaker is required to properly excite the first mode and thus obtain the desired system's resonance condition. Sinusoidal excitation in force–control mode is applied to the test specimen at the first resonance frequency of the underlying linear system. The forcing frequency is gradually tuned until the amplitude of the response rises in magnitude and is large enough to activate the nonlinearity. When the system response reaches the state–steady condition, the shaker is turned off and the decay response recorded.

The amplitude envelope and instantaneous frequency are then estimated for individual displacement records in physical space. The displacements have been calculated via double integration of the accelerometer signals. Figure 4a presents the decaying record of the store 2 originated from the steady–state condition at the first resonance frequency. The red line corresponds to the estimation of the amplitude envelope. Figure 4b shows the initial approximation of the instantaneous frequency with coloured crosses and the final estimation with black dots. The backbone curve produced is presented in Figure 4c.

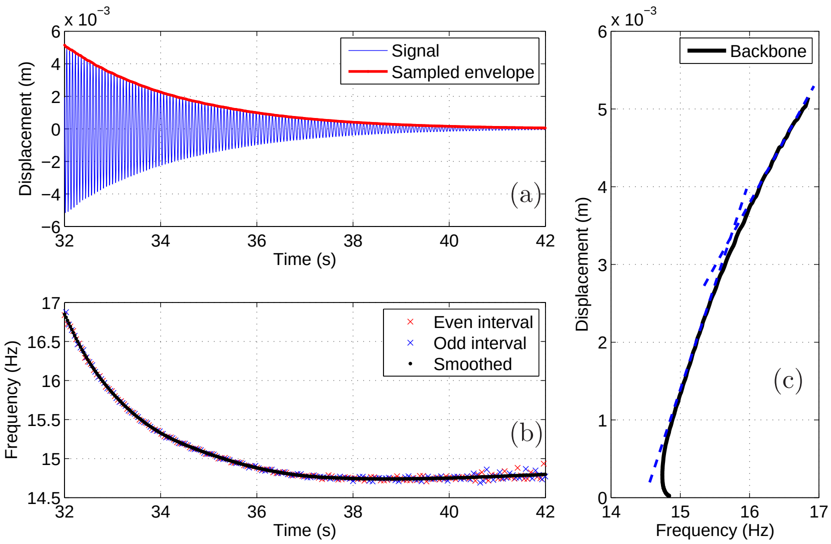

Fig. 4 Estimation of backbone curve from experimental decay data after achieving resonance condition for the first frequency (Displacements of store 2). a) Decaying signal and the estimated envelope; b) Instantaneous frequency; c) Backbone curve.

Some interesting aspects can be seen in these results. First of all, the bilinear characteristic of the nonlinear connection can be seen in the backbone curve presented in Figure 4c. Thus, the nonlinearity could be characterised by this means. With help of two auxiliary dashed lines, the inflexion point seems to be consistent with the one in Figure 2a. In addition, it can be seen that the dispersion of the estimation of the instantaneous frequency become larger when the amplitude of oscillation is small. This is due to the fact that the accuracy of the measurements is reduced at that range of vibration as the signal–to–noise ratio is critically low.

The matrix of the first four mode shapes of the underlining linear system is presented in (1). The first seven rows correspond to the accelerometers located on the wing plate, and the last two rows, to the accelerometers located in the stores as shown in Figure 1. This matrix can be used to operate a linear transformation on the decay records and thus obtain the structural response in terms of linear modal coordinates.

$$\Phi = \begin{bmatrix} -0.249 & 0.074 & -1.418 & -1.068 \\ -0.031 & 0.085 & -0.120 & 0.140 \\ 0.097 & 0.046 & 0.668 & 0.413 \\ 0.153 & 0.004 & 1.000 & 0.005 \\ 0.106 & -0.046 & 0.661 & -0.415 \\ -0.009 & -0.092 & -0.073 & -0.153 \\ -0.205 & -0.084 & -1.293 & 1.000 \\ 1.000 & 1.000 & -0.165 & -0.008 \\ -1.171 & 0.942 & 0.165 & -0.017 \end{bmatrix} \tag{1}$$

The procedure for estimating the backbone curves can then be applied independently to each decaying response of the system expressed in terms of the linear modal space. Figure 5a presents the resulting backbone curves estimated when projected onto the linear modal coordinates q_1, q_2, q_3 and q_4, from decaying signals corresponding to the first resonance frequency. Notice that the dominant backbone curve corresponds to the first linear modal coordinated q_1 (black solid line). An expected result as the structure was harmonically excited around the frequency corresponding to the first resonance. Note that this curve also shows the bilinear characteristic of the nonlinear stiffness. Furthermore, it can be seen that when the structural response is large and the influence of the nonlinearity is much more significant, the backbone curve in terms of the third modal coordinate q_3 starts to rise too. This is a nonlinear effect that can be recognised as an interaction between linear modes since the structural response can not be fully represented by a single linear mode anymore; in fact, the backbone curve relates directly to the nonlinear normal modes of the system [8]. Note that this aspect could be exploit to include additional coupled terms into the modal equations for nonlinear system identification purposes.

With the aim of verifying the aforementioned modal interaction, the RDM method is applied again, this time for the third resonance. As before a sinusoidal excitation in force–control mode is applied to the test specimen at an appropriate resonance frequency until the system reaches the state–steady response, then the shaker is turned off and the decaying signals recorded. Figure 5b presents the resulting backbone curves estimated with respect to the modal coordinates q_1, q_2, q_3 and q_4, from decaying signals that belong to the third resonance frequency. These results not only show that the structural responses can not be fully represented by the third modal coordinate only (magenta dashed line), but also confirm the interaction between modes 1 and 3, as the the response in terms of the first linear modal coordinate is significant.

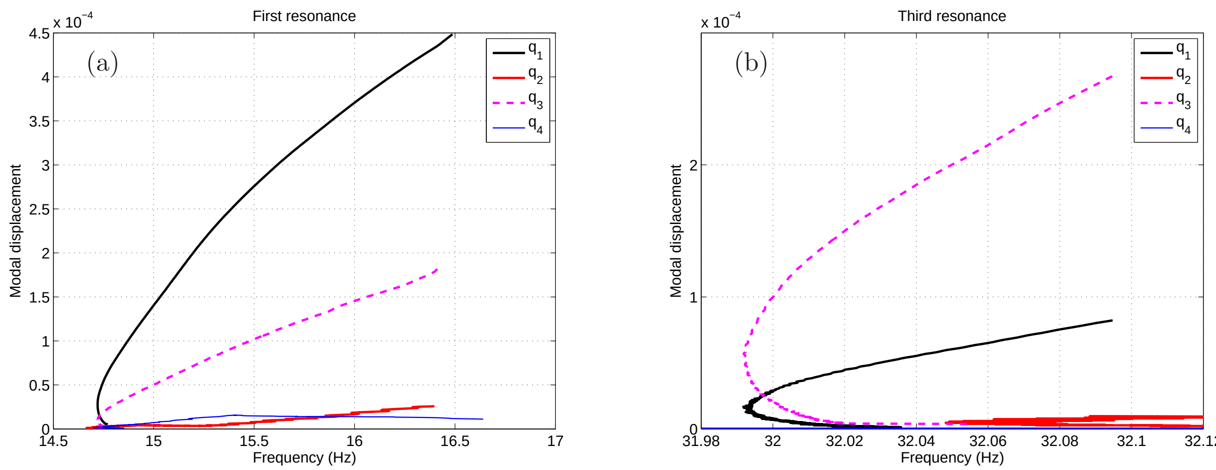

Fig. 5 Backbone curves estimated from experimental data in terms of the modal coordinates q_1 to q_4. a) From decay after achieving steady–state response at the first resonance. b) From decay after achieving steady–state response at the third resonance.

These results indicates that most of the structural response for the first and third resonance frequency (and so the information of the nonlinearity) is contained in both backbone curves q_1 and q_3. We recall that the backbone curve defines the natural frequency as a function of the amplitude of the system response when neither damping nor forcing are present. Thus, we can write for each resonance a modal equation of the form

$$\ddot{q}_i + \omega_i^2 q_i + f_{NL}(q_{1_i}, q_{3_i}) = 0 \; ; \quad \forall i = 1, 3 \tag{2}$$

where ω_i is the i-th natural frequency of the underlying linear system and f_{NL} is a nonlinear function of the first and third modal coordinates at the i-th resonance q_{1_i} and q_{3_i} that represents the nonlinear characteristics shown in Figure 5a. This can be transformed into

$$-\omega^2 + \omega_i^2 + \frac{1}{q_i} f_{NL}(q_{1_i}, q_{3_i}) = 0 \; ; \quad \forall i = 1, 3 \tag{3}$$

where ω is the instantaneous frequency in the backbone curve. By using the experimental data summarised in the backbone curves in Figure 5, Equation (4) can be curve–fitted for both resonance cases.

The contribution of quadratic and cubic stiffness in the nonlinear function f_{NL} are assumed here for illustrative purposes only. Thus, it can be written that

$$f_{NL} = a_{1i} q_i + (a_{2i} q_{1_i} + a_{3i} q_{3_i})^2 + (a_{4i} q_{1_i} + a_{5i} q_{3_i})^3 \; ; \quad \forall i = 1, 3 \tag{4}$$

We note that a specific form for the nonlinear coupling has been selected for the quadratic term, since the more general form would have been $b_{1i} q_{1_i}^2 + b_{2i} q_{1_i} q_{3_i} + b_{3i} q_{3_i}^2$. A similar consideration can be made with respect to the cubic term. Table 1 presents the results after carrying out a numerical optimisation procedure that finds the best set of coefficients that fits Equation (4) to the experimentally obtained backbone curves presented in Figure 5.

i-th		Fitted coefficients ($\times 10^6$)				
Resonance	ω_i	a_{1i}	a_{2i}	a_{3i}	a_{4i}	a_{5i}
1	14.924	−8.356	−0.989	1.928	−0.251	0.541
3	32.015	−1.954	−0.303	0.228	0.192	−0.090

Table 1 Best set of coefficients that fits Equation (4) to the backbone curves presented in Figure 5.

A similar discussion can be made with reference to the responses around the second and fourth resonance frequencies. The RDM can be applied to vibrate the structure appropriately, generate the decaying records and estimate the backbone curves. After considering the modal interactions, they can be curve-fitted to complete the model for the test structure on the linear modal space.

Conclusions

The prediction of the performance of nonlinear structural systems is an increasingly important research topic. Among other analysing tools, backbone curves stand out as they offer a better understanding of the dynamical behaviour of the nonlinear structure. They are also able to reveal the nature of nonlinearities exhibited by the structural response. This approach is well suited to investigate structures primarily linear but that contain active nonlinear elements which become significant at larger levels of excitation.

In this paper a model structure containing bilinear stiffness elements was studied. Results demonstrated that the proposed procedure is capable of achieving a sharp estimation of the backbone curves from experimental data. The backbone curves were able to reveal the bilinear characteristic of the nonlinearity from appropriately obtained experimental data. It is worth mentioning that the backbone curves can offer important information to identify the modal coupling caused by the active nonlinearities. This can be exploit to pinpoint the non–zero off diagonal terms of the matrix of modal stiffness when identifying a suitable mathematical model for the structure under test. More studies are required to establish a strategy to extract the non–linear structural parameters from a set of backbone curves. This will allow for a complete identification of the nonlinear system.

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References

[1] G. Charmbalis, J. Londono, and J. E. Cooper, *Vibration Testing of Aeroelastic Structures Containing Geometric Stiffness Nonlinearities*, American Institute of Aeronautics and Astronautics, Boston, 8–11 April 2013.

[2] D. Ewins, *Modal Testing: Theory, Practice and Application*, Engineering Dynamics Series, RSP LTD, Baldock, Hertfordshire, UK, second ed., 2000.

[3] M. Feldman, *Non–linear system vibration analysis using* Hilbert *transform–I. Free vibration analysis method 'FREEVIB'*, Mechanical Systems and Signal Processing, 8 (1994), pp. 119–127.

[4] J. M. Londono and J. E. Cooper, *Experimental identification of a system containing geometric nonlinearities*, in Topics in Modal Analysis I, Volume 7, J. De Clerck, ed., Conference Proceedings of the Society for Experimental Mechanics Series, Springer International Publishing, 2014, pp. 253–260.

[5] K. Mao and S. Billings, *Variable selection in non–linear systems modelling*, Mechanical Systems and Signal Processing, 13 (1999), pp. 351–366.

[6] S. Naylor, M. Platten, J. Wright, and J. Cooper, *Identification of multi–degree of freedom systems with nonproportional damping using the resonant decay method*, J. Vib. Acoust, 126 (2004), pp. 298–306.

[7] S. Neild, P. McFadden, and M. Williams, *A review of time-frequency methods for structural vibration analysis*, Engineering Structures, 25 (2003), pp. 713–728.

[8] M. Peeters, G. Kerschen, and J. Golinval, *Modal testing of nonlinear vibrating structures based on nonlinear normal modes: Experimental demonstration*, Mechanical Systems and Signal Processing, 25 (2011), pp. 1227–1247.

[9] M. F. Platten, J. R. Wright, K. Worden, G. Dimitrladis, and J. E. Cooper, *Non–linear identification in modal space using a genetic algorithm approach for model selection*, International Journal of Applied Mathematics and Mechanics, 3 (2007), pp. 72–89.

[10] M. F. Platten, J. R. Wright, J. E. Cooper, and G. Dimitriadis., *Identification of a nonlinear wing structure using an extended modal model*, Journal of Aircraft, 46 (2009), pp. 1614–1626.

[11] D. J. Wagg and S. A. Neild, *Nonlinear Vibration with Control*, vol. 170 of Solid Mechanics and Its Applications, Springer–Verlag, Dordrecht, The Netherlands, 2010.

[12] K. Worden and G. Tomlinson, *Nonlinearity in Structural Dynamics: Detection, Identification and Modelling*, IOP Publishing Ltd, Bristol and Philadelphia, 2001.

[13] J. Wright, J. Cooper, and M. Desforges, *Normal–mode force appropriation – theory and application*, Mechanical Systems and Signal Processing, 13 (1999), pp. 217–240.