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# Air-Fuel-Ratio Control of Engine System with Unknown Input Observer

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**Abstract**—This paper presents an alternative control to maintain the air-fuel-ratio (AFR) of port-injected spark ignition (SI) engines at certain value, i.e. stoichiometric value, to improve the fuel economy. We first reformulate the AFR regulation problem as a tracking control for the injected fuel mass flow rate, which can simplify the control synthesis when the fuel film dynamics are taken into account. The unknown engine parameters and dynamics can be lumped as an unknown signal, and then compensated by incorporating the unknown input observer into the control design. Only the measurable air mass flow rate through throttle, manifold pressure and temperature, and the universal exhaust gas oxygen (UEGO) sensor are utilized. Simulations based on a mean-value engine model (MVEM) illustrate that the proposed control can achieve satisfactory transient and steady-state performance with strong robustness when the engine is operated in varying speed conditions.

**Keywords**—Air-to-fuel ratio control, Spark ignition engine, Unknown input observer, Mean-value engine model.

## I. INTRODUCTION<sup>1</sup>

To meet strict legislative emission requirements imposed on commercial vehicles, modern port-injected spark ignition (SI) gasoline engines are usually produced with three-way catalyst converters [1]. However, an essential issue that must be addressed for such configuration is to maintain the in-cylinder air-fuel-ratio (AFR) at the ideal stoichiometric value (i.e. 14.7) [14], because the catalyst conversion efficiency and the emissions are heavily related to the injected AFR for combustion. Moreover, the engine torque generation and the stringent requirement for fuel economy also require that the AFR value should be recovered to its ideal value when engines are operated in various dynamic scenarios [2]. To achieve this objective, a common way is to control the fuel mass flow that is injected into the cylinder to fit the air mass flow entering the cylinder; this can be implemented using appropriate control for the fuel injector [3]. For this purpose, the AFR control design needs to take into account the engine modelling uncertainties, sensor noise, external loads and the fuel film effects [4, 5].

In the past decades, various control methods have been proposed, e.g. PID control [6, 7], adaptive control [3], sliding mode control [8] and predictive control [9]. In [6], a pre-filter and delay-compensation were augmented to PID control to enhance performance of SI engines. A PID control with a

parameter-varying dynamic compensator was suggested in [7] for the lean burn situation. However, linear PID controllers with fixed parameters cannot effectively account for significant nonlinearities over wide operation regimes of an engine. Thus, an observer-based sliding mode controller [5] was investigated to achieve fast convergence, and a second order sliding mode control was also studied in [8] to tackle the uncertainties in the AFR control loop. Moreover, Wong et al. [9] proposed a model predictive control for the AFR regulation of a SI engine. It is noted that the engine parameters are assumed to be known, and most internal engine variables (e.g. pressure, temperature, mass flow rate) are assumed to be measurable.

This fact motivates further work of system identification and adaptive control. In [10], the data from the in-cylinder pressure are used to estimate the AFR value and the associated model. A new adaptive control was presented to address the time-delay dynamics in the AFR control loop of SI engines [11]. However, the fuel film effect was not explicitly studied in [10, 11]. In [3], an adaptive AFR control was designed, where the model parameters are online updated, while the closed-loop stability is rigorously proved in terms of Lyapunov theory. However, the transient response of adaptive techniques heavily depends on the learning gains. Moreover, a tedious calculation was used in [3] to avoid immeasurable variables (e.g. fuel mass flow and air mass flow injected into cylinder). The applied load torque, friction torque and pumping loss should also be known, which may necessitate the use of costly torque sensors.

For a simple and robust AFR control with standard sensors, we will incorporate the idea of an unknown input observer [12] into the AFR control design. This avoids exact engine models or complex gain scheduling/adaptation, often needed with other techniques. To address the unknown dynamics and immeasurable variables, we first reformulate the regulation of AFR as a tracking control of the fuel mass flow rate injected into the cylinder. Most engine dynamics used in the control synthesis in this new framework can be merged into a lumped function. They can be taken as an unknown 'input' signal and online estimated [12]. Since the air mass flow rate entering the cylinder from the intake manifold is used in the controller, the idea of [12] is further extended to estimate this air mass flow rate based on the measured manifold pressure and the throttle air mass flow rate. Consequently, the proposed control strategy only requires the throttle air mass flow rate, manifold pressure and temperature, and AFR signal, which can be measured in commercial engines. Comparative simulations illustrate the improved transient and steady-state performance.

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## II. ENGINE DYNAMICS AND PROBLEM STATEMENT

This section will introduce essential dynamics for the port-injected SI engine. As validated in the literature [3][4, 13], the widely used ‘Mean-Value Engine Model (MVEM)’ can represent both the air and the fuel flow dynamics, and is suitable for designing various engine control strategies.

### A. Engine dynamics and modeling

In SI engines, the air mass flow into the cylinder can be manipulated by the throttle opening, and the injected fuel can be regulated at a predefined value by using the AFR control. This can improve the combustion efficiency and optimize the after-treatment condition. The major blocks of the port-injected SI engines can be found in the following Fig.1.

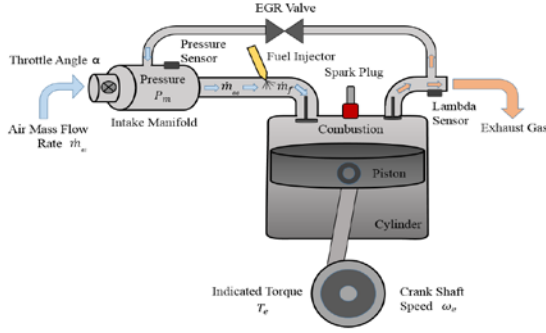


Fig. 1 Simplified sketch of SI engine systems

The engine subsystems shown in Fig.1 include the throttle dynamics, the intake manifold dynamics, the fuel injection, and the crankshaft rotation. Here, we only briefly introduce engine dynamics used in the AFR control design. For other dynamics, one may refer to [3][4, 13] for more detail.

#### A.1 Throttle body dynamics

The mass flow rate  $\dot{m}_{ai}$  past the throttle plate is determined by

$$\dot{m}_{ai} = m_{at} \frac{p_a}{\sqrt{T_a}} \cdot TC(\alpha) \cdot PRI(p_m, p_a) \quad (1)$$

where  $m_{at}$  is a constant related to the throttle area,  $p_m$  is the manifold pressure,  $p_a$  and  $T_a$  are the ambient pressure and temperature.  $TC(\alpha)$  defines the effective area of the throttle body, which is a function of the throttle opening angle  $\alpha$  and the leakage area  $\alpha'$  as

$$TC = 1 - \cos(\alpha + \alpha') \quad (2)$$

$PRI(p_m, p_a)$  is a pressure ratio, which can be described by

$$PRI = \begin{cases} \sqrt{1 - \left( \frac{p_r - p_c}{1 - p_c} \right)^2} & \text{if } p_r > p_c \text{ (choked)} \\ 1 & \text{if } p_r \leq p_c \text{ (sonic)} \end{cases} \quad (3)$$

where  $p_r = p_m / p_a$ ,  $p_c$  is a threshold.

#### A.2 Intake manifold dynamics

The intake manifold dynamics are mainly determined by the

air mass flow rate  $\dot{m}_{ao}$  going into the cylinder, the variation of the manifold pressure  $p_m$  and temperature  $T_m$  corresponding to the crankshaft speed  $n$  and the inlet mass flow  $\dot{m}_{ai}$ . Thus, assuming the manifold is adiabatic [4, 13], we obtain (based on the ideal gas law) the following differential equations

$$\dot{p}_m = \frac{\kappa R}{V_i} (\dot{m}_{ai} T_a - \dot{m}_{ao} T_m + \dot{m}_{EGR} T_{EGR}) \quad (4)$$

$$\dot{T}_m = \frac{RT_m}{p_m V_i} [\dot{m}_{ao} (\kappa - 1) T_m + \dot{m}_{ai} (\kappa T_a - T_m) + \dot{m}_{EGR} (\kappa T_{EGR} - T_m)] \quad (5)$$

where  $\kappa$  is the ratio of heat capacities,  $R$  is the ideal gas constant,  $V_i$  is the volume of the intake manifold,  $\dot{m}_{EGR}$  and  $T_{EGR}$  are the exhaust gas recirculation (EGR) mass flow rate and temperature, respectively.

Moreover, the air mass flow rate leaving the intake manifold and swept into the cylinders is given by

$$\dot{m}_{ao} = \sqrt{\frac{T_m}{T_a}} \frac{V_d \eta_{vol}}{120 RT_m} n p_m \quad (6)$$

where  $V_d$  is the displacement volume swept by the pistons, and  $\eta_{vol}$  is the volumetric efficiency, which is generally a function of engine speed and manifold pressure [4, 13].

#### A.3 Fuelling dynamics

The fuelling dynamics describe the fuel mass that enters the cylinder from the injected fuel mass. For port fuel injection engines, parts of the fuel injected at the port may be deposited on the wall of the intake runner and on the intake valves as fuel puddles, which will be inducted into the cylinder for later combustion. This is usually known as the named ‘wall-wetting’ phenomenon [3][4, 13]. The following Aquino model is used to determine the injected mass flow dynamics consisting of the fuel vapour flow and fuel film flow

$$\dot{m}_f = \chi u_f + \dot{m}_{ff}, \quad \kappa \frac{d\dot{m}_{ff}}{dt} = -\dot{m}_{ff} + (1 - \chi) u_f \quad (7)$$

where  $\dot{m}_f$  is the total fuel mass flow rate injected into the cylinder,  $\dot{m}_{ff}$  is the mass flow rate of the fuel entering the cylinder from the fuel puddles on the manifold wall, and  $\chi$  is the portion of the fuel that delivered into the cylinder directly as vapour,  $\kappa$  is the fuel lag time constant, and  $u_f$  the fuel injection command, which is the control action in this paper.

Moreover, AFR is defined as the ratio of the air mass flow rate  $\dot{m}_{ao}$  into the cylinder to the atomized fuel mass flow rate  $\dot{m}_f$  used for combustion, which can be given by:

$$\lambda = \frac{\dot{m}_{ao}}{\dot{m}_f} \quad (8)$$

Clearly,  $\lambda$  can be regulated at an ideal value by adjusting the injected fuel mass flow rate  $\dot{m}_f$  in terms of the AFR control  $u_f$  to correspond to the air mass flow rate  $\dot{m}_{ao}$ .

#### A.4 Combustion dynamics

The generated torque  $T_{ind}$  produced by the combustion can be approximated by:

$$T_{ind} = \frac{H_u \eta_{th} \dot{m}_f}{n} \quad (9)$$

where  $H_u$  is the fuel calorific value, and  $\eta_{th}$  is the thermal efficiency, which is a function of the crankshaft speed, manifold pressure, spark advance  $\theta$  and air-fuel ratio [4, 13]:

$$\eta_{th}(n, p_m, \theta, \lambda) = \eta_{th}(n) \cdot \eta_{th}(p_m) \cdot \eta_{th}(\theta, n) \cdot \eta_{th}(\lambda, n) \quad (10)$$

Detailed dynamics of  $\eta_{th}(n, p_m, \theta, \lambda)$  will be given later.

#### A.5 Crankshaft dynamics

The crankshaft dynamics denotes the transform of chemical energy into mechanical energy, which is covered by the torque equation:

$$J\dot{n}(t) = T_{ind}(n, \dot{m}_f) - T_{fric}(n) - T_{pump}(p_m) - T_{load}(n) \quad (11)$$

where  $J$  is the combined moment of inertia of the engine, and  $T_{fric}, T_{pump}, T_{load}$  refer to the friction, pumping loss and external load torques applied in the engine, respectively.

#### B. Control problem statement

The objective of AFR control design is to regulate the AFR  $\lambda(t)$  to remain at the desired stoichiometric value  $\lambda_d = 14.67$  by using the fuel injection control  $u_f$ , where both the transient and steady-state response should be satisfied in wider engine operation scenarios and in the presence of modelling uncertainties and external disturbances (e.g. sensor noise).

The inputs to the typical AFR controller generally needs  $\dot{m}_{ai}$ ,  $n$ ,  $p_m$ ,  $T_{fric}, T_{pump}, T_{load}$ , ideal AFR  $\lambda_d = 14.67$  and the UEGO sensor output  $\lambda(t)$ . The output of AFR control is the required amount of fuel injection  $u_f$ . However, a critical issue of these AFR controls (e.g. [3]) lies in that complex time-varying functions (including all engine dynamics (1)-(11)) will appear in the derivative of  $\lambda(t)$ . This will complicate the control design and analysis, and impose the assumption that accurate parameters (e.g. volumetric efficiency  $\eta_{vol}$ , combustion efficiency  $\eta_{th}$ , inertia  $J$  and friction/pumping coefficients) should be known or online estimated [3]. Another issue is that the crankshaft speed dynamics (11) are utilized in the control synthesis, such that the applied load torques  $T_{fric}, T_{pump}, T_{load}$  must be known in [3]. This may be stringent in practice. Moreover, the air mass flow  $\dot{m}_{ao}$  and fuel mass flow  $\dot{m}_f$  into the cylinder are not measurable.

To address the above issue, in what follows we will present a new control framework to regulate the AFR  $\lambda(t)$  around  $\lambda_d(t) = 14.67$  by using limited information (e.g. throttle mass

flow rate  $\dot{m}_{ai}$ , intake manifold pressure  $p_m$ , temperature  $T_m$  and the UEGO sensor output  $\lambda(t)$ ), but with enhanced robustness and transient performance.

*Remark 1:* In commercial engines, the available variables that can be measured via standard sensors/transducers and used for the control design include the throttle position/air mass flow rate, manifold pressure and temperature, crankshaft speed, and UEGO signal (lambda sensor). The principal engine actuators include the fuel injector and electronic spark.

### III. CONTROL DESIGN AND SYNTHESIS

The main idea is to reformulate the regulation of  $\lambda(t)$  as a tracking control of  $\dot{m}_f$  to simplify the control design.

#### A. Reformulation of AFR control problem

The new AFR control framework can be found in Fig. 2. It is

shown that in this case the control reference is  $\dot{m}_{fd} = \frac{\dot{m}_{ao}}{\lambda_d}$ ,

which is the required fuel mass flow rate  $\dot{m}_{fd}$  used to maintain the ideal AFR value  $\lambda_d$ . The feedback signal is the realistic fuel mass flow rate  $\dot{m}_f$ . Consequently, the error used as the control input is the difference between  $\dot{m}_{fd}$  and  $\dot{m}_f$  as

$$e = \dot{m}_{fd} - \dot{m}_f = \frac{\dot{m}_{ao}}{\lambda_d} - \dot{m}_f \quad (12)$$

It can be validated that the realistic ARF  $\lambda(t)$  can be retained at ideal value  $\lambda_d(t) = 14.67$  provided that the injected fuel mass flow  $\dot{m}_f$  can be controlled at the ideal reference  $\dot{m}_{fd}$ . The motivation for using the above mass flow rate error lies in that the derivative of error  $e$  in (12) can be presented based on (6) and (7) in such a form that has a fully known constant control input gain and lumped unknown dynamics, i.e.

$$\dot{e} = \frac{1}{\lambda_d} \frac{d\dot{m}_{ao}}{dt} - \frac{d\dot{m}_f}{dt} = \frac{V_d}{\sqrt{T_a} 120R\lambda_d} \frac{d(\eta_{vol} n p_m / \sqrt{T_m})}{dt} + \frac{1}{\kappa} \dot{m}_f - u_d \quad (13)$$

where  $u_d$  is a total fuelling amount used to determine the realistic fuelling command as  $u_f = \frac{1}{\chi S + 1/\kappa} u_d$  as [3].

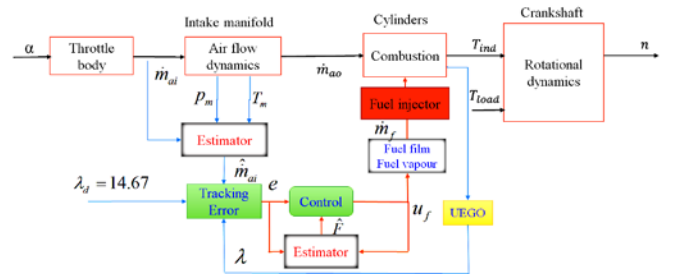


Fig. 2 Schematic of the proposed AFR control system.

It should be noted that the above reformulation is reasonable

and practically feasible because the injected fuel flow rate  $\dot{m}_f$  (though not measurable directly) can be calculated from (8) by using the measured AFR  $\lambda(t)$  and the air flow rate  $\dot{m}_{ao}$ ; the latter one can be estimated (as shown in Section III.C) based on (4) by means of the measured variables  $p_m$ ,  $T_m$  and  $\dot{m}_{ai}$ .

As shown in [3], the calculation of  $d(\eta_{vol}np_m/\sqrt{T_m})/dt$  can be very complex because all variables  $\eta_{vol}, n, p_m, T_m$  may be time-varying, so that the detailed calculation of their derivative will involve all engine dynamics given in (1)-(11). In this sense, all modelling parameters should be known or online estimated. Here, we will present a simple yet robust control method to synthesis  $u_d$  without knowing the detailed engine dynamics and model parameters involved in  $d(\eta_{vol}np_m/\sqrt{T_m})/dt$ . In particular, the crankshaft dynamics (11) will not be used, i.e. the applied loads do not need to be measured. Fortunately, this is possible because in our new formulation (13), the input gain associated with  $u_d$  is a known constant and thus we can use the idea of an unknown input observer that was recently developed by the authors [12]. The lumped unknown dynamics in (13) can be taken as an unknown 'input' and then estimated.

### B. Control design via unknown input observer

To address the unknown dynamics of (13), we will introduce an unknown input observer. For the sake of simplified notation, we rewrite (13) as

$$\dot{e} = F(n, p_m, T_m, \dot{m}_f) - u_d \quad (14)$$

where  $F(n, p_m, T_m, \dot{m}_f) = \frac{V_d}{\sqrt{T_a} 120R\lambda_d} \frac{d(\eta_{vol}np_m/\sqrt{T_m})}{dt} + \frac{1}{\kappa} \dot{m}_f$  defines the lumped unknown dynamics.

*Assumption 1:* The lumped unknown function  $F(n, p_m, T_m, \dot{m}_f)$  is continuous, and its derivative is bounded, i.e.  $\sup_{t \geq 0} |\dot{F}| \leq \hbar$  holds for a constant  $\hbar > 0$ .

Inspired by our recent work [12], the estimator of  $F$  is

$$\hat{F} = \frac{e - e_f}{k} + u_{df} \quad (15)$$

where  $e_f$ ,  $u_{df}$  are the filtered variables of  $e$  and  $u_d$  given by

$$\begin{cases} k\dot{e}_f + e_f = e, & e_f(0) = 0 \\ k\dot{u}_{df} + u_{df} = u_d, & u_{df}(0) = 0 \end{cases} \quad (16)$$

with  $k > 0$  being a design parameter.

As analyzed in [12], we know that the estimation  $\hat{F}$  is indeed the filter version of the unknown function  $F$ , i.e.  $\hat{F} = F_f$ , which is given by  $k\dot{F}_f + F_f = F$ . Thus, the estimation error of (15), i.e.  $e_f = F - \hat{F}$ , can be given as

$$\dot{e}_f = -\frac{1}{k}e_f + \dot{F} \quad (17)$$

*Lemma 1:* For system (14) with input estimator (15), the estimation error  $e_f$  is bounded by  $|e_f(t)| \leq \sqrt{e_f^2(0)e^{-t/k} + k^2\hbar^2}$ , and thus  $\hat{F} \rightarrow F$  holds for  $k \rightarrow 0$  and/or  $\hbar \rightarrow 0$ .

*Proof:* Please refer to [12] for a similar proof.

*Remark 2:* It is shown that the lumped dynamics are taken as a time-varying signal in (14), and then estimated via an input observer (15) without knowing its detailed components and concrete formulations. In this sense, tedious calculation used in [3] can be avoided.

When we obtain the estimation of  $F(n, p_m, T_m, \dot{m}_f)$ , a simple feedback control for (14) can be given as

$$u_d = k_1 e + \hat{F} \quad (18)$$

where  $k_1 > 0$  is a feedback control gain,  $e = \frac{\dot{m}_{ao}}{\lambda_d} - \frac{\dot{m}_{ao}}{\lambda}$  is the control error, and  $\hat{F}$  is the estimation of  $F$  given in (15).

Thus, the main results of this paper can be given as:

*Theorem 1:* For engine fuel injection system shown in Fig.2, the AFR control is given in (18) with estimator (15), then both the estimation error  $e_f$  and control error  $e$  will converge to a small compact set around zero, and the AFR can be regulated around the stoichiometric value.

*Proof:* By substituting the control (18) into (14), we have the closed-loop error equation

$$\dot{e} = F(n, p_m, T_m, \dot{m}_f) - k_1 e - \hat{F} = -k_1 e + e_f \quad (19)$$

Select a Lyapunov function as  $V = \frac{1}{2}e^2 + \frac{1}{2}e_f^2$ , then its time derivative is calculated along (17) and as

$$\begin{aligned} \dot{V} &= e\dot{e} + e_f\dot{e}_f = -k_1 e^2 + ee_f - \frac{1}{k}e_f^2 + e_f\dot{F} \\ &\leq -(k_1 - \frac{\eta}{2})e^2 - (\frac{1}{k} - \frac{1}{\eta})e_f^2 + \frac{\eta}{2}\hbar^2 \leq -aV + \frac{\eta}{2}\hbar^2 \end{aligned} \quad (20)$$

where  $a = \min\{2(k_1 - \frac{\eta}{2}), 2(\frac{1}{k} - \frac{1}{\eta})\}$  is a positive constant for

appropriately selected control parameters  $k_1 > \frac{\eta}{2} > \frac{k}{2}, \eta > 0$ ,

and  $\hbar$  is the upper bound of  $|\dot{F}|$ . This implies that

$V(t) \leq e^{-t/a}V(0) + \eta^2\hbar^2/2$  holds and thus the control error and estimation error will exponentially converge to a compact set

defined by  $\Omega := \{e, e_f \mid |e| \leq \eta\hbar, |e_f| \leq \eta\hbar\}$ . Clearly, we know that

$\lim_{t \rightarrow \infty} e(t) = 0, \lim_{t \rightarrow \infty} e_f(t) = 0$  holds for  $\eta \rightarrow 0$  and/or  $\hbar \rightarrow 0$ ,

(i.e.  $k$  is sufficiently small and/or  $F$  is constant so that  $\hbar = 0$ ).

Consequently, we know that the AFR  $\lambda$  can be regulated around the command value  $\lambda_d$ .  $\diamond$

### C. Estimation of air mass flow rate into cylinder

In the above control, the injected air mass flow rate into the cylinder  $\dot{m}_{ao}$  is used. However, the online measurement of this variable via transducers may be difficult due to the limited hardware configuration and increased costs. Thus, we will present an estimation to online derive this variable. To this end, we recall the intake manifold dynamics (4), which indicates that the injected mass flow rate  $\dot{m}_{ao}$  can be taken as an unknown input in (4), while the manifold pressure  $p_m$  and temperature  $T_m$  are measurable. Thus, the idea of an unknown input observer [12] can be further extended. We can design the estimator of  $\dot{m}_{ao}$  as

$$\hat{\dot{m}}_{ao} = \frac{V_i}{\kappa R T_m} \left( M_{1f} + M_{2f} - \frac{p_m - p_{mf}}{k} \right) \quad (21)$$

where  $M_{1f}$ ,  $M_{2f}$  and  $p_{mf}$  are the filtered variables of  $M_1 = \dot{m}_{ai} T_a$ ,  $M_2 = \dot{m}_{EGR} T_{EGR}$  and  $p_m$  given by

$$\begin{cases} k\dot{M}_{1f} + M_{1f} = M_1, & M_{1f}(0) = 0 \\ k\dot{M}_{2f} + M_{2f} = M_2, & M_{2f}(0) = 0 \\ k\dot{p}_{mf} + p_{mf} = p_m, & p_{mf}(0) = 0 \end{cases} \quad (22)$$

with  $k > 0$  being the filter parameter.

**Lemma 2:** For the injected air mass flow rate  $\dot{m}_{ao}$  estimation in (21), then the estimation error  $e_m = \dot{m}_{ao} - \hat{\dot{m}}_{ao}$  is bounded by  $|e_m(t)| \leq \sqrt{e_m^2(0)e^{-t/\lambda} + k^2 \tilde{\lambda}^2}$ , where  $\tilde{\lambda}$  is the upper bound of  $d\dot{m}_{ao}/dt$ , i.e.  $\sup_{t \geq 0} |d\dot{m}_{ao}/dt| \leq \tilde{\lambda}$ , and thus  $\hat{\dot{m}}_{ao} \rightarrow \dot{m}_{ao}$  holds for  $k \rightarrow 0$  and/or  $\tilde{\lambda} \rightarrow 0$ .

*Proof:* Please refer to [12] for a similar proof.

**Remark 3:** One may find that  $\dot{m}_{ao}$  can also be determined based on (6) by using the intake manifold temperature  $T_m$ , pressure  $p_m$  and crankshaft speed  $n$ . This is difficult in practice because this calculation depends on the accurate volumetric efficiency  $\eta_{vol}$ , which can be a complex *unknown* time-varying function of engine speed and manifold pressure.

## IV. SIMULATIONS

In this section, we will present simulation results based on an engine model created in Matlab/Simulink, which was built by further modifying and calibrating the MVEM model [4, 13]. Moreover, we construct new AFR control loops in this model.

The engine model parameters are appropriately determined based on experimental data sets, and the thermal efficiency coefficients are also set to represent realistic engine dynamics:

$$\begin{aligned} \eta_{th}(n) &= \eta_0 (1 - \eta_1 n^{-b}), \eta_{th}(p_m) = p_0 + p_1 p_m + p_2 p_m^2 \\ \eta_{th}(\theta, n) &= \Theta_0 + \Theta_1 (\theta - [\Theta_2 n + \Theta_3])^2 \\ \eta_{th}(\lambda, n) &= \Lambda_0 + \Lambda_1 \lambda + \Lambda_2 \lambda^2 + \Lambda_3 n \end{aligned} \quad (23)$$

where  $\eta_0$  is the efficiency of an Otto engine,  $\eta_1$  and  $b$  are

constants, and  $p_i$ ,  $\Theta_i$  and  $\Lambda_i$  are constants.

Moreover, the frictional torque loss of engine crankshaft and the pumping torque are the function of the engine speed and the manifold pressure, which are given as

$$T_{fric} = a_0 + a_1 n + a_2 n^2, \quad T_{pump} = b_0 n p_m + b_1 p_m \quad (24)$$

where  $b_0, b_1$  and  $a_0, a_1, a_2$  are all positive constants [4, 13].

To show both the transient and steady-state performance of the proposed AFR controls, the engine is operated with fair dynamic acceleration and deceleration by adjusting the throttle opening angle. The corresponding crankshaft speed, manifold temperature and pressure are shown in the following Fig.3.

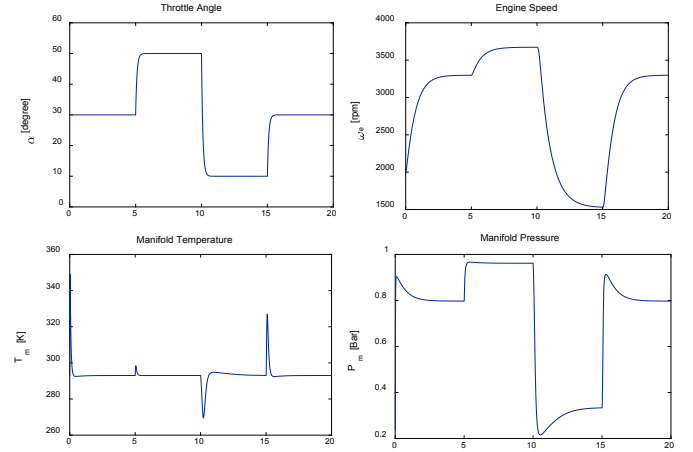
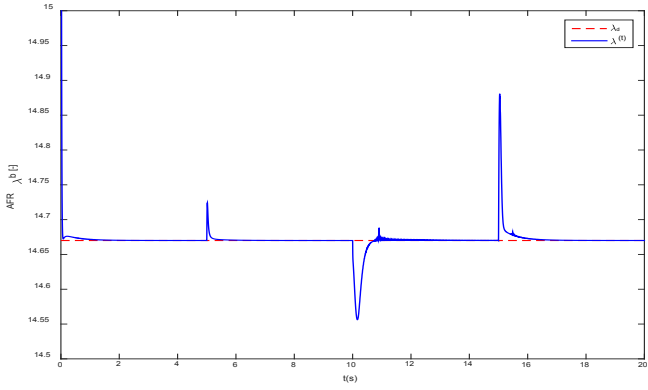


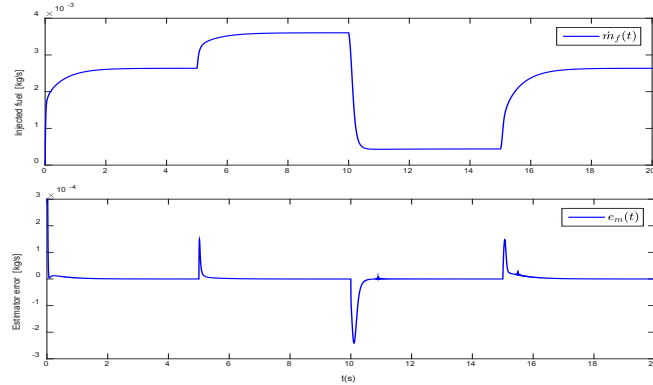
Fig.3 Operation scenario of engine simulator.

Then the proposed control with the estimator (21) will be compared with a well tuned PID control via trial-and-error method. Simulation results are provided in Fig.4 and Fig.5, where the AFR control responses are given in Fig.4 (a) and Fig.5(a), and the injected fuel mass flow rates  $\dot{m}_f$  and the estimation errors  $e_m$  with (21) are shown in Fig.4 (b) and Fig.5(b), respectively. One can find from Fig.4 that the proposed control can regulate the AFR around  $\lambda_d = 14.67$ , where both the steady-state and transient responses are satisfactory. In particular, this control scheme is able to recover the AFR to its command when the crankshaft speed (and thus the manifold pressure and injected mass flow rates) is changed via the throttle opening. This result confirms the validity of the proposed estimator (14) and control (18). The transient errors in the time instants (e.g.  $t=5, 10, 15$  sec) when the engine operation conditions are changed are due to the transient estimation error of (21) for the air mass flow rate  $\dot{m}_{ao}$  injected into the cylinder (second subplot of Fig.4 (b)).

However, as one may find in Fig.5(a), the response of PID control is more sluggish than the proposed control, and the overshoots around  $t=5, 10, 15$  sec are larger than that shown in Fig.4(a). In particular, there are oscillations in the steady-state response of specific operation conditions (e.g. 0-5sec and 17-20sec) because the fixed PID parameters are not able to cover all engine working scenarios with sufficient robustness. This also leads to a worse estimation response for  $\dot{m}_{ao}$  (Fig.7(b)).

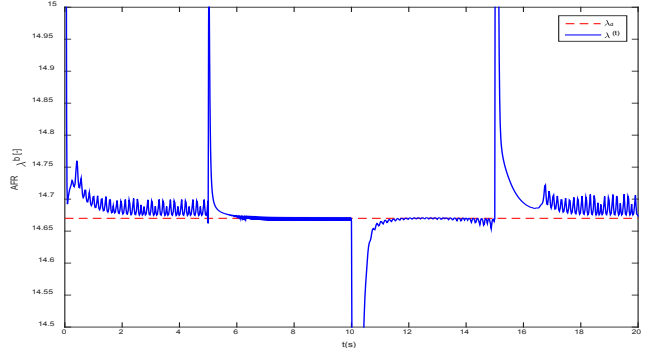


(a) AFR response with the proposed control

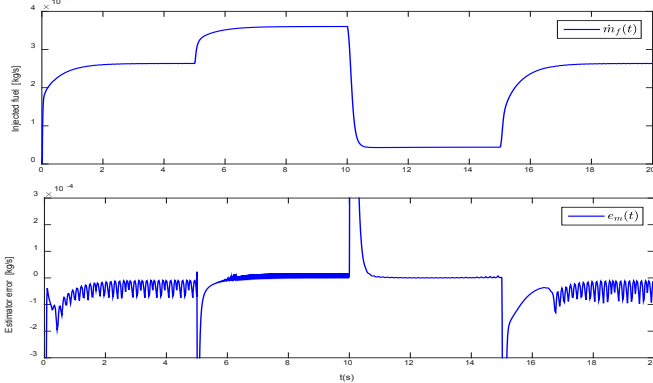


(b) Injected fuel mass flow rate  $\dot{m}_f$  and estimation error  $e_m$ .

Fig.4 Simulation results with the proposed control.



(a) AFR response with PID control



(b) Injected fuel mass flow rate  $\dot{m}_f$  and estimation error  $e_m$ .

Fig.5 Simulation results with PID control.

## V. CONCLUSIONS AND FUTURE DIRECTIONS

This paper is concerned with the AFR control for spark ignition engine systems with unknown dynamics. The idea is to reformulate the regulation of AFR as a tracking control problem, and then to incorporate unknown input observers into the control design. This can effectively compensate for the effects of the lumped dynamics and modelling uncertainties. Moreover, the proposed control can be implemented by using a few measured signals via standard sensors (e.g. air mass flow rate through throttle, manifold pressure and temperature, measured AFR at exhaust location). Numerical simulations based on a mean-value engine model are given to validate the efficacy of the suggested control and to show its capability to recover AFR value under varying operation conditions. Future work will focus on the control design by considering injection and measurement delays induced in the engine dynamics.

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