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Pitas, I., Papachristou, K., & Tefas, A. (2015). Exploiting symmetry in two-dimensional clustering-based discriminant analysis for face recognition. In 2015 23rd European Signal Processing Conference (EUSIPCO). (pp. 155-159). (Proceedings of the European Signal Processing Conference (EUSIPCO)). Institute of Electrical and Electronics Engineers (IEEE). DOI: 10.1109/EUSIPCO.2015.7362364

Peer reviewed version

Link to published version (if available):
10.1109/EUSIPCO.2015.7362364

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EXPLOITING SYMMETRY IN TWO-DIMENSIONAL CLUSTERING-BASED DISCRIMINANT ANALYSIS FOR FACE RECOGNITION

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ABSTRACT

Subspace learning techniques are among the most popular methods for face recognition. In this paper, we propose a novel face recognition technique for two dimensional subspace learning which is able to exploit the symmetry nature of human faces. We extent the Two Dimensional Clustering based Discriminant Analysis (2DCDA) by incorporating an appropriate symmetry regularizer into its objective function in order to determine symmetric projection vectors. The proposed Symmetric Two Dimensional Clustering based Discriminant Analysis technique has been applied to the face recognition problem. Experimental results showed that the proposed technique achieves better classification performance in comparison to the standard one.

***Index Terms*—** face recognition, subspace learning, symmetry regularizer, two-dimensional clustering-based discriminant analysis

1. INTRODUCTION

Face recognition is a very active topic in computer vision research [1] with applications in many fields such as information security and surveillance systems. A number of face recognition techniques employ subspace learning techniques which determine lower dimensional spaces leading to faster methods with improved classification performance [2–4]. Principal Component Analysis (PCA) [5] is a classical technique widely used on face recognition, which tries to find a subspace with the maximum data variance. Another well-known technique for face recognition is Linear Discriminant Analysis (LDA) [6], which determines a subspace where the projected data classes are optimally separated. When the classes consist of multiple clusters, the so-called Clustering-based Discriminant Analysis (CDA) [7] achieves to find the optimal subspace.

Two-dimensional versions of PCA [8], LDA [9–11] and CDA [12] have been proposed for facial image analysis. These techniques use two-dimensional data (images) instead of one-dimensional data (vectors) as input, thus avoiding vectorizing that often leads to a high-dimensional vector space, where the determination of the corresponding projection vectors is very time-consuming.

Although the above techniques perform directly on image matrices, they ignore the a-priori knowledge that facial images are (in general) symmetric. As has been shown in [13,14], enhanced facial image classification performance can be achieved by exploiting the symmetry nature of the human face. In this paper, we extend the Two-Dimensional Clustering-based Discriminant Analysis modifying its objective function by embedding an appropriate symmetry constraint. With this extension we expect that the determined projection vectors to be symmetric leading to subspaces equipped with more robustness and generalization ability.

The rest of this paper is organized as follows: In Section 2, the standard Two-Dimensional Clustering-based Discriminant Analysis (2DCDA) technique is briefly reviewed. Section 3 presents the proposed technique which incorporates symmetry constraints in 2DCDA. In Section 4, the conducted experiments are described that highlight the efficiency of the proposed technique on the face recognition problem. Finally, concluding remarks are drawn in Section 5.

2. TWO-DIMENSIONAL CLUSTERING-BASED DISCRIMINANT ANALYSIS

Let $\mathcal{X} = \{\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_N\}$ denote the image set containing N sample images $\mathbf{X}_i \in \mathcal{R}^{m\times n}$. Two-Dimensional Clustering-based Discriminant Analysis (2DCDA) [12] tries to find projection vectors $\mathbf{w}_i \in \mathcal{R}^{n\times 1}$ along which the clusters of projected data $\mathbf{y}_i = \mathbf{X}_i\mathbf{w}_i$, where $\mathbf{y}_i \in \mathcal{R}^{m\times 1}$, are well discriminated. Assuming that there c classes, d_i is the number of clusters in class i, $\boldsymbol{X}_k^{ij}$ denotes the kth image of the jth cluster in class i and, $\boldsymbol{M}_j^i$, n_{ij} are the mean vector and the number of images in the ith cluster of class i, respectively, the between-cluster scatter matrix:

$$\mathbf{S}_B = \sum_{i=1}^{c-1}\sum_{l=i+1}^{c}\sum_{j=1}^{d_i}\sum_{h=1}^{d_l}\left(\boldsymbol{M}_j^i - \boldsymbol{M}_h^l\right)^T\left(\boldsymbol{M}_j^i - \boldsymbol{M}_h^l\right), \quad (1)$$

The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement number 316564 (IMPART). This publication reflects only the authors views. The European Union is not liable for any use that may be made of the information contained therein.

and the within-cluster scatter matrix:

$$\mathbf{S}_W = \sum_{i=1}^{c} \sum_{j=1}^{d_i} \sum_{k=1}^{n_{ij}} \left(\boldsymbol{X}_k^{ij} - \boldsymbol{M}_j^i \right)^T \left(\boldsymbol{X}_k^{ij} - \boldsymbol{M}_j^i \right), \quad (2)$$

are defined.

The objective of 2DCDA is to find the transformation matrix $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ...\mathbf{w}_d]$ that maximizes the ratio of the trace of the between-cluster scatter to the trace of the within-cluster scatter matrix:

$$J(\mathbf{W}) = \arg\max_{\mathbf{W}} \frac{tr[\mathbf{W}^T \mathbf{S}_B \mathbf{W}]}{tr[\mathbf{W}^T \mathbf{S}_W \mathbf{W}]}. \quad (3)$$

subject to the orthogonal constraints $\mathbf{w}_i^T \mathbf{w}_j, \ i \neq j, \ i,j = 1, \ldots, d$.

The solution of (3) is approximated [15,16] by the following generalized eigenvalue decomposition problem:

$$\mathbf{S}_B \cdot \mathbf{w} = \lambda \cdot \mathbf{S}_W \cdot \mathbf{w}, \quad (4)$$

by keeping the first d eigenvectors. For any image $\mathbf{X}_i$, d projected vectors $\mathbf{y}_i = \mathbf{X}_i \mathbf{w}_i, \ i = 1, \ldots, d$ are obtained forming an $m \times d$ matrix $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_d]$.

3. SYMMETRIC TWO-DIMENSIONAL CLUSTERING-BASED DISCRIMINANT ANALYSIS

Human faces are typical and common examples of symmetric objects. However, in the most cases, the sample images are not strictly symmetric, resulting in overtraining and poor generalization capability. In this section, we modify the 2DCDA technique by imposing a symmetry constraint [14] into its objective function for the determination of projection vectors that are symmetric, so that the samples are projected onto symmetric discriminant subspaces in order to achieve a higher generalization capability.

A way to measure the symmetry error of a vector $\mathbf{w} = [w_1, w_2, \ldots, w_{n-1}, w_n]^T$ is given by the following equation:

$$s_{\mathbf{w}} = \sum_{i=1}^{n/2} (w_i - w_{n+1-i})^2 \quad (5)$$

It is straightforward to prove that:

$$s_{\mathbf{w}} \;=\; \mathbf{w}^T \mathbf{A} \mathbf{A}^T \mathbf{w} = \sum_{i=1}^{n/2} (w_i - w_{n+1-i})^2, \quad (6)$$

where the $n \times n$ matrix $\mathbf{A}$ is:

$$\mathbf{A} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \ldots & 0 & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \ldots & -\frac{1}{\sqrt{2}} & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -\frac{1}{\sqrt{2}} & \ldots & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \ldots & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}. \quad (7)$$

The goal of the proposed 2DCDA is to impose this symmetry constraint into the objective functions of 2DCDA by minimizing the quantity $tr[\mathbf{W}^T \mathbf{A} \mathbf{A}^T \mathbf{W}]$. Specifically, we want to maximize the trace of the quantity $\mathbf{W}^T \mathbf{S}_B \mathbf{W}$, so that the distance of the projected samples belonging to different clusters will be maximized, while minimizing the trace of the $\mathbf{W}^T \mathbf{S}_W \mathbf{W}$ and $\mathbf{W}^T \mathbf{A} \mathbf{A}^T \mathbf{W}$, where $\mathbf{W}$ contains the projection vectors $\mathbf{w}_i$. Consequently, we try to find the projection matrix $\mathbf{W}$ that maximizes the matrix trace ratio of the between-cluster scatter matrix to the within-cluster and symmetry scatter, thus obtaining the following objective function:

$$J(\mathbf{W}) = \arg \max_{\mathbf{W}^T\mathbf{W}=\mathbf{I}} \frac{tr[\mathbf{W}^T \mathbf{S}_B \mathbf{W}]}{(1-s)\, tr[\mathbf{W}^T \mathbf{S}_W \mathbf{W}] + s\, tr[\mathbf{W}^T \mathbf{A} \mathbf{A}^T \mathbf{W}]}. \quad (8)$$

subject to the orthogonal constraints $\mathbf{w}_i^T \mathbf{w}_j, \ i \neq j, \ i,j = 1, \ldots, d$. Here $s \in [0,1]$ is the symmetry factor that controls the symmetry of $\mathbf{w}$. Obviously, for $s = 0$ the proposed technique corresponds to 2DCDA, while as s is increasing to 1, the level of symmetry of the projection vectors is maximized.

The solution of (8) is given by the solution of following generalized eigenvalue decomposition problem:

$$\mathbf{S}_B \cdot \mathbf{w} = \lambda \cdot \left((1-s) \mathbf{S}_W + s \mathbf{A} \mathbf{A}^T \right) \cdot \mathbf{w}, \quad (9)$$

by keeping the d eigenvectors corresponding to the d largest eigenvalues.

4. EXPERIMENTS

In this section, we present experiments conducted in order to evaluate the performance of the proposed symmetric 2DCDA technique. We have employed four publicly available face recognition databases, namely ORL, AR, Extended YALE-B and LFW databases. In the following subsections, we describe the databases and experimental results.

4.1. Databases

4.1.1. The ORL database

The ORL database [17] contains 400 images of 40 distinct persons (10 images each). The images were captured at different times and with different variations including lighting conditions, facial expressions (smiling/not smiling) and facial details (open/closed eyes, with/without glasses). Also, the images were taken in frontal position with a tolerance for some tilting and rotation of the face of up to 20 degrees. Some example facial images from the ORL database are displayed in Figure 1.

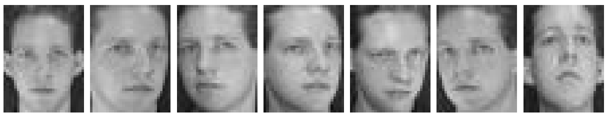

Fig. 1. Sample images from the ORL database.

4.1.2. The AR database

The AR database [18] contains over 4000 color images corresponding to 70 men's and 56 women's faces. The images were taken in frontal position with different facial expressions (anger, smiling and screaming), illumination conditions (left and/or right light on), and occlusions (sun glasses and scarf). Each person participated in two recording sessions, separated by two weeks (14 days) time. Some example facial images from the AR database are displayed in Figure 2.

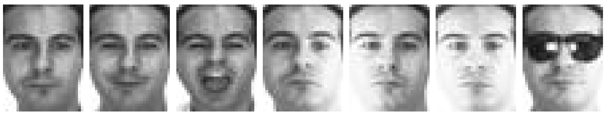

Fig. 2. Sample images from the AR database.

4.1.3. The Extended YALE-B database

The Extended YALE-B database [19] contains images of 38 persons in 9 poses and under 64 illumination conditions. Some example facial images from the Extended YALE-B database are displayed in Figure 3.

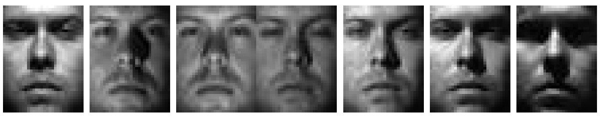

Fig. 3. Sample images from the Extended YALE-B database.

4.1.4. The LFW database

LFW [20] is an image database for unconstrained face verification. It contains more than 13,000 facial images collected from the web with large variations in pose, age, expression, illumination, etc. In our experiments, a subset with cropped images [21] was used corresponding to persons with 50 or more sample images. Some example facial images from the LFW database are displayed in Figure 4.

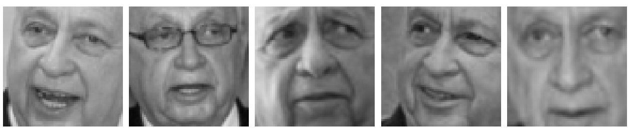

Fig. 4. Sample images from the LFW database.

4.2. Experimental results

In our experiments, each database was divided into 5 non-overlapping subsets. Each experiment includes five training-test procedures (folds). In each fold, the standard and proposed 2DCDA were trained by using 4 subsets and testing was performed on the remaining subset. The proposed 2DCDA was used for $s = 0.0, 0.1, ..., 0.9999$. For clustering, the training set was divided into a number of clusters, by applying the k-means and fuzzy c-means clustering technique. For ease of representation, we will follow the notation 2DCDA(a,n), where a denotes the clustering technique (km for k-means and fcm for fuzzy c-means) and n is the number of clusters. The projected samples were classified using the Nearest Centroid (NC) and k-Nearest Neighbor (kNN) classifiers. kNN was used for $k = 1, 3, 5$. In the following, the notation kNN(n) is adopted, where n is the number of nearest neighbors, in the case of kNN. Recognition accuracy was measured by using the mean classification rate over all five folds.

Table 1. Best recognition accuracies (mean$\pm$std %) and symmetry error of projection vectors of standard 2DCDA versus symmetric 2DCDA for the ORL database.

technique		Standard	Symmetric
2DCDA (km,2)	kNN(1)	97.75 ± 2.24	97.75 ± 2.24
	kNN(3)	96.25 ± 1.77	96.25 ± 1.77
	kNN(5)	92.75 ± 2.05	**93.50** ± 2.05
	NC	97.25 ± 1.05	**97.50** ± 0.88
	symm. error	0.973103	0.070826
2DCDA (fcm,2)	kNN(1)	97.75 ± 2.24	**98.25** ± 1.68
	kNN(3)	95.50 ± 1.90	**95.75** ± 1.43
	kNN(5)	93.00 ± 2.27	**94.25** ± 1.68
	NC	97.00 ± 1.90	**97.75** ± 1.63
	symm. error	0.971304	0.071783
2DCDA (km,3)	kNN(1)	98.00 ± 1.90	98.00 ± 1.90
	kNN(3)	95.50 ± 1.43	**96.00** ± 1.63
	kNN(5)	92.50 ± 1.98	**94.50** ± 0.68
	NC	97.50 ± 1.53	**98.25** ± 1.43
	symm. error	0.985303	0.074807
2DCDA (fcm,3)	kNN(1)	97.75 ± 1.63	**98.00** ± 1.90
	kNN(3)	95.00 ± 1.53	**96.00** ± 1.63
	kNN(5)	93.00 ± 1.90	**94.50** ± 1.43
	NC	97.50 ± 1.25	**98.25** ± 1.43
	symm. error	0.966857	0.073988

The results obtained for the ORL, AR, Extended YALE-B and LFW databases, are illustrated in Tables 1, 2, 3 and 4, respectively. For each database, the first four rows illustrate the recognition accuracies obtained by applying the standard and proposed 2DCDA technique and the kNN(1), kNN(3), kNN(5) and NC classifiers respectively, while the average symmetry error of the projection vectors is given in the fifth one. The best results are shown in bold.

As can be seen, the proposed symmetric extension of 2DCDA outperforms the standard one in the most cases. Indeed, an improvement in recognition accuracy is achieved when using symmetry constraint. Therefore, we can conclude

Table 2. Best recognition accuracies (mean±std %) and symmetry error of projection vectors of standard 2DCDA versus symmetric 2DCDA for the AR database.

technique		Standard	Symmetric
2DCDA (km,2)	kNN(1)	84.50 ± 3.19	**86.63** ± 2.27
	kNN(3)	73.96 ± 3.00	**76.43** ± 3.26
	kNN(5)	72.27 ± 2.15	**74.89** ± 2.33
	NC	63.83 ± 2.46	**64.43** ± 2.01
	symm. error	0.836865	0.070287
2DCDA (fcm,2)	kNN(1)	76.93 ± 5.46	**83.79** ± 4.41
	kNN(3)	63.81 ± 4.05	**73.01** ± 4.91
	kNN(5)	63.37 ± 4.10	**71.66** ± 1.49
	NC	59.68 ± 2.66	**61.31** ± 1.47
	symm. error	0.870080	0.072018
2DCDA (km,3)	kNN(1)	84.01 ± 3.01	**84.04** ± 2.95
	kNN(3)	71.45 ± 4.15	**71.61** ± 4.22
	kNN(5)	69.45 ± 3.59	69.45 ± 3.59
	NC	63.04 ± 3.23	**63.17** ± 3.29
	symm. error	0.798246	0.064655
2DCDA (fcm,3)	kNN(1)	83.89 ± 2.37	**84.34** ± 2.65
	kNN(3)	71.43 ± 3.15	**71.69** ± 3.21
	kNN(5)	68.75 ± 1.48	**68.99** ± 1.40
	NC	65.94 ± 1.71	**66.18** ± 1.85
	symm. error	0.838159	0.069810

Table 3. Best recognition accuracies (mean±std %) and symmetry error of projection vectors of standard 2DCDA versus symmetric 2DCDA for the Extended YALE-B database.

technique		Standard	Symmetric
2DCDA (km,2)	kNN(1)	88.59 ± 2.30	88.59 ± 2.30
	kNN(3)	87.05 ± 2.40	**87.46** ± 2.59
	kNN(5)	85.79 ± 1.98	**86.52** ± 1.65
	NC	66.01 ± 3.45	**66.10** ± 3.39
	symm. error	1.034140	0.063857
2DCDA (fcm,2)	kNN(1)	88.40 ± 2.61	**89.08** ± 2.35
	kNN(3)	86.84 ± 2.16	**87.64** ± 1.49
	kNN(5)	85.82 ± 1.78	**86.52** ± 1.94
	NC	66.88 ± 1.26	**67.38** ± 1.61
	symm. error	1.029667	0.062882
2DCDA (km,3)	kNN(1)	83.42 ± 2.02	**83.45** ± 2.00
	kNN(3)	79.44 ± 1.71	79.44 ± 1.71
	kNN(5)	77.48 ± 2.42	77.48 ± 2.42
	NC	46.80 ± 3.54	**46.93** ± 3.61
	symm. error	1.037175	0.064386
2DCDA (fcm,3)	kNN(1)	83.78 ± 2.04	83.78 ± 2.04
	kNN(3)	**79.31** ± 2.24	79.27 ± 2.22
	kNN(5)	77.68 ± 1.95	77.68 ± 1.95
	NC	**51.51** ± 2.97	51.48 ± 3.04
	symm. error	1.036090	0.063530

Table 4. Best recognition accuracies (mean±std %) and symmetry error of projection vectors of standard 2DCDA versus symmetric 2DCDA for the LFW database.

technique		Standard	Symmetric
2DCDA (km,2)	kNN(1)	45.98 ± 4.23	**49.15** ± 3.31
	kNN(3)	48.29 ± 2.16	**49.57** ± 1.71
	kNN(5)	48.05 ± 2.31	**49.63** ± 1.02
	NC	32.13 ± 2.51	**35.12** ± 2.46
	symm. error	0.954179	0.010110
2DCDA (fcm,2)	kNN(1)	47.07 ± 3.76	**49.21** ± 3.51
	kNN(3)	48.84 ± 2.94	**50.06** ± 2.34
	kNN(5)	48.96 ± 3.09	**50.37** ± 2.62
	NC	33.05 ± 1.23	**35.79** ± 0.63
	symm. error	0.984260	0.092280
2DCDA (km,3)	kNN(1)	42.07 ± 2.38	**48.11** ± 2.79
	kNN(3)	43.54 ± 1.65	**48.41** ± 3.26
	kNN(5)	44.82 ± 1.68	**49.09** ± 2.70
	NC	25.06 ± 2.55	**39.21** ± 2.35
	symm. error	1.036913	0.058583
2DCDA (fcm,3)	kNN(1)	47.07 ± 2.25	**47.80** ± 3.46
	kNN(3)	48.11 ± 2.85	**50.00** ± 1.87
	kNN(5)	48.17 ± 2.31	**48.90** ± 3.16
	NC	21.59 ± 15.60	**23.23** ± 17.04
	symm. error	1.025778	0.037836

that under various lighting conditions, expressions (happiness, sadness, surprise, etc.), facial details (open or closed eyes) and unconstrained conditions, the proposed technique achieves better generalization exploiting data symmetry. Finally, the symmetry error of the generated projection vectors decreased in the proposed 2DCDA technique, as expected.

5. CONCLUSION

In this paper a novel subspace learning technique was introduced extending the Two-Dimensional Clustering-based Discriminant Analysis (2DCDA) technique with the introduction of a symmetry constraint into its objective function. Experimental results on face recognition databases verified the superiority of the proposed technique.

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