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# How resilient is your organisation? From local failures to systemic risk

Christos Ellinas[abc], Neil Allan[c], Neil Cantle[d]

**Abstract**

Empirical evidence of reported losses suggests that insurance firms are interconnected in a non-trivial way. As a result, systemic risk is a real possibility, where the failure of a single firm can have a disproportionate effect to the market by affecting firms connected to it. Systemic risk can be viewed as the result of a cascading process, as it unravels throughout a network structure. In response, this work presents a simple analytical model that can simulate this process. The model is subsequently tested upon an empirical dataset via the means of numerical simulations. Consequently, the systemic role of individual firms, both in terms of triggering a cascade or being affect by one, is established based on two novel indices; the Criticality IDX and Sensitivity IDX respectively. This article makes three main contributions. First, it provides a novel methodology for quantitatively and objectively assessing the systemic role of individual firms within the insurance domain. Second, it exemplifies the inability of traditional, firm-based information in serving as proxies for mapping these systemic effects. Thirdly, it provides a practical example where network-based information (e.g. Criticality IDX, Sensitivity IDX) can outperform firm-based information (e.g. Admissible Assets, Excess Capital) resulting to an increased efficiency in the decision making process. These findings strengthen the need to account for the interconnected nature of the domain while showcasing some of the potential benefits that can be harvested by doing so.

*Keywords*: systemic risk, cascading process, complex networks, numerical modelling, decision making

## 1. Introduction

Systemic risk can be defined as the risk of having interdependent failures, as a result of a *cascading process* [1]. Such processes have been identified as the source of failure cascades (also known as "chain reactions", "avalanches" or "domino effect") in a wide variety of fields, ranging from epidemics to social contagion and inter-bank flows (see [2-5] and references within). In these examples, the failure of a single component (e.g. a social agent, a financial institution etc.) is capable of inducing a disproportionately large (in fact, theoretically infinite [6]) damage to the overall system. Clearly, these effects are driven by complex, non-linear relationships between these components, reflecting the intricate structures that underlie these systems. Such structure has been shown to regularly balance between order and chaos [7]. Complex network theory, a descendant of graph theory[8], has recently emerged as a unifying approach in exploring the resulting dynamical processes (such as a cascading process) that underlie the behaviour of these systems [3, 7, 9]. Under this view, a large interactive system is abstracted as a network (or graph), where single components are abstracted as nodes, and their varying interactions captured through links – for an extensive introduction, see [10]; for comprehensive technical reviews see [11-13].

[a] ce12183@bristol.ac.uk (corresponding author)
[b] Systems IDC, University of Bristol, UK
[c] Systemic Consult Ltd, UK
[d] Milliman Inc., UK

The relevance of this approach in the general area of finance has recently been voiced by both academics and policy makers [14-16], as evidence of increased interrelatedness within the finance sector have recently emerged [17]. Yet, the practical application of such concepts is still scarce [18]. This works contributes in minimising this gap by presenting a simple framework in which the damaging aspects of interconnectivity can be assessed. Clearly, for this work to be of relevance to the insurance domain, the need for such approaches must first be established. To do so, empirical evidence of loss events, as recorded within the industry, will be used. From a network's perspective, every loss event can be interpreted as a materialised *failure cascade*, where the number of involved firms corresponds to the *cascade's size.* Hence, if interconnectivity is irrelevant, one would expect an exponential drop in the probability of encountering more than a few[e] firms being affected by any such loss.

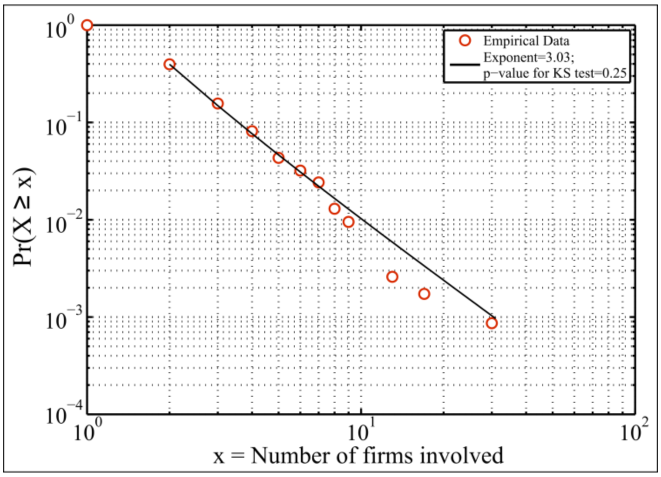


Figure 1: The cumulative probability distribution plot of number of agents involved in a single loss event, as recorder within the ORIC database, over a space of 6 years. Empirical data (circles) can be closely matched by a straight fit line (dotted), suggestive of a power law distribution. In the context of this work, single loss event are considered to be materialised cascades, where the number of firms involved represents the size of the cascade.

Evidently, this is not the case; Figure 1 presents the cumulative probability distribution of the number of firms involved in losses, as recorded by ORIC over a period of approximately 6 years[f]. Although the average number of involved firms (*or* cascade size) is merely 1.78, a divergence of up to 18 standard deviations is noted – clear evidence of a heavy-tail. In fact, the noted straight line on the log-log plot suggests[g] a power law distribution [19], a signature feature of a wide range of complex systems [10, 19, 20]. Evidence of this sort make a strong case for the need to account for the emergence of systemic risk as surprisingly large failure cascades do happen. As such, the main research question (RQ) can now be posed:

RQ: *Can we quantitatively asses the capacity of a local firm in triggering a failure cascade?*

Naturally, the capacity of traditional, firm-based information in capturing these effects will also be assessed – such information includes the Admissible Assets ($AA$) and Excess Capital ($EC$) for each firm. For assessing them, an empirical dataset of 90 life-insurance firms will be used. Due to the

[e] Strictly speaking, this number should in fact be limited to one unique firm per loss event i.e. a δ distribution
[f] Data captured from the ORIC loss database, capturing losses within the insurance industry, between 12/01/2009 and up to 05/02/2015. A total of 1155 loss events are considered.
[g] Further analysis of the tail suggest that a power law with an exponent of 3 is the best fit, yet results are non-conclusive (goodness of fit using the KS test results to a p-value of 0.25; substantially larger than 0.05 yet still far from a perfect score of 1).

confidential nature of the industry, lack of data around the nature of the interconnections between the firms has led to an indirect way of constructing the underlying network (Figure 2).

The contribution of this work is three-fold: Firstly, it provides a novel approach in assessing the systemic role of individual firm it arises from their interconnectivity. Though approaches of this sort have been introduced in various domains [21-26], they are largely underrepresented within the insurance domain. Secondly, as the impact of interconnectivity increases, traditional firm-based information is shown to be inadequate in serving as proxies for assessing the systemic role of each firm. In response, two novel indices are introduced; the Criticality IDX and the Sensitivity IDX. The former captures the capacity of each firm in inducing failure cascades; the latter capturing the probability of being affected by one such cascade. Thirdly, a comparative analysis in the performance of firm-based and network-based information is undertaken in the context of a firm acquisition example. As a result, the latter are shown to outperform the former, leading to more efficient decision making.

The remaining paper is structured as follows: Section 2 provides the methodological background of this work; Section 3 presents the results around the resulting failure cascades, as triggered by local ones, along with an assessment on the capacity of firm-based information in serving as proxies for them. Section 4 discusses on theoretical implications of the results, before expanding on the practical implications of this work through an example. Finally, Section 5 will provide concluding comments and a perspective on aspects worth further development.

# 2. Methodology

## 2.1 Network Model

A network can be defined as $G = \{\{N\}\{E\}\}$, where every firm $i$ is abstracted as node $i$, where $i \in N$, and a link between node $i$ and $j$ is denoted as $e_{i,j}$, where $e_{i,j} \in E$.

In describing the attributes of every node $i$, an empirical dataset of 90 firms[h] actively competing in the life insurance market of the UK will be used. Specifically, information regarding the grand total of admissible assets ($AA$) and excess of capital resource to cover LT business ($EC$), as recorded in 2013 – see Table 1 for a representative sample. Note that firms have been anonymised.

Table 1: A sample of the empirical information used

| **Node ID** | **Firm** | **EC (GBP, thousands)** | **AA (GBP, thousands)** |
|---|---|---|---|
| 1 | $\text{Firm}_1$ | 13733 | 2523863 |
| 2 | $\text{Firm}_2$ | 14035 | 47499 |
| 3 | $\text{Firm}_3$ | 435816 | 53504847 |
| | | ⋮ | |
| 88 | $\text{Firm}_{88}$ | 3809 | 16748 |
| 89 | $\text{Firm}_{89}$ | 198403 | 3628497 |
| 90 | $\text{Firm}_{90}$ | 365884 | 35814080 |

Due to the competitive nature of the industry, there is a notable lack of information around the nature of interactions between firms. As a response, an assumption needs to be made in order to construct set $\{E\}$. Specifically, it is assumed that firms with comparable $AA$ invest in assets with similar

[h] Information around subsidiaries have been aggregated under their respective parent firm

characteristics (i.e. volatility, risk etc.). In other words, larger firms have the capacity to invest in higher-risk, higher-return assets and hence, are more likely to be linked (Figure 2). As such, the resulting network is expected to have an assortative mix (i.e. highly connected nodes are more likely to be connected between each other) [27] and of heterogeneous nature, in terms of individual node degree. It is worth noting that recent work within the general domain of finance has focused on the use of trivial network architectures [9] as models for capturing a system's connectivity pattern (e.g. in [18, 28] random graphs [29] are used; in [30] complete and ring networks are used). Under these models, a number of important assumptions are present, including that of degree homogeneity [31] and null mixing patterns[i] [27, 33]. These assumptions are clear idealisations and cannot account for heavy-tail cascade distribution sizes [34]. Hence, we argue that even simple assumptions such as the one noted above brings these methodologies closer to reality and thus, increases their practical utility.

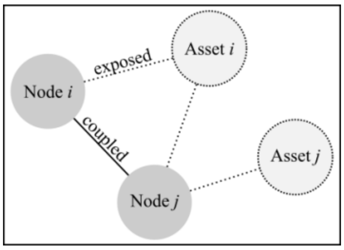


Figure 2: Simple example on how the network of interactions between firms (solid line) is inferred based on the assets that they are exposed to (dotted line).

Mathematically, the network generation process is driven by the probability of firm $i$ and $j$ being connected. The magnitude of this probability is considered to be a function of a firm's $AA$ and the maximum $AA$ that exists within the system. Hence, it can be defined as:

$$\mathbf{P}(i,j) = \Pr(\mathbf{A}(i,j) = 1) = \frac{\mathrm{AA}_i}{\max_{i \in N} AA} \times \frac{\mathrm{AA}_j}{\max_{j \in N} AA}$$

where **A** is the adjacency matrix, used to capture the structure of the network – it can be defined as

$$\mathbf{A}(i,j) = \begin{cases} 1 \text{ if there is a link between } i \text{ and } j \\ 0 \text{ otherwise} \end{cases}$$

Due to the stochastic nature of the network generation mechanism, a Monte-Carlo approach will be adopted, resulting to an ensemble of subtly different network architectures. As a result, a total of 1,000 networks has been generated and subsequently tested.

## 2.2. Cascading process

The impact of interconnectivity on the global system is to be assessed by artificially failing each node $i$ and capturing the maximum number of nodes (i.e. cascade size) that are subsequently affected. Such failure cascades can be seen as the result of a cascading process as it unravels across the network. The threshold model, first introduced by Granovetter [21] to explain social unrest, has been widely used in simulating such processes across a wide range of domains, ranging from epidemiology [22-25] and finance [35, 36] to infrastructure management [37, 38]. The flexibility and universal

[i]The use of non-trivial architectures, such as a scale-free network (see [32] B. DasGupta and L. Kaligounder, "On global stability of financial networks," *Journal of Complex Networks,* vol. 2, pp. 313-354, 2014. for an example) may still suffer from the null-mixing hypothesis, depending on the algorithm used to generate the network. For example, networks generated under the BA algorithm [32] leads to scale-free structures with a Pearson coefficient of 0 [34], indicating that the null-hypothesis still stands, even though the structure is far from being trivial.

applicability of this model suggests that it may provide the grounds for a unified modelling framework for exploring such processes [35]. As such, it is adopted throughout this work.

The underlying principle of the threshold model is simple – every node $i$ is assigned a state ($s_i$) and a threshold ($\theta_i$). A set of local rules is then formulated, dictating the impact of every interaction. In the context of the insurance market, the threshold of node $j$, at time $t$, can be defined as:

$$\theta_j^{\mathrm{t}-1} = EC_j$$

Consequently, a firm with $EC_i > 0$ is considered to be in a healthy state, captured by$s_i = 0$. If at any one point $EC_i < 0$ (e.g. due to a harmful interaction), then firm $i$ is considered to have failed ( $s_i = 1$ ) and can subsequently affect its neighbouring node(s) $j$. The magnitude of this impact is proportional to the size of the failed node $i$ (i.e. its $AA_i$) and the number of nodes that will absorb the impact of its failure i.e. it's degree $k_i$. As it follows, the *new* threshold of node(s) $j$ can now be defined as:

$$\theta_j^{\mathrm{t}} = EE_j - \frac{\alpha}{100} \times \frac{AA_i}{k_i}, \text{ where } \alpha \in [1{,}100]$$

In the case of $\theta_j^{\mathrm{t}} \leq 0$, node $j$ is considered to have failed and can sustain the cascading process by affecting its own neighbours. Conversely, if $\theta_j^{\mathrm{t}} > 0$, then the cascading process is stopped and the sized consequent failure cascade recorder. Control parameter $\alpha$ provides the means of controlling the impact felt by every node $j$, assuming $s_i = 1$. In other word, it provides the means to artificially vary the extent upon which interconnectivity affects the capacity of firm $j$ to operate. For example, increased exposure between firm $i$ and $j$ would correspond to a high $\alpha$ value. Naturally, the smallest failure cascade is expected to occur at $\alpha = 1$; largest at $\alpha = 100$. It is worth emphasizing that this mechanism is clearly a simplification – it is not the aim of this paper to provide a comprehensive set of conditions that may lead to an *individual* firm failing. Rather, it is to uncover the effect of (increased) interconnectivity on the robustness of the overall system under an individual failure. By doing so, the focus shifts from local failures with a local impact (*risk*) to local failures with a global impact (*systemic risk*).

## 2.3. Network-Based Indices

Matrix **R** is used to record the size of each cascade, where entry $\mathbf{R}(i, \alpha)$ corresponds to the cascade size triggered by the failure of firm $i$, under a given $\alpha$ value, $\alpha \in [1{,}100]$ – see below for a small extract of the actual output:

$$\begin{array}{c} \mathbf{Firm_1} \\ \mathbf{Firm_2} \\ \mathbf{Firm_3} \\ \vdots \\ \mathbf{Firm_{88}} \\ \mathbf{Firm_{89}} \\ \mathbf{Firm_{90}} \end{array} \begin{bmatrix} \boldsymbol{\alpha_1} & \boldsymbol{\alpha_2} & \boldsymbol{\alpha_3} & & \boldsymbol{\alpha_{98}} & \boldsymbol{\alpha_{99}} & \boldsymbol{\alpha_{100}} \\ 1.07 & 1.12 & 1.43 & \cdots & 14.73 & 14.73 & 14.82 \\ 1 & 1 & 1 & & 1.08 & 1.08 & 1.08 \\ 3.21 & 5.12 & 8.97 & & 36.25 & 36.26 & 36.29 \\ & \vdots & & \ddots & & \vdots & \\ 1 & 1 & 1 & & 1 & 1 & 1 \\ 1.11 & 1.27 & 1.53 & \cdots & 20.99 & 21.00 & 21.07 \\ 2.56 & 3.28 & 7.97 & & 30.26 & 30.27 & 30.41 \end{bmatrix}$$

Based on this raw set of results, two indices are constructed in order to provide the means of capturing the systemic role of each firm. Specifically, the Criticality IDX captures the capacity of firm $i$ to impact the entire system by triggering large failure cascades. In other words, it provides a measure of how important a firm is, based on its capacity to impact the entire system. The Sensitivity IDX is further introduced in order in order to capture the probability of firm $i$ being affected by one such cascade. Note that both indices are normalised values by definition and hence, are conveniently bound between 0 (min) and 1 (max).

A high-level, algorithmic description for computing the Criticality IDX is as follows:

1. Create $k$ copies of a zero matrix $\mathbf{T_{CI}}$, where $k$ corresponds to the number of simulations – in this case, $k = 1000$
2. For simulation run $k$ and for a given $\alpha$ value, artificially fail firm $i$ and record the size of resulting cascade in the $\mathbf{T_{CI}}(i, \alpha)$ entry – repeat for all $i, i \in N$ and for all $\alpha, \alpha \in [1{,}100]$
3. Assign a rank to every $\mathbf{T_{CI}}(i, \alpha)$ entry, based on the set of values across its respective column $\alpha$ e.g. Consider a trivial example where the resulting set of cascade sizes, in the case of $\alpha = 5$, is stored as $\mathbf{T_{CI}}(i, 5) = \{2,3,20,3,67\}$. Subsequently, the resulting rank is given by the set $\{1,2,3,2,4\}$.
4. Divide every $\mathbf{T_{CI}}(i, \alpha)$ entry with the maximum rank noted under its respective column $\alpha$ – this is the normalised rank of every firm $i$. Continuing the previous trivial example, $\mathbf{T_{CI}}(i, 5)$ is now equal to $\{0.25,0.5,0.75,0.5,1\}$.
5. Average each entry across all simulation runs i.e. across all $k$ copies of the matrix $\mathbf{T_{CI}}$
6. Average all normalised ranks across the 2$^{nd}$ dimension of $\mathbf{T_{CI}}$ matrix i.e. across all $\alpha$ values

Equally, the steps for computing the Sensitivity IDX are as follows:

1. Create $k$ copies of a zero matrix $\mathbf{T_{SI}}$, where $k$ corresponds to the number of simulations – in this case, $k = 1000$
2. For simulation run $k$ and for a given $\alpha$ value, artificially fail firm $i$ and record the identity of firms affected in a temporary matrix – repeat for all $i, i \in N$ and for all $\alpha, \alpha \in [1{,}100]$
3. If firm $i$ is affected, under a given $\alpha$ value, then $\mathbf{T_{SI}}(i, j)$ entry switches from 0 to 1
4. Compute the probability for each firm $i$ to be affected in all $k$ runs.
5. Average this probability across 2$^{nd}$ dimension of $\mathbf{T_{SI}}$ matrix i.e. across all $\alpha$ values.

# 3. Results

## 3.1. Cascade Sizes

The cascade process was simulated across an ensemble of 1,000 networks – results were subsequently averaged and captured in matrix **R** - Figure 3a and 3b present the results in a color-coded fashion. As expected, the size of the cascade induced by the failure of each firm $i$ follows a monotonic increase as $\alpha \uparrow$. Interestingly, a relatively large portion of firms induces no cascades (colour coded as blue), though this portion is substantially reduced as the impact of interconnectivity increases. Specifically, and at the point where $\alpha = 1$, a total of 35 firms have no capacity in triggering *any* failure cascades; this number drops to 10 as the impact of interconnectivity reaches its maximum (i.e. $\alpha = 100$). Hence, the capacity of a firm to impact the entire system is not only conditional to the underlying network structure and its own individual attributes but also on the degree upon which its function may be affected by the operation of its neighbouring firms.

In other words, non-critical firms (i.e. incapable of triggering failure cascades) may switch to critical if the nature of the interaction between other firms becomes stronger. In fact, it appears that the impact of interconnectivity (i.e. $\alpha$) has an abrupt effect ov the number of nodes capable of inducing cascades of a given size. In the case of small failure cascades (Figure 3c, grey and green markers), a continuous and incremental increase in the number of nodes is noted, until a saturation point is reached. Interestingly, and in the case of larger failure cascade sizes (Figure 3c, blue and purple markers) a bursty behaviour is noted, where increasing the value of α has no effect until a certain point is reached. Beyond this point, the number of firms that can induce large failure cascades jumps, suggesting that the number of critical firms *abruptly* change as the coupling between firms increases (i.e. $\alpha \uparrow$).

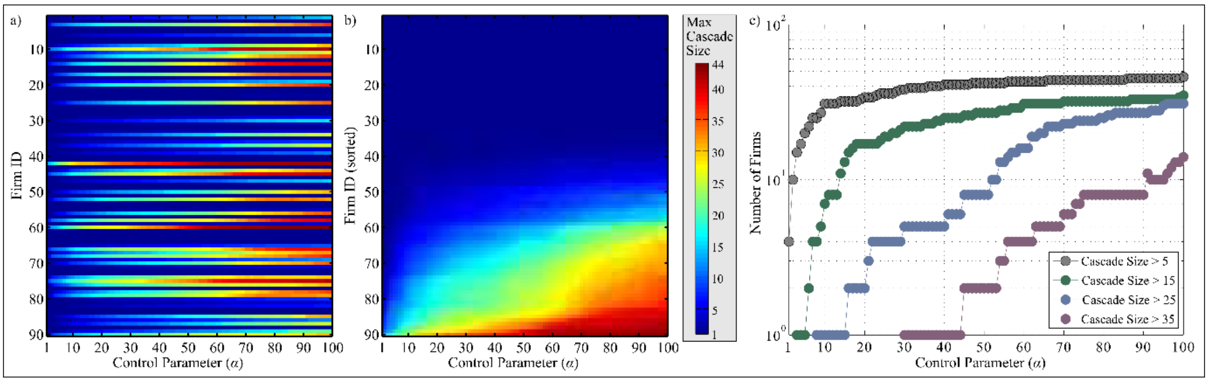


Figure 3: a) Cascade size (colour) induced by the failure of firm $i$ (y-axis) across the entire range of the control parameter $\alpha, \alpha \in [1{,}100]$ (x-axis); b) same as a) though firms are now sorted in an ascending manner, based on the cascade size that their failure can induce under each $\alpha$ value; c) number of firms (y-axis) capable of inducing a failure cascade greater than a given size (marker) under a given $\alpha$ value (x-axis).

As captured in **R**, the model output is a function of two distinct variables – firm identity and control parameter $\alpha$. For simplicity, the dimensionality of these results can be reduced by aggregating either one. Firm identity information may be omitted by simply considering the maximum and mean cascade obtained under every $\alpha$ value – see Figure 4a. Equally, by averaging the results across the entire range of the $\alpha$ parameter $\left(\frac{1}{100}\sum_{a=1}^{100}\mathbf{R}(i,a)\right)$, one can deduce the *average* impact of firm $i$ failing, omitting the need to present results for each distinct $\alpha$ value. Such information can be subsequently used to compute the probability of encountering a cascade of any given size – see Figure 4b.

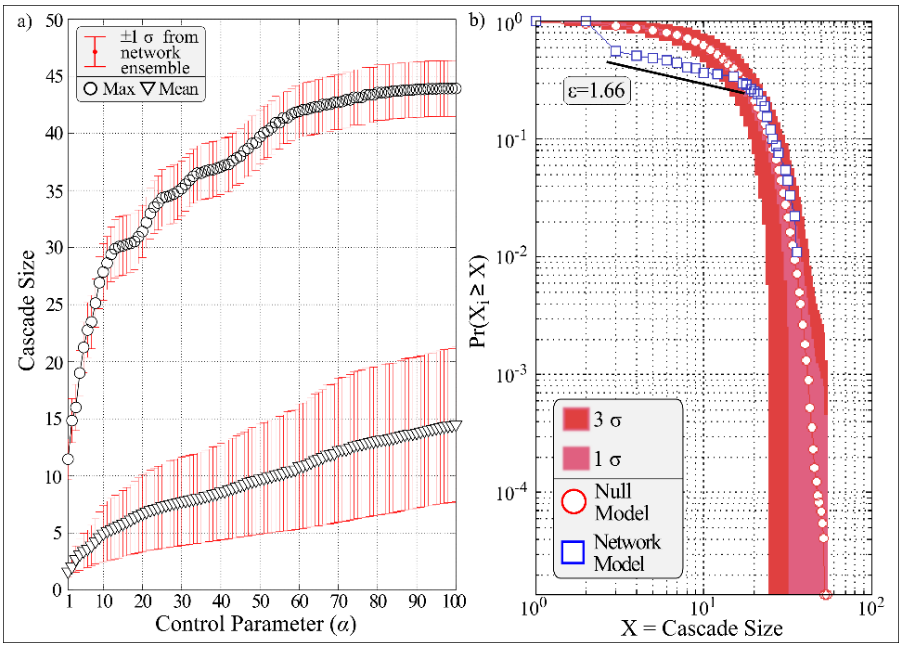


Figure 4: a) Maximum (circle) and mean (triangle) cascade size (y-axis) noted under each $\alpha$ value, $\alpha \in [1{,}100]$. Error bars indicate 1 standard deviation across the ensemble of 1,000 networks; b) Cumulative probability distribution of cascade size, averaged across all α. Results are for the cascade model (circle) and for a null model (square). 1 and 3 standard deviations are also shown for the latter.

With respect to Figure 4a, and as expected, the same monotonic increase is noted for both maximum and mean cascade size, though both exhibit a degree of variation as the structure of each network within the network ensemble is bound to be subtly different. Specifically, and with respect to the maximum cascade size, this variation (quantified by the standard deviation) remains relatively constant, ranging from a minimum of 1.76 to a maximum of 2.61. In other words, the uncertainty of knowing the maximum cascade size is relatively low and fairly independent of $\alpha$. On the other hand, and when considering the mean cascade size (Figure 4b), the degree of variation significantly changes with $\alpha$, ranging from a minimum of 0.44 to a maximum of 6.77. In other words, the uncertainty of knowing the size of the mean cascade increases with the impact of interconnectivity.

With respect to the cumulative probability distribution, two set of results are shown – one from the cascade model, and one from a null model. The latter essentially modifies the results of the former in order to satisfy the condition of being normally distributed. Focusing on the results produced by the cascade model, a straight line is observed for a given set of x values, followed by a sharp cut-off. Interestingly, this cut-off converges to the distribution produced by the null model, highlighting its exponential character[j]. The combination of a straight line in a log-log plot, followed by an exponential cut-off suggests the existence of a truncated power-law distribution, where $f(\mathrm{x}) \propto \mathrm{x}^{-\varepsilon}$ applies for a given set of x values. Similar distributions have been noted in several real-world systems and are thought to arise due to the finite size of the system[k]. The character of any power law distribution can be described by a single parameter, $\varepsilon$, referred to as the exponent (under a log-log plot, this corresponds to the gradient, though in the case of a cumulative plot, a translation is required [19, 20]). Following a methodology proposed by Clauset, et al. [19], and by excluding the exponential tail (reducing the sample size to 76, from the original 90) $\varepsilon$ is estimated to be 1.66.

## 3.2. Firm-based information as Cascade Size Proxy

To justify the need for network-based information during a decision making process, the ability of traditional, firm-based information is serving as proxies for effects that arise through this interconnectivity (such as failure cascades) will be assessed. Specifically, the capacity of $AA_i$ and $EC_i$ to approximate the systemic impact of firm $i$ failing will be evaluated.

The resulting cascade size of 72 randomly chosen firms was used to construct a simple, 2nd order linear regression model. In the spirit of cross validation, $AA_i$ of the remaining 18 firms was used as an input to the model in order to compute the theoretical cascades – this process was repeated for every $\alpha$ value and the goodness of fit measured. The predictive power of the model was then assessed by evaluating the absolute percentage error between actual and theoretical cascade size. Equally, the process was repeated by using $EC_i$ as the predictor value – see Figure 5.

Interestingly, the goodness of fit, as measured by the $\mathrm{R}^2$ value, gradually decreases as the impact of interconnectivity (i.e. $\alpha$) increases – this trend is encountered under both $AA$ (Figure 5a) and $EC$ (Figure 5c). Trivially, $AA$ appears to serve as a better predictor compared to $EC$, though both $\mathrm{R}^2$ are rather low. With respect to the absolute percentage error, both $AA$ and $EC$ perform poorly – see Figure 5b and 5d respectively. Interestingly, large values are more frequently encountered at high α values, strengthening the argument that as the impact of interconnectivity increases, the ability of firm-based information to adequately serve as a proxy diminishes

[j] Results from the null model are drawn from a normal probability density function and thus, are of an exponential nature.
[k] A power law distribution with an exponent ($\varepsilon$) greater than 1 implies an infinite variance of values (and depending on the value of $\varepsilon$, infinite mean), requiring a system of an infinite size.

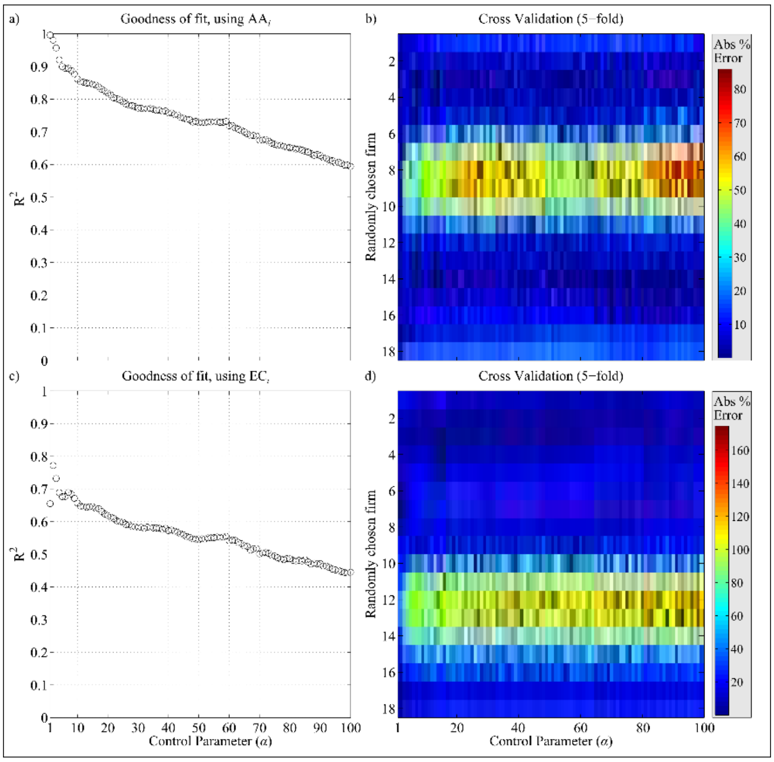


Figure 5: a) and c) map the goodness of fit ( $R^2$ value) between the regression model and the actual cascades, where $AA_i$ and $EC_i$ were used as the predictor values respectively. Note the decrease in $R^2$ as the impact of interconnectivity increases; c) and d) present the results of a 5-fold cross validation, capturing the absolute percentage error between actual and predicted cascade sizes, under $AA_i$ and $EC_i$ respectively.

It is worth noting that models of up to the 4$^{th}$ order have been tested, where $R^2$ showed a significant improvement, suggesting a much better fit. However, the resulting absolute percentage error between the predicted and actual cascade sizes grew rapidly (in the case of a 4$^{th}$ order model, exceeding 3000% - see Supplementary Information), clearly indicating that the predictive power of the model had evaporated. This further analysis highlights that the diminished capacity of both $AA_i$ and $EC_i$ to serve as proxies to systemic effects are fundamental[1] rather than being model-dependent.

# 4. Discussion

Increasing interconnectivity between insurance firms poses a new challenge in a) understanding the individual role of a firm, with respect to the system that contains it and b) choosing the right information for decisions where systemic effects are important. In response, this paper has presented a simple analytical model that may be used to explore point a) via the means of numerical simulation. Hence, a number of theoretically important questions may be explored – see Section 4.1. In response to b), results of the model can be subsequently used to derive network-based information for individual firms (Section 2.3.), supplementing traditional, firm-based information. By doing so, the former may be used to both assess and drive the decision making process, when systemic effects are to be taken into account – see Section 4.2.

[1] Further explorative analysis indicates that one may translate firm-level information to network-based information in a rather straightforward, yet coarse, way. Specifically, AA may be used to approximate the corresponding Criticality and Sensitivity IDX for firm i. Interestingly, EC fails to provide a similar translation as its correlation with both Criticality and Sensitivity IDX is weak – see Appendix

## 4.1. Theoretical Implications: A Brief Exposition

Evidence of a power law distribution of cascade sizes (Figure 4b) are of great importance, as they draw parallels with self-organising criticality – a theory that describes critical phenomena that underpin various systems, ranging from earthquakes to financial crisis [39, 40]. Furthermore, as the exponent of the distribution is lower than 2, it implies that both its variance and mean value are (theoretically) infinite, negating the very notion of a*n averaged-size* failure. In other words, the size of a possible failure cascade is only limited by the size of the system rather than its reduced probability of occurrence – the latter would have been the case if the cascade distribution was normally distributed (e.g. Figure 4b, null model). This observation has three main consequences: a) both small and large failure cascades follow the same dynamics; b) large failure cascades may occur without any large exogenous force, and c) the expected impact of a local failure cannot be estimated *a priory*.

One would naturally expect that large failure cascades are the inevitable conclusion of an unfortunate set of conditions aligning. Though this may indeed be the case, it is of no surprise that such extraordinary conditions would greatly impact the system. Perhaps more surprising is the emergence of such failures that do *not* require these conditions, as they may arise via the exact same cause as local failures i.e. the failure of a single node. Assuming that resource spent in risk mitigation is proportional to the envisioned impact of the risk materialising, the combination of the aforementioned points (a), (b) and (c) challenge the very notion of the risk management process. Specifically, as one may not reasonably assess whether a local failure is bound to trigger a cascade or not, assigning a suitable amount of resource for mitigating against its impact is bound to be a challenging task.

These observations need to be taken with a word of caution as a number of limitations around the data sample exist such as the nature of the network structure itself along with the impact of finite size bias. Nonetheless, the evidence of a power law distribution in actual loss events (Figure 1) are in agreement with the aforementioned implications and hence, form a solid argument on the utility of this mode, at least on a qualitative basis.

## 4.2. Practical Implications: An Example

The practical utility of the cascade models, along with the performance of network-based measures will be illustrated via the use of a decision making example. Specifically, the impact of firm acquisition on the entire system will be assessed. Choosing which firms are allowed to be merged is based on either firm-based ($AA_i$ or $EC_i$) or network-based (Cascade IDX or Sensitivity IDX) information. Once a merger is completed, the system is stressed using the cascade model and the impact of the largest failure cascade is subsequently recorded.

The method for doing so is as follows. First, firms are ranked in an ascending manner, based on a given set of information $-AA_i, EC_i$, Criticality or Sensitivity IDX. The top-two *or* bottom-two ranking firms (firm $i$ and $j$) are subsequently merged, forming a new firm $k$. This new firm preserves the exposure of the original two firms (i.e. its connections, hence $k_k = k_i + k_j$). Similarly, its economic status is defined as $AA_k = AA_i + AA_j$ and $EC_k = EC_i + EC_j$. At this point, the robustness of the resulting system is evaluated by running the cascade model, evaluating the impact of the largest failure cascade possible within the given state of the system. Note that the size of the system has now been reduced by 1. Once the impact is recorded, the respective set of information is updated to reflect the new state of the system and the process is subsequently repeated, until the limit of merger iterations is reached (i.e. $N - 1$). Results for merging the top ranking and bottom ranking firms are shown in Figure 6a and 6b respectively.

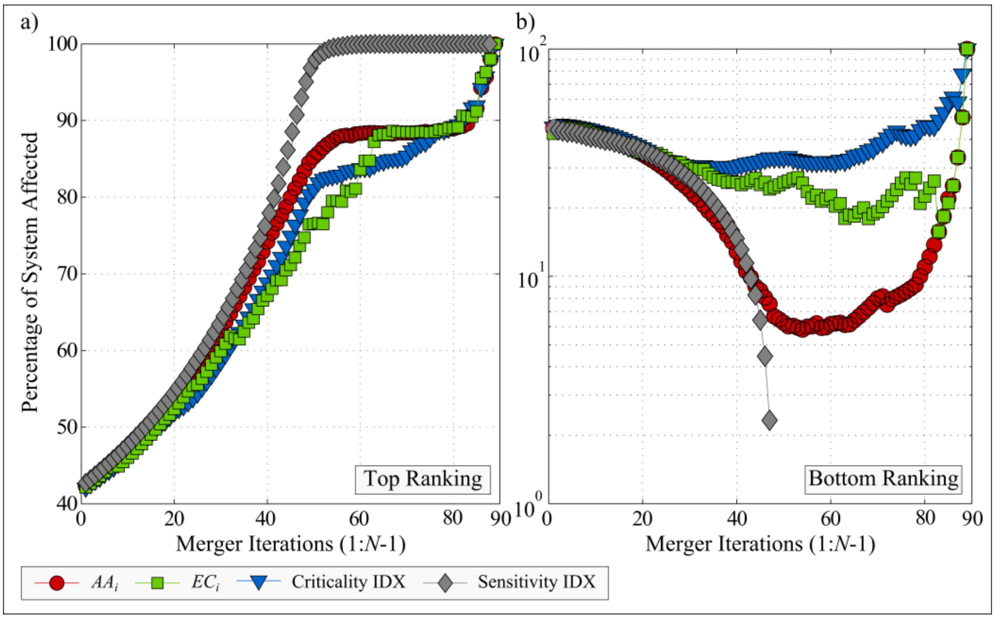


Figure 6: Robustness of the system as firms are iteratively merged, measured as the percentage of firms affected by the maximum failure cascade noted under failing each firm $i,\ i \in N$; in a) the top two ranking firms are merged; in b) the two lowest-ranking firms are merged; note the logarithmic nature of the y-axis. Information used to enable this ranking is based on their Criticality IDX (triangle), Sensitivity IDX (diamond), $AA_i$ (circle) or $EC_i$ (square). Note that results are averaged across $\alpha$.

This approach provides insights on both the utility of firm acquisition as a way of varying the robustness of the overall system, as well as identifying which information can provide for the most efficient way in taking such decisions. Note that as this section aims to provide an example of how the cascade model, as proposed within this paper, can be of practical use – an in-depth analysis of the results will be presented in future work.

#### *4.2.1. Systemic impact of firm acquisition*

Let us focus on the case where top-ranking firms are allowed to merge (Figure 6a). Clearly, there is a decrease in the robustness of the system (seen as an increase in the percentage of system affected) as firms becomes larger, and hence, their individual failure resonates deeper within the system. Importantly, the monotonic nature of this reduction is *irrespective* of the set of information used to rank firms. This insight has important implications as the merger of individually healthy firms (i.e. high $EC_i$) still has a negative impact to the robustness of the overall system.

In the case of merging bottom ranking firms (Figure 6b), an improvement in the robustness of the system (seen as a decrease in the percentage of the system affected) can be achieved, the extent of which depends on the set of information used to rank the firms. Alas, this gradual improvement is negated as the number of merger iterations increase, highlighting the fact that as soon as firms reach a critical value (which depends on the set of information used to rank them) a loss in the systems' robustness is inevitable. Interestingly, this trend is not reflected when the Sensitivity IDX is used – see Section 4.2.2.

#### *4.2.2. Efficiency in decision making*

In this context, efficiency in the decision making process may be measured by the rate in which a change in the system's robustness is achieved. In other words, the steeper the change noted, the more efficient the decision of merging the two given firms is.

With respect to Figure 6a, the use of the Sensitivity IDX results to increased efficiency as the system is quickly brought to a point where the occurrence of the largest impact will affect the *entire* system.

In other words, the use of Sensitivity IDX as the means to identify mergers that need to be avoided significantly outperforms any other set of information as it can bring the system to a critical state with the fewer number of mergers. Interestingly, merging the firms that are most capable in triggering large cascades (i.e. ranking high in the Criticality IDX) underperforms when compared to $AA$, though still outperforms $EC$. This behaviour suggests that a firm's sensitivity (in terms of being affected by the actions of other firms) is more important than its individual capacity in triggering failure cascades. Such insight can be used to focus mitigation strategies in reducing individual firm exposure (e.g. by restricting the number of connections individual firms may have) rather than controlling individual firms' characteristics that drive the cascading process itself (such as $AA$ and $EC$).

With respect to Figure 6b, and by using the Sensitivity IDX, the state of system exhibits the greatest improvement. In fact, its use was capable in bringing the system at a stable state, where *no* cascade was sustained. In other words, merging firms that ranked low in sensitivity gradually led to an increasingly robust system, outperforming the remaining set of information. On the other hand, by using the Criticality IDX, the system exhibits a rather stable state, where the impact of the largest cascade consistently lingers at the highest level. In fact, its use outperforms the remaining set of information in terms of achieving the lowest level of robustness at *any* given number of merger iterations. In other words, by merging firms that rank low in terms of their capacity to trigger large cascades, a transient stage of improved robustness is achieved. This state is conserved until new firms with an *increased* capacity to trigger large cascades emerges. At this point, each new merger leads to a further decrease in the robustness of the system.

#### *4.2.3. Concluding Remarks*

This section has briefly illustrated how the cascade model may be used to support decision making process when effects at a global level are to be expected. Specifically, it has been shown that firm acquisition between high-ranking firms increases the exposure of the entire system to systemic risk. Interestingly, when low-ranking firms are considered the converse is true, within a limited regime, before the same damaging behaviour is reached. Evidence of this sort suggest that attempts to save individual firms via firm mergers (e.g. the acquisition of Meryl Lynch by Bank of America, saving the latter from an eminent collapse) may be unsubstantiated at best (and dangerous at worst), as the exposure of the system to systemic risk greatly increases. Equally, a merger may become more viable by changing the role of the individual firms within the network e.g. increase the degree of similarity across various investments between the involved parties, decreasing the number of connections. Finally, it was also shown that examples of network-based information can significantly outperform firm-based information, leading to increased efficiency in the decision making process.

# 5. Conclusion

Empirical evidence within the insurance industry suggests that loss events can affect a surprisingly large number of firms (Figure 1), indicating that interconnectivity within the industry is a central aspect. From a complex network perspective, these events may be interpreted as failure cascades, a direct result of a cascading process. As such, a simple network model capable of replicating this qualitative behaviour was constructed, and later tested on an empirical dataset. By doing so, the capacity of each firm in triggering a failure cascade (Figure 3), along with the probability of being affected by one (Figure 4b), was assessed. Traditional, firm-based information was subsequently shown to be a poor predictor of the systemic impact of each firm. In response, two network-based indices were constructed, enabling the ranking of firms based on their systemic role. Finally, the utility of the cascade model and the resulting indices was illustrated through a decision making example. Specifically, the model was used to assess the systemic impact of a firm acquisition process, while the network-based information was shown to significantly outperform traditional firm-based information in terms of their efficiency in driving the decision making process (Figure 6).

Though the aim of this work is to introduce a novel way in assessing systemic risk in the insurance industry, along with raising a number of questions around the means of mitigation actions and regulatory frameworks, numerous challenges remain yet untackled. For one, both individual firms and regulators are left with the considerable task of capturing meaningful interactions between firms. By doing so, one may begin to explore the impact of various network structural features on the robustness of the system. Although some work around these aspects already exists (e.g. [18, 28]), it is yet immature as numerous idealisations need to be lifted before their output can be put into practical use. Increasing data quality by directly observing the network of relationships is expected to increase the practical relevance of similar approaches – notable examples of include [41-43]. Finally, the insurance community is faced with the challenge of keeping up with the ever increasing pace in which the field of complex networks is being developed.

In an era of increased interconnectivity, non-linearity becomes the rule rather than the exception, fostering local events with a global impact. Academia has risen to the challenge by developing novel frameworks that can account for these effects, complex networks being one of the most successful. It is now up to the industry to harvest this insight and engineer the conditions for a robust financial system.

# 6. Acknowledgments

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# 7. References

[1] D. Helbing, "Systemic risks in society and economics," in *Social Self-Organization*, ed: Springer, 2012, pp. 261-284.

[2] M. Newman, "Complex systems: A survey," *arXiv preprint arXiv:1112.1440*, 2011.

[3] A. Vespignani, "Modelling dynamical processes in complex socio-technical systems," *Nature Physics*, vol. 8, pp. 32-39, 2012.

[4] A. Barrat, M. Barthelemy, and A. Vespignani, *Dynamical processes on complex networks*: Cambridge University Press, 2008.

[5] A. Vespignani, "Predicting the behavior of techno-social systems," *Science*, vol. 325, p. 425, 2009.

[6] D. Helbing, "Globally networked risks and how to respond," *Nature*, vol. 497, pp. 51-59, 2013.

[7] S. H. Strogatz, "Exploring complex networks," *Nature*, vol. 410, pp. 268-276, 2001.

[8] D. J. Watts, "The" new" science of networks," *Annual review of sociology*, pp. 243-270, 2004.

[9] A.-L. Barabási, "Scale-free networks: a decade and beyond," *Science*, vol. 325, pp. 412-413, 2009.

[10] M. E. Newman, *Networks: an introduction*: Oxford University Press, 2009.

[11] M. E. Newman, "The structure and function of complex networks," *SIAM review*, vol. 45, pp. 167-256, 2003.

[12] R. Albert and A.-L. Barabási, "Statistical mechanics of complex networks," *Reviews of modern physics*, vol. 74, p. 47, 2002.

[13] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: Structure and dynamics," *Physics reports*, vol. 424, pp. 175-308, 2006.

[14] A. G. Haldane, "Rethinking the financial network," *Speech delivered at the Financial Student Association, Amsterdam, April*, pp. 1-26, 2009.

[15] R. M. May, S. A. Levin, and G. Sugihara, "Complex systems: Ecology for bankers," *Nature,* vol. 451, pp. 893-895, 2008.

[16] F. Schweitzer, G. Fagiolo, D. Sornette, F. Vega-Redondo, A. Vespignani, and D. R. White, "Economic networks: The new challenges," *Science,* vol. 325, p. 422, 2009.

[17] M. Billio, M. Getmansky, A. W. Lo, and L. Pelizzon, "Econometric measures of connectedness and systemic risk in the finance and insurance sectors," *Journal of Financial Economics,* vol. 104, pp. 535-559, 2012.

[18] P. Gai and S. Kapadia, "Contagion in financial networks," in *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 2010, p. rspa20090410.

[19] A. Clauset, C. R. Shalizi, and M. E. Newman, "Power-law distributions in empirical data," *SIAM review*, vol. 51, pp. 661-703, 2009.

[20] M. E. Newman, "Power laws, Pareto distributions and Zipf's law," *Contemporary physics,* vol. 46, pp. 323-351, 2005.

[21] M. Granovetter, "Threshold models of collective behavior," *American journal of sociology,* pp. 1420-1443, 1978.

[22] M. J. Keeling and K. T. Eames, "Networks and epidemic models," *Journal of the Royal Society Interface,* vol. 2, pp. 295-307, 2005.

[23] S. Melnik, J. A. Ward, J. P. Gleeson, and M. A. Porter, "Multi-stage complex contagions," *Chaos: An Interdisciplinary Journal of Nonlinear Science,* vol. 23, p. 013124, 2013.

[24] M. E. Newman, "Spread of epidemic disease on networks," *Physical Review E,* vol. 66, p. 016128, 2002.

[25] R. Pastor-Satorras and A. Vespignani, "Epidemic dynamics in finite size scale-free networks," *Physical Review E,* vol. 65, p. 035108, 2002.

[26] J. Lorenz, S. Battiston, and F. Schweitzer, "Systemic risk in a unifying framework for cascading processes on networks," *The European Physical Journal B-Condensed Matter and Complex Systems,* vol. 71, pp. 441-460, 2009.

[27] M. E. Newman, "Mixing patterns in networks," *Physical Review E,* vol. 67, p. 026126, 2003.

[28] E. Nier, J. Yang, T. Yorulmazer, and A. Alentorn, "Network models and financial stability," *Journal of Economic Dynamics and Control,* vol. 31, pp. 2033-2060, 2007.

[29] P. Erdős and A. Rényi, "On random graphs," *Publicationes Mathematicae Debrecen,* vol. 6, pp. 290-297, 1959.

[30] D. Acemoglu, A. Ozdaglar, and A. Tahbaz-Salehi, "Systemic risk and stability in financial networks," National Bureau of Economic Research2013.

[31] A.-L. Barabási and R. Albert, "Emergence of scaling in random networks," *Science,* vol. 286, pp. 509-512, 1999.

[32] B. DasGupta and L. Kaligounder, "On global stability of financial networks," *Journal of Complex Networks,* vol. 2, pp. 313-354, 2014.

[33] M. E. Newman, "Assortative mixing in networks," *Physical review letters,* vol. 89, p. 208701, 2002.

[34] D. J. Watts, "A simple model of global cascades on random networks," *Proceedings of the National Academy of Sciences,* vol. 99, pp. 5766-5771, 2002.

[35] J. Lorenz, S. Battiston, and F. Schweitzer, "Systemic risk in a unifying framework for cascading processes on networks," *The European Physical Journal B,* vol. 71, pp. 441-460, 2009.

[36] C. J. Tessone, A. Garas, B. Guerra, and F. Schweitzer, "How big is too big? Critical Shocks for Systemic Failure Cascades," *Journal of Statistical Physics,* vol. 151, pp. 765-783, 2013.

[37] S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin, "Catastrophic cascade of failures in interdependent networks," *Nature,* vol. 464, pp. 1025-1028, 2010.

[38] G. A. Pagani and M. Aiello, "The Power Grid as a complex network: A survey," *Physica A: Statistical Mechanics and its Applications,* vol. 392, pp. 2688-2700, 2013.

[39] P. Bak and K. Chen, "Self-organized criticality," *Scientific American,* vol. 264, 1991.

[40] P. Bak and M. Paczuski, "Complexity, contingency, and criticality," *Proceedings of the National Academy of Sciences,* vol. 92, pp. 6689-6696, 1995.

[41] R. Cont and A. Moussa, "Network structure and systemic risk in banking systems," *Edson Bastos e, Network Structure and Systemic Risk in Banking Systems (December 1, 2010),* 2010.

[42] S. Battiston, M. Puliga, R. Kaushik, P. Tasca, and G. Caldarelli, "Debtrank: Too central to fail? financial networks, the fed and systemic risk," *Scientific reports*, vol. 2, 2012.
[43] R. Cont, A. Moussa, and E. B. Santos, "The Brazilian interbank network structure and systemic risk," 2010.

# Supplementary Information

This section presents the results of examine the capacity of $AA_i$ in serving as a proxy for the cascade size cause by the failure of firm $i$. In contrast to the 2nd order model used in the main text (Figure 5a), a 4th order model is used to examine whether this failure was due to the model being simplistic (i.e. smaller number of parameters).

With respect to Figure 1SI, the same trend of a diminishing $R^2$ value as the impact of interconnectivity increase is noted, yet the decrease is much more subtle. Such evidence would suggest that indeed, failure to correlate, as described in Section 3.2., was due to under-fitting the model. However, by examining the absolute percentage error of the model, it is clear that the predictive power of the model has completely deteriorates, with values in the range of thousands. Hence, one can attribute the increase $R^2$ value as being the result of over-fitting, validating the fact that $AA_i$ is a poor predictor in evaluating the impact of firm $i$ failing. Similar behaviours was also noted when $EC_i$ was considered as the predictive value.

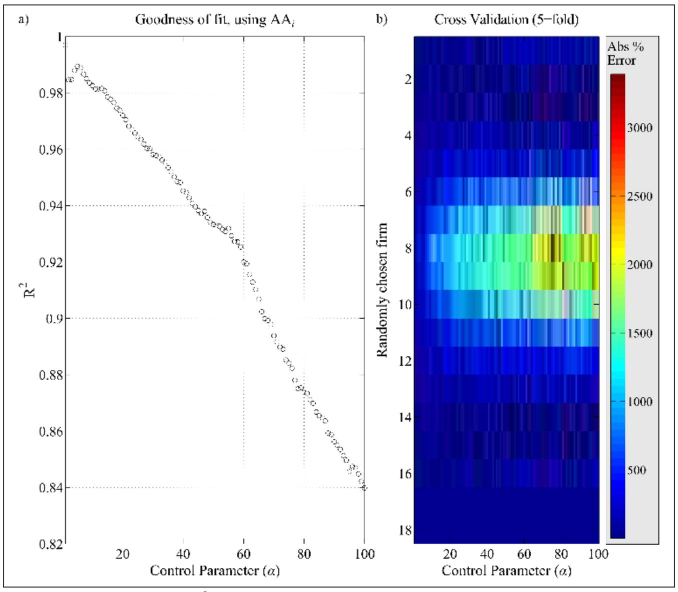


Figure 1SI: As Figure 5a and 5d, where a 4th order, linear regression model is used, compared to the previous 2nd order model. a) Similar to the 2nd order model, a decrease in the $R^2$ value is noted as the impact of interconnectivity increases ($a$), though its impact is substantially less; b) the absolute percentage error

# Appendix

By exploring the nature of the relationship between network-based (such as the Criticality and Sensitivity IDX) and firm-based proxies (such as AA and EC) one can evaluate the extent upon which the latter capture effects that arise from firm interconnectivity. Figure 1A drives this comparison forward by presenting a set of scatter plots between the two set of information.

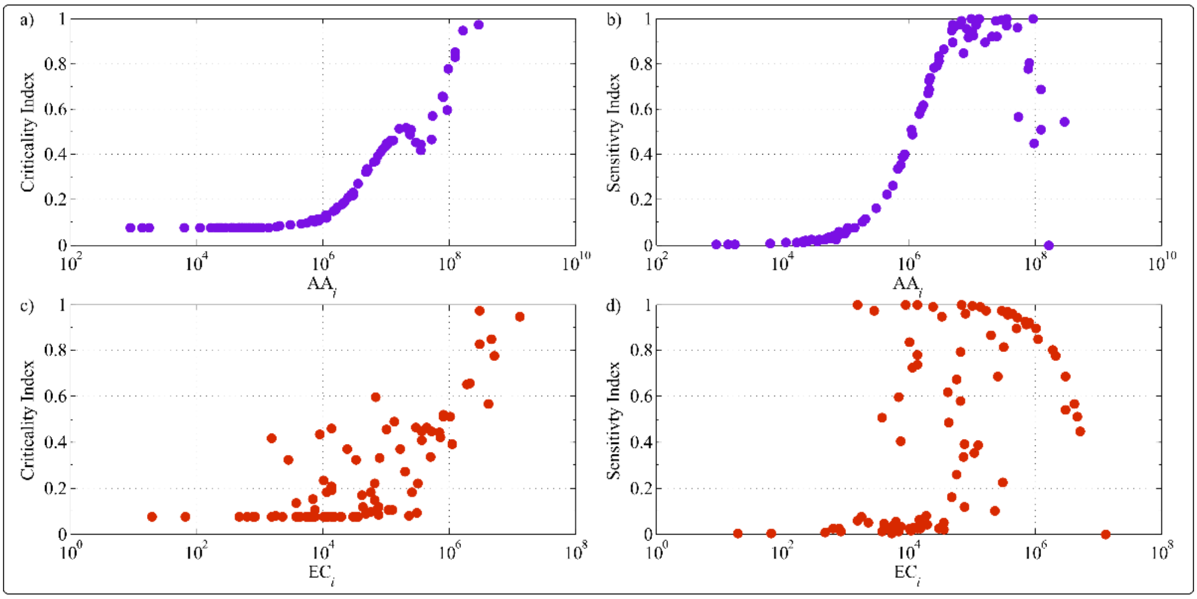


Figure 1A: a) and b) explore the relationship between AA and the two network-based proxies, Criticality and Sensitivity IDX respectively. Similarly, c) and d) map the relationship between EC and the network-based proxies.

One could reasonably postulate that EC would correlate in a negative manner with the Sensitivity IDX, simply because a greater excess suggests an increased capacity in absorbing the impact of other firms failing. In fact, Figure 1A(d), indicates that this relationship is not as trivial as one would expect. Specifically, a subset of firms indeed appears to follow this pattern, where the Sensitivity IDX drops quickly as EC increases. However, a 2$^{nd}$ subset of firms exhibit no correlation between Sensitivity IDX and EC, with the former remaining constant as the latter increases. Finally, a 3$^{rd}$ subset appear to be scatter between these two behaviours. Shifting the focus on the relationship between EC and Criticality IDX, there appears to be some sort of weak and positive correlation, where the latter increases with the former. Nonetheless, this relationship is too noisy to attach any significant confidence to the observed behaviour.

In stark comparison to EC, AA exhibits clear relationships between both Criticality and Sensitivity IDX. Yet again, both relationships illustrate non-trivial features such as non-monotonicity and threshold behaviour. Non-monotonicity is evident in a small range between the relationship of Criticality IDX and AA. Within that region, an increase in AA leads to a decrease in the obtained Criticality IDX, a contrasting trend to the overall positive correlation between the two. Focusing on the relationship between the Sensitivity IDX and AA, the former appears to increase with the former, until a threshold value of $\sim 10^7$ is reached. Beyond this point, the Sensitivity IDX becomes negatively correlated, rapidly decreasing in value as AA continues to increase.