



Delli Carri, A., Di Maio, D., Lucchetti, A., & Sever, I. A. (2015). Experimental model validation of a non-linear structure with lap-joint using the physical space approach. In P. Sas, D. Moens, & H. Denayer (Eds.), 26th International Conference on Noise and Vibration Engineering (IMSA 2014): Held in Conjunction with the 5th International Conference on Uncertainty in Structural Dynamics (USD 2014). (Vol. 1, pp. 1185-1198). Katholieke Universiteit Leuven.

Peer reviewed version

[Link to publication record in Explore Bristol Research](#)
PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/pure/about/ebr-terms.html>

Experimental model validation of a non-linear structure with lap-joint using the physical space approach

A. delli Carri¹, D. Di Maio¹, A. Lucchetti¹, I.A. Sever²

¹ University of Bristol, Department Mechanical Engineering
Queens' building, University walk, Bristol, BS8 1TR, UK
e-mail: dario.dimaio@bristol.ac.uk

² Rolls-Royce plc
SinA-33, PO Box 31, Derby, DE24 8BJ, UK

Abstract

It is commonly accepted that bolted structures can present nonlinearities which can be caused by the contact conditions and that these are severely exercised for large vibration amplitudes. Several researches have targeted these joint's nonlinear behaviours and experimental model validations have demonstrated to be successful.

The scope of this paper is to demonstrate that nonlinearities which are quantified in the modal space can be correlated with the same ones which are identified in the physical space by using reverse path identification method. A dumbbell test rig was designed and manufactured so as to have a lap joint connecting the two weights placed at either ends of the bolted beam. The test structure will be modelled by both one degree of freedom lump parameter model and Finite Element model. The nonlinearities will be identified by the frequency and damping nonlinear functions in the modal space and then correlated with the equivalent laws identified by reverse path method in the physical space.

1 Introduction

The structural nonlinearities often arise from the presence of joints, which are likely to cause contact and friction conditions, severely exercised for large amplitudes of vibration. There are many types of joints, but the mechanics of interfaces of built-up structures often most closely resemble those of lap joints. These configurations involve normal compressive loads holding components together combined with dynamic lateral and/or normal loads inducing some amount of shear slip in the interface. The actual physics taking place at a lap joint interface of a vibrating structure is complex and not fully understood. The reason is that there exist many parameters and uncertainties that prevent an accurate understanding and modelling of the joint physics: these include interface area, distribution of normal and shear forces at the interface, surface finishing and history-dependent and non-linear effects in the dynamics.

The stiffness of the structure — and thus its eigen-frequencies — as well as the amount of damping and the occurrence of nonlinearities are strongly influenced by the location and the nature of its joints. Consequently, special attention has to be paid when the properties of joints are estimated from measurements and the results are used to model the joint's behaviour in numerical simulations.

Grooper [1] defined mathematical equations for the frictional force and slip in both stages- partial slip and full slip and evaluated the dissipation energy. The small, localized motions during microslip result in energy losses at the joint, which is perceived as localized damping of the structure. Indeed, most damping effects encountered in practical structures are taking place at jointed interfaces. Beards [2] carried out a series of experiments to show that an optimum joint clamping force exists for maximum energy

dissipation due to the slip and that the resonances frequencies of the structure can be controlled to some extent by adjusting the clamping force, hence, the slip in joints. Gaul and Lenz [3] investigated the dynamic transfer behaviour of bolted single-lap joint located between two large masses, with a mixed experimental and numerical strategy. The behaviour of the joint was measured first for different modes and then analyzed by using a detailed FE models which included the Valain model for simulating micro and macro-slip in joints. The studies revealed that the hysteresis loops measured for different excitation force levels had different slopes as the force was increased, the slope decreased, and indicating softening effect. Goodman [4] analytically studied interfaces and examined several analytic solutions for elastic-frictional contact problems under oscillating shear loads and noted that they all manifested a power-law dependence of dissipation on the amplitude of the shear force. When the interface was described using Coulomb friction model the exponent of the power law was equal to 3. Sandia National Laboratories [5] have undertaken further investigations into axial joints. Series of closely controlled experiments were able to establish that exists a power-law relationship between input force and energy dissipated per cycle, and further experimental results identified the regions of microslip. A highly detailed Finite Element (FE) model for a single shear lap joint containing hundreds of thousands of Degrees of Freedom (DOF) was constructed at Sandia that seems to predict joint physics with some degree of precision and confirmed the power-law relation, although the predicted slope was different from the measured one.

1.1 Non-Linear Modal Testing (NLMT) background

In recent years, there has been an increased interest in the study of nonlinear phenomena in the field of structural dynamics. Many approaches have been developed in order to get more insights into the complex dynamics of nonlinear systems, but one can categorise them in two main branches: modal space approaches and physical space approaches. The methods that exploit the modal space are usually founded on the concept that a system can still be decomposed into its modal components, although renouncing to all that properties that made this approach appealing in the first place like superposition and homogeneity, and the arise of issues like modal coupling and interactions. On the other hand, the methods that exploit the physical space of a system have the clear advantages to retain all the information about the physical locations of the nonlinear elements and to describe such elements with mathematical laws that could be implemented in lumped parameters models or FEM, but they often lead to extensive computational time and the need for a thorough way to validate their estimations.

Among other advantages, the physical space approach has the capability to deal with nonlinearities in a modular fashion, not seeking for a jack-of-all-trades monolithic algorithm but relying on a toolbox of different methods, each of which capable of answering to one or more of the four main questions about the nonlinear elements (see Figure 1):

- Is there nonlinear behaviour? (**D**etection)
- What kind of nonlinearity? (**C**haracterisation)
- Where are the nonlinear elements located? (**L**ocalisation)
- How strong are these nonlinearities? (**Q**uantification)

In this work the authors seek to validate the non-linear forced response of a mechanical system with localized nonlinearities by means of a recently developed toolbox of physical-space algorithms [8]. A test structure identified in a dumbbell with a lap joint has been selected for this purpose.

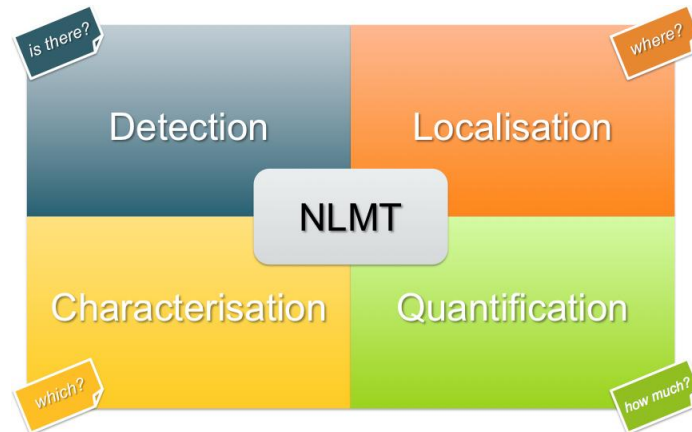


Figure 1 - A breakdown of Non Linear Modal Testing

Since there is the need for a thorough validation of the results, the authors developed a procedure that will assess the goodness of the estimation, based on the modal properties of the test piece extracted using both standard linear modal analysis techniques and nonlinear C-FRF approach. This procedure (see Figure 2) relies on the presence of a linearly validated model in order to implement the estimated nonlinear elements. The structure can be tested using both modal analysis techniques (left branch) and the previously discussed nonlinear identification toolbox (right branch) leading to a set of estimated nonlinear elements, which will subsequently be added to the model. The validation is finally carried out in the modal space: from the left branch one can track the changes in modal properties of the test piece using tools like the frequency-amplitude plots, while on the right branch one must retrieve the same plots via simulation of the nonlinear model. This procedure is powerful enough to provide an updating loop in order to minimise some estimation errors.

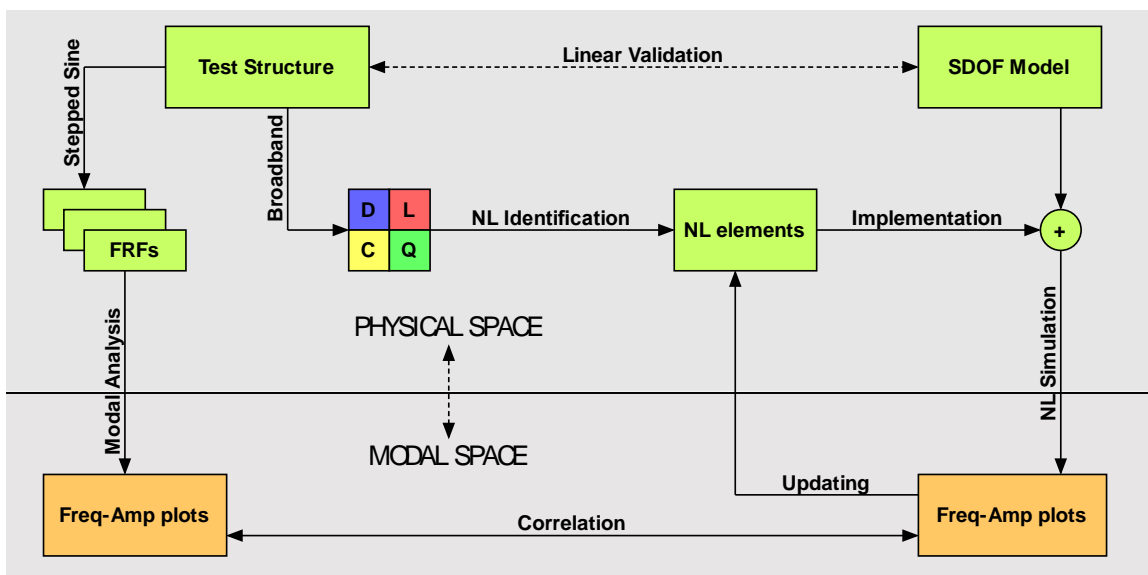


Figure 2 - Experimental nonlinear model validation method

In this work, the structure was selected in order to minimise the modelling effort: since the main mode of vibration of the structure is the counter-phase motion of the big cylindrical masses, a SDOF model will suffice to describe its basic dynamics. The changes in the modal parameters (left branch) were measured using the C-FRF method [13], while the identification in physical space was carried out using the approach and methods developed by the authors in their previous work [8], [9].

There are several different approaches that can be followed toward investigating joint dynamics. One is to try to identify the actual physics taking place within the joint on a microscale. An alternative direction is to look at the effect of the joint on the overall dynamics of the structure. In this latter approach, the microphysics need not necessarily be considered (or modelled) in detail, but rather the overall dynamical effect of the full joint needs to be examined.

The next sections of this paper will focus on:

- The test campaign followed in order to provide the measurements
- Nonlinear analysis for the identification of the order of the nonlinear elements
- Mathematical model of the lap-joint dynamics and forced response predictions
- Correlation between the modal parameters by the means of frequency-amplitude tracking plots

2 Experimental measurements

The dumbbell test structure was used for carrying out both theoretical and experimental work. This latter was divided in several steps so as to capture as much data as possible in order to recover a full understanding of the lap joint physics.

2.1 Test setup

Figure 3 shows the setup of the dumbbell hardware and shaker. The test structure was designed to have two stainless steel masses the weight of which was approximately 4 kg. The lap joint is made of two square aluminium sections each connected to the mass with a single 10/32 UNF bolt and coupled to each other with two M5 bolts. Two torque levels of 6 Nm and 10 Nm were applied to the bolts in order to track the differences in behaviour. The structure was instrumented with four mono-axial accelerometers placed at the ends of the cylinders and a force gauge was installed along the axial direction (see Figure 3). The dumbbell was then supported by two belts and suspended in free-free conditions by elastic. This configuration aimed as much as possible to avoid any undesired strains at the joint but the only one acting along the axial direction.

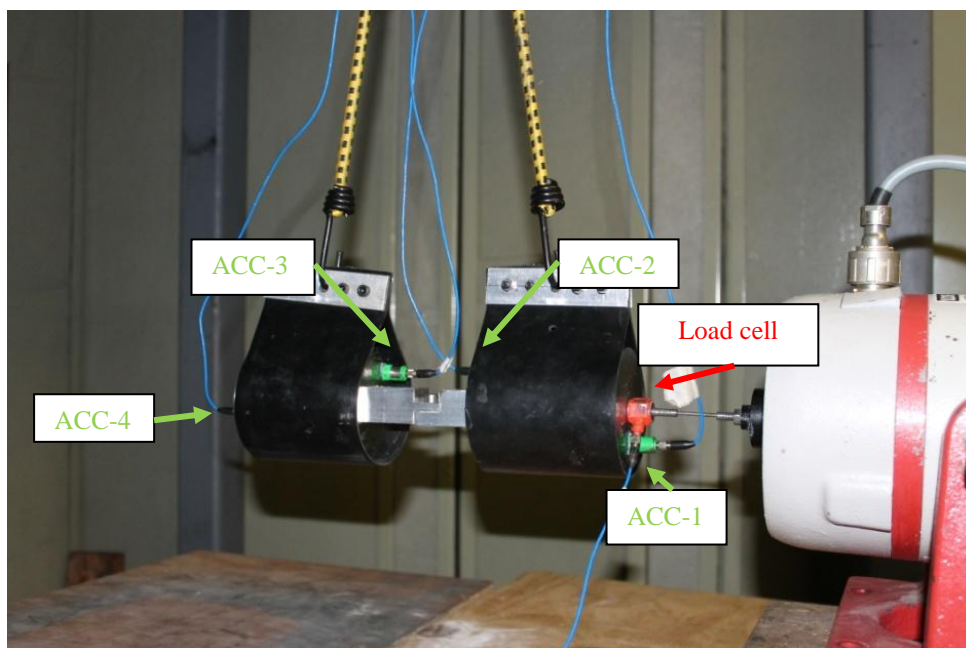


Figure 3 - Experimental setup

2.2 Experimental testing

Due to the unknown nature of the nonlinearity, it has been decided to test with a wide range of excitation signals and force levels in order to get more insights into the phenomenon: from broadband to harmonic excitation, including impact test to study the transient characteristics of the system. The thorough list of tests can be found in Table 1.

		# Force levels
Broadband	Random	4
Narrow band	Stepped sine Up	7
	Stepped sine Down	7
Transient	Impact test	1

Table 1 - Table of the excitation signals used for testing

2.2.1 Broadband excitation

Broadband excitation was accomplished using an electrodynamic shaker. Due to the energy of the signal being spread over a wide frequency range, the broadband excitation deals a somewhat linearized FRF; this linearization can make it difficult to detect if a system behaves nonlinearly. Although the FRFs could be of little use, the time histories of both excitation and responses still carry all the information needed to the experimentalists in order to successfully characterise the nonlinear behaviour. These time histories were fed into the Reverse Path method [1] in order to characterise and localise the nonlinear elements (see section 3.2).

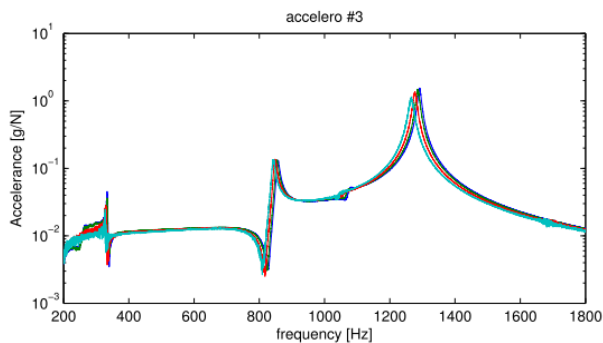


Figure 4 - FRF of ACC-3; torque 10 Nm

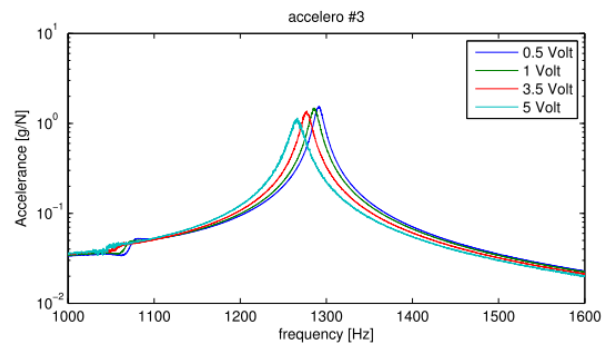


Figure 5 - Mode of interest; torque 10 Nm

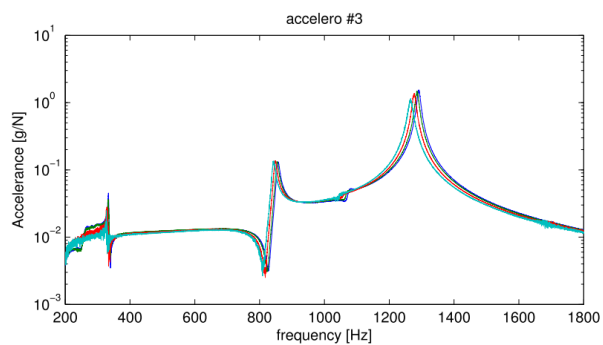


Figure 6 - FRF of ACC-3; torque 6 Nm

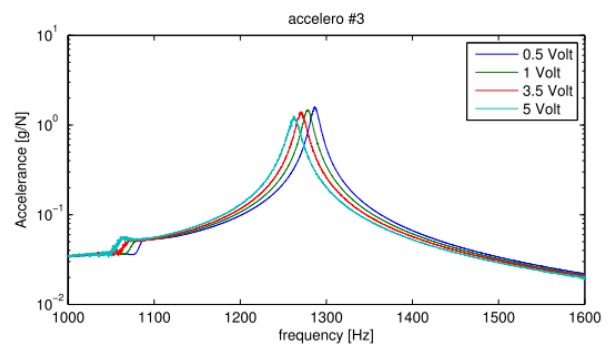


Figure 7 - Mode of interest; torque 6 Nm

Four excitation levels were used for measuring the FRFs. Figure 4 and Figure 6 show the FRF measured at 10 Nm and 6 Nm, respectively. Figure 5 and Figure 7 show a zoom around the mode of interest. Although

the small energy involved into the excitation, it is clear from the FRFs that the system is nonlinear due to the evident shift in frequency obtained increasing the amplitude levels. After a preliminary modal analysis, the third resonance peak was identified as being the main axial mode of interest. Another important matter is that the modes are sufficiently far from each other, thus making the application of modal techniques much more reliable.

2.2.2 Stepped sine

The test presented hereafter regards the stepped sine excitation, which was carried out at pure tone excitation in a frequency range between 1200 [Hz] and 1360 [Hz] with a frequency resolution of 0.2 [Hz]. As a result of this type of testing, the entire energy of the shaker is transferred to the structure in a single tone, leading to a more defined nonlinear behaviour.

This approach becomes time consuming when applied to a nonlinear structure: the complex interaction between the shaker and the test piece leads to the generation of undesired harmonic content in the excitation signal, which a standard controller could find difficult to account for. The stepped sine was carried out both increasing (up-sweep) and decreasing (down-sweep) the harmonic frequencies, in order to have a better representation of phenomena like jumps in the responses.

These campaigns were carried out using several excitation forces, which are listed in the Table 2. Figure 8 and Figure 9 show the FRFs measured for 6 Nm and 10 Nm, respectively.

	Force level [N]					
Up-sweep	1	3	7	10	15	20
Down-sweep	1	3	7	10	15	20

Table 2 - Test planning for stepped sine

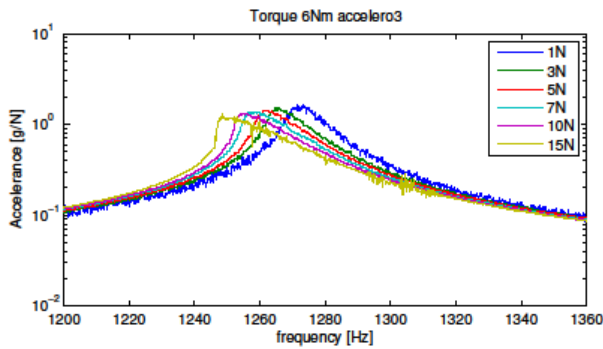


Figure 8 - Stepped sine test for torque 6Nm

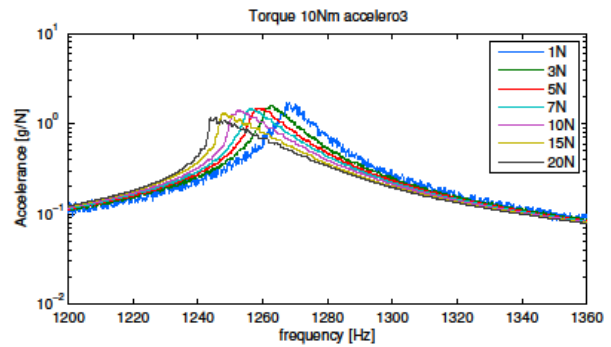


Figure 9 - Stepped sine test for torque 10Nm

The measured FRFs show a clear distortion of the response curves as the excitation force is increased and therefore demonstrating the amplitude dependency of this nonlinear phenomenon under study. Linear modal analysis would not be suitable for these types of FRFs but when it is carried out the results show a clear softening trend, as shown in Figure 10. The section will present a more rigorous analysis of the nonlinear responses.

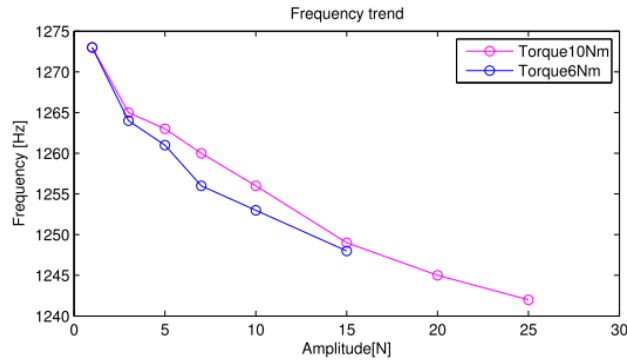


Figure 10 - Frequency trend from linear modal analysis

2.2.3 Impact excitation

Eventually, an impact test was performed in order to define the loss factor of the structure to use in the SDOF model, Figure 11 shows the envelope of the response, calculated via Hilbert transform. An exponential fit of the envelope was then performed (Figure 12) and a single damping coefficient retrieved. The exponential fit is presented in Figure 12. In this case f_0 is equal to 1235 [Hz] and this gives a value of the loss factor equal to $\zeta = 0.0051$.

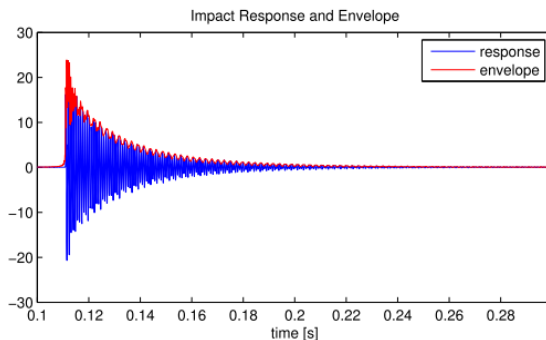


Figure 11 - Decay Envelope

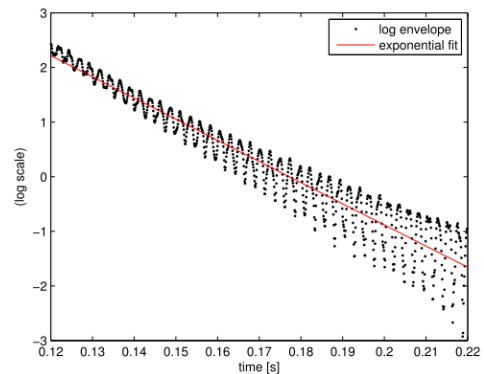


Figure 12 - Exponential fit

3 Nonlinear analysis and correlation

The main characterisation and localisation steps were performed using the Reverse Path Method. Bendat [3] first introduced this method in 1990 as a tool for random data analysis, and the authors included it into the nonlinear analysis toolbox after it proved good potentialities [2].

3.1 Modal space approach; nonlinear SDOF analysis

The modal space analysis will be accomplished by using the C-FRF method, which was developed by Lin [11] and eventually coded into ICATS modal analysis software [12]. Carrella et al used the same basic algorithm and developed a method for identification and quantification of nonlinearity in engineering structures; the code was refined and implemented in stand-alone code called CONCERTO [13]. The method is rather simple and works on the assumption that the mechanical system can be treated as freedom SDOF system. Clearly, there are not many mechanical systems as such and therefore the

assumption was relaxed by assuming that the modal contributions of nearby modes can be neglected. The mode of interest can be analysed as SDOF system. Figure 13 and Figure 14 show the results of the SDOF NL analysis for identification of the natural frequency of the dumbbell for increasing amplitude of vibrations. Figure 15 and Figure 16 show the results for the loss factor for 6 Nm and 10 Nm, respectively.

It is suspected that the modal contribution from nearby modes is showed by the offsets between the frequency-amplitude curves (see both plots, in Figure 13 and Figure 14) It was noticed that these offsets were cancelled when the SDOF analysis was carried out by using a smaller number of frequency points, such as the ones around the resonance peak, rather than the full frequency range (between 1200 and 1360Hz).

It would be expected that the natural frequency of the dumbbell for 10 Nm torque level would be higher than the ones for 6 Nm level, which presents an initial natural frequency higher than the one for the test at 10Nm. The reasons of this behaviour are not fully investigated yet.

The damping plots presented in Figure 15 and Figure 16 do not show clear trends.

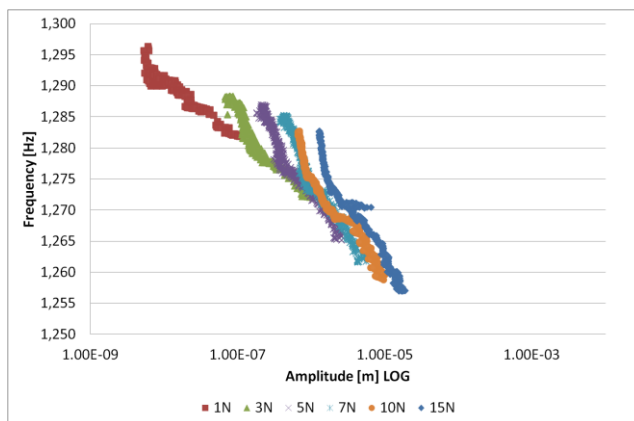


Figure 13 - Natural frequency trend for 6Nm

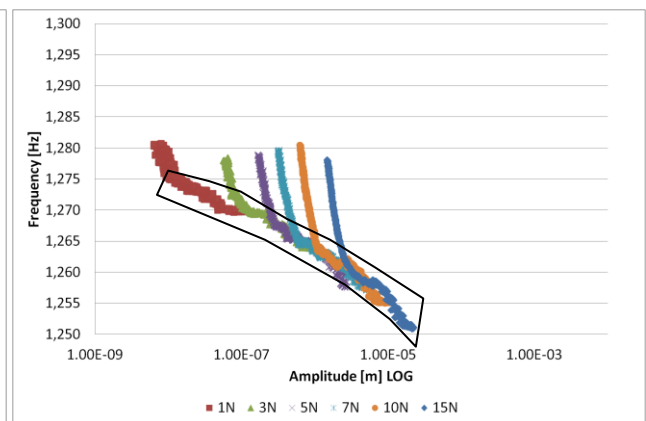


Figure 14 - Natural frequency trend for 10Nm

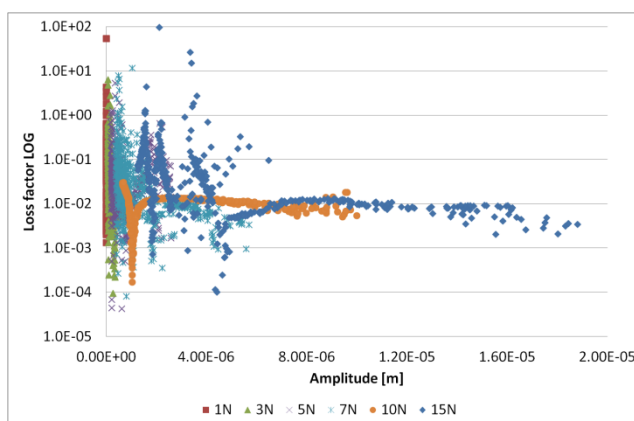


Figure 15 - Loss factor trend for 6Nm

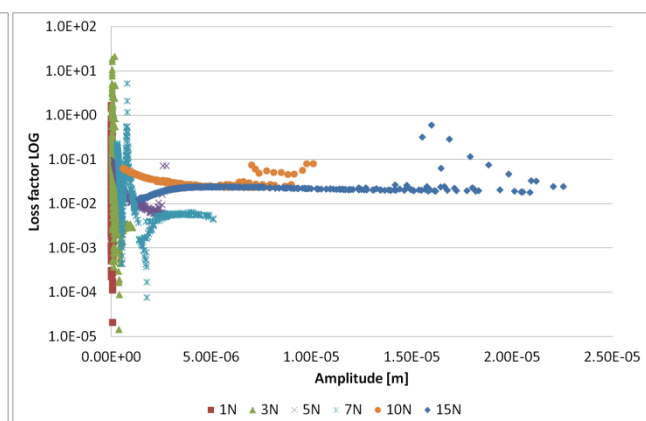


Figure 16 - Loss factor for 10Nm

3.2 Physical space approach

This section regards all the analyses involved in the right branch of the flowchart in Figure 2. It is clear from visual inspection of Figure 8 and Figure 9 that the response functions are distorted. A homogeneity analysis was performed to quantify the loss of coherence between the first FRF at 1 N and the last one at

20 N. The selection of the first FRF is based on the fact that it shows the least curve distortion, if compared with all others. Figure 17 shows the result of the analysis.

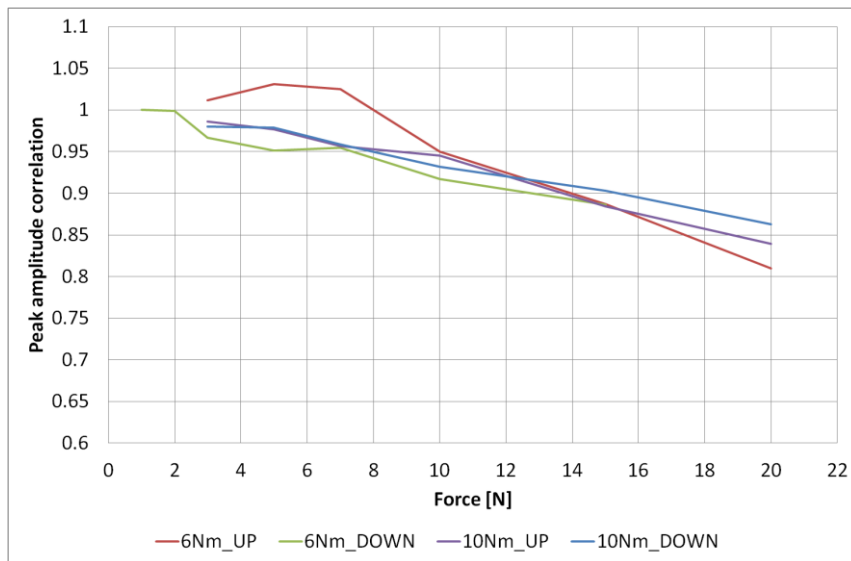


Figure 17 - Homogeneity plot

The localisation step is performed via Reverse Path method. Treating the nonlinearities as feedback terms acting on an underlying linear system, the concept is to search for a series of nonlinear functions that maximize the coherence of this linear system. Figure 18 shows the coherence error associated with all the potential nonlinear locations. It is important to notice that the error is minimised when the nonlinearity is acting between the two cylindrical masses.

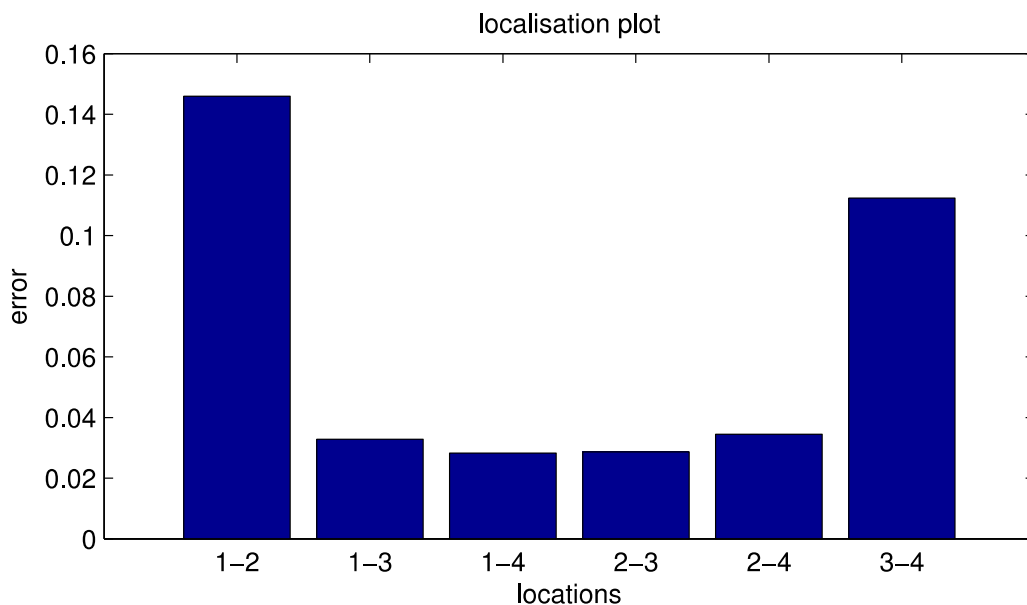


Figure 18 – Reverse Path method, localisation plot.

Once the nonlinear element has been localised, it is possible to seek for a nonlinear relationship that will improve the coherence of the underlying linear model. To do so, a monomial model with variable exponent has been fed to the system and the resulting coherences calculated. Figure 19 shows the results of this process: one can notice that the best result is achieved with an exponent of 2.1.

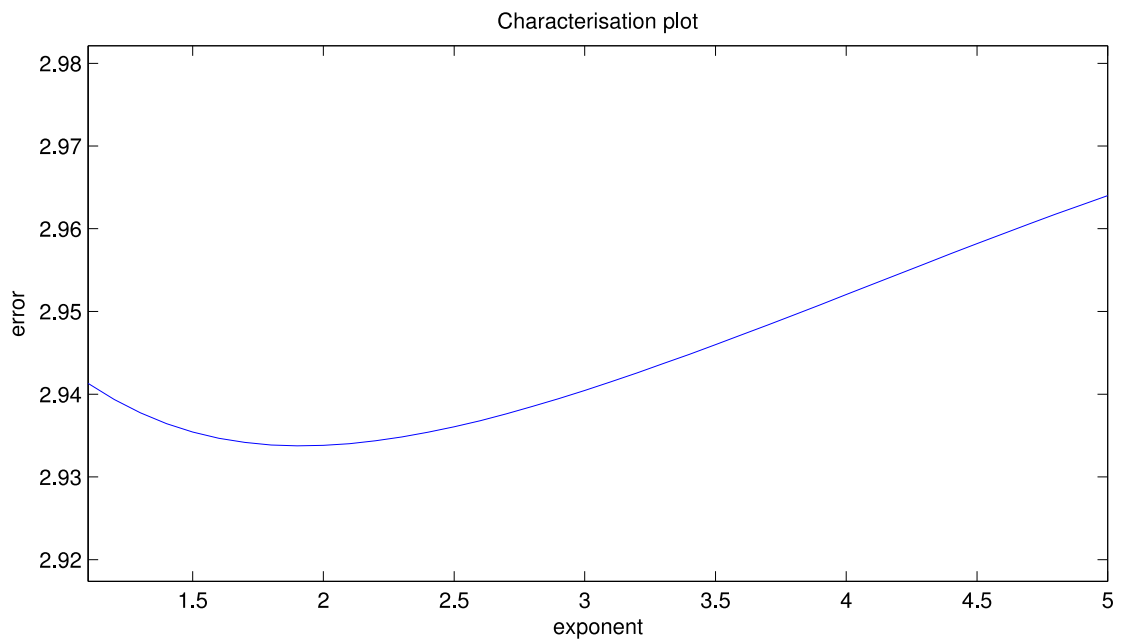


Figure 19 – Reverse Path method, characterisation plot

Finally, Figure 20 shows the real part of the coefficient retrieved using the information of the underlying linear system. This coefficient is just a rough approximation of the real one, but it serves as a starting value upon which a model could be updated.

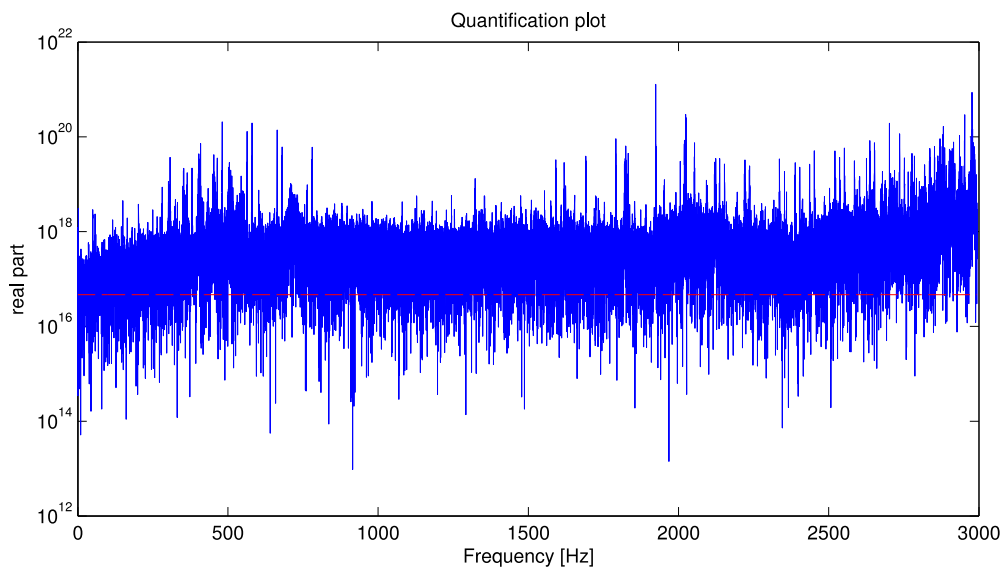


Figure 20 – Reverse Path method, quantification plot

3.3 Correlation

Simulations of FRFs were produced using the classical equation of motion with the previously identified nonlinear term, shown as follows:

$$m\ddot{x} + c\dot{x} + kx + k_{NL}\text{sign}(x)|x|^{2.1} = F\cos\Omega t \quad (1)$$

Where $m=4$ Kg, $c=645$ Ns/m, $k= 2.67452E+8$ N/m and $k_{NL}= -1E+17$ N/m^{2.1}. All simulations were performed using the same levels of excitation force used for the experimental measurements (see Table 2) assuming a sweep-up configuration. The simulations for single tone excitation were performed by means of Runge-Kutta fixed time integration. Amplitude and phase of the each time response were calculated so as to produce a stepped sine FRF. Both the theoretical and experimental response functions are plotted in

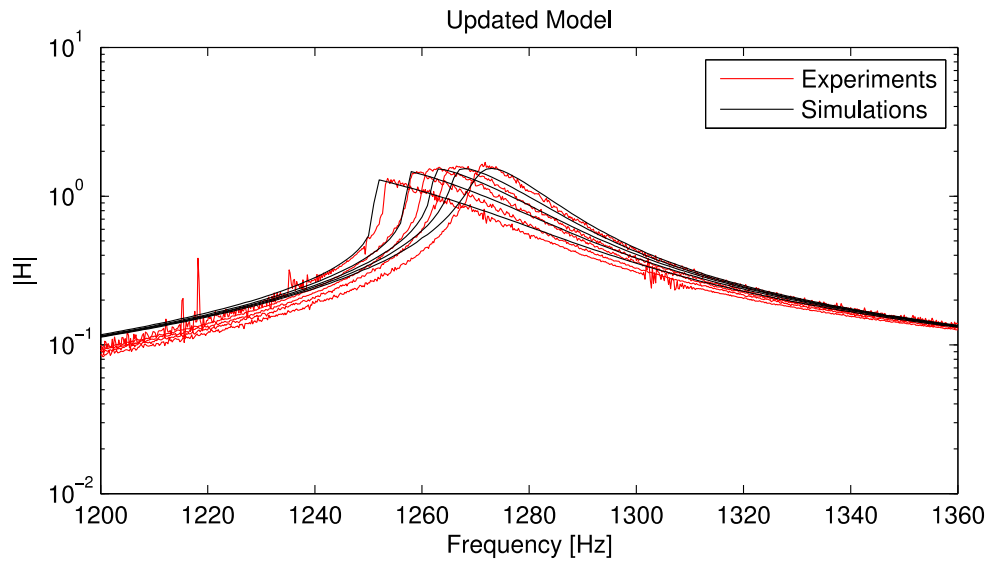


Figure 23.

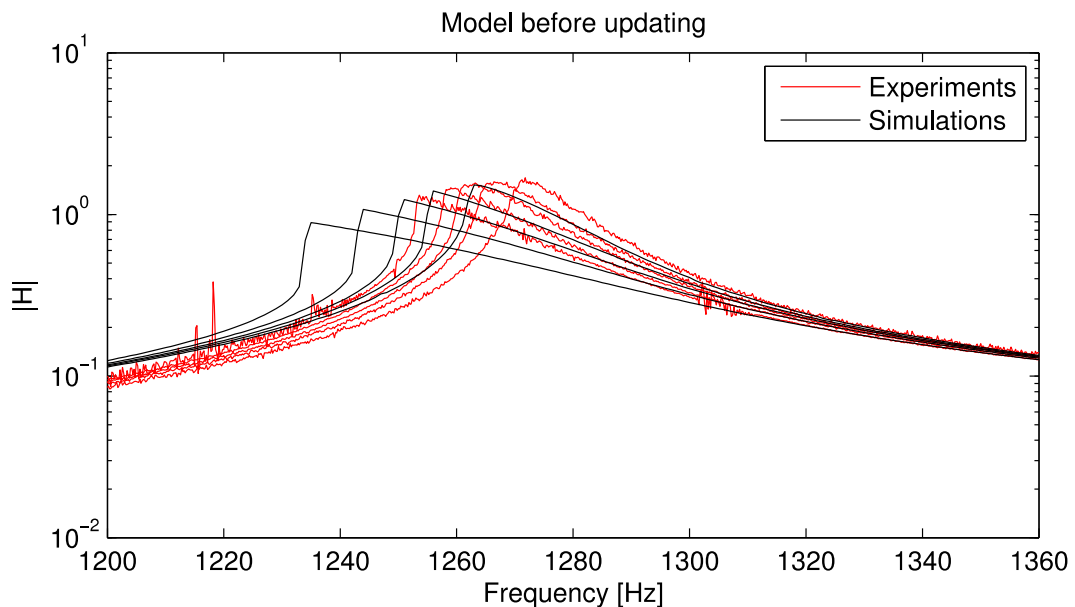


Figure 21 - Measured and simulated FRFs for the SDOF model

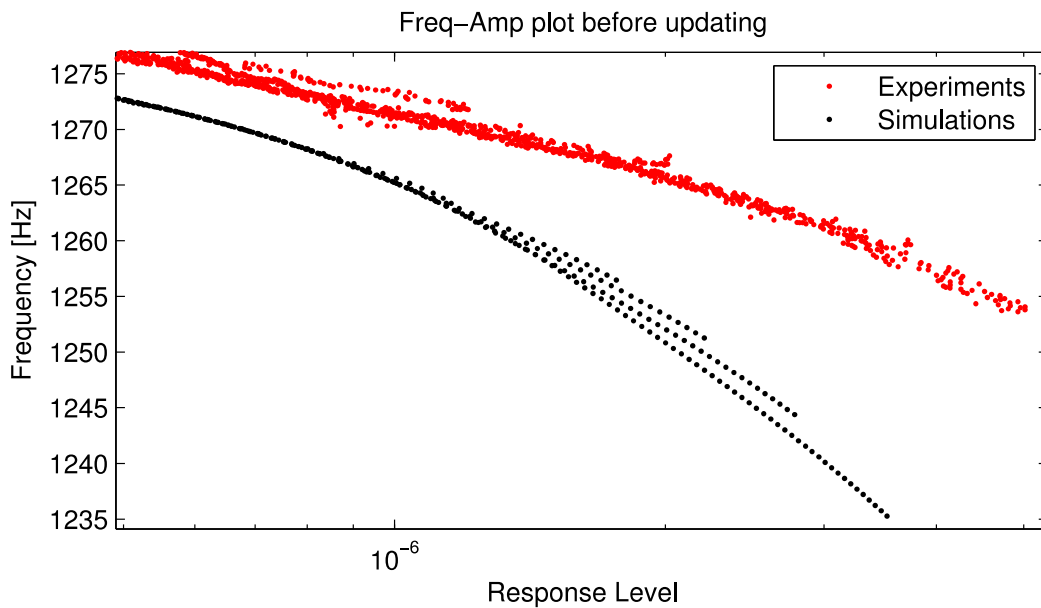


Figure 22 - Frequency-Amplitude plot for the SDOF model

The final step was to correlate the nonlinearity identified in the physical space by the Reverse Path method and the one identified in the modal space by the C-FRF method. In order for this correlation to happen, the FRFs retrieved via nonlinear simulations have been processed with the same C-FRF method, dealing frequency-amplitude plots. Figure 22 (in log scale) shows the result: it is clear that the model requires some updating process in order to match the experimental tests.

3.4 Updating

Since the weak link in the analysis chain is to be found in the quantification step of the Reverse Path method, the estimated coefficient k_{NL} has been chosen as the only parameter to update.

As one can notice from Figures 23 and 24, the updating step successfully turns the model into a more reliable representation of the physical phenomenon, setting the k_{NL} coefficient to a value of $-2E+12$.

The authors believe that by including a more sophisticated model – like a polynomial one – the nonlinear behaviour can be even better described.

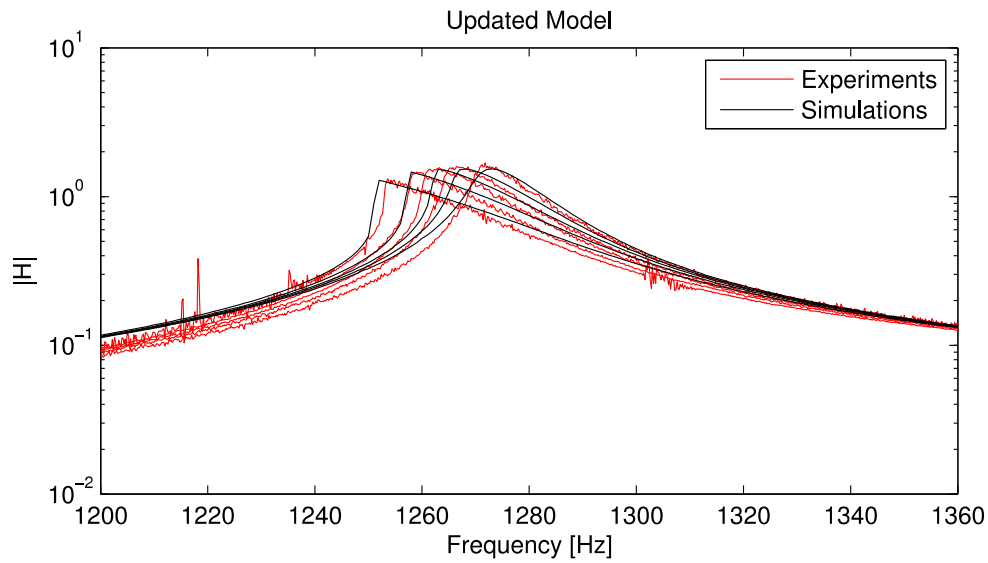


Figure 23 Measured and simulated FRFs for the updated model

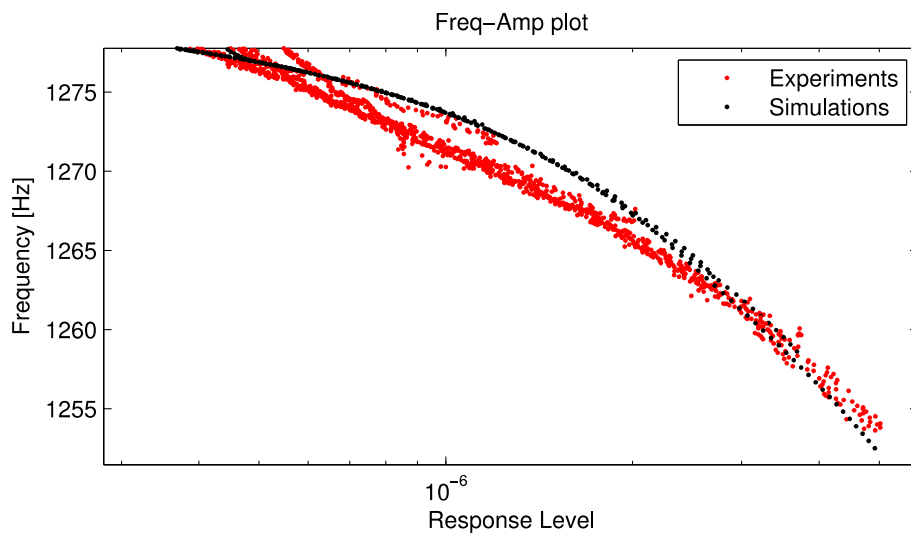


Figure 24 – Frequency-Amplitude plot for the updated model

4 Conclusions and future work

This paper presented for the first time a research work focussed on the correlation between physical and modal space when modal testing is performed on nonlinear systems.

A lap-joint test structure was designed and made with two weights either ends of the bolted structure. The structure was tested using several excitation methods and both spatial and modal information were captured from the specimen.

Nonlinear identification was focussed on the stiffness term only; the damping was temporary neglected. The nonlinear term was identified in the modal space as a relationship between resonant frequency and amplitude of vibrations. The same identification was carried out in the physical space by using the Reverse Path method. This latter term was used for simulating frequency response functions which were compared with the measured ones. A C-FRF analysis was performed on the simulated data and the frequency-amplitude relationship was obtained and compared with the same of the experimental

measurements. Lastly, an updating process was required in order to tune the coefficients and match the experimental tests.

This exercise helped to clear the path towards the application of this novel approach to the modal testing of nonlinear structures, without losing sight of the overall objective of this kind of analysis: to get a reliable mathematical model of structures that behave outside of the linear standards.

The industry has the need to model nonlinear structures in a systematic and reliable way, and the authors believe that this approach may prove valuable once refined and applied with the aid of more powerful tools like FE modelling and updating techniques, which represents the industry standard in structural dynamics.

References

- [1] M.Gooper. (1985). *Microslip and macroslip in bolted joints*. Experimental mechanics 25.
- [2] C.F.Beards. (1983). *The damping of structural vibration by controlled interfacial slip in joints*. Journal of Vibration, Acoustics, Stress and Reliability in Design.
- [3] L. Gaul, J. (1997). *Nonlinear dynamics of structures assembled by bolted joints*. Acta Mechanica.
- [4] L.E.Goodman. (1980). *Contributions of continuum mechanics to the analysis of the sliding of unlubricated solids*. AMD (Symposium Series of the ASME Applied Mechanics Division).
- [5] Daniel J. Segalman, D. L. (2009). *Handbook on Dynamics of Jointed Structures*. Sandia National Laboratories.
- [6] Jeong Kim, Y. (2006). *Finite Element Analysis and modelling of structure with bolted joints*.
- [7] H. Ahmadian, M. Ebrahimi, J.E. Mottershead, M.I. Friswell (2002). *Identification of bolted-joint interface model*. Proceedings of ISMA2002 - volume IV p. 1741-1748.
- [8] A. Delli Carri and D. Ewins (2014). *Assessment and Validation of Nonlinear Identification Techniques Using Simulated Numerical and Real Measured Data*, Top. Modal Anal. II, Vol. 8.
- [9] A. Delli Carri and D. Ewins (2014). *A Systematic Approach to Modal Testing of Nonlinear Structures*, Top. Modal Anal. Vol. 7.
- [10] J. S. B. and A. G. Piersol (2000). *Random Data Analysis and Measurement Procedures*, Measurement Science and Technology, vol. 11. pp. 1825–1826.
- [11] R. Lin, *Identification of the dynamic characteristic of nonlinear structures*, Ph.D. Thesis, Department of Mechanical Engineering, Imperial College, London, 1990.
- [12] ICATS, modal analysis software, ICON Suite, 58 Prince's Gate Exhibition Road, London SW7 2PG.
- [13] Carrella, A., Ewins, D. J., *Identifying and quantifying structural nonlinearities in engineering applications from measured frequency response functions*, Mechanical Systems and Signal Processing 25 (2011).