OPEN ACCESS

University of BRISTOL

Peer reviewed version

Link to published version (if available):
10.1109/CONTROL.2012.6334614

Link to publication record in Explore Bristol Research
PDF-document

University of Bristol - Explore Bristol Research
General rights

An Adaptive Observer-based parameter estimation algorithm with application to Road Gradient and Vehicle's Mass Estimation

Muhammad Nasiruddin Mahyuddin*, Jing Na**, Guido Herrmann***, Xuemei Ren**** and Phil Barber*****

Abstract—A novel observer-based parameter estimation algorithm with sliding mode term has been developed to estimate the road gradient and vehicle weight using only the vehicle's velocity and the driving torque from the engine. The estimation algorithm exploits all known terms in the system dynamics and a low pass filtered representation to derive an explicit expression of the parameter estimation error without measuring the acceleration. The proposed algorithm which features a sliding-mode term to ensure the fast and robust convergence of the estimation in the presence of persistent excitation is augmented to an adaptive observer and analyzed using Lyapunov Theory. The analytical results show that the algorithm is stable and ensures finite-time error convergence to a bounded error even in the presence of disturbances. A simple practical method for validating persistent excitation is provided using the new theoretical approach to estimation. This is validated by the practical implementation of the algorithm on a small-scaled vehicle, emulating a car system. The slope gradient as well as the vehicle's mass/weight are estimated online. The algorithm shows a significant improvement over a previous result.

I. INTRODUCTION

In the automotive industry, reliable online vehicle parameter estimation is important to reduce emissions, improve fuel efficiency and enhance the safety of the vehicle. The vehicle's mass and the road grade are two parameters that largely influence a vehicles performance. This is particularly true for heavy duty vehicles where the loadings due to the mass and the grade can be significant [1]. The road gradient and mass estimation provides useful information to a vehicle in improving the transmission shift scheduling and vehicle longitudinal control, including cruise control, hill holding and traction control [2].

Having road inclination measured by a dedicated sensor such as an inclinometer may be inaccurate. Inclinometers are in fact accelerometers which can distort road gradient measurement due to its susceptibility to noise in dynamic conditions of a vehicle [3]. Therefore, the road grade should be accurately estimated [4-7]. It is evident that the transmission control unit and the anti-lock brake system can benefit from mass and road gradient estimates [7]. To address this issue, there has been significant interest in the estimation of the road gradient and vehicle mass [2-9]. Some of these results were developed based on on-board sensors and the use of

*Muhammad Nasiruddin Mahyuddin is currently a PhD student in the Department of Mechanical Engineering, University of Bristol, BS8 1TR, UK. and being sponsored by University Sains Malaysia. Email: memnm@bristol.ac.uk or nasiruddin@ieee.org

**Jing Na is with Plant Engineering Division, ITER Organization, St Paul Lez Durance, 13115, France. Email: najing25@163.com

***Guido Herrmann is a Senior Lecturer with the Department of Mechanical Engineering, University of Bristol, BS8 1TR, UK. Email: g.herrmann@bristol.ac.uk

****Xuemei Ren is a Professor at School of Automation, Beijing Institute of Technology, Beijing, 100081,China, E-mail: xmren@bit.edu.cn

*****Phil Barber is with Jaguar and Land Rover Research, W/2/021 Engineering Centre, Abbey Road, Whitley, Coventry CV 4LF, UK. E-mail: pbarber2@jaguarlandrover.com

sensor/data fusion methods. For instance, in [1], a GPS or barometer sensor is utilized in addition to torque and velocity sensors to obtain absolute road height information, while Barrho, et al. in [9] require accurate information of the vehicle mass which is not always possible. In recent work by [10], [11] and [12], low-cost sensors were placed in the vehicle to estimate the road grade ahead. In [13], the vehicle's mass in addition to vehicle speed and torque information is required. Bae et al. [4] suggest a recursive least squares approach which essentially requires acceleration information and then assumes the existence of sufficient data points to solve for the missing parameters, i.e. vehicle mass and gradient, by inverting a regressor matrix in a batch process. Similarly, the work in [7] and [2] estimate the road grade using the position, velocity and driving torque or force signal. In [8], a combination of an observer is suggested which provides for known mass the exact estimation of the road gradient. In all of the aforementioned approaches, however, some of the required information, e.g. acceleration, mass and vehicle's current location through GPS, may not be readily available. Although most of the approaches show good results, the convergence speed and complexity may cast some problems.

In this paper, we revisit the online road gradient and mass estimation of vehicular systems using only the vehicle's velocity and the driving torque. This is achieved based on a novel adaptive nonlinear observer design. Compared to previous results (e.g. [14]) concerning the parameter estimation, some appropriate information of the parameter error is derived, and then incorporated into the parameter adaptation for the observer design. In our work, the parameter estimation scheme uses a filtered regressor matrix. Measurable system states, a regressor vector and the known dynamics are collected and filtered to form auxiliary variables. Moreover, vehicle acceleration is not required in our estimation algorithm. Owed to the special feature of a sliding mode term, the adaptation algorithm guarantees robust finite-time convergence to a compact set, provided that there is a Persistent Excitation (PE) condition fulfilled so that the regressor matrix remains positive definite. In contrast to [15], our scheme calculates the inverse of the filtered and integrated regressor matrix without prior invertibility checking of the matrix and direct matrix inverse computation. The parameter error information can be explicitly formulated by virtue of the filtered auxiliary variables. The possible instability and infinite growth found in [15] due to the existence of an unstable integrator (as a result of auxiliary matrix and vector) are prevented in this paper. We also show robustness of our adaptive scheme and *we can verify* the PE condition in a straightforward and practical manner. The proposed method is verified experimentally in a reduced-scale vehicular system, which provide a significant improvement over a previous algorithm.

II. SYSTEM FORMULATIONS

Consider a nonlinear system of the following structure:

$$\dot{x} = Ax + B_1 u_1 + B_2 f(x, u_2) + \zeta, \quad y = Cx \qquad (1)$$

where $A \in \mathbb{R}^{n \times n}$ is the known system matrix, $B_1 \in \mathbb{R}^{n \times m_1}$ and $B_2 \in \mathbb{R}^{n \times m_2}$ are known input matrices, $u_1 \in \mathbb{R}^{m_1}$ and $u_2 \in \mathbb{R}^{\bar{m}_2}$ are known inputs, whilst $C \in \mathbb{R}^{p \times n}$ is the corresponding output matrix and $\zeta \in L_\infty$ is bounded disturbance. The function $f(x, u_2) : \mathbb{R}^n \times \mathbb{R}^{\bar{m}_2} \to \mathbb{R}^{m_2}$ is partially unknown for which the detail will be outlined below and the pair (A, B_1) is controllable. It is assumed that $p \geq m_2$. The following assumptions are made:

Assumption 1 (C, A, B_2) is minimum phase and (CB_2) is full rank.

Assumption 2 The function $f(x, u_2)$ can be represented in a linear parameterized form: $f(x, u_2) = \varphi(x, u_2)\Theta$, where $\varphi : \mathbb{R}^n \times \mathbb{R}^{m_2} \to \mathbb{R}^{m_2 \times l}$ is a known Lipschitz continuous function, while $\Theta = const., \Theta \in \mathbb{R}^l$ is an unknown parameter vector which is to be estimated.

Assumption 3 The signals x, u_1 and u_2 are measurable and bounded.

Assumption 3 is a common assumption for observer design and can be easily achieved by suitable choice of the control signal u_1 (e.g. [15] [17]).

Under these conditions, the system is assumed to take the following structure

$$
\begin{array}{rcl}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} & = & \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \\
& & \begin{bmatrix} B_{11} \\ B_{12} \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ \bar{B}_2 \end{bmatrix} \varphi\Theta + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} \\
y & = & \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\end{array} \quad (2)
$$

where $\bar{B}_2 \in \mathbb{R}^{p \times m_2}$, $I \in \mathbb{R}^{p \times p}$ and $x_2 = Cx$. Note that this reformulation is always possible from Assumptions 1 and 3 using Proposition 6.3 in [17]. Moreover, we make the following assumption.

Assumption 4 $A_{21} = 0$, the second state equation in 2 is decoupled.

Assumption 4 is possibly a strong assumption but it will fit the generic practical system structures (e.g. vehicular) investigated in this paper.

III. ADAPTIVE OBSERVER DESIGN

We will design an adaptive observer to estimate the state vectors which will be suitably combined with a novel parameter estimation algorithm. The adaptive observer takes the following form:

$$
\dot{\hat{x}} = A\hat{x} + B_1 u_1 + B_2 \varphi\hat{\Theta} + L(y - C\hat{x}) \quad (3)
$$

where $\hat{x}$ is the estimated state vector, $\hat{\Theta}$ is the estimated parameter vector. L is the observer gain matrix such that $A_c = A - LC$ is a stable matrix and there exist, according to Proposition 6.3 in [17], positive definite matrices, P and Q so that,

$$
A_c^T P + P A_c = -Q, \quad P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0 \quad (4)
$$

$$
Q = \begin{bmatrix} Q_1 & Q_{12} \\ Q_{12}^T & Q_2 \end{bmatrix} > 0, \quad PB_2 = C^T F^T \quad (5)
$$

and $F \in \mathbb{R}^{m_2 \times p}$ is a positive definite matrix. From (4), it follows,

$$
\begin{bmatrix} A_{c11}^T P_1 + P_1 A_{c11} & P_1 A_{c12} \\ A_{c12}^T P_1 & P_2 A_{c22} + A_{c22}^T P_2 \end{bmatrix} = -Q \quad (6)
$$

Let $\tilde{x} = x - \hat{x}$ and $\tilde{\Theta} = \Theta - \hat{\Theta}$. We can then use (1) and (3) to define the error dynamics $\tilde{x} = x - \hat{x}$ as

$$
\begin{array}{rcl}
\dot{\tilde{x}} & = & (A - LC)\tilde{x} + B_2 \varphi\tilde{\Theta} + \zeta \\
& = & A_c \tilde{x} + B_2 \varphi\tilde{\Theta} + \zeta
\end{array} \quad (7)
$$

where $\tilde{\Theta} = \Theta - \hat{\Theta}$ is the estimated parameter error vector.

In the next section, the adaptive laws that will update the estimated parameter vector, $\hat{\Theta}$ will be developed.

IV. ADAPTIVE LAW FORMULATION

In this section, we shall define the adaptive law for our parameter estimator.

A. Filter design

From (2), the second state equation can be expressed as,

$$
\dot{x}_2 = (A_{22}x_2 + B_{12}u_1) + \bar{B}_2 \varphi\Theta + \zeta_2 \quad (8)
$$

Let,

$$
\psi = A_{22}x_2 + B_{12}u_1, \quad \phi = \bar{B}_2 \varphi \quad (9)
$$

then, the following filtered variables can be defined as,

$$
\begin{array}{rclcl}
k\dot{x}_{2f} + x_{2f} & = & x_2, & x_{2f}(0) = 0 \\
k\dot{\psi}_f + \psi_f & = & \psi, & \psi_f(0) = 0 \\
k\dot{\phi}_f + \phi_f & = & \phi, & \phi_f(0) = 0
\end{array} \quad (10)
$$

In addition, we may introduce an auxiliary filter for the bounded disturbance (which is only used for analysis),

$$
k\dot{\zeta}_{2f} + \zeta_{2f} = \zeta_2, \quad \zeta_{2f}(0) = 0 \quad (11)
$$

i.e. $\zeta_{2f} \in L_\infty$. Consequently, we can obtain from (8) and (10) that,

$$
\dot{x}_{2f} = \frac{x_2 - x_{2f}}{k}, \qquad \frac{x_2 - x_{2f}}{k} - \psi_f = \phi_f \Theta + \zeta_{2f} \quad (12)
$$

B. Auxillary integrated regressor matrix and vector

The filtered variables introduced above will be used in the definition of a filtered regressor matrix, $M(t)$, and a vector, $N(t)$ as,

$$
\dot{M}(t) = -k_{FF}M(t) + k_{FF}\phi_f^T(t)\phi_f(t), M(0) = 0 \quad (13)
$$

$$
\dot{N}(t) = -k_{FF}N(t) + k_{FF}\phi_f^T(t)\left(\frac{x_2 - x_{2f}}{k} - \psi_f\right), \quad (14)
$$

where, $k_{FF} \in \mathbb{R}^+$, can be implemented as a forgetting factor and the initial condition of $N(t)$ is $N(0) = 0$. Note that (14) is equivalent to:

$$
\dot{N}(t) = -k_{FF}N(t) + k_{FF}\phi_f^T(t)(\phi_f(t)\Theta + \zeta_{2f}) \quad (15)
$$

Consequently, we can find the solution to (13), (14) and (19),

$$
\begin{array}{l}
M(t) = \int_0^t e^{-k_{FF}(t-r)} k_{FF}\phi_f^T(r)\phi_f(r)dr \\
N(t) = \int_0^t e^{-k_{FF}(t-r)} k_{FF}\phi_f^T(r)\left(\frac{x_2 - x_{2f}}{k} - \psi_f\right)dr
\end{array} \quad (16)
$$

and

$$
N(t) = M(t)\Theta + \zeta_{2N} \quad (17)
$$

where $\zeta_{2N} = \int_0^t e^{-k_{FF}(t-r)} k_{FF}\phi_f^T(r)\zeta_{2f}dr$. Note that ϕ is bounded since it is Lipschitz continuous and x, u_2 are bounded (Assumption 1). Thus, ϕ_f is bounded. Since $\zeta_{2f} \in L_\infty$, it follows that $N(t)$, $M(t)$ and ζ_{2N} are bounded.

Lemma 1: The auxiliary regressor matrix $M(t) \in \mathbb{R}^{l \times l}$ is positive definite, $M(t) > 0$, if and only if $\int_0^t \phi_f^T \phi_f > 0$. •

Proof : It can be easily shown that

$$\int_T^t \phi_f^T(r)\phi_f(r)dr \geq \int_T^t e^{-k_{FF}(t-r)}\phi_f^T(r)\phi_f(r)dr \quad (18)$$

$$\geq e^{-k_{FF}t}\int_T^t \phi_f^T(r)\phi_f(r)dr$$

when $T < t$. For $T = 0$, the claim follows. ∎

Thus, if ϕ_f is persistently excited, $M(t) > 0$ is positive definite. Clearly, if ϕ is persistently excited, then ϕ_f is also persistently excited and $M(t) > 0$ [22], [20] (as derived from the linear system (10) and definition (9)). Thus, if ϕ is persistently excited then $M(t) > 0$ and $\int_T^t \phi_f^T \phi_f > 0$. In this paper, it is important to achieve $M(t) > 0$ for our adaptation algorithm to work. This can be achieved through persistent excitation of ϕ:

Remark 1: The Persistent Excitation (PE) condition for the regressor ϕ can be achieved in the experiment through an appropriate control signal, u. For instance, the control signal can be augmented by a noise signal or the controller can introduce for the system states, x, a tracking demand which achieves 'sufficient richness' (SR) of x and guarantees $M(t) > \lambda_m I, \lambda_m > 0$ as in [19]. Suitable analytical detail is avoided here due to space reasons. ○

Another auxillary matrix $K(t)$ may be defined as,

$$\dot{K}(t) = k_{FF}K(t) - k_{FF}K(t)\phi_f^T(t)\phi_f(t)K(t), \quad (19)$$

where the initial condition $K(0) > 0$ is specified as a diagonal matrix, $K(0) = \frac{1}{\lambda}I$ with $\lambda > 0$ being constant. It will be seen that $K(t)$ is an approximation of the inverse of $M(t)$ where $\lim_{t \to \infty} K(t)M(t) = I$. With the help of the following derivative matrix identity,

$$\frac{d}{dt}KK^{-1} = K\frac{d}{dt}K^{-1} + K^{-1}\frac{d}{dt}K = 0 \quad (20)$$

we can obtain,

$$K(t) = [e^{-k_{FF}t}K^{-1}(0) + M(t)]^{-1} \quad (21)$$

This also implies boundedness of $K(t)$, if $M(t) > 0$. To show the invertibility of $K(t)$ as well as $K(t)M(t)$ approaches unity, we are to employ the singular value decomposition for the matrix, $M(t)$,

$$M(t) = U(t)S(t)V^T(t) \quad (22)$$

where $S(t) = diag(s_1, \ldots, s_n)$ is the matrix with s_i being the singular values of matrix, $M(t)$ whilst, $U(t)$ and $V(t)$ are unitary matrices.

We know that $K(0) = \frac{1}{\lambda}I$ is a diagonal matrix, thereby,

$$K(t) = V(t)(S(t) + e^{-k_{FF}t}\lambda I)^{-1}U^T(t) \quad (23)$$

Then,

$$K(t)M(t) = V(t)diag(\frac{s_1}{s_1 + e^{-k_{FF}t}\lambda}, \cdots, \frac{s_n}{s_n + e^{-k_{FF}t}\lambda})V^T(t) \quad (24)$$

and $\lim_{t \to \infty} \frac{s_i}{s_i + e^{-k_{FF}t}\lambda} = 1, \quad if \quad M(t) \geq \lambda_m I > 0,$ for $\lambda > 0$, adhering to the Persistent Excitation (PE) condition [14]. Since $V(t)$ is unitary, the matrix $K(t)M(t)$ can be represented as,

$$K(t)M(t) = I - \Delta(t) \quad (25)$$

where Δ converges to zero in infinite time. This shows that $K(t)$ is indeed a representation of the inverse of $M(t)$ where $\Delta(t)$ denotes the effects of the initial condition $K(0)$. Hence, the parameter estimation error vector can be written as,

$$\tilde{\Theta} = \Theta - \hat{\Theta} = [K(t)M(t) + \Delta(t)]\Theta - \hat{\Theta} \quad (26)$$

where $\lim_{t \to \infty} \Delta(t) \to 0$

Remark 2: Note that a practical test for $M(t) > 0$ is to verify in an experiment if $K(t)M(t) \approx I$ holds. This implies non-singularity of $K(t)$ and $M(t)$. Again, this condition can be achieved through PE of ϕ (see Remark 1) which can be verified experimentally as will be seen in Section VII on Practical Application Results. ○

C. Parameter Estimation

We shall write our adaptive law as,

$$\dot{\hat{\Theta}} = \Gamma[\varphi^T F(y - C\hat{x}) - \Omega R(t)] \quad (27)$$

In (27), Γ and Ω are positive definite and diagonal design matrices, i.e. $\Gamma = diag(\gamma_1, \ldots, \gamma_l)$ and $\Omega = diag(\omega_1, \ldots, \omega_l)$ respectively. The term $R(t)$ contains a sliding mode type term to ensure fast parameter convergence,

$$R(t) = \Omega_1 \frac{\hat{\Theta} - K(t)N(t)}{\delta + \left\|\hat{\Theta} - K(t)N(t)\right\|} + \Omega_2(\hat{\Theta} - K(t)N(t)) \quad (28)$$

where Ω_1 and Ω_2 are diagonal positive definite matrices, whilst δ is a positive constant. It will be proven that the parameter error matrix, $\tilde{\Theta}$, converges to a small residual set around zero, $\left\|\tilde{\Theta}\right\| \leq c$, in finite time, where $c > 0$ is a positive constant.

Remark 3: Compared to previous results (i.e. the parameter adaptation is only driven by the observer error in (27)), the extra term $R(t)$ taking parameter error information, $\hat{\Theta} - K(t)N(t)$ is employed, which could enhance the parameter convergence performance [21]. In particular, we incorporate the sliding mode technique in (28) such that the finite-time convergence to a set of ultimate boundedness is guaranteed as stated in the next section. ○

V. STABILITY AND PERFORMANCE

Theorem 1: Given a system (1), which satisfies Assumption 1-4, an adaptive observer (3) with adaptation law (27) using (13) - (19), (28) can be designed for persistently excited ϕ (9) so that the unknown parameter vector Θ can be estimated via $\hat{\Theta}$ within finite time satisfying an ultimate bounded stability characteristic for $\tilde{\Theta}$ and the estimated state $\tilde{x}$. The set of ultimate boundedness can be arbitrarily small for $\zeta = 0$. ◇

Proof : The following Lyapunov candidate shall be employed,

$$V(t) = \frac{1}{2}\tilde{x}^T P\tilde{x} + \frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta} \quad (29)$$

For ease of analysis, we shall decompose (29) as,

$$V(t) = \frac{1}{2}\tilde{x}_1^T P_1\tilde{x}_1 + \frac{1}{2}\tilde{x}_2^T P_2\tilde{x}_2 + \frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta} = V_1 + V_2 + V_3 \quad (30)$$

We now analyse the functions of $V_1 = \frac{1}{2}\tilde{x}_1^T P_1\tilde{x}_1$ and $\tilde{V} = V_2 + V_3 = \frac{1}{2}\tilde{x}_2^T P_2\tilde{x}_2 + \frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta}$ separately for convenience. The derivative of $\tilde{V} = \frac{1}{2}\tilde{x}_2^T P_2\tilde{x}_2 + \frac{1}{2}\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta}$ can be verified as,

$$\dot{V} = \frac{1}{2}[\tilde{x}_2^T P_2(A_{c22}\tilde{x}_2 + \bar{B}_2\varphi\tilde{\Theta}) + (A_{c22}\tilde{x}_2 + \bar{B}_2\varphi\tilde{\Theta})^T P_2\tilde{x}_2] + \tilde{\Theta}^T\Gamma^{-1}\dot{\tilde{\Theta}} + 2\tilde{x}_2^T P_2\zeta_2$$

Using (27), it follows:

$$\dot{V} = -\frac{1}{2}\tilde{x}_2^T Q_2\tilde{x}_2^T + \tilde{x}_2^T P_2\bar{B}_2\varphi\tilde{\Theta} \qquad (31)$$
$$-\tilde{\Theta}^T\Gamma^{-1}[\Gamma(\varphi^T FC\tilde{x} - \Omega R(t))] + 2\tilde{x}_2^T P_2\zeta_2$$

The observer error, $C\tilde{x}$ can be written as $\tilde{x}_2$ from (2), $Cx = x_2$. From (5), knowing that $P_2 B_2 = (FC)^T$, equation (31) can be further simplified as,

$$\dot{V} = -\frac{1}{2}\tilde{x}_2^T Q_2\tilde{x}_2^T + \tilde{\Theta}^T\Omega R(t) + 2\tilde{x}_2 P_2\zeta_2 \qquad (32)$$

Taking care of the diagonal positive definite matrices, i.e. $\tilde{\Omega}_1 = \Omega\Omega_1$ and $\tilde{\Omega}_2 = \Omega\Omega_2$ with $\zeta_{2KN} = K(t)\zeta_{2N}$ for ease of analysis, equation (32) can be written with the sliding-mode term, $R(t)$ using (26),

$$\dot{V} = -\frac{1}{2}\tilde{x}_2^T Q_2\tilde{x}_2 + (\Theta - \hat{\Theta})^T\tilde{\Omega}_1\frac{\hat{\Theta} - K(t)N(t)}{\delta + \|\hat{\Theta} - K(t)N(t)\|}$$
$$+(\Theta - \hat{\Theta})^T\tilde{\Omega}_2[\hat{\Theta} - K(t)N(t)] + 2\tilde{x}_2 P_2\zeta_2$$
$$\leq -\frac{1}{2}\tilde{x}_2^T Q_2\tilde{x}_2 - \lambda_{min}(\tilde{\Omega}_1)\frac{\|\hat{\Theta} - K(t)N(t)\|^2 + \delta\|\hat{\Theta} - K(t)N(t)\|}{\delta + \|\hat{\Theta} - K(t)N(t)\|}$$
$$+\lambda_{min}(\tilde{\Omega}_1)\frac{\delta\|\hat{\Theta} - K(t)N(t)\|}{\delta + \|\hat{\Theta} - K(t)N(t)\|} + 2\tilde{x}_2 P_2\zeta_2$$
$$+(K(t)N(t) - \hat{\Theta})^T\tilde{\Omega}_2[\hat{\Theta} - K(t)N(t)]$$
$$+(\Delta(t)\Theta - \zeta_{2KN})^T[\tilde{\Omega}_2\left(\hat{\Theta} - K(t)N(t)\right)$$
$$+\tilde{\Omega}_1\left(\frac{\hat{\Theta} - K(t)N(t)}{\delta + \|\hat{\Theta} - K(t)N(t)\|}\right)]$$
$$\leq -\frac{1}{2}\lambda_{min}(Q_2)\lambda_{min}(P_2^{-1})V_2 - \lambda_{min}(\tilde{\Omega}_2)\frac{\lambda_{min}(\Gamma)}{2}V_3$$
$$-(\lambda_{min}(\tilde{\Omega}_1)\lambda_{min}(\Gamma^{1/2})$$
$$-\lambda_{max}(\tilde{\Omega}_2)\|\Delta(t)\Theta|\lambda_{max}(\Gamma^{1/2}))\sqrt{V_3}$$
$$+2\lambda_{max}(\tilde{\Omega}_2)\|\Delta(t)\Theta - \zeta_{2KN}\|^2 + \lambda_{min}(\tilde{\Omega}_1)\delta$$
$$+2\lambda_{max}(\tilde{\Omega}_1)\|\Delta(t)\Theta - \zeta_{2KN}\| + 2\|\tilde{x}_2\|\|P_2\|\|\zeta_2\|$$

$$(33)$$

There are suitable positive scalars c_1, c_2, c_3 for large enough time, $t > 0$ such that:

$$\tilde{V} \leq -c_1\tilde{V} - c_2\sqrt{V}_3 + c_4 \qquad (34)$$

Therefore, $\tilde{x}_2$ and $\tilde{\Theta}$ converge to a compact set bounded by parameter δ, $\|\Delta(t)\|(\lim_{t\to\infty}\Delta(t) \to 0)$ and $\|\zeta_{2KN}\|$. The term $\Delta(t)$ denotes the effect of the initial conditions of $K^{-1}(0)$. For $\zeta_{2KN} = 0$, the size of the compact set can be adjusted to be smaller by reducing δ and the elements λ_i in matrix $K^{-1}(0)$. Note that ultimate bounded stability for $\tilde{x}_1$ and subsequently for $\tilde{x}$, now trivially follow. Again, for $\zeta = 0$, the set of ultimate boundedness can be arbitrarily small for suitable choice of δ. ∎

Remark 4: The result in Theorem 5.1 in fact is quite generic. It also allows for analysis of measurement errors of x_2 and ϕ. For this reason, we may have in the observer some measurement errors affecting both x_2 and also $\phi(x, u_2)$ measurement and in reality $\tilde{x}_2$ and $\check{\phi}(x, u_2)$ are provided in the practical system. Thus, the observer equation is

$$\dot{\hat{x}} = A\hat{x} + B_1 u_1 + B\check{\phi}\hat{\Theta} + L(\tilde{x}_2 - C\hat{x}) \qquad (35)$$

The plant dynamics in (8) can be rewritten as,

$$\dot{\tilde{x}} = (A\tilde{x} + B_1 u_1) + B_2\check{\phi}\Theta + \zeta + (\dot{\tilde{x}} - \dot{x})$$
$$+A(x - \tilde{x}) + B_2(\phi - \check{\phi})\Theta \qquad (36)$$

where $\check{x} = [x_1^T \quad \check{x}_2^T]^T$. *Assuming the measurement errors and its derivative are bounded, (i.e. $(x-\check{x}), (\dot{x}-\dot{\check{x}}), (\phi-\check{\phi}) \in L_\infty$), so that $\tilde{x}_2, \dot{\tilde{x}}_2 \in L_\infty$, then the plant dynamics (8) are,*

$$\dot{\tilde{x}} = (A\check{x} + B_2 u_1) + B_2\check{\phi}\Theta + \check{\zeta}, \qquad (37)$$

where $\check{\zeta} = \zeta + (\dot{\tilde{x}} - \dot{x}) + A(x - \tilde{x}) + B_1(\phi - \check{\phi})\Theta$ *can be regarded as a bounded disturbance. Defining the error dynamics as $\tilde{\check{x}} = (\check{x} - \hat{x})$ it follows that,*

$$\dot{\tilde{\check{x}}} = A_c\tilde{\check{x}} + B_2\check{\phi}\Theta + \check{\zeta} \qquad (38)$$

Under the assumption that $\check{\zeta}$ is bounded, we can continue the analysis as for Theorem 5.1. Boundedness of $\check{\zeta}$ might be achieved under suitable assumptions on the measurement errors affecting x_2 and an additional assumption on the nonlinear functions ϕ. ○

VI. PARAMETER ESTIMATION IN THE VEHICLE DYNAMICS

In this section, we will discuss the previously formulated parameter estimation algorithm in the context of its application to road gradient and vehicle's weight estimation. Figure 1 shows the simplified model of the small-scaled model car used in the experiment to validate the parameter estimation algorithm.
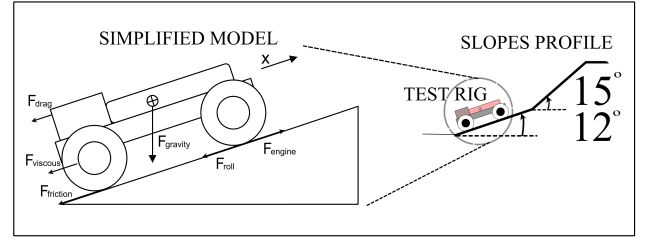

Fig. 1. Simplified model of the small-scaled model car and the slope profile

A. Vehicle model

The parameters to be estimated are the road inclination, θ, on which the vehicle traverses, the mass of the vehicle, m and the viscous friction coefficient, C_{vf}. Referring to Figure 1, assuming the air drag, F_{drag} and the rolling friction, F_{roll} are negligible, and the braking force, F_{brake} is subsumed in the driving force, F_{engine}, we may model the small-scaled model car using Newton's Second Law in the longitudinal direction to yield,

$$m\ddot{x} = F_{engine} - mg\sin(\theta) - C_{VF}\dot{x} \qquad (39)$$

where m is the mass of the vehicle, θ is the road gradient on which the vehicle traverses, $\dot{x}$ is the vehicle's velocity and C_{VF} is the viscous damping coefficient.

B. Observer Design

Following the general structure presented in (3), the adaptive observer with finite-time parameter estimation can be written as,

$$\dot{\mathbf{x}} = A\hat{\mathbf{x}} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}[g \quad F_{engine} \quad \dot{x}]\begin{bmatrix} \hat{s} \\ \hat{b} \\ \hat{f} \end{bmatrix} + L(y - \hat{y}) \quad (40)$$

where A is the system matrix (adheres to the Assumption 4), $\hat{s}$, $\hat{b}$, $\hat{f}$ are the estimated parameters of $-\sin\theta$, $\frac{1}{m}$ and $-\frac{C_{VF}}{m}$ respectively whereas L is the observer gain chosen to deliver the positive definite Lyapunov matrix, P, such that it satisfies (4). The engine driving force, F_{engine} is assumed

to be bounded to ensure that the system states, x remains bounded. The vector $\hat{y} = C\hat{x}$ will be the corresponding observer output and $\hat{\mathbf{x}} = [\hat{x} \quad \dot{\hat{x}}]$ is the observed state vector. Thus, using this structure it follows,

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \hat{\Theta} = \begin{bmatrix} \hat{s} \\ \hat{b} \\ \hat{f} \end{bmatrix}, \quad \varphi = \begin{bmatrix} g \\ F_{engine} \\ \dot{x} \end{bmatrix}^T \quad (41)$$

The observer adaptive weights are lumped such that,

$$\Gamma = diag(\gamma_1, \gamma_2, \gamma_3), \quad \Omega = \Gamma^{-1} diag(r_s, r_b, r_f) \quad (42)$$

VII. PRACTICAL APPLICATION RESULTS

The small-scale model car, previously built by Foreman *et al.* [18],was used in the experiment to evaluate the estimation algorithm. The vehicle's mass (nominally weighs 10kg) and the road gradients on which it traverses were the two parameters to be estimated. Figure 2 shows the implemented controller system network and architecture which emulates the system network of a road vehicle. Together with Matlab^{TM},
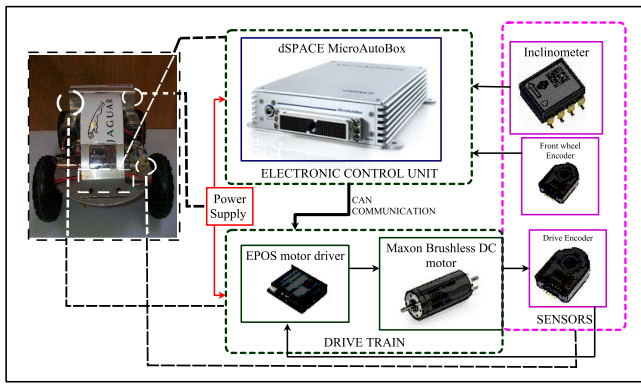

Fig. 2.   Functional Structure of the System Onboard

dSPACE MicroAutoBox, used in the experiment, is a dedicated Rapid Prototyping embedded system suited to test the proposed estimation algorithm. The drive train comprises of an EPOS 24/5 motor driver and the brushless DC motor (EC-i 40 Maxon) representing the vehicle's engine. The motor is current-controlled via the MicroAutoBox which subsequently provides the driving force, F_{engine}, proportional to the current signal being controlled. Gradient measurements provided by the installed SCA61T inclinometer is entirely for reference purpose and not to be used in the algorithm. In our experiment, we would avoid the occurrence of significant slippage as this would invalidate our estimation effort. The test slope was constructed using three stiff wooden planks of 2 m in length each. They are tilted and bolted together to give a slope profile with tilting angle of $12°$ for the first slope, $15°$ for the second slope and the last slope is horizontal i.e. parallel to the ground. The small-scaled model car was required to traverse up the designated slope at a constant speed of 0.2 m/s (Figure 1).

A. Parameters tuning

In our experiment, there were important parameters (see Table I) of the adaptive observer algorithm needed to be tuned to achieve satisfactory results. Realistic and acceptable physical bounds/limits were considered and shown in Table II. The corresponding values of the parameters displayed in the table are used in the experiment. Noise injected to the velocity and the control signal was kept constant in terms of power so that the actuating signal applied to the motor exerts sufficient and persistent excitation.

TABLE I
PARAMETERS TO BE TUNED

Parameter Description	Symbols	Values
Observer Adaptive weights, Γ	γ_1	0.01
	γ_2	0.001
	γ_3	0.001
Sliding-Mode Adaptive weights, Ω	r_s	0.01
	r_b	1
	r_f	0.00001
Forgetting Factor	k_{FF}	0.6
Filter Poles	k	0.005
Regressor Matrix, K initial condition	$K(0)$	diag(0.4,0.4,1)

TABLE II
SATURATION LIMITS

Plant Parameters	Estimation	Lower Limit	Upper Limit
Mass(m,kg)	$\hat{m}$	0.1	20
	$\hat{b} = 1/\hat{m}$	0.05	10
Gradient($\theta,°$)	$\hat{\theta}$	-20	20
	$\hat{s} = sin(\hat{\theta})$	-0.4	0.4
Friction Coefficient	$-C_{VF}$	240	0.1
(C_{VF},kg/s)	$\hat{f} = C_{VF}/\hat{m}$	-12	-1

B. Results

The proposed adaptive observer algorithm with sliding-mode term performance is compared with that of the recent adaptive observer (without the term in (28)) previously carried out by [18]. Referring to Table III, the Integral Absolute Error (IAE), $\int_0^\infty |e(t)|dt$, is used as the performance index measuring the difference between the actual and the estimated road gradient. The low value of IAE translates to good parameter estimation as the estimated value converges to the true value. Figure 3(a) versus Figure 5(a) shows an excellent estimation performance of the new algorithm, having the estimated gradient converging close to the actual gradient during the climbing of the slope although there is a slight offset during the vehicle traversing the flat ground. A very consistent mass estimation of the new estimator is evident in Figure 3(b) as compared to the previous algorithm as shown in Figure 5(b). The estimated mass value settles at approximately 14 kg throughout the test slope profile and slightly descends to 12 kg towards reaching the flat ground at the end. The recorded IAE for the road gradient estimation for the proposed algorithm is 178.2402. In contrast, the previous algorithm lacks in performance when it exhibits a large increase in IAE of 229.9736. The estimated parameters, $\hat{s}, \hat{b}, \hat{f}$ in Figure
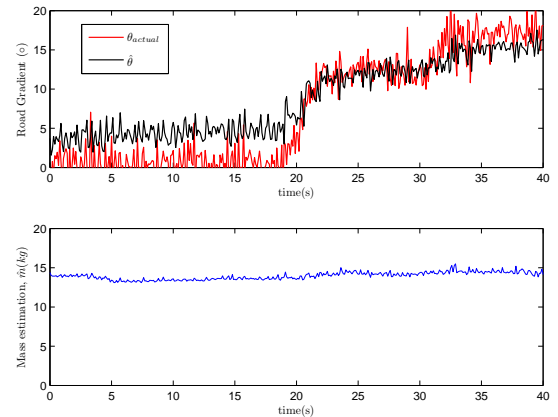

Fig. 3.   (a) Gradient comparison(in Degree) between the actual, θ and estimated, $\hat{\theta}$ (b) Mass Estimation (kg) of the proposed novel algorithm
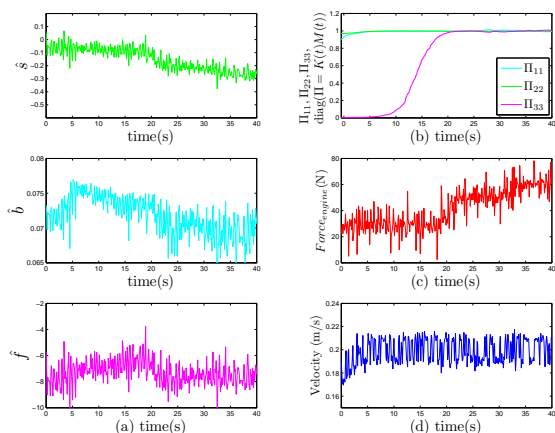
Fig. 4. (a) Estimated parameters($\hat{\Theta}$) consisting of $\hat{s}, \hat{b}, \hat{f}$, (b) proof of PE by which $K(t)M(t) \approx I$ holds experimentally, (c) Driving force,F_{engine} and (d) vehicle's velocity, V

4(a) also show excellent behaviour as they remain within the bound of the given physical limit without saturation whereas in Figure 5(b), the estimated parameters saturate at the given bounds. Interestingly, the engine driving force, F_{engine} signal in Figure 4(c) exhibits persistent excitation (PE) throughout the test slope profile which assures the finite-time convergence to the true value. Note that persistent excitation has been also verified via the approach of Remark 3. The product $K(t)M(t)$ has converged to unity as evident in the experimental data shown in Figure 4(b). Table III sums up the performance of the proposed algorithm compared with the previous one in terms of IAE.
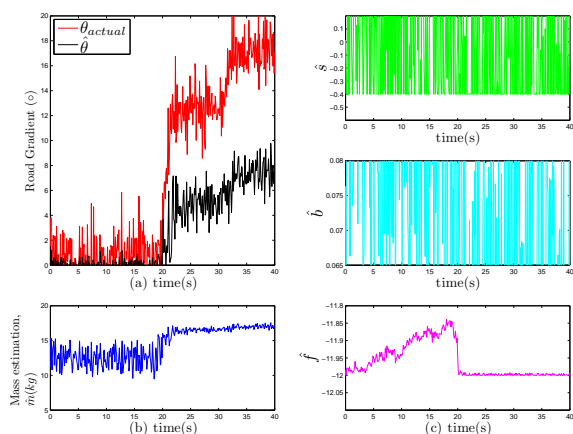

Fig. 5. (a) Gradient comparison between the actual, θ and the estimated, $\hat{\theta}$, (b) Mass Estimation (kg) and (c) Estimated parameters(Θ) consisting $\hat{s}, \hat{b}, \hat{f}$ of algorithm from [18]

VIII. CONCLUSION

An adaptive observer with novel sliding-mode based parameter estimation algorithm to estimate the vehicle's mass and the road gradient is presented. The proposed parameter estimator with the sliding-mode term has been proven analytically to be finite time convergent to an error of well defined bound. The algorithm shows significant levels of robustness to disturbances and a particular class of measurement errors. The analytical results are further supported and validated by the practical implementation in a form of experiments conducted on a small-scale vehicle traversing a designated test slope profile with certain parameters tuned. The practical results show a significant improvement over the previous algorithm in terms of persistent excitation (PE), realistic values within the physical bound, estimation accuracy and convergence.

REFERENCES

[1] H. Jansson, E. Kozica, P. Sahlholm, and K.-H. Johansson, "Improved road grade estimation using sensor fusion," in *Proceedings of the 12th Reglermote in Stockholm, Sweeden*, 2006.

[2] V. Winstead and I. Kolmanovsky, "Estimation of road grade and vehicle mass via model predictive control," in *Control Applications, 2005. CCA 2005. Proceedings of 2005 IEEE Conference on*, 2005, pp. 1588 –1593.

[3] S. Mangan, J. Wang, and Q. Wu, "Measurement of the road gradient using an inclinometer mounted on a moving vehicle," in *IEEE International Symposium on Computer Aided Control System and Design*, 2002.

[4] H. Bae, J. Ryu, and J. Gerdes, "Road grade and vehicle parameter estimation for longitudinal control using gps," in *IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC*, 2001, pp. 166–171.

[5] P. Lingman and B. Schmidtbauer, "Road slope and vehicle mass estimation using Kalman filtering," *Vehicle System Dynamics*, vol. 37, pp. 12–23, 2002.

[6] P. Sahlholm, *Iterative road grade estimation for heavy duty vehicle control*. Elektrotekniska system, Kungliga Tekniska hogskolan, 2008.

[7] A. P. H. Vahidi, A.; Stefanopoulou, "Experiments for online estimation of heavy vehicle's mass and time-varying road grade," in *Proceedings IMECE, Jan 2003*, 2003.

[8] A. Vahidi, M. Druzhinina, A. Stefanopoulou, and H. Peng, "Simultaneous mass and time-varying grade estimation for heavy-duty vehicles," in *American Control Conference, 2003. Proceedings of the 2003*, vol. 6. IEEE, 2003, pp. 4951–4956.

[9] J. Barrho, M. Hiemer, U. Kiencke, and T. Matsunaga, "Estimation of Elevation Difference Based on Vehicle's Inertial Sensors."

[10] P. Sahlholm and K. Johansson, "Segmented road grade estimation for fuel efficient heavy duty vehicles," in *IEEE Conference on Decision and Control*, 2010.

[11] ——, "Road grade estimation for look-ahead vehicle control," in *Proceedings of the 17th IFAC World Congress*, 2008.

[12] ——, "Road grade estimation for look-ahead vehicle control using multiple measurement runs," *Control Engineering Practice*, vol. 18, no. 11, pp. 1328 – 1341, 2010.

[13] S. Mangan and J. Wang, "Development of a novel sensorless longitudinal road gradient estimation method based on vehicle can bus data," *Mechatronics, IEEE/ASME Transactions on*, vol. 12, no. 3, pp. 375 – 386, 2007.

[14] P. Iaonnou and J. Sun, *Robust Adaptive Control*. Prentice Hall, 1996.

[15] V. Adetola and M. Guay, "Finite-Time Parameter Estimation in Adaptive Control of Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 53, no. 3, pp. 807–811, Apr. 2008. [Online]. Available: http://dx.doi.org/10.1109/TAC.2008.919568

[16] ——, "Performance Improvement in Adaptive Control of Linearly Parameterized Nonlinear Systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 9, pp. 2182–2186, Sep. 2010. [Online]. Available: http://dx.doi.org/10.1109/TAC.2010.2052149

[17] C. Edwards and S. Spurgeon, *Sliding mode control: theory and applications*. CRC, 1998.

[18] J. Foreman, M. Yazdouni, S. Russo, G. Chan, A. Spiers, G. Herrmann, and P. Barber, "Hardware in the loop validation of a gradient and weight estimation algorithm and longitudinal speed control using a laboratory model car," in *International Conference on Systems Engineering (ICSE)*, 2009.

[19] J. Na, G. Herrmann, X. Ren, M. Nasiruddin, and P. Barber, "Robust adaptive finite-time parameter estimation and control of nonlinear systems," in *Multi-Conference on System and Control 2011, MSC2011*, 2011.

[20] N. Shimkin and A. Feuer, "Persistency of excitation in continuous system," *System and Control Letters*, pp. 225–233, 1987.

[21] J.-J. Slotine and W. Li, *Applied Nonlinear Control*. Prentice Hall, Oct. 1990. [Online]. Available: http://www.worldcat.org/isbn/0130408905

[22] T. Soderstrom and P. Stoica, *System Identification*, M. Grimble, Ed. Prentice Hall, 1989.