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# Theoretical and experimental stability maps of optically injected semiconductor laser

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*Abstract: We found very good agreement between theoretical and experimental bifurcation diagrams of an optically injected diode laser. Identifying regions of various dynamics opens new possibilities for applying this system to all optical signal manipulation.*

## Introduction

Semiconductor lasers with optical injection are known to produce an enormous wealth of dynamics ranging from stable operation (CW) via periodic and quasiperiodic oscillations to chaos [1-3]. On top of this they exhibit multistability between different types of operation such as coexistence of periodic and chaotic oscillations. Although most applications took advantage of stable operation, chaotic laser output has recently become of great interest too. This is because semiconductor lasers were proposed for use in chaotic communication schemes [4,5]. Adding the possibility of switching between different types of output makes an optically injected semiconductor laser a very interesting candidate for applications in all-optical signal processing. For these kind of applications it is crucial that the dynamics are well understood so that theoretical predictions can nicely explain experimental results.

## Model

A single-mode diode laser with monochromatic optical injection can be modeled by three-dimensional rate equations for the slowly varying complex electric field  $E = E_x + iE_y$  and the normalized inversion  $n$  inside the laser [2]:

$$(1) \quad \begin{aligned} \dot{E} &= K + \frac{1}{2} \{ (1 + i\alpha)n - i\omega \} E \\ \dot{n} &= -2\Gamma n - (1 + 2Bn)(|E|^2 - 1) \end{aligned}$$

Eqs.(1) are scaled for convenience and the connection between the scaled quantities used here and experimental quantities is given in Ref.2. The most important parameters are the strength  $K$  of the field, which is injected from an external source into the laser, and its detuning frequency  $\omega$  from the free-running laser. Both can easily be changed during an experiment and, therefore, are natural operational

parameters. The quantities  $B$  and  $\Gamma$ , on the other hand, represent material properties of the laser and are fixed to the realistic values  $B = 0.015$  and  $\Gamma = 0.035$ . The parameter  $\alpha$  is called the linewidth enhancement factor, and it quantifies how much the refractive index and, hence, the instantaneous laser resonance frequency, changes with the population inversion  $n$ . Throughout this study we focus on  $\alpha = 3$  which fits best the results obtained in the experiment [2,3]. The detuning  $\omega$  is expressed in the units of the relaxation oscillation frequency which is 3.5 GHz.

## Theoretical stability map

The model described by Eqs.(1) allows for bifurcation analysis which brings new insight and understanding of the dynamics of a diode laser with optical injection. Depending on the parameter settings, Eqs.(1) have different solutions which relate to different types of laser output. As the parameters are tuned, solutions of Eqs.(1) change. Transitions between different types of laser output, due to the change of parameters, are called bifurcations. We use special techniques to detect bifurcations and follow them in the  $(K, \omega)$ -plane. Computed bifurcation curves divide the parameter space into regions of qualitatively different dynamics of the laser. The resulting stability map is presented in Fig.1. The two curves, SN and H, confine the region where the laser locks to the injected signal. Outside of the locking region the output of the laser oscillates in several different fashions. When approaching the locking region from below, a stable equilibrium appears along SN in a so-called saddle-node bifurcation. This stable equilibrium exists for parameter settings above SN, up to H. This is where the relaxation oscillations are excited via a so-called Hopf bifurcation. For parameters above H the laser output oscillates periodically. These periodic oscillations may undergo instabilities such as period doubling denoted by PD1. Period doubling bifurcations usually appear as infinite sequences leading to chaos. In Fig.1 we show secondary period doubling bifurcations PD2 too. Regions of chaotic dynamics are nested inside PD bubbles. On top of this there are more bifurcations of

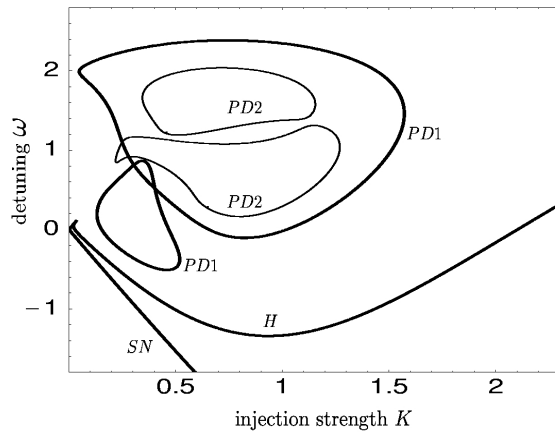


Figure 1: Theoretical stability map

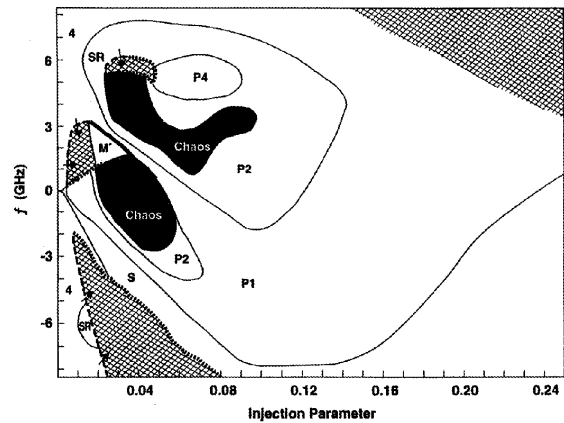


Figure 2: Experimental stability map

periodic oscillations. For example, a torus bifurcation introduces an extra frequency in the laser output and creates quasiperiodic oscillations that may transform into chaos. We do not show these extra bifurcations in Fig.1 for clarity reasons but they all connect and interact with each other making a consistent dynamical picture [2].

#### Experimental stability map

Fig.2 shows the experimental stability map obtained using (as an injected laser) a conventional Fabry-Perot edge-emitting laser pumped around 65% percent above threshold [3]. The dynamics of the laser were determined by observation of spectra. The region of stable locking is marked with s. Regions of periodic oscillations with 'basic' period are indicated by P1 and oscillations with higher period created in period-doubling bifurcations by P2 and P4 for twice and four times the basic period of oscillation. Chaotic islands created due to period-doubling cascades are in black. Hatching represents parameter settings for which either the spectra did not clearly indicate any specific type of operation or the laser switched to multimode operation.

Comparison of the theoretical and the experimental stability maps strikes with great similarity. Not only the topology is the same but also the agreement is close to being quantitative. The Hopf bifurcation curve H provides a good reference and starting point. The boundaries between the regions P1 and P2 from the experimental map (Fig.2) resemble well the two period-doubling bubbles PD1 predicted in Fig.1. Regions of chaos detected in the experiment (Fig.2) coincide with the area confined by the secondary period-doubling curves PD2 from Fig.1. Moreover, theory predicts some quasiperiodic oscillations and chaos in the hatched regions as well as some new effects that are to be confirmed by forthcoming measurements.

#### Conclusions

Our results show that there is very good agreement between theory based on the rate equations model and real experiment for an optically injected diode laser. This agreement regards simple locking operation as well as regions of chaotic dynamics that can now be understood better and eventually exploited in applications. Most importantly, mapping out the global dynamics of the injected laser with all its nonlinear phenomena opens new possibilities for using this configuration for all-optical communication applications.

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