COMPUTER EXPERIMENTS WITH COHORT FERTILITY IN BIRTH PROJECTIONS

by

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ABSTRACT

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Reduction of the fertility of women in order to stem the growth of population is considered essential to the continuance of economic growth in many developing nations. The effectiveness of programs for reduction of fertility can be increased if especially sensitive sectors of the potential clientele can be defined by appropriate characteristics. In addition to age, the parity of women, or number of children they have already borne, is thought to be such a characteristic. A mathematical model for projecting births, using cohort fertility by age and parity, provides a means for experimentation relevant to the definition of clientele sectors.

The model is a stochastic process which uses an initial frequency distribution of a cohort of women by parity, plus birth probabilities estimated from age-and-parity-specific birth rates, to project for successive years the frequency distribution of the cohort, the distribution of cohort annual births by change of parity of mother, and cohort cumulative births. Graphical smoothing of data from the India-Harvard-Ludhiana field study of fertility in the rural Punjab, provides the required computer input. Cohort cumulative births at age forty-five are used to measure the response of the model to changes in age-and-parity-specific birth probabilities.

The birth rate estimators of these probabilities are chosen for experimentation in highly simplified combinations. The ability to synthesize the response of the system to larger realistic combinations is found to be difficult on the basis of the experiments reported. However, alternative clientele sectors can be defined by age and parity which will produce relatively greater impact on cumulative births over a thirty year projection span than other possible choices. One of these sectors ranges in age from about 25 to about 35, in parity from 2 to 4 or 3 to 5. The other one is limited to parities 3 and 4 but covers all ages of these parities. These program designs are quite sensitive to loss of potential impact by delay in achieving decreases in targeted birth rates. Conversely, their sensitivities imply relatively greater impact for given decreases achieved.

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I Introduction

Purpose, Scope, Method

A high rate of population growth threatens economic growth in many developing nations. This population growth results from a high fertility, or birth rate, coupled with a decreasing mortality, or death rate. To supplement their economic development programs, some of these nations are increasing the resources applied to reduction of their birth rates in order to reduce their population growth. This thesis describes a set of experiments on a mathematical model for birth projections. A digital computer is the "laboratory". The purpose of the experiments is to explore the structure of the model for implications useful in the design of research, policies and programs for reduction of fertility in the high fertility populations of developing nations.

The model uses cohort fertility by age and parity to project births. A cohort of women is a subset of all women containing just those born in a given year. It is referenced by the year of birth of the cohort. Equivalently, in a given year all members of a cohort will attain the same age. The parity of a woman is the number of children she has borne to date. With cohort birth year and age and parity as parameters, the model is distinguished by the addition of parity to the other two, conventional, parameters of cohort fertility models. Parity is an essential parameter in studies for the design of fertility reduction programs. For example, if elderly parents are traditionally supported by grown children, couples may be uninterested in family limitation until reasonably assured of the survival of three or four children.

The model is built around the notion that to women of each parity of each childbearing age of each cohort can be assigned a unique probability of giving birth to an additional child during the current year. The model is a stochastic process which projects for each age of any number of childbearing ages remaining to a cohort in an initial year, the frequency distribution of the cohort by parity, the distribution of cohort annual births by parity change of mother, and cohort cumulative births.

Two FORTRAN computer programs written for the model actually project for all cohorts of childbearing age in the initial year, births to completion of childbearing. They also project for all younger cohorts which become of childbearing age after the initial year, births through the year in which the youngest initial year cohort completes childbearing. Since the childbearing span is taken as ages fifteen through forty-five, the projection span of the programs is thirty-one years.

Two measures of output or response of the system are used in the experiments. One is cohort cumulative births at completion of childbearing at age forty-five, for individual cohorts. The other is the average of such cohort cumulative births at forty-five, projected one cohort per year for thirty-one years, for the thirty-one cohorts of childbearing age in the initial year.

Model data requirements for each cohort include its frequency distribution by parity in the initial year, plus its birth probabilities, one for each parity at each childbearing age remaining to the cohort. Age-and-parity-specific birth rates, which are annual births per one-thousand women of the specified combinations of age and parity, can serve to estimate the probabilities. Previously the only source of such rates was the time series developed for the United States by the late P.K. Whelpton. We have been fortunate to obtain data from the India-Harvard-Ludhiana field study of fertility in the rural Punjab, also known as the Khanna study, from which such rates could be calculated, effectively for a cross-section point in time.

Graphical smoothing of the Khanna data provides the required computer input for a high fertility population. Use of cross-section data implies the assumption that the birth probabilities are constant in time and therefore the same for successive cohorts. While undoubtedly not accurate for populations in developed economies, the approximation is probably closer for the high fertility populations of the developing nations in which we are interested.

The experiments are limited in scope to observation of the response of the model to changes in cohort birth probabilities. These probabilities, or their birth rate estimators, are thus treated as "policy" variables, the manipulation of which produces changes in cohort births, Cohort frequency distributions could also be manipulated to simulate the effects of mortality and marriage. This is not done as we are interested, presently at least, in the effects of age-and-parity-specific fertility on births independent of other effects.

The basic method used in the experiments is that of sensitivity analysis. A first computer run is made using the smoothed data without change. This "base run" gives a result in the manner of a singular, unconditional prediction, against which are compared the results of other runs. Each of these other runs is made with a different change in the birth rate portion of the input, according to a systematic schedule of such changes. Differences within the set of these results reflect the structure of the model and the form of the data.

Results of other runs similarly measured against the base run are used for other specific tests. One is for linearity of response of the system to different types of changes in input birth rates. Another is for the possibility of aggregating the response to a compound change in cohort fertility from responses to simpler changes. Finally, the effect to time delay in the achievement of targeted fertility changes is examined. This effect relates to the study of different rates of diffusion of changed fertility behavior into clientele targeted by a fertility reduction program.

The simple experiments we have conducted are not realistic in the sense that the fertility changes whose effects we have examined could actually occur. Individual age-and-parity-specific birth rates are treated as subject to change, independent even of rates in adjacent ages and adjacent parities. Our justification for this is a conviction that a simplification even to the point of "unrealistic" distortion can sometimes provide insight into the nature of a problem.

The overall problem to which these experiments are related is to find a lever by which to move a socio-biological process off in a new direction. We have a representation of that process in the birth projection model. Our task is to observe the action of the simplest components of that process, singly and together, for characteristics relevant to the construction of a fulcrum.

Summary of Results

1. Decrease of cohort cumulative births with decrease of individual age-and-parity-specific birth rates over the thirty-one year projection span, expectedly, tends to be greatest for the greatest birth rates. Not as obvious, is the observation that the sensitivity of cumulative births is nearly uniform for individual maximum rates along the "diagonal" of a table of such rates, with the exception of the high age-high-parity end of this "diagonal".
2. Decrease of cumulative births over the projection span with decrease of age-and-parity-specific birth rates taken an age at a time, is greatest for the middle of the range of childbearing years.
3. Decrease of cumulative births over the projection span with the birth rates taken a parity at a time, is greatest for the lower and middle parities and is much more sensitive than to either individual rates or rates taken an age at a time.
4. The joint implication of these results is that a program of fertility reduction would tend to have greater impact by focusing on ages from about 25 to 35 and parities 2 to 4, or alternatively, by focusing on parities 3 and 4 and all ages in those parities, than by focusing on other birth rates.
5. Response of cumulative births to decrease of individual

age-and-parity-specifc birth rates, and to decrease of such rates taken an age at a time, is linear.

6. Response to such rates taken a parity at a time, is non-linear.

7. Response to rates taken an age at a time can be synthesized from responses to individual age-and-parity specific rates.

8. Response to rates taken several ages at a time, at least if those ages are a few years apart, can be synthesized moderately closely from responses to rates taken one age at a time.

9. Response to rates taken a parity at a time cannot be synthesized from individual rates.

10. Response to rates taken several parities at a time cannot be synthesized from rates taken one parity at a time.

11. It might be possible to contradict the last two results using a table of relations empirically derived with further computer experiments.

12. Sensitivity of cumulative births over the projection span to individual age-and-parity-specific birth rates and to such rates taken an age at a time, decreases linearly with increase of time delay in the achievement of drops in birth rate.

13. Sensitivity to rates taken a parity at a time, decreases slightly non-linearly but greatly, with

increase of diffusion time.

14. The implication of this result is that the potential impact of a program of fertility reduction within a span of about thirty years, will be considerably lessened by slow diffusion, but the relative advantage of the alternative programs described in 4., will be retained during the process of diffusion of the new behavior patterns.

II Two Contexts

The Population Problem[1]

The current rapid growth of population in the developing nations has attracted much attention in recent years. High birth rates conjoined with relatively low and falling death rates contribute ever larger cohorts of children to populations already burdened with high ratios of dependents to labor force. Concern has been voiced by national governments and others that the progress of economic development, painfully slow at best, may not be able to accelerate and achieve a "take off" into modernization and industrialization. This concern arises from the possibility that expanding production may have to be devoted to output for consumption, at continued minimal levels by burgeoning populations, rather than being devoted to output for capital investment at increasing levels of capital-to-labor ratio, productivity and per capita income.

Classical Malthusian social theory held that population tended naturally to outrun resources. Therefore improvements in general welfare would soon be caught up, first by expanding population, then by increased deaths as pressure grew upon resources. Improved welfare would be crushed in cyclical recurrence of poverty, and social reform efforts on behalf of the poor were futile.

The classical Marxian argument held that a surplus of poverty stricken industrial labor was a condition of capitalist society which would disappear with the advent of socialism. The socialist change in ownership of the means of production would result in the redistribution of wealth, the disappearance of the class system previously supported by property and wealth, and the disappearance of the labor surplus promoted by the propertied classes to keep wages low.

In contrast to Malthusian under-estimation of possible progress in technology, a common contemporary view is that technology will probably continue to advance sufficiently rapidly for resource expansion in developing nations to keep pace with population growth. Current levels of living will probably be maintained, or perhaps the slow gains of recent decades may continue. In contrast to Marxian historical inevitability, this contemporary view couples with its expectations for technology, an advocacy of policies and programs of fertility reduction to enable economic growth to become more rapid than the slow gains of the past.

A "revolution of rising expectations" has spawned economic development plans and programs in many developing countries. In this "revolution", fatalism towards poverty has been widely replaced by the belief that substantial improvement in material welfare is possible, largely by industrialization, and in less time than was required for industrialization in the developed countries. Thus disaster in the form of massive starvation may not be anticipated from lack of resources. But social upheaval and major violence are distinct possibilities if these aspirations for accelerated development are thwarted. Thus the "population problem" is not simply population growth, nor outrunning resources, nor improper organization of the means of production or technology, but rather it is to reduce fertility to the levels of mortality sufficiently rapidly as to prevent plans for economic development from being thwarted by rapid population growth.

Most discussions of population growth in the developing countries contrast their experience with that of Europe or the Western countries as summarized in the scheme known as the "demographic transition". The "transition" describes the passage of vital rates from high to low levels, and its results are observed in the size of a population. The scheme is as follows. European populations for centuries were small and fluctuating as both birth and death rates fluctuated at high levels; in the nineteenth century death rates declined rapidly resulting in rapid population growth; then late in the nineteenth century birth rates declined to the low levels of the death rates resulting in relatively large but stable or slowly growing national populations.

Many developing nations are thought to be in the second stage of their own version of the demographic transition. Thus, while death rates have fallen and are still falling because of public health measures, birth rates have not yet followed. But the resulting current rates of population growth in these countries of 2.5 and 3.5 or even 4 per cent per year, substantially exceed European second stage growth rates of somewhat over 1 percent per year. The safety-valve of large scale outmigration to other continents, without which the European rates would have been larger, is not available to the developing nations. After falling to near replacement levels, post-transitional European population growth rates are now somewhat less than 1 percent per year without large outmigration.

The historical socio-economic processes of industrialization and urbanization are often associated with the Western demographic transition as causal elements. Industrial and urban life is suggested to have initiated and aided the spread of values supporting smaller families and the adoption of the birth control technology which considerable evidence aducces was the means to this end. Thus the ideal of smaller family size to enable attainment of economic and social aspirations, and the corollary use of contraception, spread from higher class to lower and from urban center to rural area, aided by the increase of economic and social mobility which accompanied industrialization and urbanization.

The longer the developing nations remain in the second stage of the demographic transition, the greater the magnitude of their social, economic and political problems. Yet efforts to promote fertility reduction in these countries in advance of, or apart from, a more general transformation to Western industrial and urban technology and values, have so far proved largely unrewarding.

The results of these efforts indicate that Western experience cannot always be exported by simple communication. In particular it is not enough merely to make contraceptive technology, at least current technology, available in these countries. "----We must view social systems from the perspective of their permeability to technological change, and the selectivity of their permeability to different aspects of technology. For herein lies the difference between the rate of effective adoption of mortality reducing technology and fertility reducing technology in the non-Western or 'under-developed' nations.

In considering any particular technological innovation we must raise the classical questions of means and ends. We ask, what values or purposes do the people seek through adoption of technology? Mere availability of technology is an insufficient explanation for its use."[2]

Either values and purposes must be identified which will support an ideal of smaller family size, or values must be transformed before the technology will be adopted. It is also possible that some new technological means for contraception yet to be developed, may prove acceptable to indigenous values without a prior massive transformation of those values from traditional to modern.

But whatever the extent of value transformation necessary before fertility reduction is generally accepted, it is wise to try to discover whether there are some sectors of the client population which, if they accepted any form of birth control, old or new, would have an especially large or important impact on population growth. These sectors could then be used as target clientele in the initial design of a field program. Through other research including field testing, it might develop that sectors other than those with the largest potential impact would be more receptive to the program in practice. In this event the more receptive sectors might then be made the target clientele.

In any event the importance of achieving as rapid a reduction of overall fertility as possible, underscores the importance of the relative sensitivity of population growth to differential fertility changes. In this context this thesis explores the relative sensitivity of cohort births to differential fertility reduction in clientele sectors characterized by age and parity.

Simulation[3]

Socio-economic processes can be represented mathematically and these representations or models may vary in type, in qualities of design and in usage. By simulation we mean the construction and use of these models. By type, models are deterministic or statistical or a combination of both. In deterministic models certain events or dependent variables are mechanistically related to other events or independent variables, and the laws of dependence express certainty in causal relationship or outcome. In statistical models dependent events are probabilistically related to other events, and the rules of dependence include either probabilities of alternative outcomes or statistically inferred relationships with associated measures of uncertainity. Models are also typed by the level of aggregation of the phenomena represented by their variables, and by whether they provide descriptions of static states of equilibrium, of dynamic paths of change of state, or of system states at points in time along such paths.

In qualities of design, models vary in size and realism measured by the number and variety of variables they include. They also vary in predictive accuracy, depending partly on the quality of the representation and partly on the regularity or inherent predictability of the phenomenon represented. Predictive accuracy of statistical models of socio-economic processes is improved the greater the aggregation of the quantity being predicted, the longer the historical record of data, and the more frequent the distribution of samples. The accuracy with which empirical curves are fitted to such data can be improved by incorporating more independent variables and associated constants in the fitted equations, but the scope for such improvement is limited. Relationships estimated from data for only one point in time can only be used on the assumption the relationships do not change with time. An effort to improve the accuracy of fitted curves in this case, actually risks lesser predictive accuracy as the risk of instability with time rises with more constants to be estimated. The final design quality of models to be mentioned is the capability of being manipulated. The larger and more complex a model is, the more manipulation of it may be impeded, especially manipulation with comprehension. Conversely, manipulation and interpretation may be facilitated by simpler models, even though chosen at the expense of realism.

Models, or the information resulting from their operation, are used sometimes to verify the accuracy of their representations of reality or to test hypotheses embodied in them, and sometimes to study the consequences of alternative policies. Highly specific forecasts of magnitudes and less specific predictions of general directions or ranges of change may both be used for both these purposes. Unconditional forecasts based on explicit anticipation of particular conditions may be used for both verification and operational decision. Conditional predictions, however, based on experimental assumption of conceivable conditions without assertion of their relative likelihood are used for study of policy alternatives and obviously cannot be used for verification unless the conditions actually come to pass.

While many socio-economic models strive for realism through recognizability of detail and use of many components in the simulation, a case can be made for greater abstraction from reality through idealization and simplification. Further, the suggestion has been made that "----one might be more interested, at first, in the penalty that unrealism exacts, and therefore seize upon simulation as the chance to try the outlandish conjecture and the pathological case. The penalty might not be large, and if it is, the insight gained may compensate."[4] If all simulation is a form of experimentation and as such can be placed at one end or the other of the cycle of observation-hypothesis-verification, then the spirit of this approach to simulation is one of observation rather than verification. It seems likely that experiment in this vein, while moving away from a large concern with theorizing and verification, should still be useful to the design of policy. This is the approach we have adopted in exploration of the cohort fertility birth projection model.

III The Equipment and Its Use

The Model

The birth projection model is a non-stationary Markov process and as such, in terms of the characteristics just described, is a statistical model, having a considerable degree of dis-aggregation, which produces cross-sections at points along a time path of system change. The Markov process is simpler in schema than many socio-economic models. It admits varying degrees of "realism", especially as component models are added to it for generation of certain of its variables. It is moderately easy to manipulate but a few computer runs are capable of producing a volume of numerical output which is best digested by degrees. The proposal of its use for birth projections is described in detail by J. M. Beshers[5] and therefore only an outline of the formal structure is given here.

That structure consists of the multiplication of a row vector, $m(t)$, by a square matrix, $P(t)$, to produce a new row vector, $m(t+1)$. Formally, $m(t)P(t) = m(t+1)$, where t and $t+1$ are successive points in time. The vectors are the frequency distributions of a given cohort by parity at some initial time, t, and at one unit of time or year later, $t+1$. The matrix contains conditional transition probabilities. Multiplication of the vector $m(t+1)$ by a new transition matrix, $P(t+1)$, gives another vector,

$m(t+2)$. A succession of such matrix multiplications generates the fertility "history" of the given cohort as a sequence of frequency distributions by parity, from whatever year in the span of childbearing ages it is chosen to start, through any desired year or corresponding age of the cohort. Formally, $m(t)\prod P(i) = m(t+n+1)$, where $i = t, t+1, t+2, \ldots, t+n$ and $\prod$ indicates the product of all matrices $P(i)$.

The transition matrix for the case where the possible states or parities considered are zero, one, two, and three-and-up is as follows:

$$P(t) = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3up \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3up \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & 0 & 0 \\ 0 & p_{22} & p_{23} & 0 \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & 0 & p_{44} \end{bmatrix} \end{array}$$

Each element in the matrix represents the probability, p_{ij}, that a member of the cohort, given the condition that she was in parity i at time t, will be in parity j by time $t+1$. Each row of the matrix contains all of the transition probabilities p_{ij} for going from a given state or parity i to each of the other states j. Eliminating multiple births and two single births within a year admits as possibilities only no birth and single birth per woman per year. Therefore all probabilities in a row are zero except for p_{ii}, the probability of staying in the same parity, and p_{ij} $(j = i+1)$, the probability of moving to the next higher parity during the year between t and t+1. (Note that $p_{ii} = 1-p_{ij}$.)

As we have defined the possible states in our example, once state three-and-up is entered it cannot be left. Therefore $p_{44} = 1$. Such a state is called an "absorbing" state. Since there is a biological limit to the number of possible parities, the parity corresponding to that limit is an absorbing state, as is any intermediate but upwardly inclusive terminal parity state defined as in the example. In our computer programs we have taken parity seven-and-up as the terminal state.

The complete model as elaborated by Professor Beshers includes not only the stochastic process projection model just described, but suggestions for deduction of effects of mortality and marriage on the frequency distributions, and for deduction of the transition probabilities for the successive matrices as well. In effect the stochastic process, the deduction of transition probabilities, and the modification of frequency distributions can be treated as three component models of the complete model. Transition probabilities could be deduced from a combination of probabilities for non-family planners with those for family-planners. The latter could be obtained using time dependent social and economic parameters relevant to family planning.

Appropriate proportions in which to combine the two sets of probabilities could be deduced from the manner of diffusion of birth control technology and values through the social structure. In general the transition matrix at any time t+1 is some function of the matrix at the previous time t.

A basic assumption of the complete model is that women of each cohort at each childbearing age and of each parity have a unique probability of having an additional child during a year. In other words history bears differently on different cohorts. For the United States much of the data required to estimate parameters for deduction of accurate transition probabilities is thought to be available at some cost. But for the high fertility populations of the developing nations in which we are interested, most of the historical record of such data does not exist.

Following the approach of lesser concern for strict realism, two other avenues to present use of the model are open. One is to use whatever guidance is easily obtainable from both non-Western and appropriate Western experience to develop a set of simple "imaginary" numbers from which at least equally "imaginary" yet not entirely unreasonable frequency distributions and probabilities can be deduced. In this way the fully articulated model could be utilized. The second avenue is to work not with the complete model but with only the Markov process component, which has previously been tested against United States data.[6] Direct empirical estimates of frequency distributions and probabilities can be made from whatever sources might help suggest their forms, without attempting to deduce them from another component. These numbers can then be used as the basis for exploration of the projection component of the complete model. The results of such experiments may in turn have implications for the later design of experiments with the other components of the complete model. This second avenue is the one we have adopted.

The Computer Programs

Two computer programs in the FORTRAN language have been used in our experiments. The differences between them are not great and will be made clear later in this section. We introduce them with a brief discussion of the basic algorithm common to them both.

Age-and-parity-specific birth rates, if available, can be treated as transition probabilities p_{ij} where $i \neq j$, that is as probabilities of change to higher parity. Such birth rates therefore are a basic input to the computer programs. Referring to the example of a transition matrix given above, this input gives the non-zero off-diagonal entries. The zero entries are fixed, and the diagonal entries, or probabilities of remaining in the same parity, are given by $p_{ii} = 1-p_{ij}$. Therefore the input birth rates determine the matrix.

The new frequency distribution at time t+1 may be computed by a normal matrix multiplication working with full columns. It may also be computed by simpler schemes giving the same results working either with both the diagonal and non-zero off-diagonal entries or with just the latter. We chose to work with just the non-zero off-diagonal entries. Eight states are used in the frequency distributions and transition matrices: parities zero, one, two, etcetera, through seven-and-up.

The basic algorithm is as follows. For a given cohort at a given age the probability that a woman in the highest parity state, seven-and-up, will have another child during the year is multiplied by the number of women in the seven-plus state. This gives the annual births to women of parity seven-plus in the year from t to t+1. Next, the birth probability for women in the second highest state, parity six, is multiplied by the number of women of that parity. This gives the annual births to women of parity six or, equivalently, the number of women changing parity from six to seven. Then, this number of women changing parity is added to the number already in parity seven-and-up, and is subtracted from the number initially in parity six. This gives the new frequency of women in seven-and-up at time t+1, and leaves a residual of non-changers in six in preparation for the next lower parity change. Finally, this number of women changing from six to seven, now identified as its equivalent in annual births, is added to the annual births to women of seven-plus.

Cycling through again, the number of women in parity five is multiplied by their birth probability, giving the number changing from five to six. This change is added to the previous residual not changing out of six, and is subtracted from those initially in five. This gives the new frequency in parity six, and again prepares for the next lower change. Finally, the change from five to six is added to the sum of the annual births to women of parity six and seven-plus.

This cycle continues until women changing parity from zero to one are added to the residual not changing out of one, and are subtracted from those initially in zero. This gives the new frequencies in parities one and zero, completing the new frequency distribution at t+1. Addition of this last parity change to the sum of annual births to women of the higher parities, gives total cohort annual births in the projection year t to t+1. Addition of these cohort annual births to cohort cumulative births as of time t, gives cumulative births at time t+1. This completes the basic algorithm.

The computer programs actually project to completion, one at a time, the birth "histories" of all those cohorts which are of childbearing age as of some initial year. With a childbearing span from age fifteen through forty-five, there are thirty-one such cohorts. Thus the computer input includes thirty-one initial cohort frequency distributions, in addition to the birth rates described above.

Cohort cumulative births are cumulated from the start of childbearing for only the youngest of these initial cohorts. For each of the other initial cohorts the cumulative births as of the start of the initial year are computed by summing the products resulting from multiplying each of the parity state numbers by its respective frequency in the initial distribution of the cohort. This gives an accurate number only for those cohorts which do not yet have any members in parity seven-and-up.

Cumulative births as of the initial year for all cohorts with members in seven-plus, are understated by the numbers of births of order eight and above which have already occurred prior to the initial year. This understatement is progressively greater from the youngest cohort with members in seven-plus through the initial cohort aged forty-five. Since this bias is generated by events prior to the initial year, it is identical in all our experiments and thus does not affect any conclusions. If desired, _ad hoc_ correction factors could be estimated for any particular data.

At the end of the initial year the oldest cohort, having completed childbearing, drops out of the system, all other cohorts increase in age by one year, and a new cohort rises into the first childbearing age entering the system for the start of the next year. There are always thirty-one cohorts in the system. In each successive projection year after the initial year there is one less initial year cohort and one additional younger, replacement cohort. Thus in the second projection year there are thirty initial cohorts and one replacement cohort, while in the thirty-first projection year there is one initial cohort remaining and thirty younger replacement cohorts which have entered after the initial year. The computer programs do not project the replacement cohorts to completion of fertility, but only through the year which ends with the departure from the system of the youngest initial cohort.

The birth projection model is a "non-stationary" process because the birth probabilities in the transition matrix change for each successive age of a given cohort. With the assumption of unique probabilities for each cohort, they can also change for a given age as different cohorts pass through that age. By this assumption it would be possible for each cohort to have all age-and-parity-specific birth probabilities for its entire history, different from the corresponding probabilities for every other cohort. Similarities of social values due to proximities in historical experience, however, are likely to make probabilities for adjacent cohorts only moderately, not radically, different.

The first of the two computer programs is based on the simplifying assumption that probabilities, while differing for different ages and parities for a given cohort, are identical for different cohorts. They are assumed non-stationary within each cohort but stationary between cohorts. We refer to this as the "fixed probability" program.

The second program is similar to the first except that provision is made for changing probabilities between cohorts in line with the uniqueness assumption. This provision is necessary to study the effect of changes, specifically of fertility reductions, which take place over time. A single probability can be changed in a single year after the initial year. Or probabilities for as many as eight parities per age, for twenty-five ages per projection year, for fifteen projection years can be changed. We refer to this as the "variable probability" program.

Printed output from both programs includes two tables for each initial cohort. The first contains the projected sequence of frequency distributions of the cohort by parity with corresponding cohort annual and cumulative births. The second contains the sequence of distributions of cohort annual births by parity change. In the "fixed" program the input age-and-parity-specific birth rates are printed after all cohort projection tables have been printed. In the "variable" program they are printed as a third table for each cohort. The "variable" program also prints the same three tables for each replacement cohort entering at the bottom of the system.

A component for adjustment of cohort frequency distributions for mortality, marriage or migration has not been included in the computer programs. Therefore there are two alternatives for treatment of total cohort size. The first is to use the relative distribution of total cohort sizes prevailing among the initial cohorts, and add all replacement cohorts at an estimated total size, all in parity zero. The second is to use a uniform total cohort size for all cohorts, and add all replacements at that size. We have done the latter, using a uniform size of one thousand. With this procedure, cohort cumulative births at age forty-five produced by the "fixed" program, is a measure for age-and-parity-specific projections which corresponds to the "total fertility rate" used by demographers in conjunction with simple age-specific projections. This procedure also produces birth projections for replacement cohorts in the "fixed" program which are identical to those of the youngest initial cohort. This is not necessarily the case in the "variable" program, which is why projection tables for replacement cohorts are printed by the "variable" but not by the "fixed" program.

The final output table printed by both programs lists by projection year the total of annual births for the declining number of initial cohorts, for the increasing number of younger cohorts, and for the constant number of all childbearing cohorts. It also lists the birth rate per thousand women of childbearing age, called by demographers the "general fertility rate". As computed here it is somewhat different from the usual form in that in any given projection year, it has an equal number of women contributing to it from each cohort of childbearing age. The last two numbers output by both programs are the average of this annual rate over the thirty-one projection years, and the average of the cumulative birth rate at age forty-five over the thirty-one initial cohorts.

A single computer run with either program produces a volume of numerical output which is not indigestible. With several runs, however, the volume soon mounts to enough for more than one meal. The different output measures of the programs may make suitable a variety of experiments with the programs. Different measures may also be useful to different aspects of the same general problem. Thus the projected cohort frequency distributions and distributions of annual births resulting from the fertility changes in our experiments may contain information relevant to our concern with programs of fertility reduction. But study of these distributions is beyond the scope of this thesis. They must be left for another sitting.

For the present we are interested in generalized measures of system response. Therefore we have considered only two program outputs. The first is cohort cumulative births at age forty-five. This measures the total volume of births to a cohort on completion of childbearing. This volume is the ultimate concern of a program of fertility reduction, regardless of details of spacing within cohorts and overlap between them. The second output is the average of the cumulative births at forty-five for the thirty-one initial cohorts. This average provides a single number to summarize the output or response of the total system of initial cohorts on a given run. It is the most general measure by which to compare results of a set of runs.

The Data

The computer programs are well suited to the data made available to us from the Khanna study.[7] Though from a longitudinal study, they are in this instance treated as cross-section data from a point in time. The data are from a population of slightly over 2000 women, then currently married with husband present, drawn from eleven villages in the rural Punjab. The Khanna study began in 1953 and ended in 1960. The data were graciously made available to us by Dr. J. B. Wyon of the Department of Demography and Human Ecology, Harvard School of Public Health, who was field director for the study.

An age-and-parity-specific birth rate is calculated by dividing births in a given year to women of the specified age and parity, by the number of such women in the population during the year, and then multiplying the result by 1000 to obtain the usual form as a rate per thousand specified women. The data provided to us include live births by age and parity of mother for women aged fifteen through forty-five for 1957, 1958 and 1959. There were no live births in those years to women of the sample population aged forty-six or more. The data also include frequency distributions by age and parity for all women in the sample population in 1959. Thus we have birth rate numerators for three years and denominators for one year. Using numerators for 1959 only, corresponding to the denominators, would waste too much information from 1957 and 1958. Ways of estimating denominators for the two earlier years have been considered and rejected. Finally, we have calculated birth rates, effectively for 1959, using the averages of the three numerators with the corresponding single year denominators. Frequency distributions of thousand-member cohorts required by the computer programs have been obtained by proportioning the distributions in the data provided to us, and multiplying the results by one thousand.

An earlier set of computer runs, in the nature of tests of the experimental method, were made using the unsmoothed results of these calculations. Results from these runs indicated that it would be easier to work with simpler, generalized input producing more easily observable results. Therefore the calculated birth rates and frequency distributions have been graphically smoothed. Some reassurance as to the forms which should be generated in the smoothing is obtained from the limited comparisions possible of the Khanna data with data from other studies.

An earlier stage of work with the projection model unearthed several statistical sources on fertility in India. Of these, two contain data adequate for comparison with the Khanna data. In both cases the data is not distributed by single age but by five-year groupings of cohorts (ages 15-19, 20-24, etc.). Also, birth rates can only be compared for age-specific rates in which women are grouped solely by age without distinction of parity. These qualifications required regrouping of the Khanna data and calculation of new rates and distributions for comparison at a more aggregated level than that being smoothed.

Birth rates are compared first. Estimates of age-specific birth rates by five-cohort groups for India, based on both rural and urban surveys, are relatively plentiful in published sources. However, differences in the characteristics of the respondents and other incomparabilities result in all but one being eliminated from comparison with Khanna. The Mysore Population Study[8] of 1952 reports sample sizes and respondent characteristics, especially women currently married with husband present, such as to make its data the most comparable to the Khanna data. It has the advantage, also, of containing data for five sample areas, including three rural zones, towns and Bangalore City.

Khanna and Mysore five-cohort age-specific birth rates are plotted in Figure 1. It is interesting to note that rates for both rural zones and towns in Mysore are generally consistent with those for Khanna, while the Bangalore City curve is quite different, being straighter and lower. The Khanna data give a noticeably smoother curve than the Mysore data.

To use whatever guidance can be obtained from the shape of these curves, the Khanna single cohort age-and-parity-specific birth rates have also been plotted versus age, giving eight graphs, one for each parity, for graphical smoothing. (See Appendix)

Two constraints for consistency among the curves are introduced in this smoothing. These include that the maximum value for any parity-curve be lower than, and occur at a greater age than the maximum value for the preceeding parity-curve. The latter constraint is violated only for parities six and seven-plus which are both given maximum birth rates at age thirty-three. The resulting set of smoothed curves is shown in Figure 2. The age-and-parity-specific birth rates used in the experiments are read from these curves and are shown in Table 1. Each smooth curve is a graphic display of one column of the table. (Note: The reader is warned that the ordinate scales are sometimes not the same for graphs of similar data versus different abcissae for both birth rate and frequency data in this section and the Appendix.)

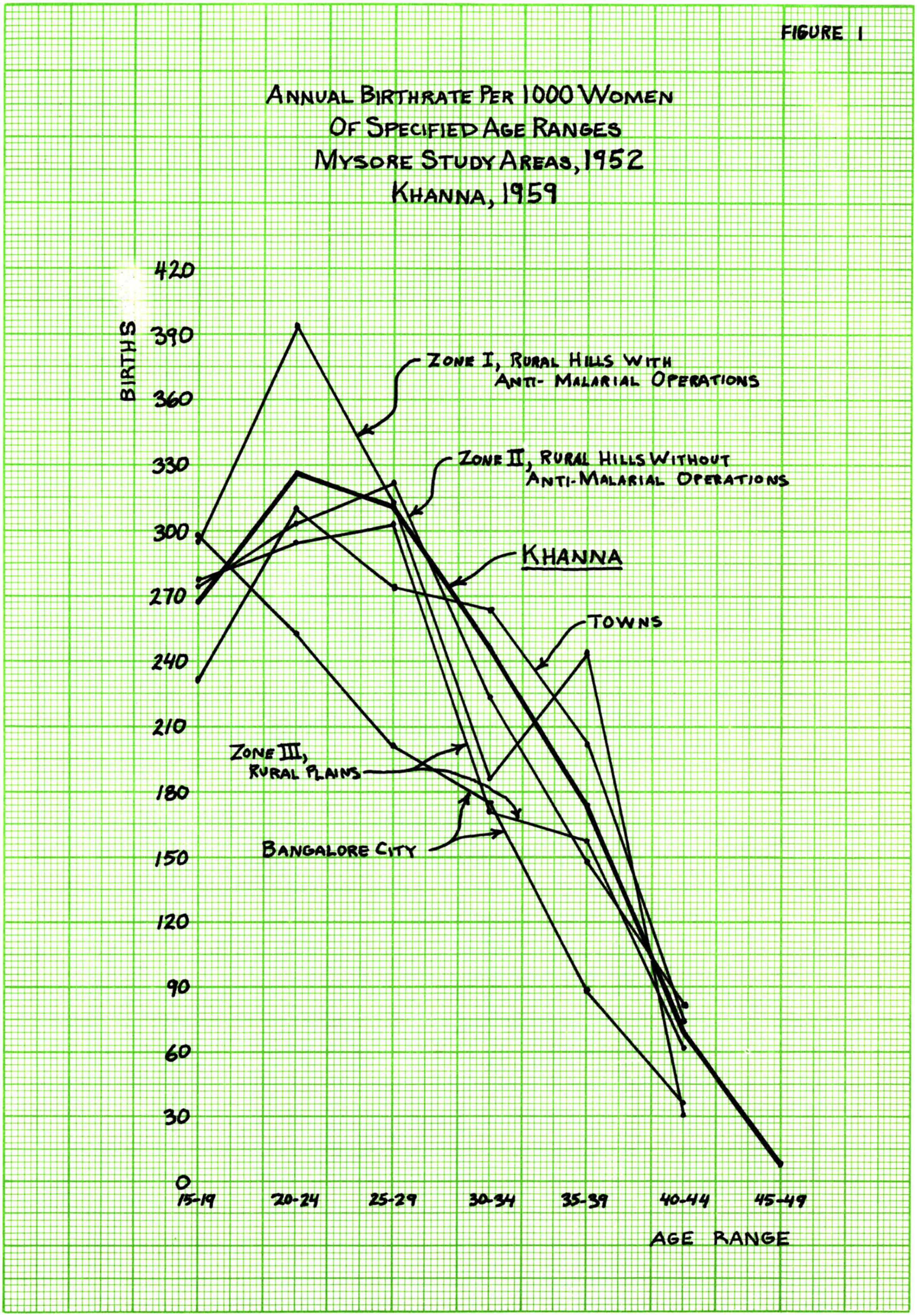


FIGURE 1
ANNUAL BIRTHRATE PER 1000 WOMEN
OF SPECIFIED AGE RANGES
MYSORE STUDY AREAS, 1952
KHANNA, 1959
BIRTHS
420
390
360
330
300
270
240
210
180
150
120
90
60
30
0
ZONE I, RURAL HILLS WITH
ANTI-MALARIAL OPERATIONS
ZONE II, RURAL HILLS WITHOUT
ANTI-MALARIAL OPERATIONS
KHANNA
TOWNS
ZONE III,
RURAL PLAINS
BANGALORE CITY
15-19
20-24
25-29
30-34
35-39
40-44
45-49
AGE RANGE

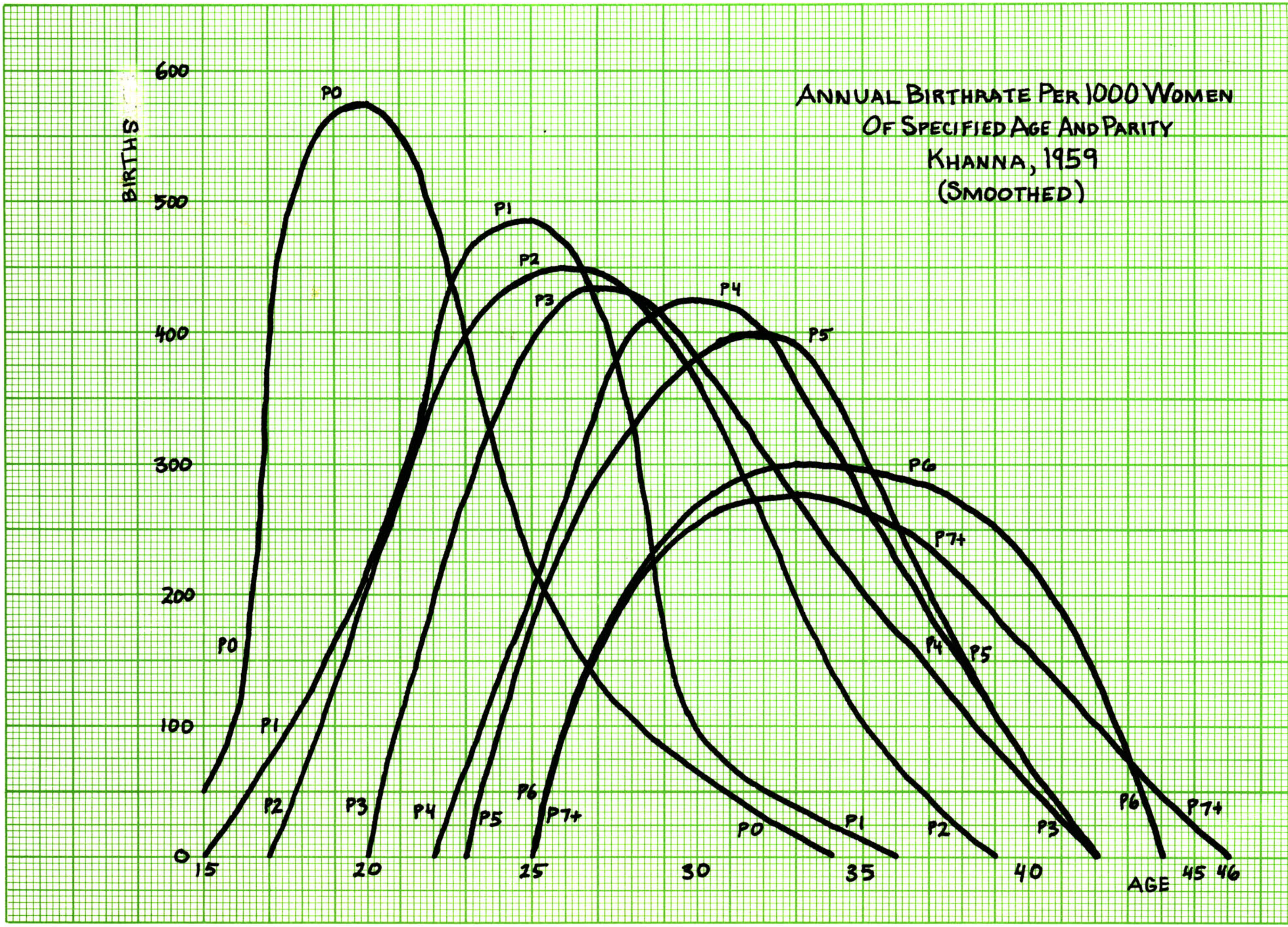

ANNUAL BIRTHRATE PER 1000 WOMEN
OF SPECIFIED AGE AND PARITY
KHANNA, 1959
(SMOOTHED)
BIRTHS
600
500
400
300
200
100
0
15
20
25
30
35
40
45
46
AGE
P0
P1
P2
P3
P4
P5
P6
P7+

FIGURE 2

BIRTH RATE PER 1000 WOMEN OF SPECIFIED AGE AND PARITY

AGE	PARITY=0	1	2	3	4	5	6	7+
15	50.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	100.0	35.0	0.0	0.0	0.0	0.0	0.0	0.0
17	350.0	75.0	0.0	0.0	0.0	0.0	0.0	0.0
18	525.0	115.0	65.0	0.0	0.0	0.0	0.0	0.0
19	565.0	160.0	135.0	0.0	0.0	0.0	0.0	0.0
20	575.0	215.0	225.0	0.0	0.0	0.0	0.0	0.0
21	550.0	275.0	295.0	110.0	0.0	0.0	0.0	0.0
22	500.0	375.0	355.0	200.0	0.0	0.0	0.0	0.0
23	400.0	460.0	400.0	280.0	70.0	0.0	0.0	0.0
24	300.0	480.0	430.0	350.0	140.0	100.0	0.0	0.0
25	225.0	485.0	445.0	395.0	205.0	180.0	0.0	0.0
26	175.0	470.0	450.0	425.0	280.0	240.0	100.0	100.0
27	130.0	425.0	445.0	435.0	350.0	290.0	160.0	160.0
28	105.0	340.0	430.0	430.0	400.0	325.0	210.0	205.0
29	85.0	175.0	400.0	410.0	420.0	355.0	245.0	235.0
30	65.0	95.0	365.0	355.0	425.0	380.0	270.0	255.0
31	45.0	70.0	315.0	345.0	420.0	395.0	282.0	267.0
32	30.0	50.0	260.0	310.0	400.0	400.0	297.0	273.0
33	15.0	35.0	200.0	270.0	365.0	390.0	300.0	275.0
34	0.0	25.0	150.0	235.0	320.0	355.0	299.0	272.0
35	0.0	12.0	100.0	200.0	270.0	310.0	297.0	265.0
36	0.0	0.0	70.0	170.0	225.0	255.0	292.0	250.0
37	0.0	0.0	45.0	140.0	185.0	200.0	282.0	235.0
38	0.0	0.0	20.0	110.0	145.0	155.0	270.0	210.0
39	0.0	0.0	0.0	80.0	105.0	105.0	250.0	185.0
40	0.0	0.0	0.0	55.0	70.0	70.0	225.0	155.0
41	0.0	0.0	0.0	25.0	35.0	30.0	190.0	130.0
42	0.0	0.0	0.0	0.0	0.0	0.0	140.0	100.0
43	0.0	0.0	0.0	0.0	0.0	0.0	80.0	75.0
44	0.0	0.0	0.0	0.0	0.0	0.0	0.0	45.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	20.0

The other data source comparable with Khanna is a fertility survey taken in 1955 in the industrial city of Kanpur in the state of Uttar Pradesh.[9] It is the only published source we found which includes parity as a characteristic. The data consist of frequency distributions by parity of five-cohort groups. Certain entries in the body of the published table are incorrect. Corrections have been made consistent with printed row and column totals and with information in other tables in the report. Both the Kanpur and Khanna data have been proportioned and multiplied by 1000 to obtain distributions by parity of five-cohort groups of one-thousand women per group.

The Khanna five-cohort distributions are graphed versus parity in Figures 3 and 4. The Kanpur distributions are shown in Figures 5 and 6. Strong consistency is noted in the shapes of the two sets of curves as the frequency maxima for succeeding age groups march across the parities.

The Khanna single cohort frequency distributions of 1000-women cohorts aged fifteen through forty-five are graphed versus parity (see Appendix) but have not been smoothed directly. Instead, in order to examine the progression of curve shapes, the five-cohort frequencies for Khanna and Kanpur have been plotted versus age, holding parity constant, in Figures 7 through 10.

Finally, to reduce the number of single cohort curves to be smoothed, the Khanna single cohort frequencies have also been plotted versus age, holding parity constant. It is these curves which have been directly smoothed. (See Appendix) Although the computer input extends only through age forty-five, these curves have been extended through age forty-nine in order to assist smoothing the tails of the distributions. A consistency constraint similar to that for birth rates is maintained, namely a maximum frequency which declines steadily with parity and with age. The smoothing process here was considerably more difficult than for the birth rates, since the sums of ordinates of the smoothed curves across all parities at each age must equal 1000, the total cohort size at each age. The resulting set of smoothed curves is shown in Figure 11. The frequency distributions by parity used in the experiments are read from these curves and are shown in Table 2. Each smooth curve is a graphic display of one column of the table.

The dominant shape of both frequency and birth rate data is an association of increasing parity with increasing age such that data maxima tend to shift from low age-low parity to high age-high parity. This association is one basis for the consistency constraints imposed. This form in the frequency distributions is a reflection of the same form in the birth rates which generate the distributions. This is demonstrated conclusively in the base run with the smoothed data, in which the complete set of thirty-one frequency distributions generated for the initial cohort aged fifteen, which starts with 1000 members in parity zero, shows similarity in form to the complete set of smoothed Khanna initial distributions.

The fact that Khanna and Kanpur City frequency distributions of five-cohort groups by parity are so similar, indicates that their age-and-parity-specific birth rates are also similar. This conclusion is reinforced by comparison of Khanna and Kanpur five-cohort age-specific birth rates. Though not graphed, the Kanpur five-cohort rates[10] are close to those for Khanna. This constrasts with the considerable difference between Khanna five-cohort rates and those for Bangalore City, shown above. An explanation may lie in the fact that population samples, in the Mysore study were carefully constructed to represent the several groups in the social structure of each area, including Bangalore City. In the Kanpur study, however, the fertility survey was added to the household schedule of a larger study after it was well underway. As a result the Kanpur fertility data cover principally lower income households. These households include large numbers of relatively recent migrants from rural villages. Therefore, the Kanpur study would be expected to be a better source for comparison with Khanna than broadly representative urban samples such as Bangalore.

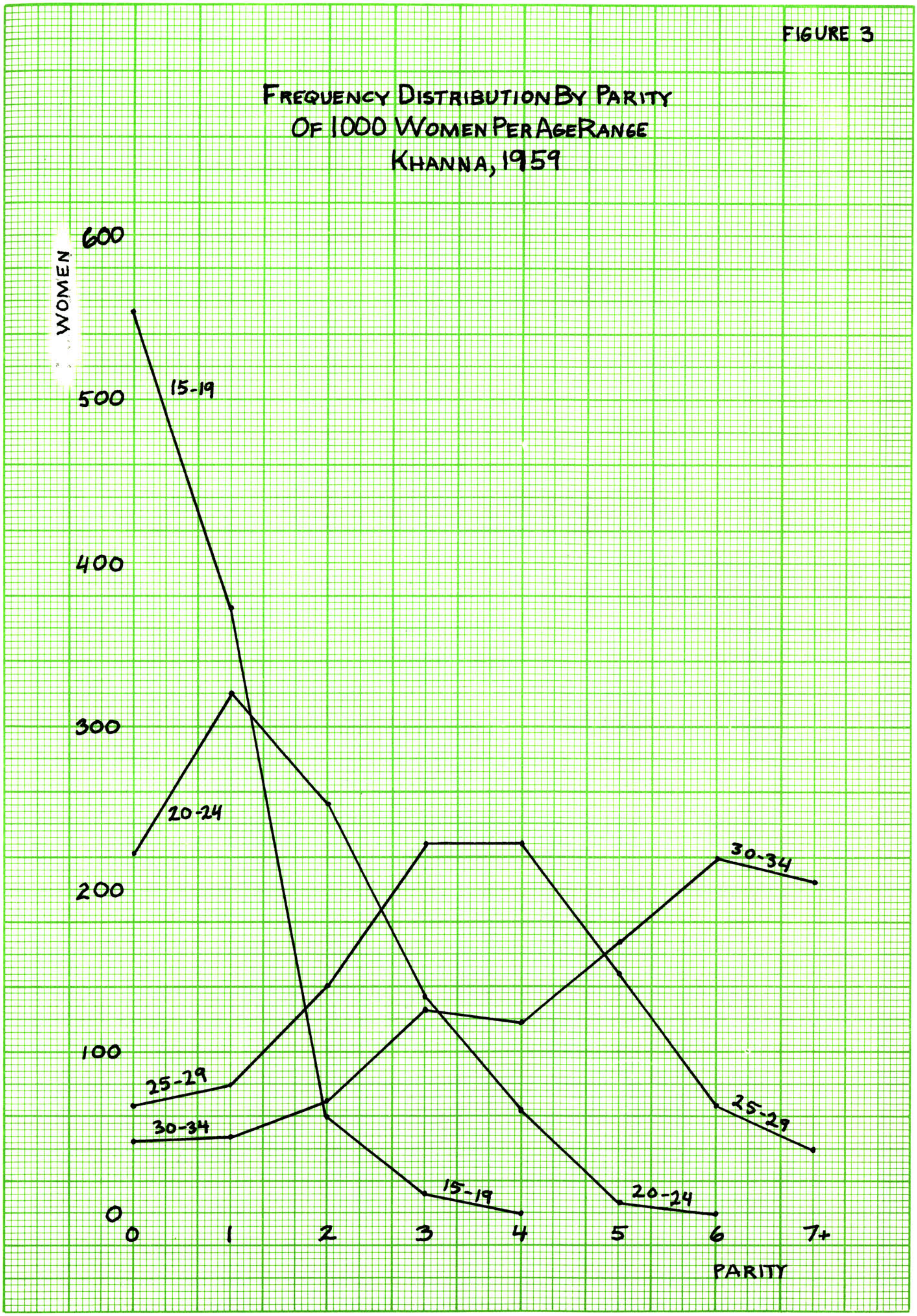


FIGURE 3
FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE RANGE
KHANNA, 1959
WOMEN
600
500
400
300
200
100
0
15-19
20-24
25-29
30-34
30-34
25-29
15-19
20-24
0
1
2
3
4
5
6
7+
PARITY

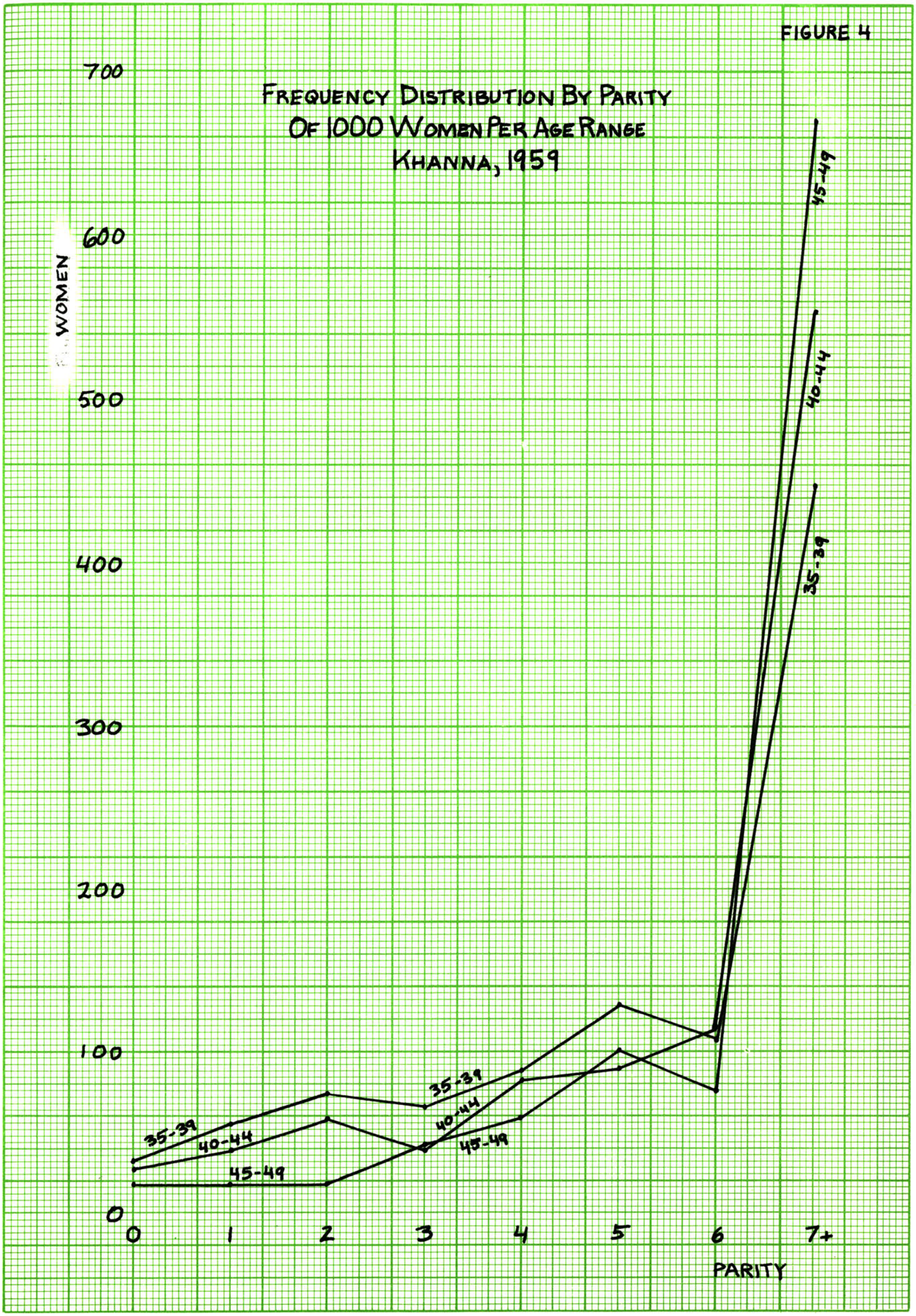


FIGURE 4
FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE RANGE
KHANNA, 1959
WOMEN
700
600
500
400
300
200
100
0
0
1
2
3
4
5
6
7+
PARITY
35-39
40-44
45-49
35-39
40-44
45-49
45-49
40-44
35-39

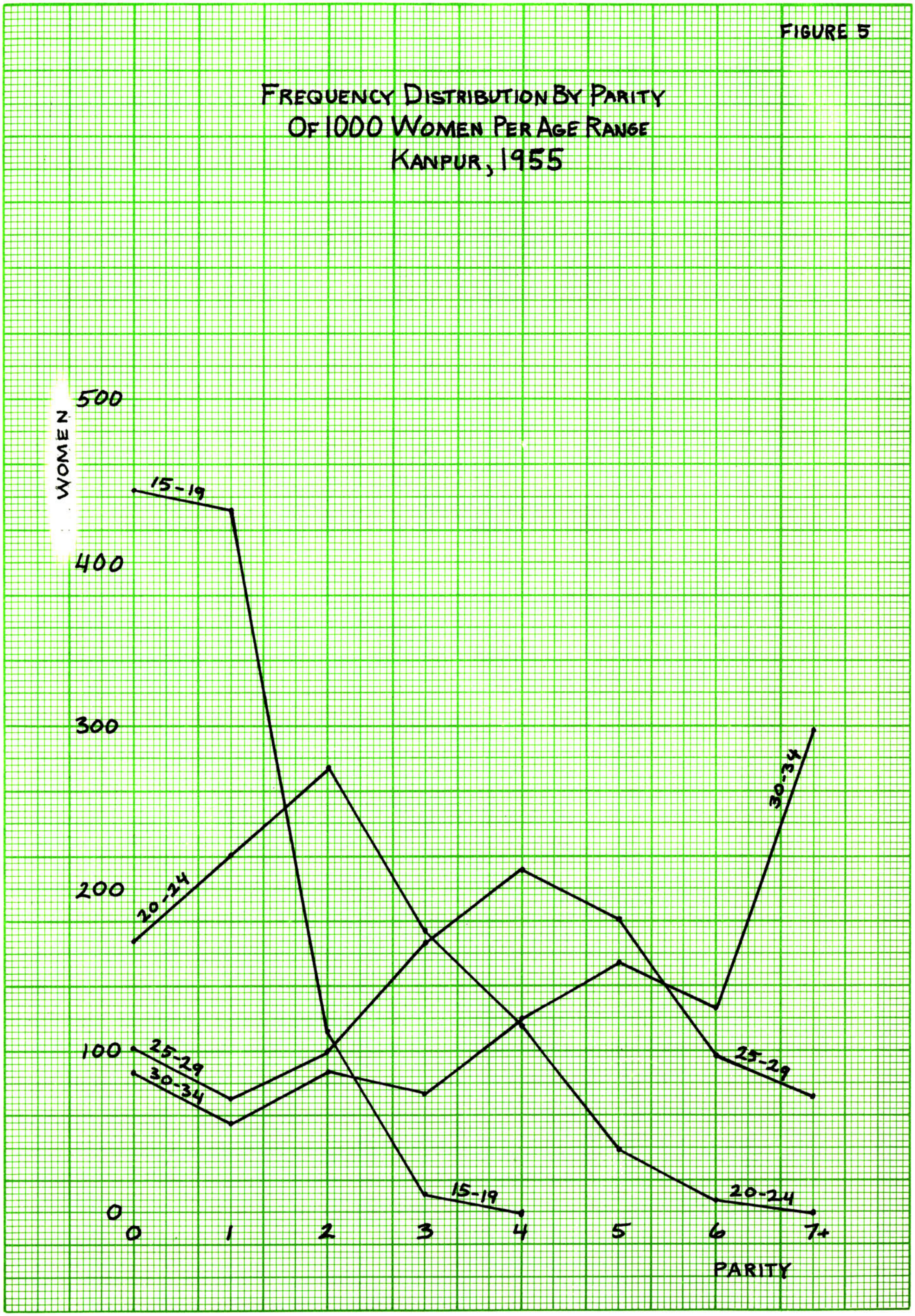
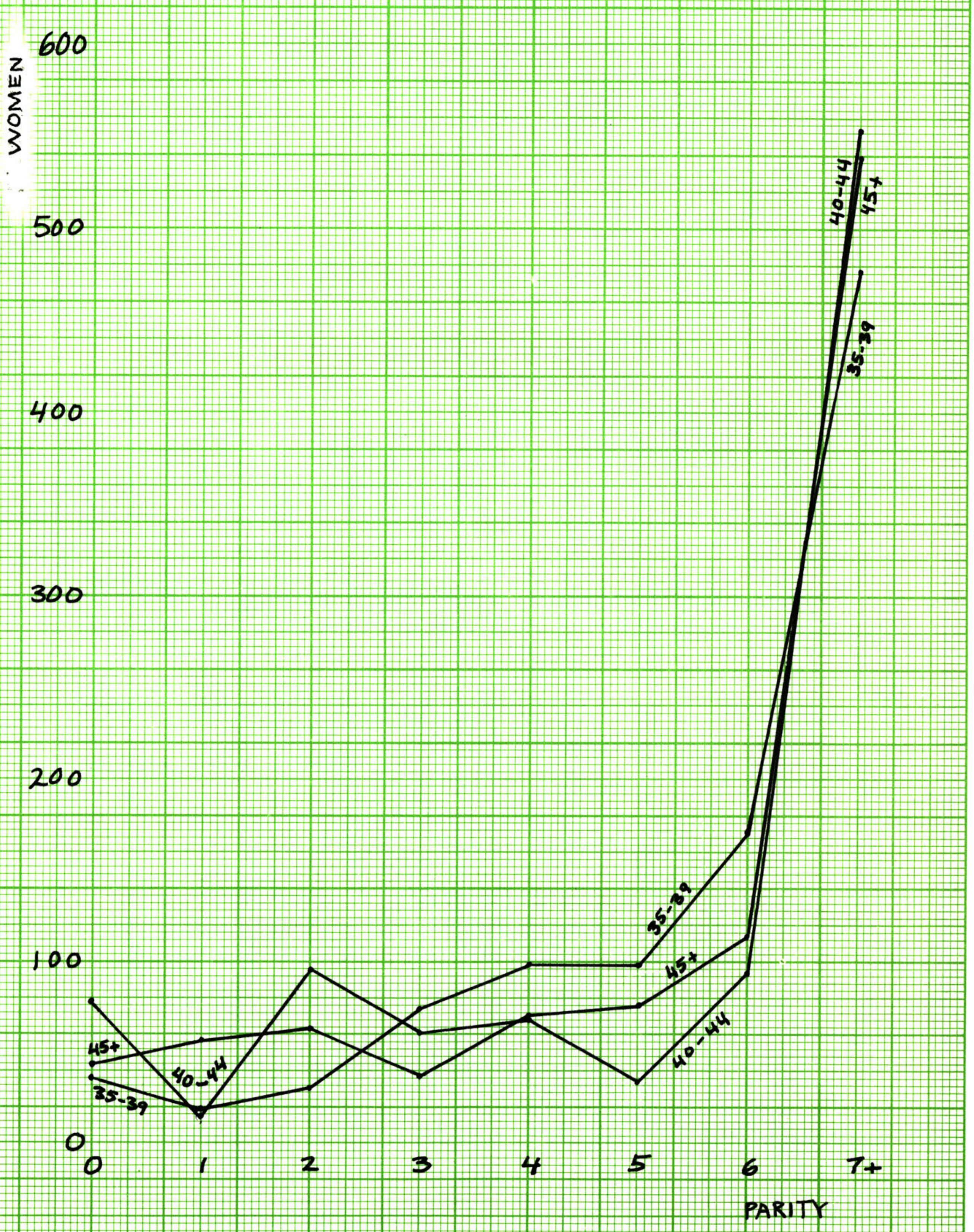
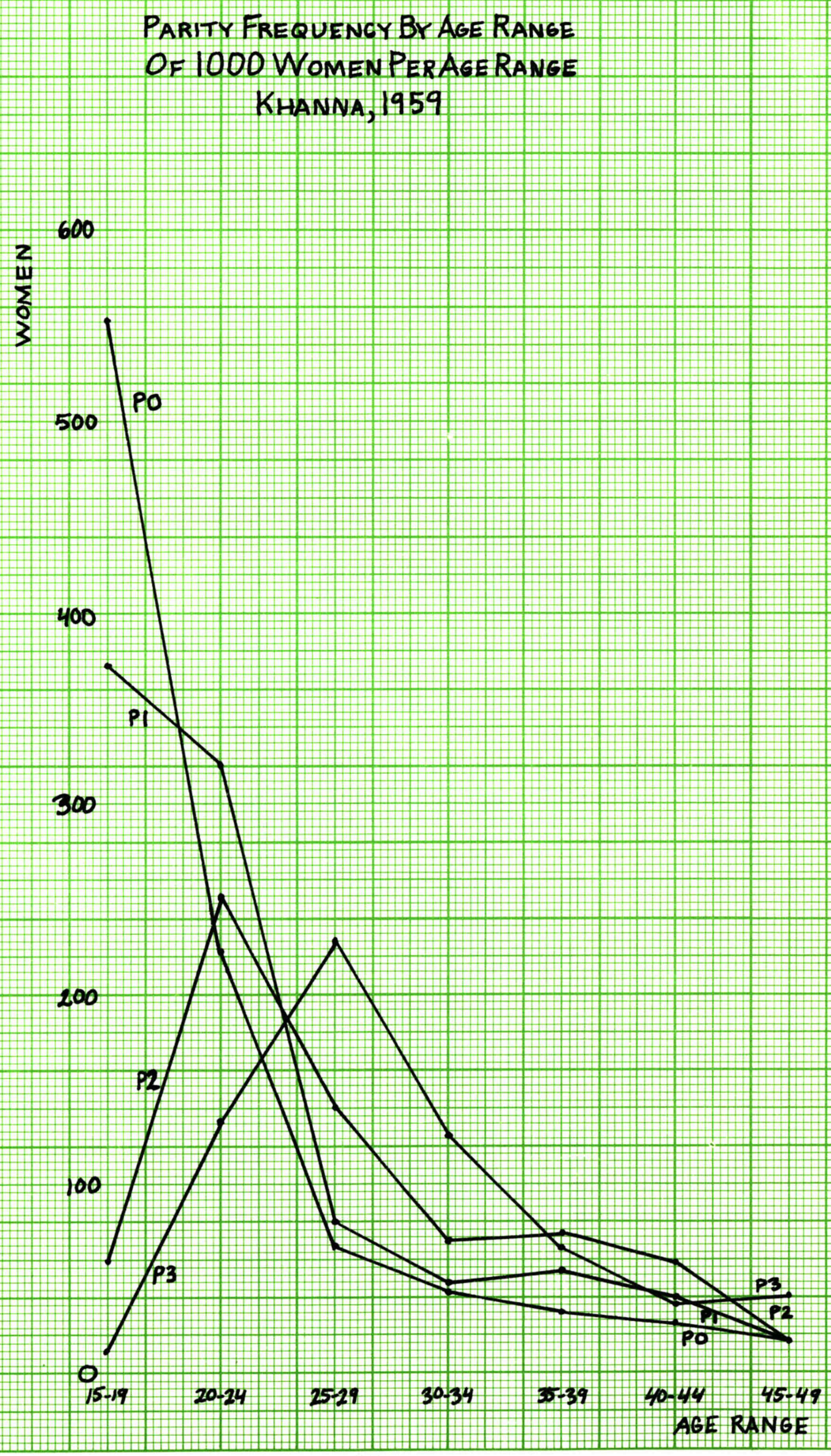


FIGURE 5
FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE RANGE
KANPUR, 1955
WOMEN
500
400
300
200
100
0
15-19
20-24
25-29
30-34
15-19
20-24
25-29
30-34
0
1
2
3
4
5
6
7+
PARITY

FIGURE 6

FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE RANGE
KANPUR, 1955




PARITY FREQUENCY BY AGE RANGE
OF 1000 WOMEN PER AGE RANGE
KHANNA, 1959
WOMEN
600
500
400
300
200
100
0
P0
P1
P2
P3
15-19
20-24
25-29
30-34
35-39
40-44
45-49
AGE RANGE

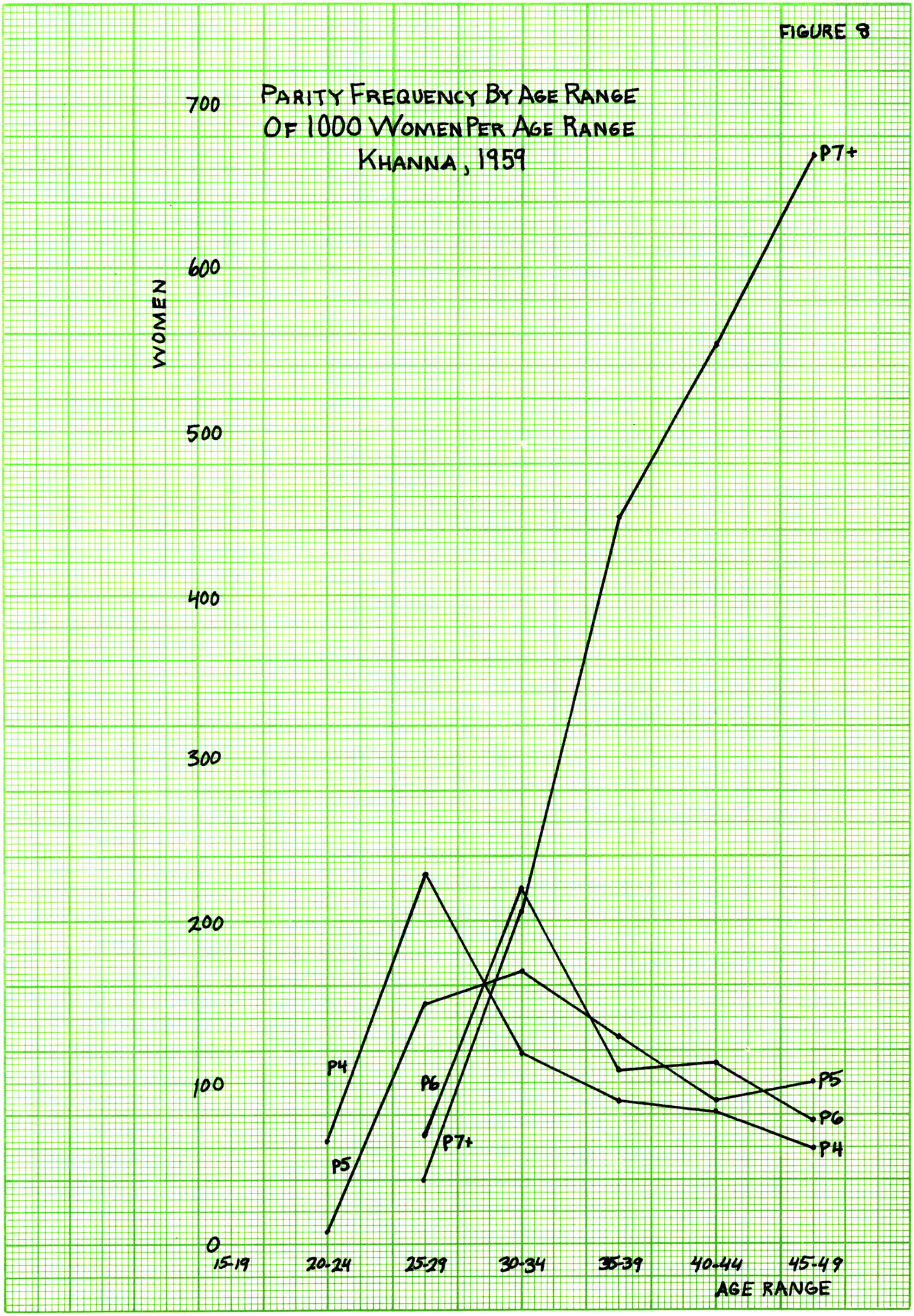


FIGURE 8
PARITY FREQUENCY BY AGE RANGE
OF 1000 WOMEN PER AGE RANGE
KHANNA, 1959
WOMEN
700
600
500
400
300
200
100
0
15-19
20-24
25-29
30-34
35-39
40-44
45-49
AGE RANGE
P4
P5
P6
P7+

FIGURE 9

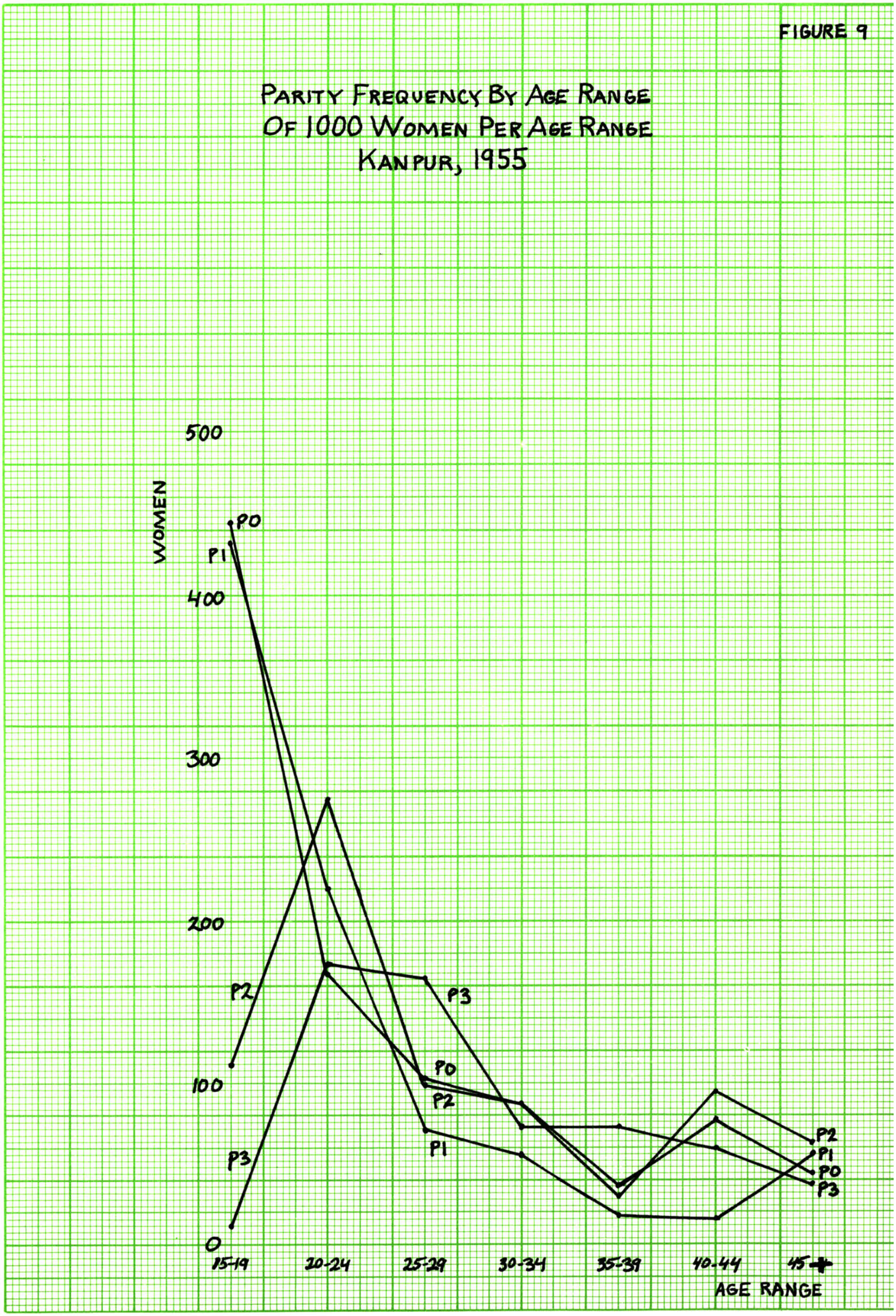
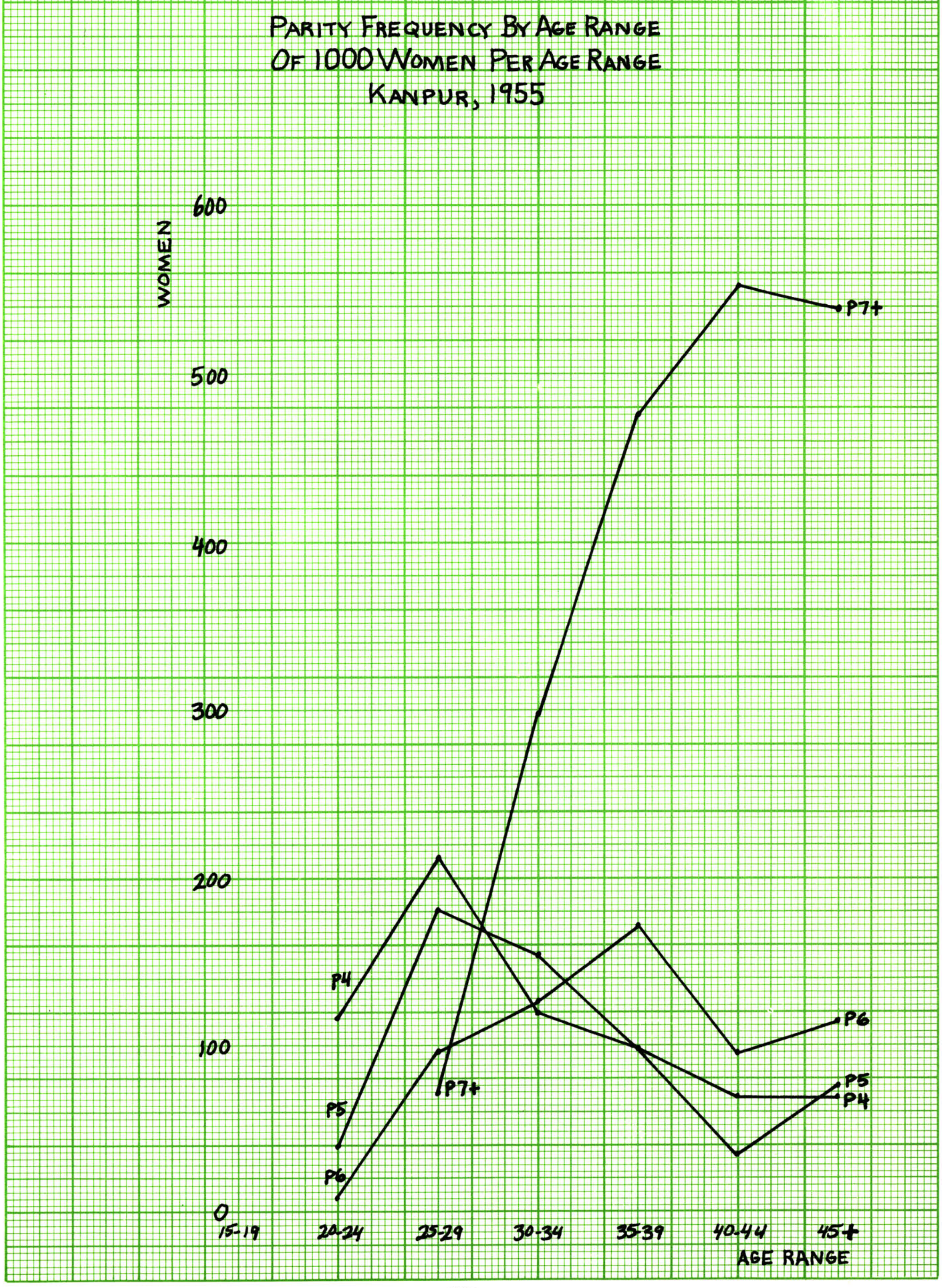

PARITY FREQUENCY BY AGE RANGE
OF 1000 WOMEN PER AGE RANGE
KANPUR, 1955
WOMEN
500
400
300
200
100
0
P0
P1
P2
P3
15-19
20-24
25-29
30-34
35-39
40-44
45 +
AGE RANGE

FIGURE 10


PARITY FREQUENCY BY AGE RANGE
OF 1000 WOMEN PER AGE RANGE
KANPUR, 1955
WOMEN
600
500
400
300
200
100
0
15-19
20-24
25-29
30-34
35-39
40-44
45+
AGE RANGE
P7+
P7+
P6
P5
P4
P4
P5
P6

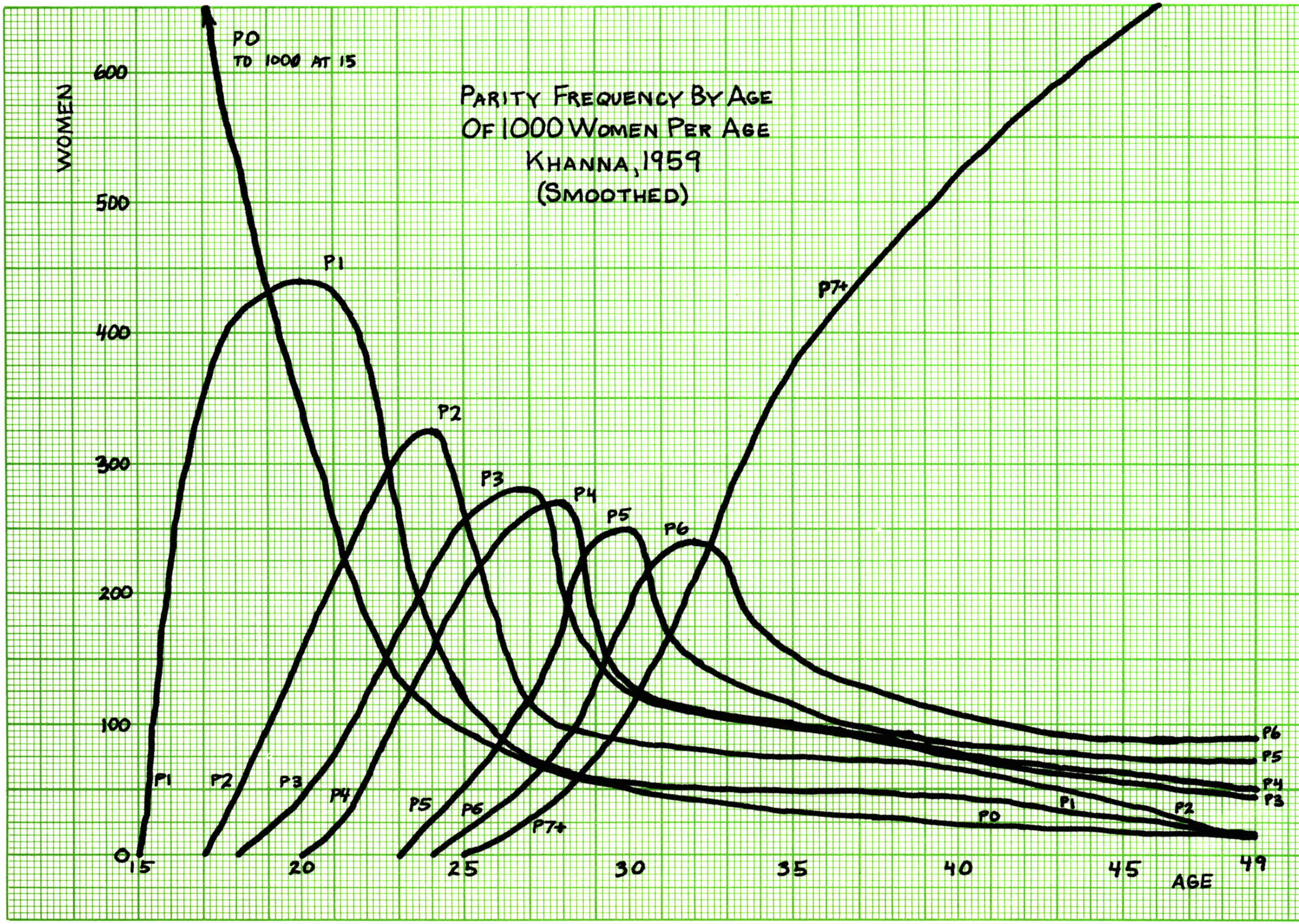

PARITY FREQUENCY BY AGE
OF 1000 WOMEN PER AGE
KHANNA, 1959
(SMOOTHED)
WOMEN
AGE
P0
TO 1000 AT 15
P1
P2
P3
P4
P5
P6
P7+
0
100
200
300
400
500
600
15
20
25
30
35
40
45
49

FIGURE II

FREQUENCY DISTRIBUTION OF 1000 WOMEN PER COHORT BY PARITY IN 1959

COHORT/AGE	PARITY= 0	1	2	3	4	5	6	7+
194415	1000.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
194316	780.0	220.0	0.0	0.0	0.0	0.0	0.0	0.0
194217	665.0	335.0	0.0	0.0	0.0	0.0	0.0	0.0
194118	540.0	410.0	50.0	0.0	0.0	0.0	0.0	0.0
194019	445.0	430.0	105.0	20.0	0.0	0.0	0.0	0.0
193920	350.0	440.0	165.0	45.0	0.0	0.0	0.0	0.0
193821	265.0	435.0	210.0	75.0	15.0	0.0	0.0	0.0
193722	185.0	385.0	255.0	120.0	55.0	0.0	0.0	0.0
193623	145.0	260.0	310.0	175.0	110.0	0.0	0.0	0.0
193524	112.0	173.0	325.0	210.0	155.0	25.0	0.0	0.0
193425	97.0	120.0	260.0	255.0	200.0	50.0	18.0	0.0
193326	83.0	90.0	175.0	275.0	245.0	82.0	38.0	12.0
193227	71.0	72.0	115.0	280.0	265.0	115.0	55.0	27.0
193128	63.0	64.0	97.0	200.0	270.0	181.0	80.0	45.0
193029	57.0	61.0	93.0	150.0	185.0	245.0	135.0	74.0
192930	50.0	57.0	88.0	125.0	130.0	250.0	190.0	110.0
192831	46.0	54.0	85.0	115.0	117.0	190.0	234.0	159.0
192732	43.0	52.0	80.0	110.0	113.0	150.0	240.0	212.0
192633	40.0	50.0	75.0	105.0	108.0	135.0	225.0	262.0
192534	37.0	50.0	75.0	101.0	105.0	125.0	177.0	330.0
192435	34.0	50.0	75.0	97.0	102.0	115.0	155.0	372.0
192336	32.0	50.0	75.0	93.0	97.0	105.0	142.0	406.0
192237	30.0	50.0	75.0	89.0	93.0	98.0	130.0	435.0
192138	28.0	48.0	72.0	85.0	87.0	93.0	122.0	465.0
192039	26.0	45.0	68.0	81.0	83.0	90.0	115.0	492.0
191940	24.0	43.0	65.0	76.0	79.0	87.0	108.0	518.0
191841	23.0	41.0	61.0	71.0	75.0	84.0	102.0	543.0
191742	22.0	39.0	57.0	66.0	71.0	81.0	98.0	566.0
191643	21.0	35.0	52.0	62.0	68.0	79.0	95.0	588.0
191544	20.0	31.0	45.0	59.0	65.0	77.0	93.0	610.0
191445	19.0	27.0	39.0	57.0	62.0	76.0	92.0	628.0

The Method

As described in the introduction, the basic method used to map results of the experiments is simple comparison with the result of a base run which uses the data of Tables 1 and 2 without change. The structure of cohort births is more or less sensitive to a given change of cohort fertility from base run values, as cumulative births at age forty-five, or the average of this measure for the initial cohorts, deviate by a greater or lesser amount from the base run values of these output measures.

Changes in age-and-parity-specific birth rates are made in one of three basic modes in any experiment, whether with "fixed" or "variable" computer program. The difference between these programs is that birth rate changes with the former represent instantaneous diffusion of fertility behavior different from the base run. Changes with the latter, however, are not fully in effect in the initial projection year but instead are achieved gradually over varying numbers of years of time delay.

Since we are concerned with reduction of fertility, all birth rate changes in the experiments are decreases from base run values. The three modes of change are referred to as "point drops", "row drops" and "column drops". Consider first the "fixed" program. In a "point drop" a single age-and-parity-specific birth rate is decreased. For example in Table 1 the 430.0 rate for age 28 parity 3 might be set at 0.0, and the program run on the computer, generating cohort projections slightly different from those of the base run. In a "row drop" all non-zero birth rates across a row of Table 1 corresponding to a given age are decreased simultaneously. For example the parity 0, 1 and 2 rates of 525.0, 115.0 and 65.0 for age 18 might all be dropped to 0.0 like those of parities 3 through 7+, and the program run. In a "column drop" all non-zero rates across a column of Table 1 for a given parity are decreased simultaneously. For example the non-zero rates of ages 21 through 41 for parity 3 might all be decreased by twenty percent, and a run made, again generating cohort projections different from those of the base run.

With the "variable" program such point, row and column drops are achieved over time. For example with a five year delay in achieving a row drop to zero rates for all parities at age 28, the birth rate input changes for five successive years. The initial cohort aged 28 runs with the base run values. The initial cohort aged 27, when being projected for age 28, runs with eighty percent of the base run values. The initial cohort aged 26, runs at age 28 with sixty percent of base run values, and so forth. Finally the initial cohort aged 23 and all younger cohorts, initial and replacement, run at age 28 with zero birth rates at all parities. Although non-linear rates are permissable, linear rates of reduction of fertility, such as in this example, are used in all time delay experiments with the "variable" program.

IV Experiments and Results

Sensitivity

Four types of experiments have been performed: basic sensitivity to point, row and column drops; linearity of system response to these three modes of change of birth rates; aggregation of responses to compound changes of birth rates from these simple modes; and effect of time delay or rate of "diffusion" of behavior change on response to these modes of change. The frequency distributions in the Khanna data are for 1959. Therefore, in all computer runs, 1959 is recorded as the initial projection year and 1959 through 1989 as the thirty-one year projection span of the programs. However, the particular years are irrelevant to the nature of our experiments.

The sensitivity observations cover a systematic set of birth rate changes. Fifteen point drop runs, each with one of the following age-and-parity-specific birth rates set at zero rather than at its base run value, are made for age 18 parity 1, age 23 parities 1 and 3, ages 28 and 33 parities 1, 3, 5, and 7-plus, age 38 parities 3, 5 and 7-plus, and age 43 parity 7-plus. These effectively cover the non-zero entries in Table 1. Resulting from these runs are fifteen values of the average of cohort cumulative births per thousand women at age forty-five taken over the thirty-one initial cohorts. These values are shown by bar-charts in both Figures 12 and 13. Zero entries occur in Table 1 at some of the intersections of the six ages and four parities which appear in combination in the birth rates chosen for point drops. For example age 18 parity 7-plus and age 43 parity 1 both have birth rates of zero. Since no drop in birth rate and therefore no deflection from base run output is possible with these points, the base run value of average cumulative births is shown at these and similar points in both Figures 12 and 13. All values in Figure 13 are the same as in Figure 12. Only the order of presentation is different. Figure 12 shows values for experimental and zero-deflection point drops ordered by parity. In Figure 13 the point drops are ordered by age.

In the bar-charts of these Figures the height of the bar corresponding to any age-parity intersection point in Table 1 is the value of average cumulative births produced by a computer run with the birth rate for that point set at zero. Both sets of bar-charts reflect the association of increasing parity with increasing age in the input table of birth rates. The higher the parity, the higher the age at which maximum response to a point drop occurs, and vice versa.

In each bar-chart except that for parity 7-plus in Figure 12, maximum deflection from the base run value of average cumulative births occurs for that experimental point drop in the bar-chart for which the base run value in Table 1 is a maximum. However, it is not certain that this matching of maxima in the bar-charts excepting parity 7-plus, holds for all possible point drops in a row or column of Table 1, in addition to the experimental points.

The sizes of deflections or of average cumulative births shown in Figures 12 and 13 are not related in any simple direct fashion to the sizes of the corresponding base run birth rate values. Thus a larger birth rate at one age and parity than at another does not mean a larger deflection will necessarily occur for a point drop run at the first point than for one at the second. Neither are these deflections simply related to the initial frequencies at these points, nor to the products of initial frequencies and base run birth rates. Rather, average cumulative births or its deflection is a function of the number of cohorts affected by a point drop, and of the sizes of deflections of the individual cumulative birth rates for those cohorts which are affected.

Of the initial cohorts, the only ones affected by a point drop are those which are of the same age as, or younger than, the age of the point drop. If a birth rate changes it cannot affect cohorts already older than its specific age. Thus, as the specific age of a point drop changes, the number of initial cohorts affected changes. This can be seen in Figure 14, in which the upper curve shows base run values of cohort cumulative births at forty-five for each of the initial cohorts. The lower curves show the effects of selected point drops. For a given point drop run, the curve remains the same as the base run curve from 1959 up to the year in which childbearing is completed by the initial cohort of the same age as the point drop. For that year and on through 1989, the curve is lower than the base run curve. Thus setting a point drop for age 43 parity 7-plus soon has an effect, in 1961, while a drop for age 18 parity 1 is not seen in the measure of completed childbearing until 1986.

The average of the thirty-one values in the curve for a selected point drop in Figure 14 gives one bar of a bar-chart in Figure 12 and in Figure 13. All fifteen point drop runs are not shown in Figure 14. Only the run for the parity producing the lowest curve for a given age is plotted. These are thereby also the runs which produce maximum deflections in Figures 12 and 13.

From Figure 14 it is seen that, although cohort cumulative births vary over a considerable range for a given run, the range of variation of deflections for point drops, within and even between runs, is not large. The size of deflection of cumulative births for a single cohort for a given point drop, is a function of most of the several age-and-parity-specific birth probabilities remaining to the cohort which are of the age of the point drop and older, and of the parity of the point drop and higher. It is also a function of the number of members of the cohort of the age and parity of the point drop, that is of the first frequency affected by the drop. In these sensitivity experiments with the "fixed probability" computer program, probabilities are the same for successive cohorts

FIGURE 12

Average Of Cumulative Births At Age 45 For 31 Cohorts: Sensitivity To Individual Age-And-Parity-Specific Birth Rates Set At Zero Births Per Year

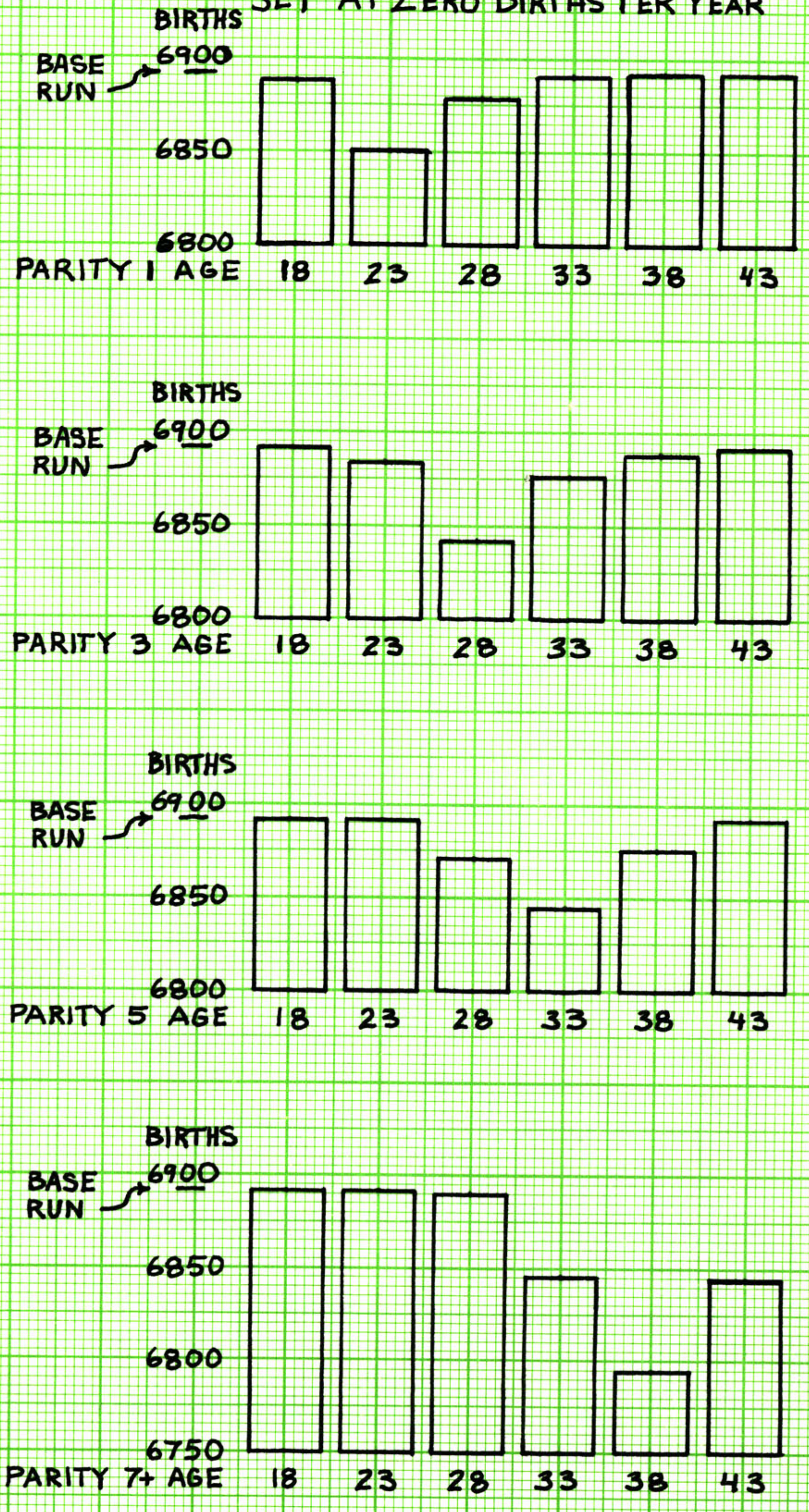
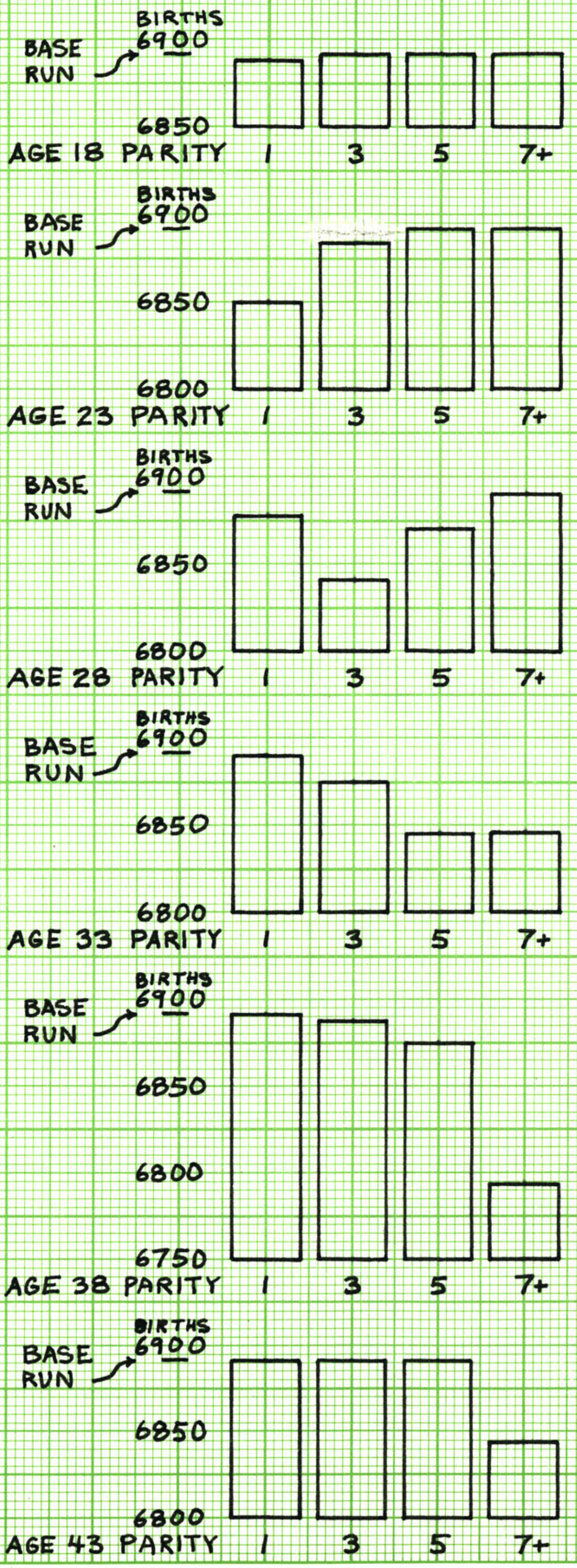


FIGURE 13

AVERAGE OF CUMULATIVE BIRTHS AT AGE 45 FOR 31 COHORTS : SENSITIVITY TO INDIVIDUAL AGE-AND-PARITY-SPECIFIC BIRTH RATES SET AT ZERO BIRTHS PER YEAR



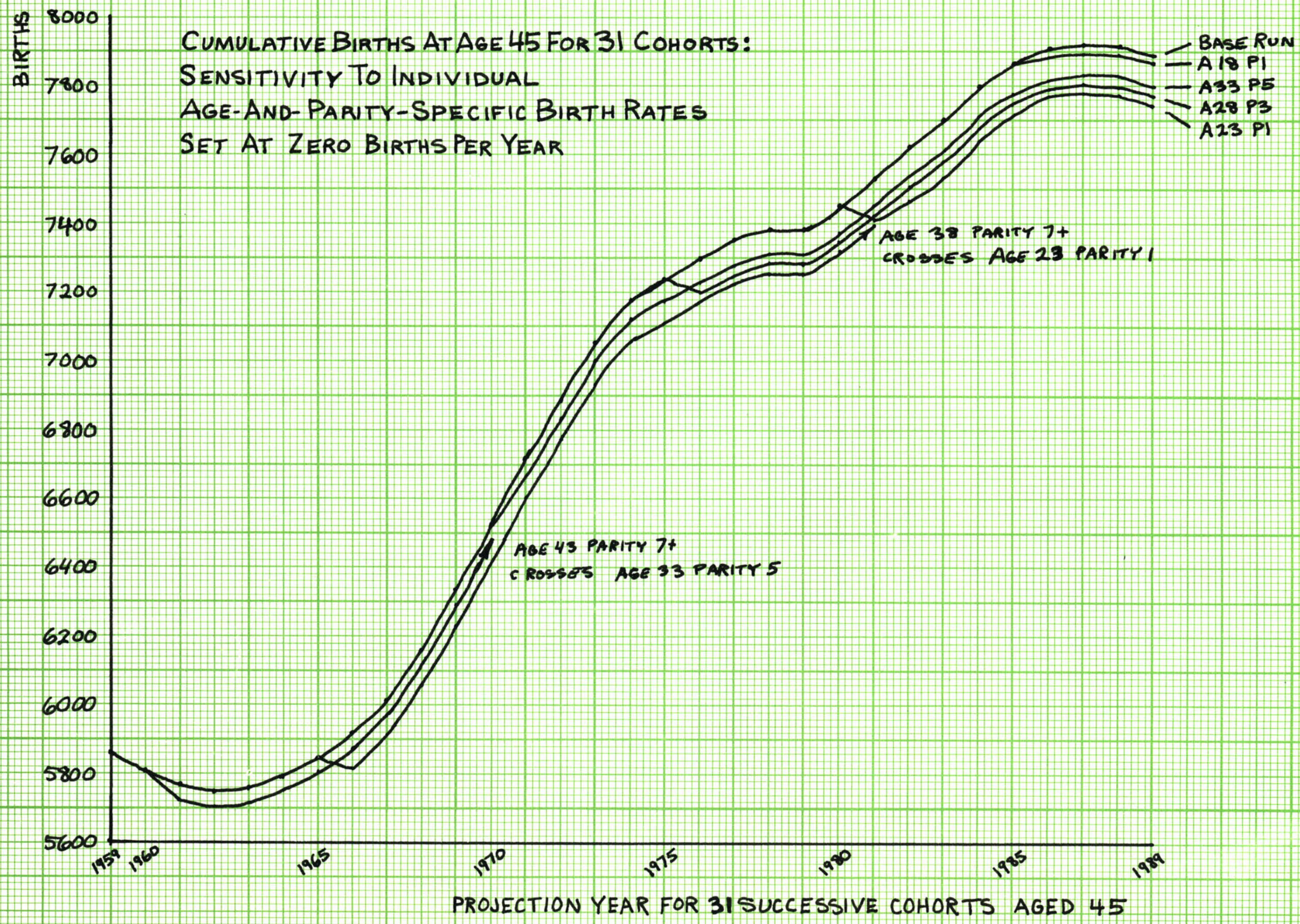

CUMULATIVE BIRTHS AT AGE 45 FOR 31 COHORTS:
SENSITIVITY TO INDIVIDUAL
AGE-AND-PARITY-SPECIFIC BIRTH RATES
SET AT ZERO BIRTHS PER YEAR
BIRTHS
8000
7800
7600
7400
7200
7000
6800
6600
6400
6200
6000
5800
5600
1959
1960
1965
1970
1975
1980
1985
1989
BASE RUN
A 18 P1
A 33 P5
A 28 P3
A 23 P1
AGE 38 PARITY 7+
CROSSES AGE 23 PARITY 1
AGE 43 PARITY 7+
CROSSES AGE 33 PARITY 5
PROJECTION YEAR FOR 31 SUCCESSIVE COHORTS AGED 45

FIGURE 14

within a run. Since the range within each cohort of the first frequency affected by the drop is not likely to be large over the affected cohorts, the range of deflection sizes within a point drop run is not likely to be large.

We have no simple formula for determining cumulative births or their deflections, although a complex one can be derived. Therefore we resort to the simple graphics of response generated by the computer runs, to observe the pattern of interaction of the data with the structure of the model. We are interested in whether this pattern has implications for the design of programs of fertility reduction.

A program may be measured by its utility or results alone, and also by its efficiency, which relates results or output to input and thereby introduces consideration of costs. A measure of efficiency for different decreases of birth rate could be devised for the cohort birth projection model. It would probably best be of the form "cumulative births not occuring per targeted client". However, a preliminary attempt to sketch out a definition of such a measure proved unsatisfactory. Therefore, although it is perhaps a less definitive procedure, we discuss the outcome of the experiments without an added assessment in terms of efficiency or cost of possible program design choices.

With the given data, the relative deflections of the point drop responses are not surprising. They indicate that the birth rates having the most effect on cumulative births are those that lie generally near the diagonal of maximum values in the table of age-and-parity-specific birth rates. What is perhaps not as predictable is that they would be as uniform as they are for the first three parities in Figure 12, or that deflection would deepen at parity 7-plus.

If a program of fertility reduction for a population with the characteristics of our data, and concerned only with the thirty-one year projection span, were to be restricted in its design to the attack of a single age-and-parity-specific birth rate, the rate chosen should be that for age 38 parity 7-plus. If the design were expanded, it would be an essentially indifferent matter with respect to deflection produced, which of the "maximum diagonal" birth rates were added, except for age 18 parity 1 which has little effect.

If, however, the program were concerned with a shorter or longer span from 1959 than the thirty-one projected years, the choice would not be indifferent. For a shorter span the rates for older ages and higher parities must be chosen. Rates at younger ages may not evenbegin to affect cumulative births within the shorter span. For a span extending beyond 1989, a program initiated in 1959 should favor the middle of the range of ages and parities. If the "fixed" computer program were extended to project beyond 1989, projected cumulative births in Figure 14 would remain at their 1989 levels, and average cumulative births from 1959 would tend to those levels in the limit. Since the middle of the age and parity ranges, still on the "maximum diagonal", provide the largest constant deflection beyond 1989, they would be favored as birth rates were added to the program design.

In addition to the fifteen point drop computer runs, the sensitivity experiments also include six row drop runs and four column drop runs. In the former, each run is made setting the non-zero birth rates at all parities of a given age, that is, across a row of Table 1, equal to zero. In the latter, each run has rates at all ages of a given parity, down a column of Table 1, equal to zero. The ages for which row drops are made include 18, 23, 28, 33, 38 and 43. The parities of column drops are 1, 3, 5 and 7-plus. These are the same ages and parities that are combined to define the point drop runs. Average cumulative births for both row and column drops are shown in Figure 15. Since one of the two birth rate parameters is held constant in each of the two series, the results are both single bar-charts rather than sets of bar-charts as in Figures 12 and 13. Cumulative births for the initial cohorts are plotted in Figures 16 and 17.

It is readily apparent that the effect of a column drop is much greater than that of a row drop. With a row drop, births to women of a given age are suspended, but once a cohort has passed through the given age, births to its women are resumed in all parities to the end of the childbearing span. The cohort resumes childbearing with the same frequency distribution for which it was suspended one year earlier. The size of deflection of cumulative births for a single cohort for a given row drop, is a function of all of the several age-and-parity-specific birth probabilities remaining to the cohort. It is also a function of the frequencies in the distribution of the cohort at the age of the row drop.

With a column drop, births to women with a given number of children are suspended, and the only women permitted to have children of higher birth order than the suspended parity are those in each initial cohort who are already of a higher parity in the frequency distributions of the initial year. If they are not already of a higher parity they cannot pass through the suspended parity of the column drop the way they can pass through the row drop, merely by waiting for another year. In the language of the model, the parity for which births are suspended becomes an "absorbing" state for all ages. Parities greater than the one of the row drop continue to have cohort members initially in those higher parities pass through them at successive ages. The size of cohort deflection for a column drop, is a function of most of the probabilities remaining to the cohort at its initial age which are of the parity of the column drop and higher. It is also a function of the first frequency of the cohort to which zero probability of the column drop applies.

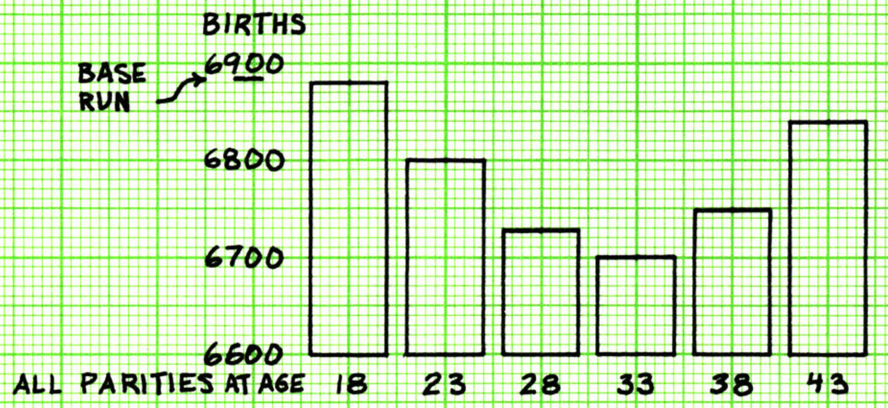
The results of the row and column drop experiments are quite straightforward. If a program of fertility reduction aimed at the 1959 to 1989 span were limited to choosing a single age for which all parity-specific birth rates could be decreased to zero, from Figure 15, age 33 would be the age to choose. If the d esign were expanded, other ages would be chosen with the first two or three younger and the next one or two older than 33, then alternating one or more at a time. If the choices were limited to the experimental row drops, the succession would be 33, 28, 38, 23, 43 and 18. If the span of interest stopped well short of 1989, the older ages would be favored rather than this succession. If the span extended well beyond 1989, from Figure 16, the middle of the range of ages, from 23 to 33, would prove the most effective choices to start work on in 1959.

If the program design choice were limited to a single column drop, parity 3 would be the most effective over the thirty-one year projection span. Expansion of the design would be made, from Figure 15, in the sequence of parities 1, 5 and 7-plus. For a span of nearly twenty years or less, parity 7-plus is the most effective single choice, from Figure 17. With a shift in span of concern from nearly twenty years from 1959 to the full thirty-one year projection span, the single most effective parity drop shifts from parity 7-plus towards parity 3. Beyond 1989, successively lower parities come to dominate the average of cumulative births.

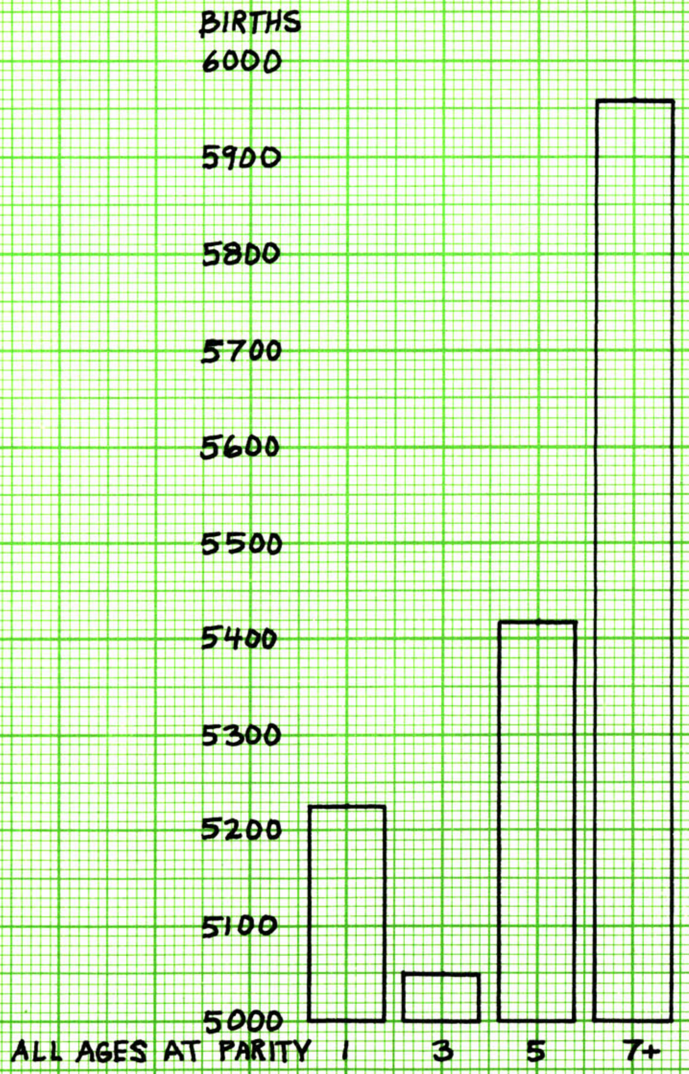
If we attempt to piece together the results of all three modes of change of birth rates to form a more realistic picture, they seem jointly to imply that the design of a fertility reduction program, intended to have its greatest impact over a period of thirty or more years, should focus initially on a set of age-and-parity-specific birth rates near the center of a table of such rates like Table 1. With the data of our experiments, such an area in Table 1 might include rates in a short range of ages in each of three parities. These ranges might best include ages 23 through 29 in parity 2, ages 25 through 31 in parity 3, and ages 27 through 33 in parity 4. An alternative might favor inclusion of a range of ages, say 29 through 35, in parity 5 instead of those in parity 2. This alternative might be preferred in the initial stages of the program or if greater resistance were anticipated from women in the lower parity. Another program alternative, inferred from the column drop results, would focus on just parities 3 and 4 but over the full range of ages with non-zero birth rates.

FIGURE 15

AVERAGE OF CUMULATIVE BIRTHS AT AGE 45 FOR 31 COHORTS:
SENSITIVITY TO AGE-AND-PARITY-SPECIFIC BIRTH RATES BY AGE
SET AT ZERO BIRTHS PER YEAR



SENSITIVITY TO AGE-AND-PARITY-SPECIFIC BIRTH RATES BY PARITY
SET AT ZERO BIRTHS PER YEAR



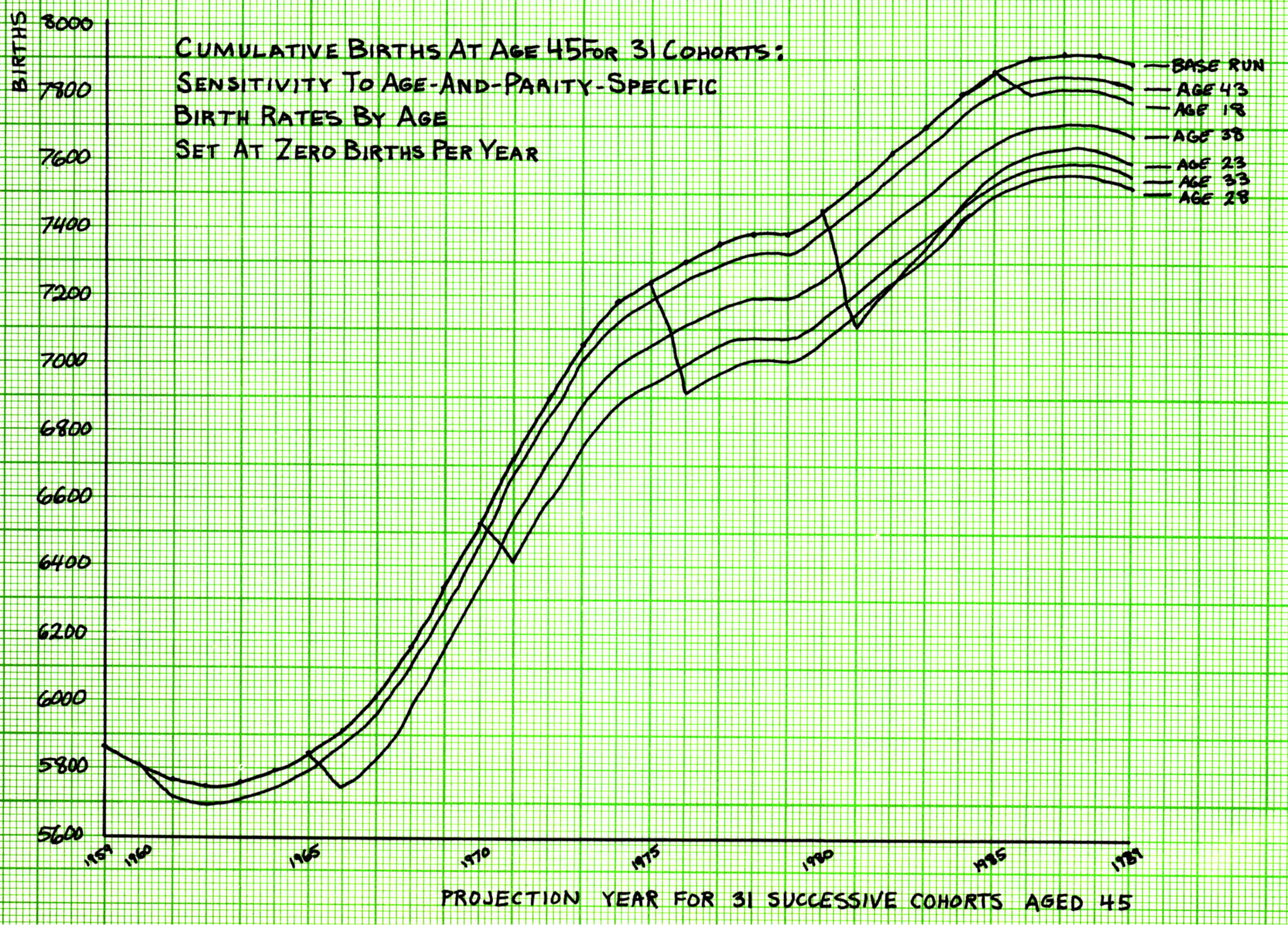

Cumulative Births At Age 45 For 31 Cohorts:
Sensitivity To Age-And-Parity-Specific
Birth Rates By Age
Set At Zero Births Per Year
BIRTHS
8000
7800
7600
7400
7200
7000
6800
6600
6400
6200
6000
5800
5600
1959
1960
1965
1970
1975
1980
1985
1989
BASE RUN
AGE 43
AGE 18
AGE 38
AGE 23
AGE 33
AGE 28
PROJECTION YEAR FOR 31 SUCCESSIVE COHORTS AGED 45

FIGURE 16

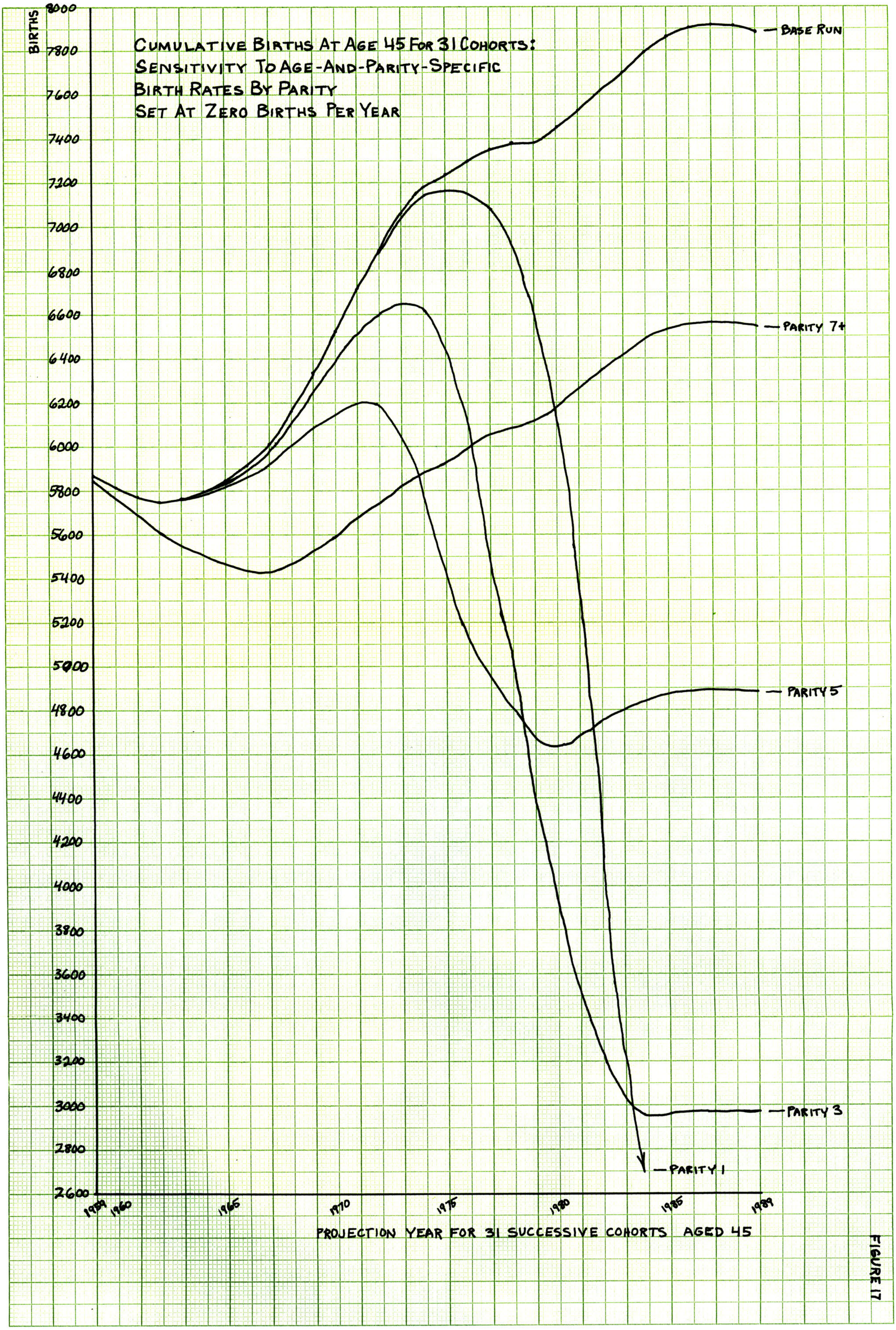

CUMULATIVE BIRTHS AT AGE 45 FOR 31 COHORTS:
SENSITIVITY TO AGE-AND-PARITY-SPECIFIC
BIRTH RATES BY PARITY
SET AT ZERO BIRTHS PER YEAR
BIRTHS
8000
7800
7600
7400
7200
7000
6800
6600
6400
6200
6000
5800
5600
5400
5200
5000
4800
4600
4400
4200
4000
3800
3600
3400
3200
3000
2800
2600
1959
1960
1965
1970
1975
1980
1985
1989
PROJECTION YEAR FOR 31 SUCCESSIVE COHORTS AGED 45
BASE RUN
PARITY 7+
PARITY 5
PARITY 3
PARITY 1

FIGURE 17

Introducing another note of realism, we can perhaps guess at the effect of introducing death probabilities into the model instead of using a uniform cohort size of one-thousand. Naturally, total cohort sizes would steadily diminish from youngest to oldest, both among cohorts within a given year, and within a given cohort through the years. If we assume, as for birth probabilities, that death probabilities are stable for successive cohorts, though varying with age within a cohort, and if we further assume death probabilities are identical for all parities of a given age, then it is probably safe to assume that the shapes of the bar-charts and curves generated in our experiments are not radically altered by use of realistic cohort sizes. The decrease of cohort size with age would result in relatively smaller deflections from base run values than those shown here, with increasing age and parity. The maximum deflections in Figures 12 and 13 would likely tend to lessen with increasing age and parity, rather than being nearly uniform. With increasing age and increasing parity in the respective portions of Figure 15, the bars would be increasingly higher. These shifts would probably not radically affect the description of alternative program designs suggested by joint implication of the three modes of sensitivity experiment.

The next set of experiments examines the effects of point, row and column drops made with birth rate values intermediate between the base run values and the zero values of the sensitivity experiments.

Linearity

The nature of the mathematics in the model plus a faint indication from the first computer runs with the unsmoothed Khanna data, mentioned earlier as in the nature of trial runs, suggested that the system response to some changes of birth rates might be linear. Totest this possibility, a set of runs is made for one sample of each mode of change. The samples used areage 28 parity 3 for point drop, age 28 for row drop, and parity 3 for column drop. For each sample the set of runs is made with birth rates decreased by 20, 40, 60 and 80 percent from base run values. Together with the b ase run itself, or no decrease, and the appropriate run from the preceeding section, at zero birth rates or 100 percent decrease, these four runs give adequate coverage to the full range of response to possible values of birth rates in the samples.

The results in terms of average cumulative births for the initial cohorts are plotted in Figure 18. The response to the point and row drops is obviously linear and to the column drop is not. A sample of individual initial cohorts young enough to be affected in cumulative births, not surprisingly shows the same results in Figures 19, 20 and 21.

These results mean that average cumulative births for point or row drops, in Figures 12 and 13 and in Figure 15, will rise by the same proportion of the total deflection that the corresponding birth rates might be raised in any new computer runs. The portions of the corresponding curves of cumulative births, as in Figures 14 and 16, which are below the base run curve, will be raised in the same proportion. Bars in the bar-chart for column drops in Figure 15 will not move in proportion to changes in column drop settings. As column drop settings are moved towards zero, the responses are increasingly, not proportionately, greater.

FIGURE 18

Average Of Cumulative Births At Age 45 For 31 Cohorts: Linearity With Age-And-Parity-Specific Birth Rates

CUMULATIVE BIRTHS AT AGE 45 FOR SELECTED COHORTS: LINEARITY WITH INDIVIDUAL AGE-AND-PARITY-SPECIFIC BIRTH RATES (AGE 28 PARITY 3)

FIGURE 20

CUMULATIVE BIRTHS AT AGE 45 FOR SELECTED COHORTS: LINEARITY WITH AGE-AND-PARITY-SPECIFIC BIRTH RATES BY AGE (AGE 28)

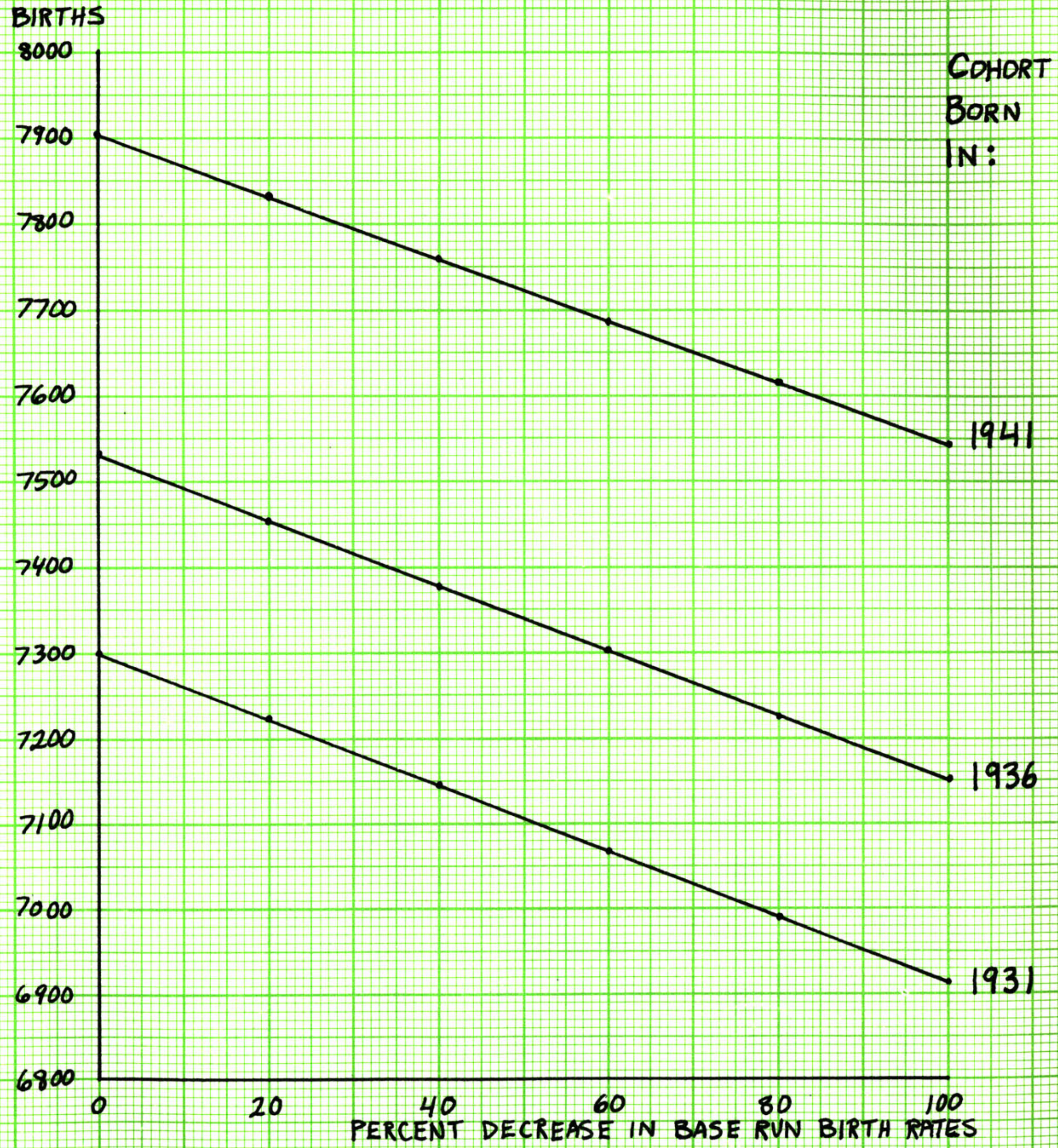
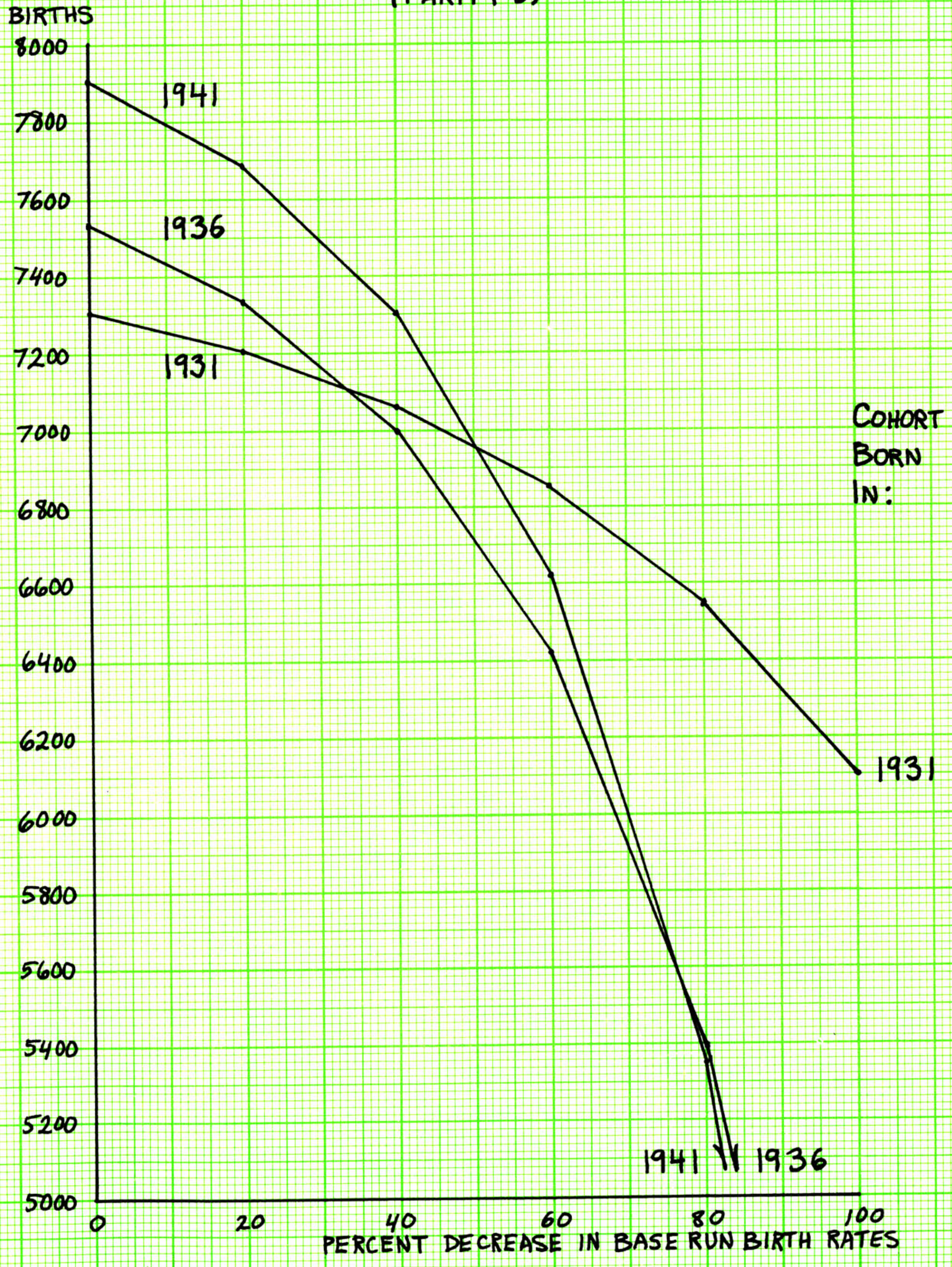


FIGURE 21

CUMULATIVE BIRTHS AT AGE 45 FOR SELECTED COHORTS: LINEARITY WITH AGE-AND-PARITY-SPECIFIC BIRTH RATES BY PARITY (PARITY 3)



Aggregation

The sensitivity and linearity experiments are designed to explore the response of the cohort birth system to changes in the simplest components which govern that system. However, in the more realistic alternatives for design of a program of fertility reduction, inferred from that response, these components are necessarily treated in larger combinations. It would be extremely difficult in practice to focus a program on the sorts of choices of birth rate reduction with which we have experimented. Nor would it be desirable to do so since the system response, at least to individual point and row drops, is so small.

Both to simplify administration and to produce decreases of worth while magnitude in cumulative births, a practical program must focus on larger combinations of birth rates. But rather than experiment with realistic combinations directly, we are interested to observe whether system response to larger combinations can be synthesized from responses to the simpler components. To test this possibility the aggregation experiments were performed.

Six aggregations are examined. The first two are from point drops to a row drop for only part of a row, and from point drops to a column drop for only part of a column. The second two are from point drops to full row and column drops. The third pair are from row drops to a multi-row drop and from column drops to a multi-column drop. In all runs the chosen birth rates were set at zero.

In the first aggregation, deflections from average cumulative births for point drops at age 28 parities 4, 5, 6 and 7-plus, are summed. This sum is compared with the deflection resulting from a partial row drop over the same age and parities. The sum of deflections from cumulative births for a single cohort is also compared with the corresponding partial row deflection for the cohort. In both cases the sums differ from the aggregated deflections only by amounts attributable to computer round-off procedure.

The same is not true in the second aggregation. Deflections from cumulative births, both average and for a single cohort sample, are summed for point drops at parity 3 in each of ages 32 through 41. These two sums are compared with the corresponding single deflections from a partial column drop over the same parity and ages. Both for the average and for the sample cohort, the single deflection in the aggregate run is considerably greater than the sum of the corresponding point drop deflections.

The same results occur in the third and fourth aggregations. Both in the average and for a sample cohort, deflections for point drops at all parities in age 28 sum to the two corresponding single deflections resulting from a full row drop at age 28. In contrast, the single deflections for the average and for a sample cohort for an aggregate run with a full column drop at parity 3, are several times larger than the sums of deflections for all possible point drops in the parity.

In the fifth aggregation, the deflections of average cumulative births and of cumulative births for a sample cohort, which result from the six row drops of the sensitivity experiments, are summed. The two sums are found to be less than the single deflections, which result from a multi-row drop in which all birth rates at all six ages are set at zero. The deflections from the aggregate, multi-row drop are not much greater than the sums of deflections from the single row drops. But the differences are not attributable to round-off procedure and may be significant. The differences are nearly the same for both average cumulative births and for the sample cohort. Why this is so, and why the differences arise in the first place, cannot be determined without more detailed comparison of the cohort frequency distributions, and the distributions of annual births by parity change, between the base and other runs involved, than is within the scope of the present effort.

In the sixth aggregation, the average and sample cohort deflections for the column drop runs of the sensitivity experiments are summed. The two sums are nearly double the single deflections resulting from an aggregate run in which all birth rates in parities 1, 3, 5 and 7-plus are set at zero.

In summary, we can synthesize system response to a partial or a full row drop from the results of corresponding point drops. If the component rows are not adjacent, thus avoiding possible small column drop effects, we can synthesize system response to a multi-row drop fairly closely from corresponding row drops, and therefore from corresponding point drops.

Our six experimental ages are spaced five years apart. It is uncertain whether column drop effects would be more noticeable as rows are chosen closer together, or only when they are actually adjacent. If such column drop effects do increase with proximity of age-row, then synthesis will become more difficult.

Apparently we cannot synthesize either partial or full column drops from component point drops. On the basis of these experiments at least, neither can multi-column drops be synthesized from component column or point drops.

The various types of row drops can be synthesized for birth rates set in between zero and base values, by using the linearity property of point and row drops. The non-linearity of column drops is what makes it difficult at best, to synthesize the various types of column drops. Such synthesis might be possible using a table of empirical relations of column drops to components, which might be established with a larger systematic set of computer experiments directed to that end. Most larger, more realistic combinations of birth rates are properly viewed as sets of adjacent column or partial column drops in birth rate. Thus the non-linearities can be expected to prevail. Therefore to synthesize target drops in such programs, other than in quite crude fashion, will require the development of such an empirical table.

Diffusion

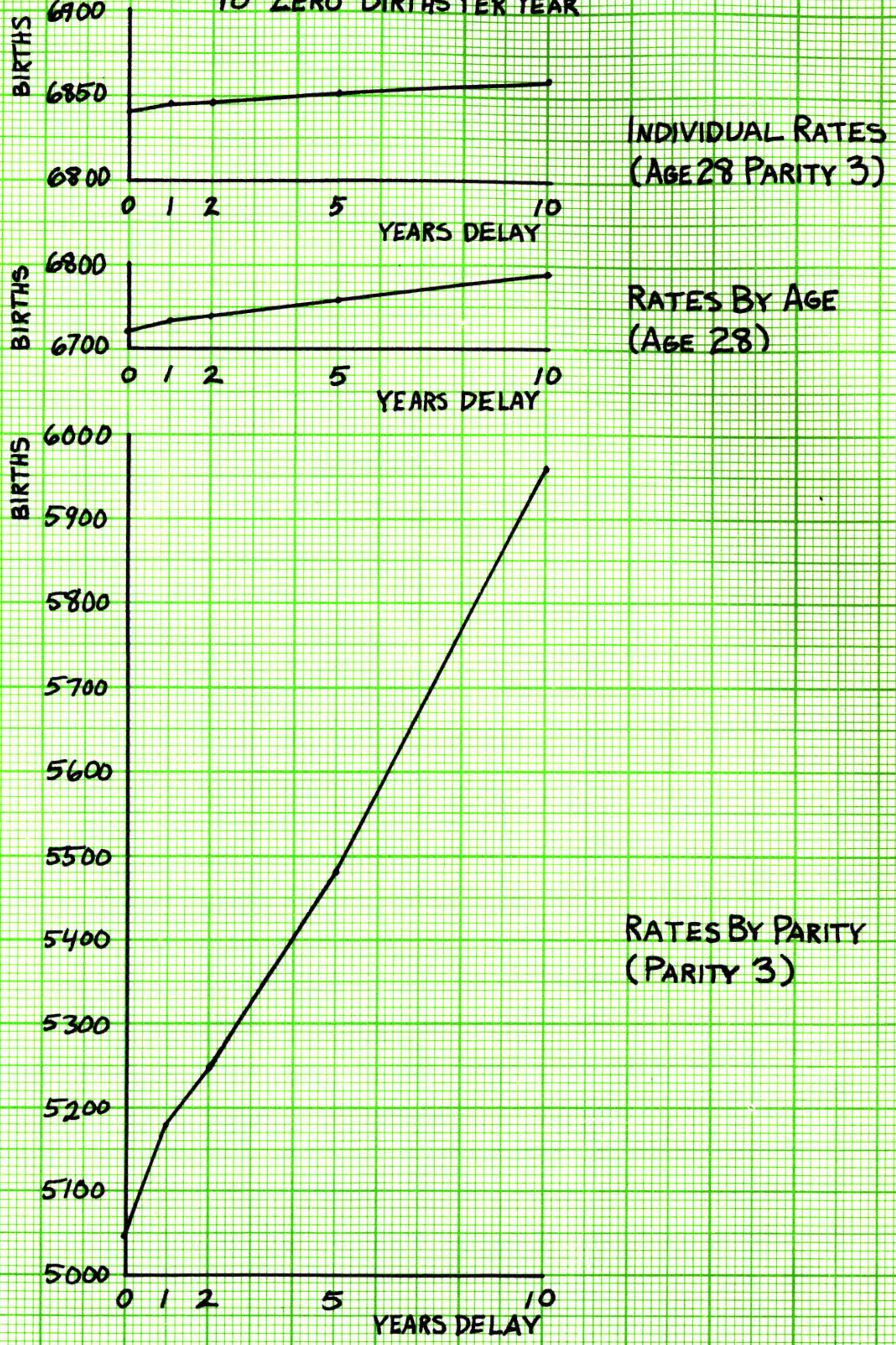
Decrease of birth rate in the real world, whether by spontaneous adoption of birth limitation practices as in Western history or under the aegis of a government program as in some of the developing nations, takes place over time. It does not happen instantaneously, as is represented in the preceeding experiments using the "fixed probability" computer program. Yet those experiments give us some insight into the relations between certain of the output of the model and the parameters of its principal data.

The purpose of the final set of experiments is to observe the effect of various rates of diffusion of fertility control, using the "variable probability" program. The same samples for the three modes of change of birth rate are used here as in the tests for linearity. Average cumulative births resulting from point drops at age 28 parity 3, from row drops at age 28, and from column drops at parity 3, are plotted in Figure 22. The results of five runs are plotted for each mode. The first run for each mode is the fixed program run, that is zero years of time delay in achievement of a drop in the selected birth rates to zero values. The other runs are for time delays of one, two, five and ten years in reaching zero birth rates.

No matter how birth rates are combined, whether in one of our three modes or not, the longer the delay in attaining

FIGURE 22

AVERAGE OF CUMULATIVE BIRTHS AT AGE 45 FOR 31 COHORTS:
EFFECT OF SELECTED TIME DELAYS IN ACHIEVEMENT OF
DROP IN AGE-AND-PARITY SPECIFIC BIRTH RATES
TO ZERO BIRTHS PER YEAR



a given decline in those birth rates, the higher will be the average cumulative birth rate over a period covering the decline. Thus the average rises with time delay for all three curves in Figure 22. The rise is linear with time for the point drop and row drop starting from one year of delay. It is slightly non-linear for the column drop from the one year point. Just as the deflection for column drops in the sensitivity experiments is much greater than for the other modes of change, the deflection decreases or the average of cumulative births rises much more rapidly with time delay for the column drop.

We have not examined the effect of time delay on individual cohorts and their cumulative births. We can describe what it might bb by an example. Referring to the row drop for age 28 in Figure 16, the full effect is seen in 1976 with zero delay. With one year delay, we would expect a partial deflection in 1976 and full deflection in 1977. With two years delay, 1976 and 1977 should show partial deflections, with full deflection in 1978. With ten years delay, full deflection should occur in 1986 and partial deflections from 1976 through 1985.

The slight non-linearity at the start of each of the three curves in Figure 22 is probably due to the way the birth rate decreases are scheduled with the "variable" program. In each run the initial year, 1959, is given the full base run birth rate. The first fraction of a decrease is given to 1960, the second, if any, to 1961 and so forth until the zero birth rate is reached. If the first fraction of a decrease, rather than the full base run rate, were given to the initial year, 1959, the slight non-linearity would probably not appear.

The effects of time delay on the response patterns of the sensitivity experiments can be estimated fairly easily. Bars in the bar-charts of average cumulative births for point drops and for row drops will rise linearly with time delay in achievement of the respective drops in birth rate. Bars in the chart for column drops will rise nearly linearly, but much more rapidly. This latter rise is such that average cumulative births are about the same with a ten year delay using a column drop at parity 3, as they are with an instantaneous drop at parity 7-plus.

The implications of time delay in achievement of birth rate targets are not reassuring for a program of fertility reduction. Realistic combinations of birth rates will generate response in the manner of a multi-column drop. That is, the sensitive column drop effects can be expected to predominate over the less sensitive row drop effects. Thus such combinations can be expected to show the disadvantage of rapid loss of potential deflection with time delay. Countering this, when a given drop in birth rates is finally achieved, a program designed around large portions of adjacent parities or entire parities will show the sensitivities inherent in this design, by producing relatively greater results than designs aimed at women of particular ages, irrespective of parity, as target clientele.

FOOTNOTES

1. This section is drawn from several sources in the Bibliography. Much of the material is presented in more than one source. Therefore, rather than distinguishing among them with repeated references, they are cited here as a group by item munber in the Bibliography: 2, 6. 7, 12, 14, 17.

2. J. M. Beshers, Population Processes in Social Systems, manuscript, reproduced, to be published 1966, Chapter 1, p. 10-11

3. This section is also drawn from several sources with some of the material being presented in more than one source. The relevant Bibliography item numbers are: 2, 5, 8, 9, 10, 11, 15, 16.

4. A Fleisher, "The Uses of Simulation", in J. M. Beshers, ed. Computer Methods in the Analysis of Large-Scale Social Systems, 1965, p. 145

5. See Bibliography items 1, 2, 3, 4

6. See J. M. Beshers, "Birth Projections with Cohort Models", in Demography, Vol. 2, 1965, p. 594, footnote 2

7. See Bibliography items 19, 20

8. United Nations, Department of Economic and Social Affairs, The Mysore Population Study, 1961, p. 84, Table 8.9

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10. ibid, p. 171, Table IV

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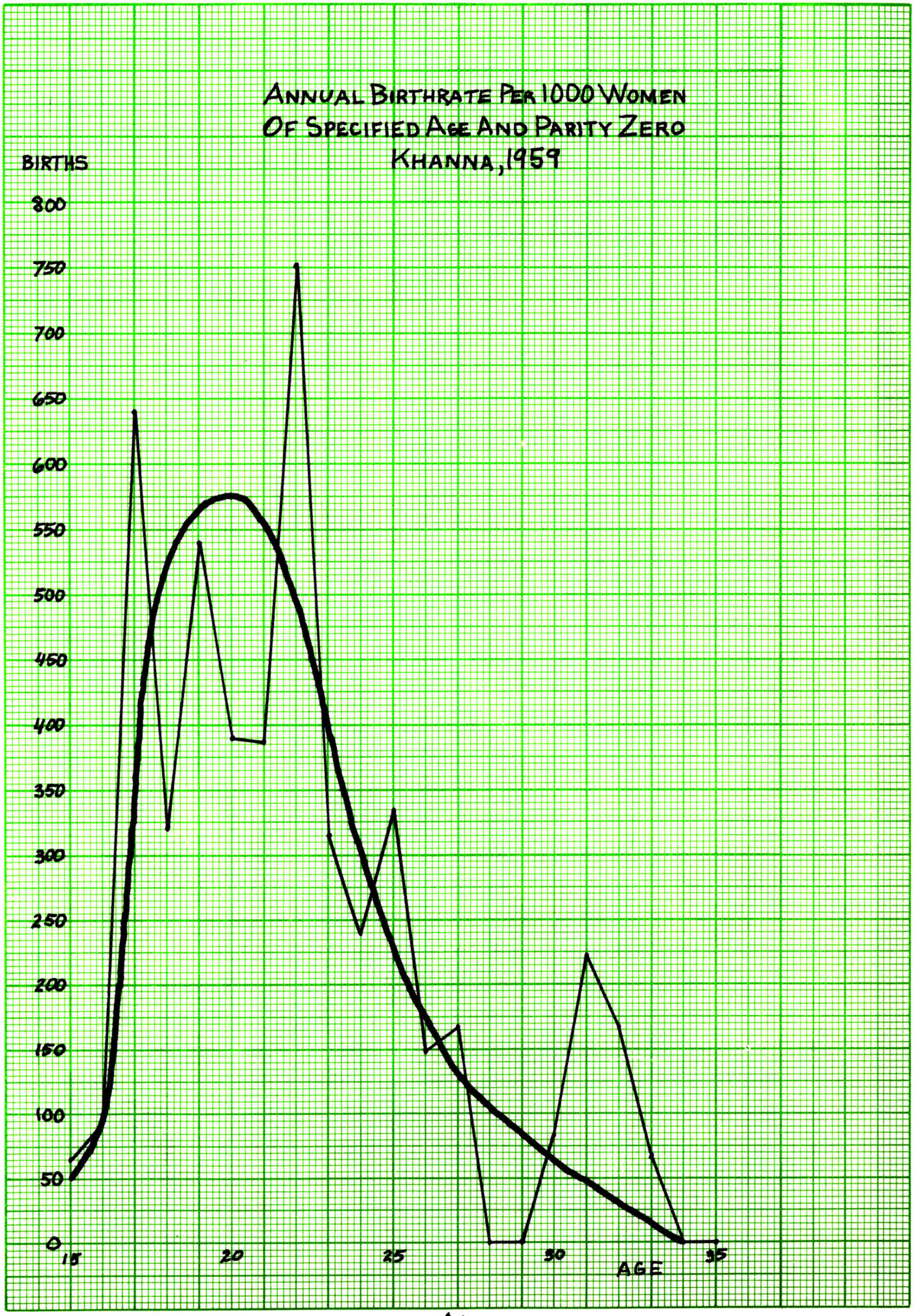
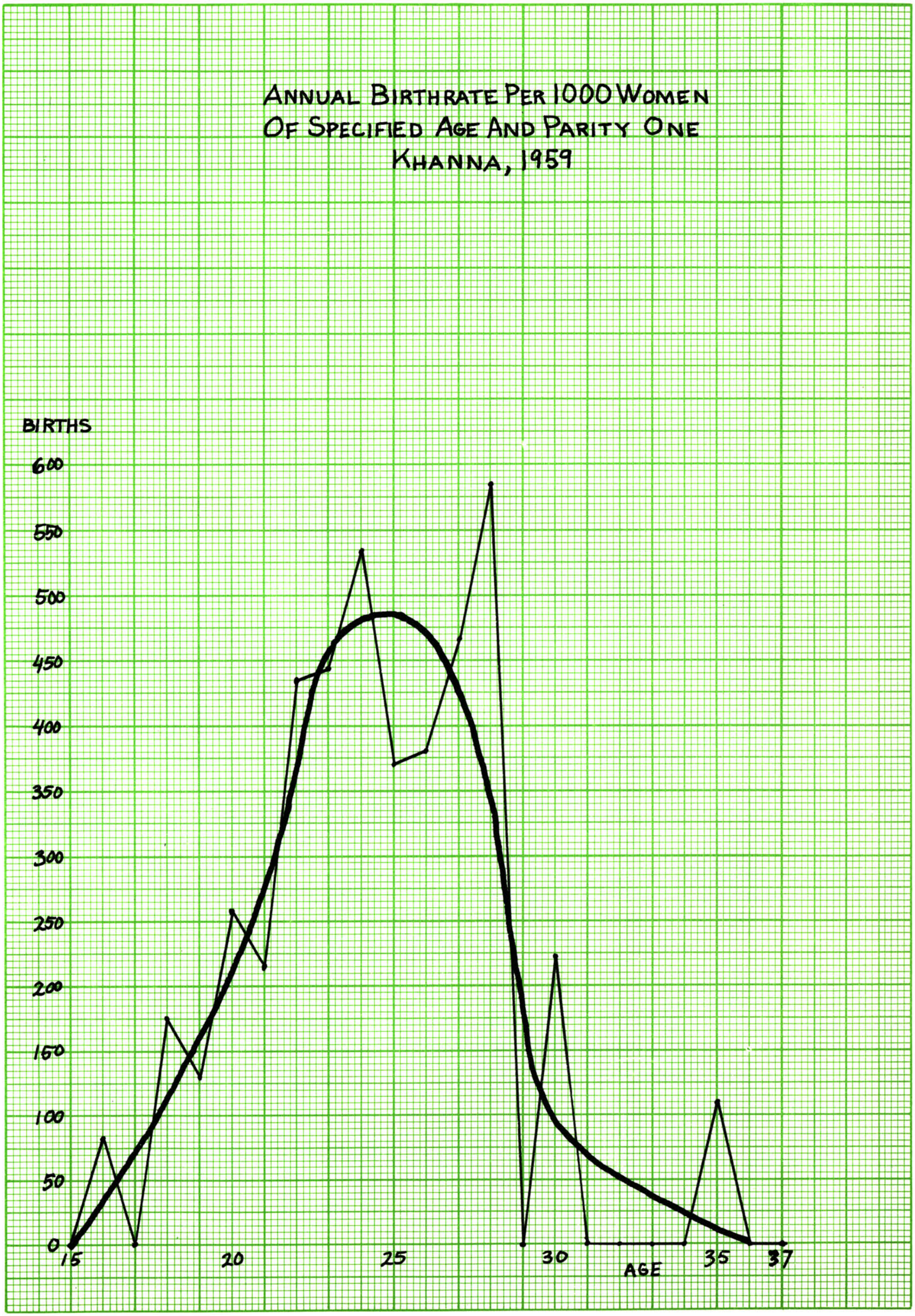
APPENDIX


ANNUAL BIRTHRATE PER 1000 WOMEN
OF SPECIFIED AGE AND PARITY ZERO
KHANNA, 1959
BIRTHS
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
15
20
25
30
35
AGE

A1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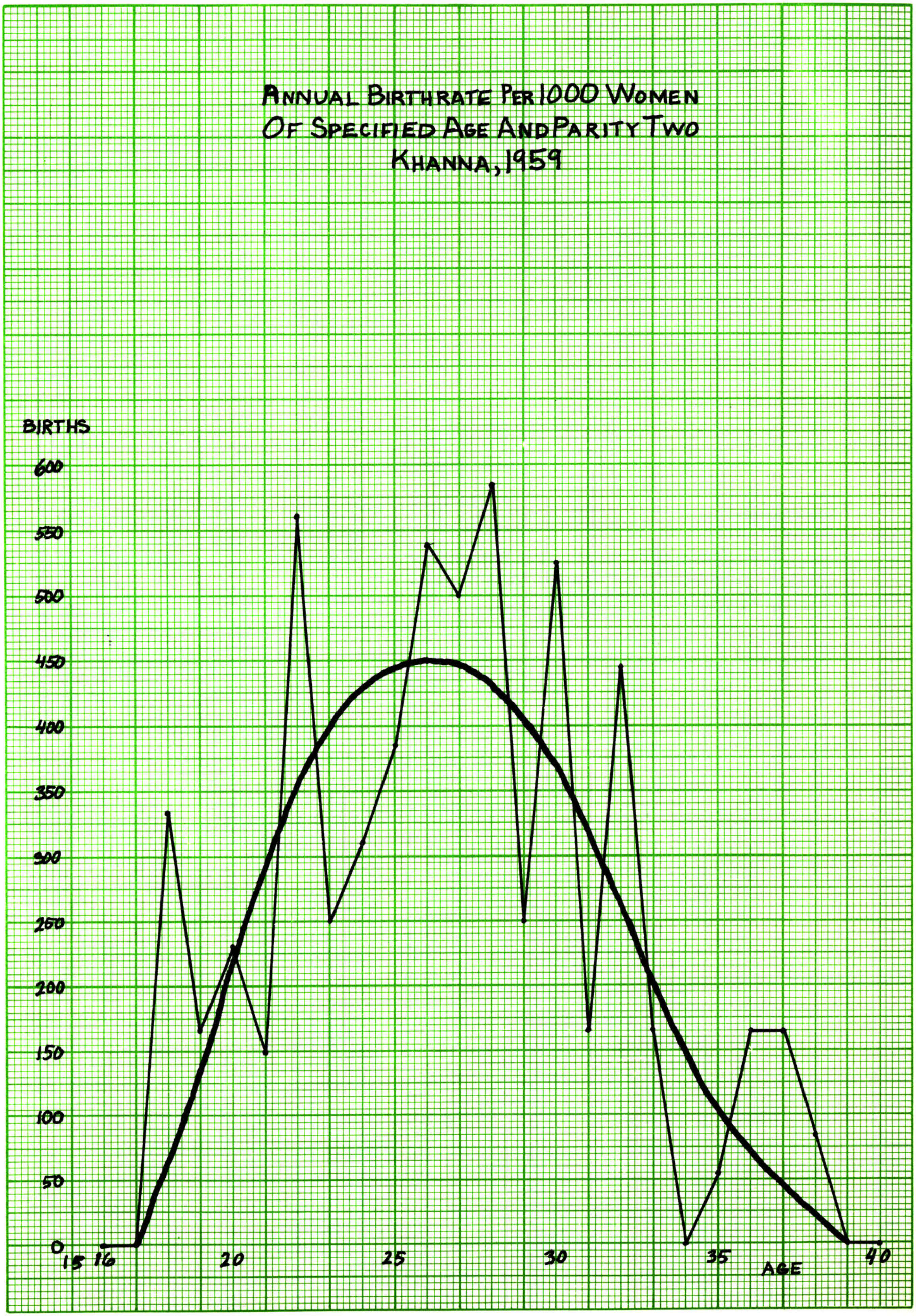
OF SPECIFIED AGE AND PARITY ONE
KHANNA, 1959
BIRTHS
600
550
500
450
400
350
300
250
200
150
100
50
0
15
20
25
30
35
37
AGE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OF SPECIFIED AGE AND PARITY TWO
KHANNA, 1959
BIRTHS
600
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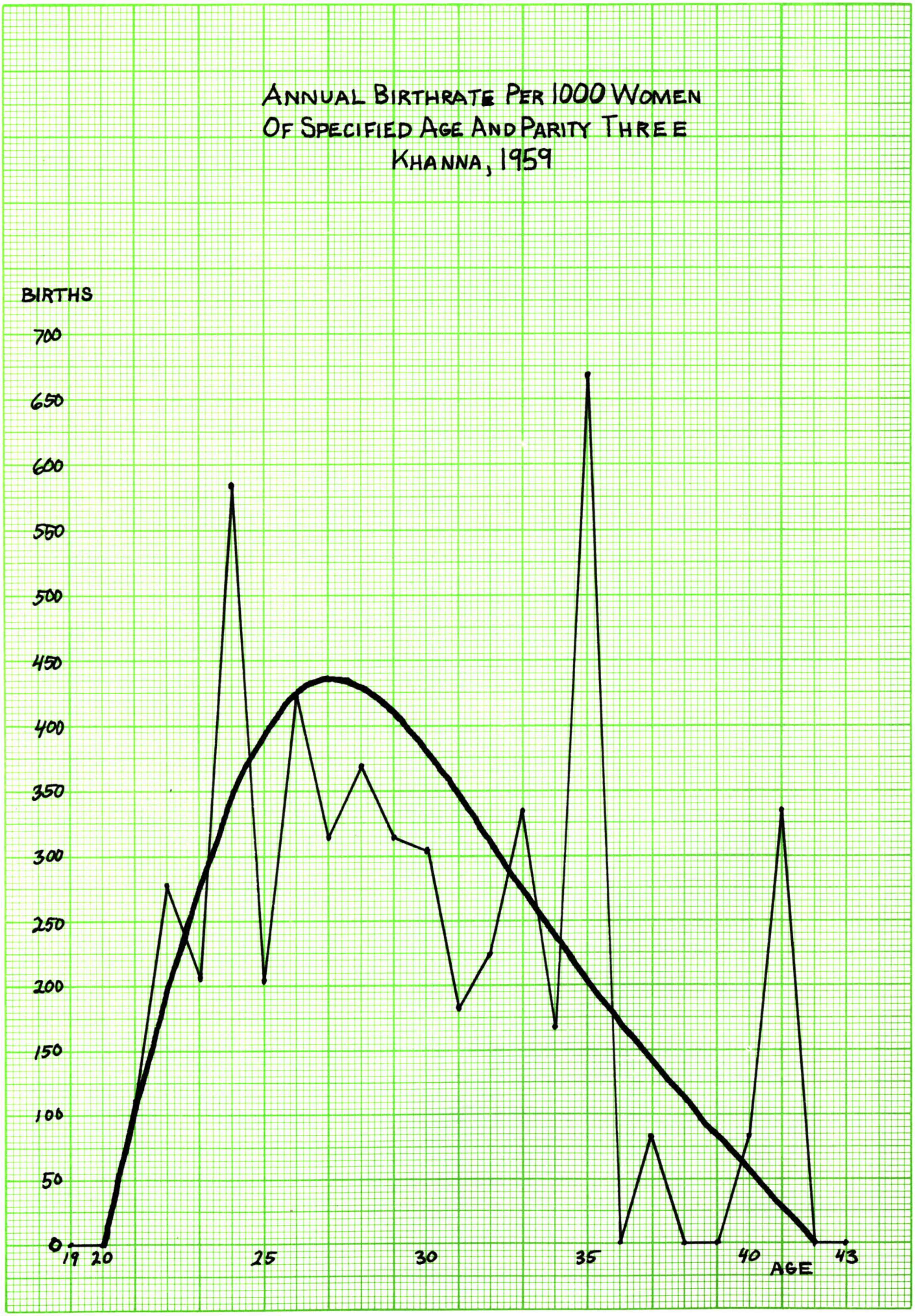
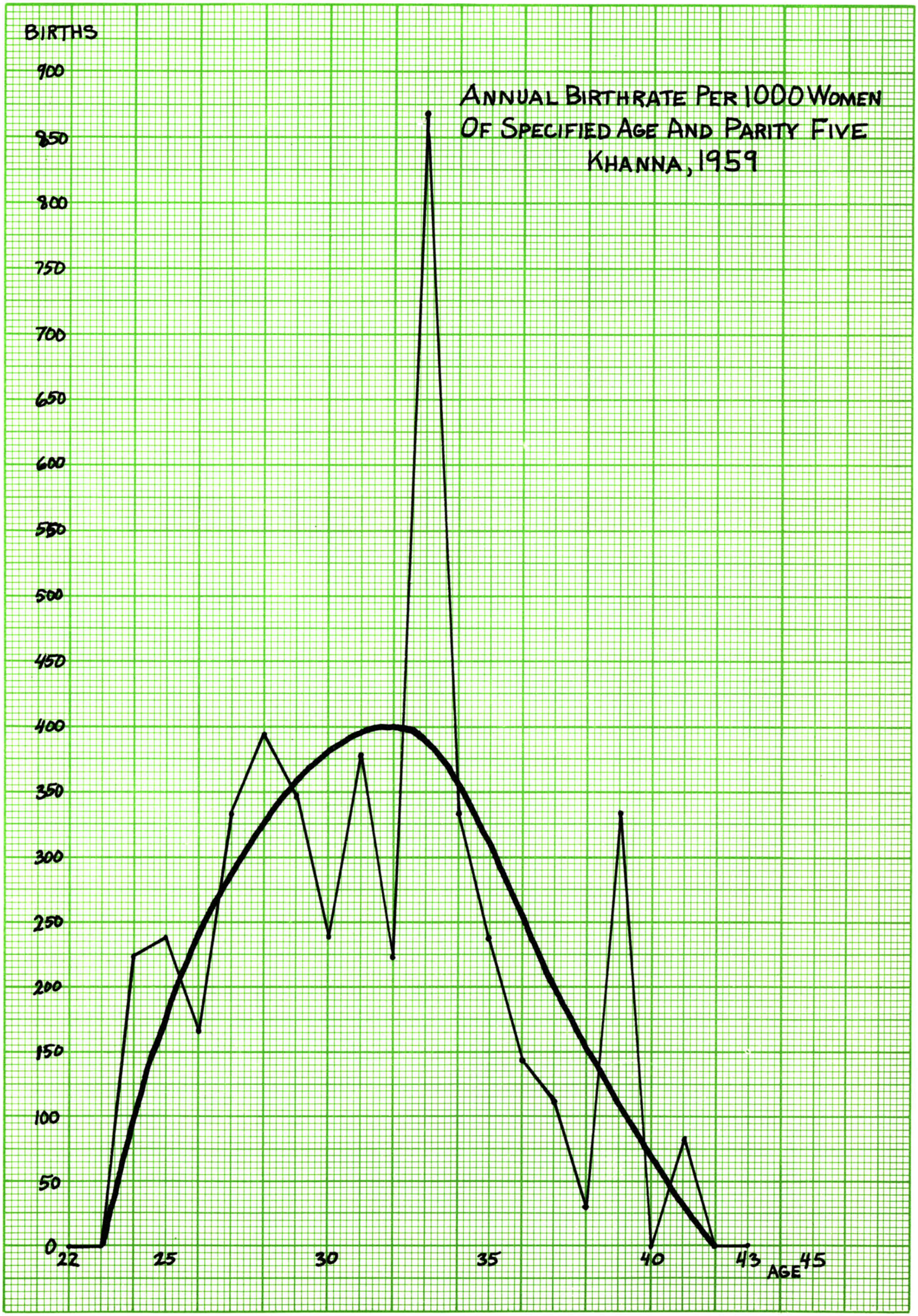
450
400
350
300
250
200
150
100
50
0
15
16
20
25
30
35
40
AGE

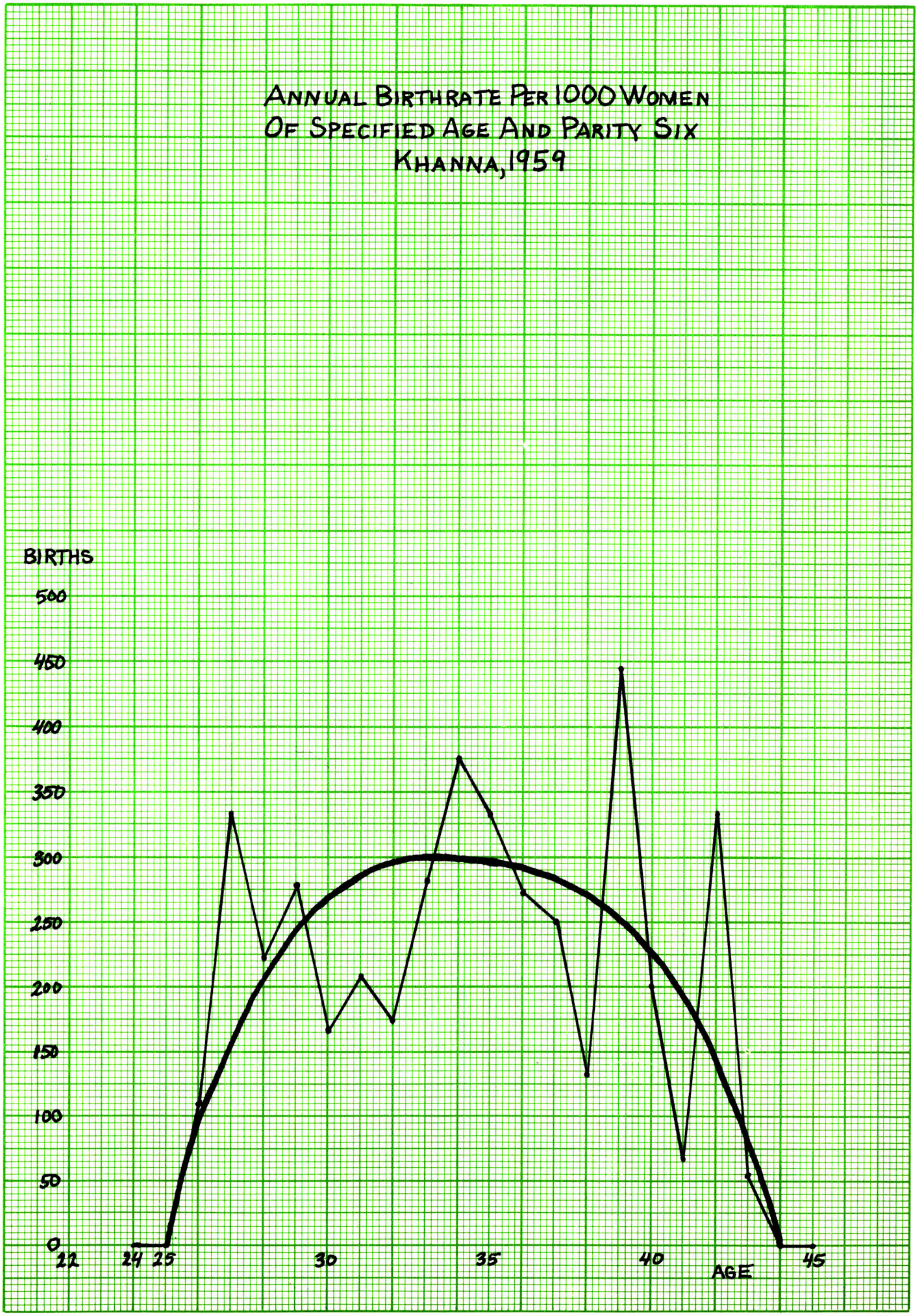


ANNUAL BIRTHRATE PER 1000 WOMEN
OF SPECIFIED AGE AND PARITY THREE
KHANNA, 1959
BIRTHS
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
19
20
25
30
35
40
43
AGE


ANNUAL BIRTHRATE PER 1000 WOMEN
OF SPECIFIED AGE AND PARITY FOUR
KHANNA, 1959
BIRTHS
950
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
22
25
30
35
40
42
AGE
45



BIRTHS
900
850
800
750
700
650
600
550
500
450
400
350
300
250
200
150
100
50
0
Annual Birthrate Per 1000 Women
Of Specified Age And Parity Five
Khanna, 1959
22
25
30
35
40
43
AGE
45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OF SPECIFIED AGE AND PARITY SIX
KHANNA, 1959
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100
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24
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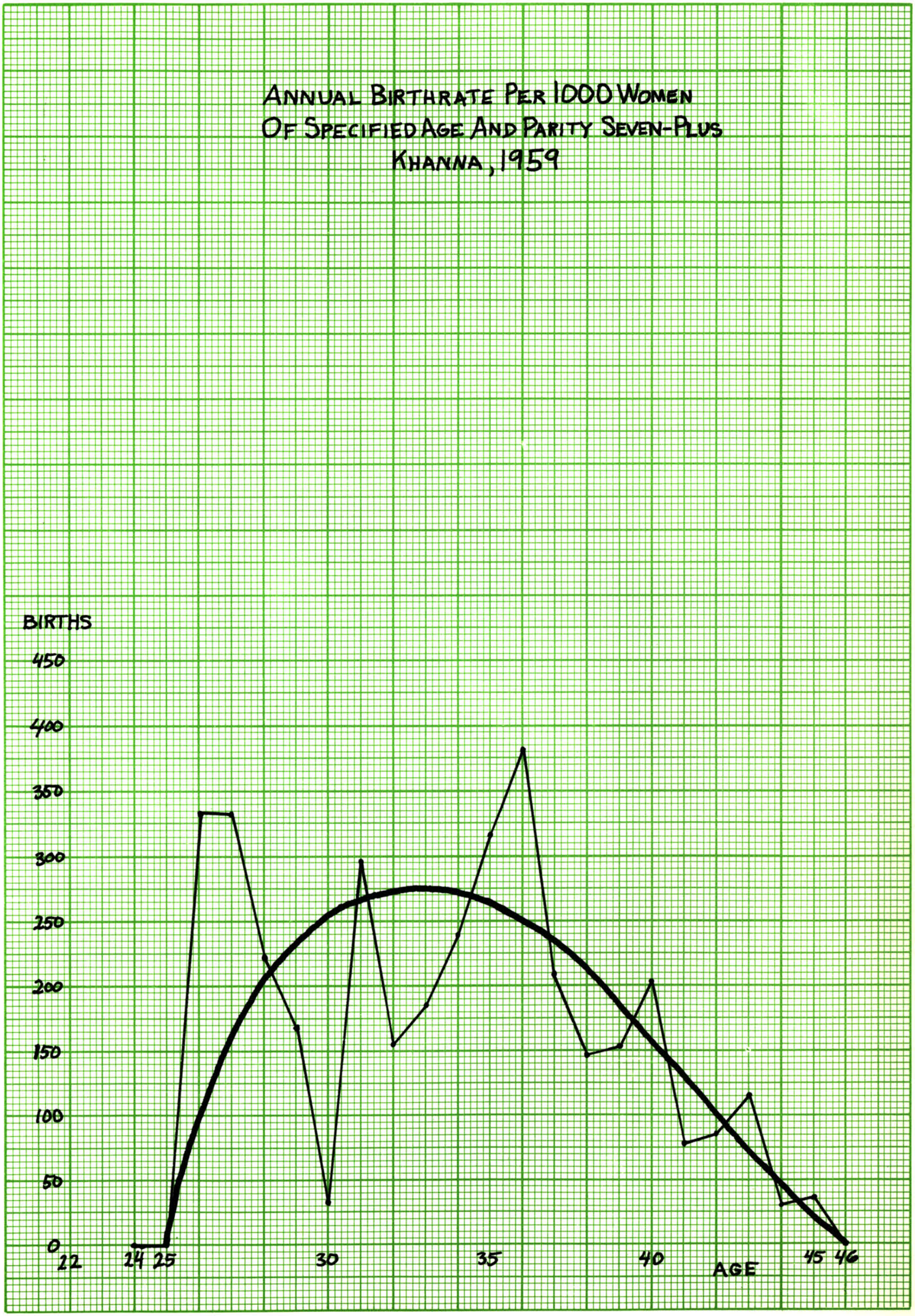
35
40
45
AGE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OF SPECIFIED AGE AND PARITY SEVEN-PLUS
KHANNA, 1959
BIRTHS
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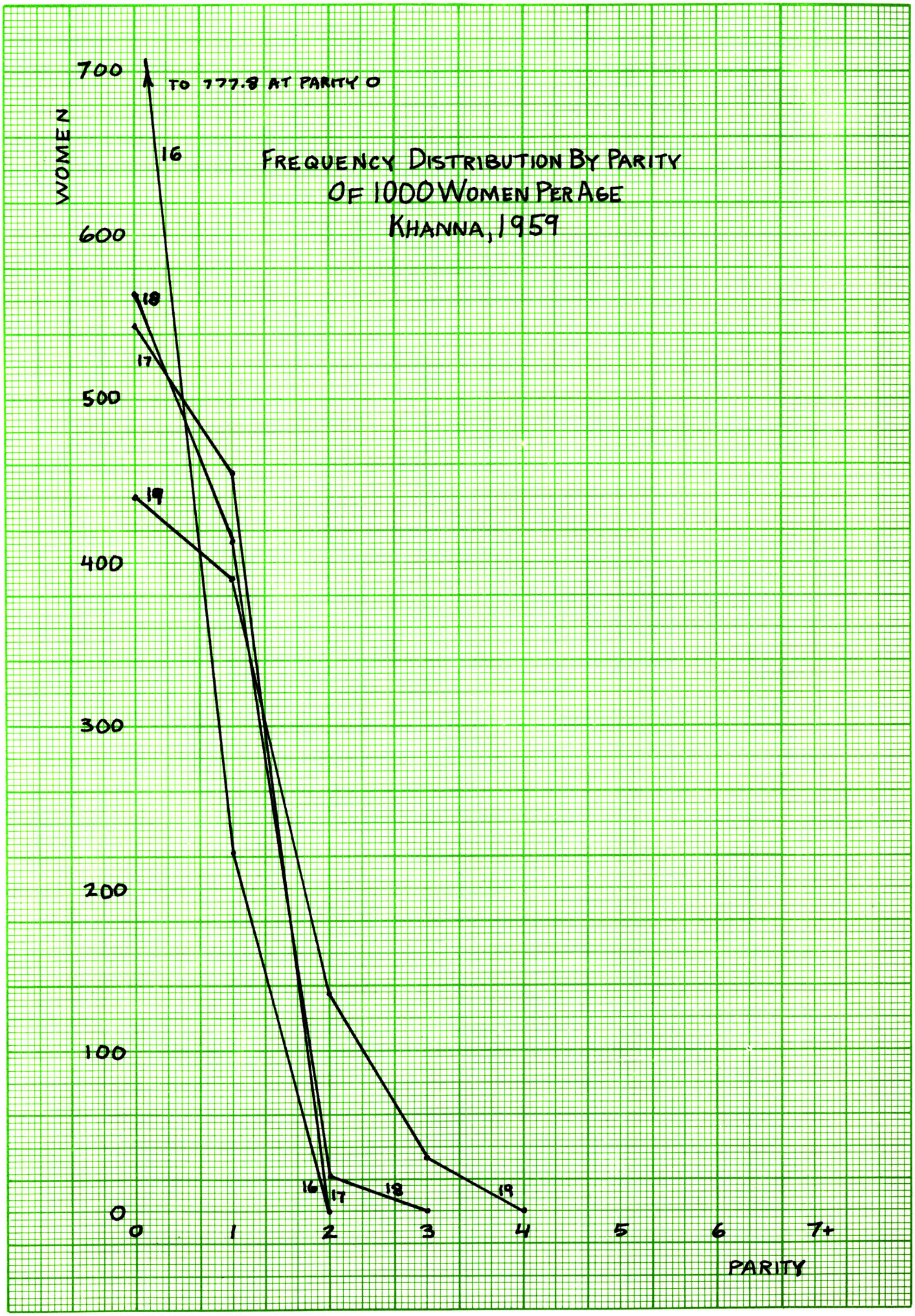
250
200
150
100
50
0
22
24
25
30
35
40
AGE
45
46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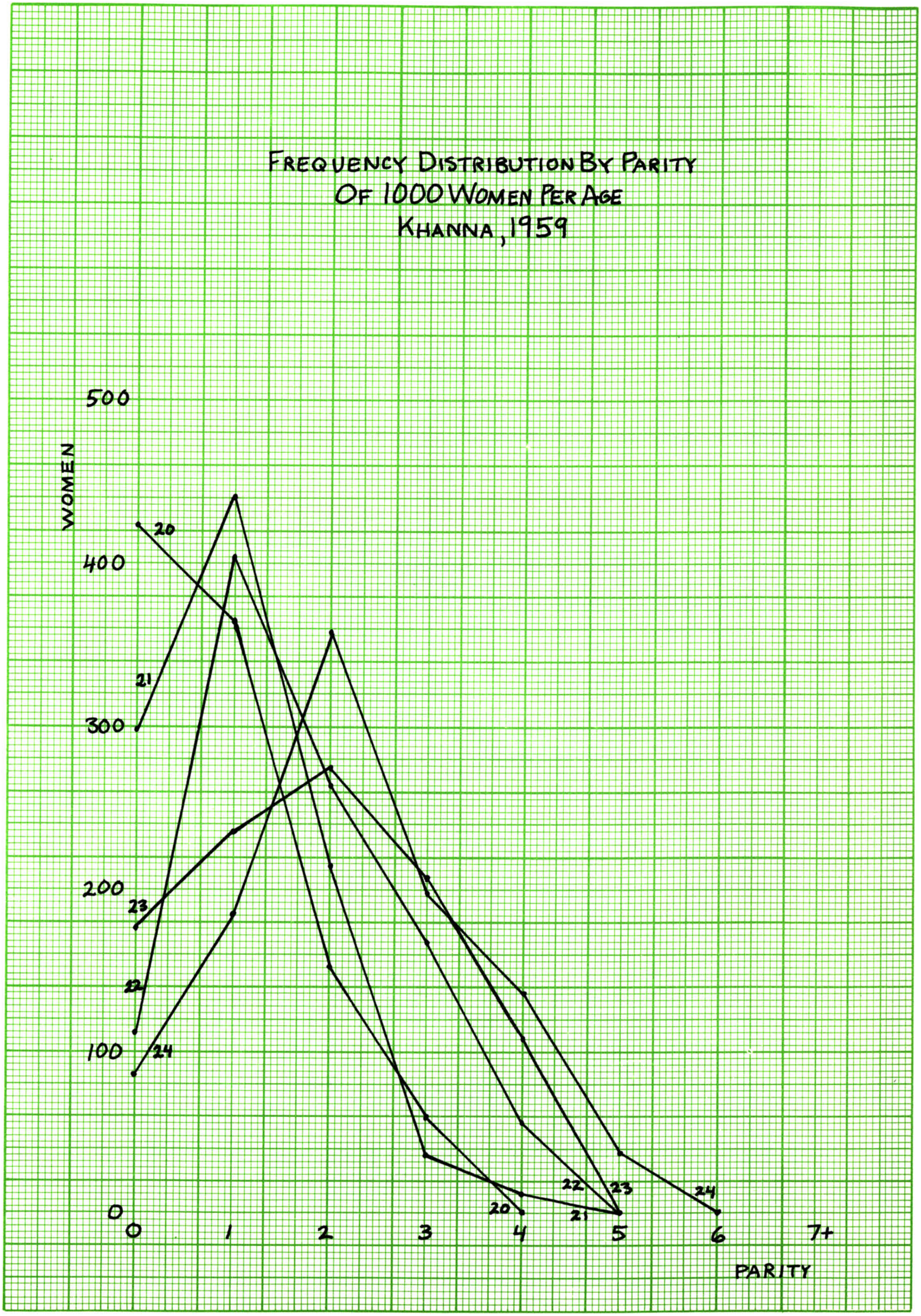
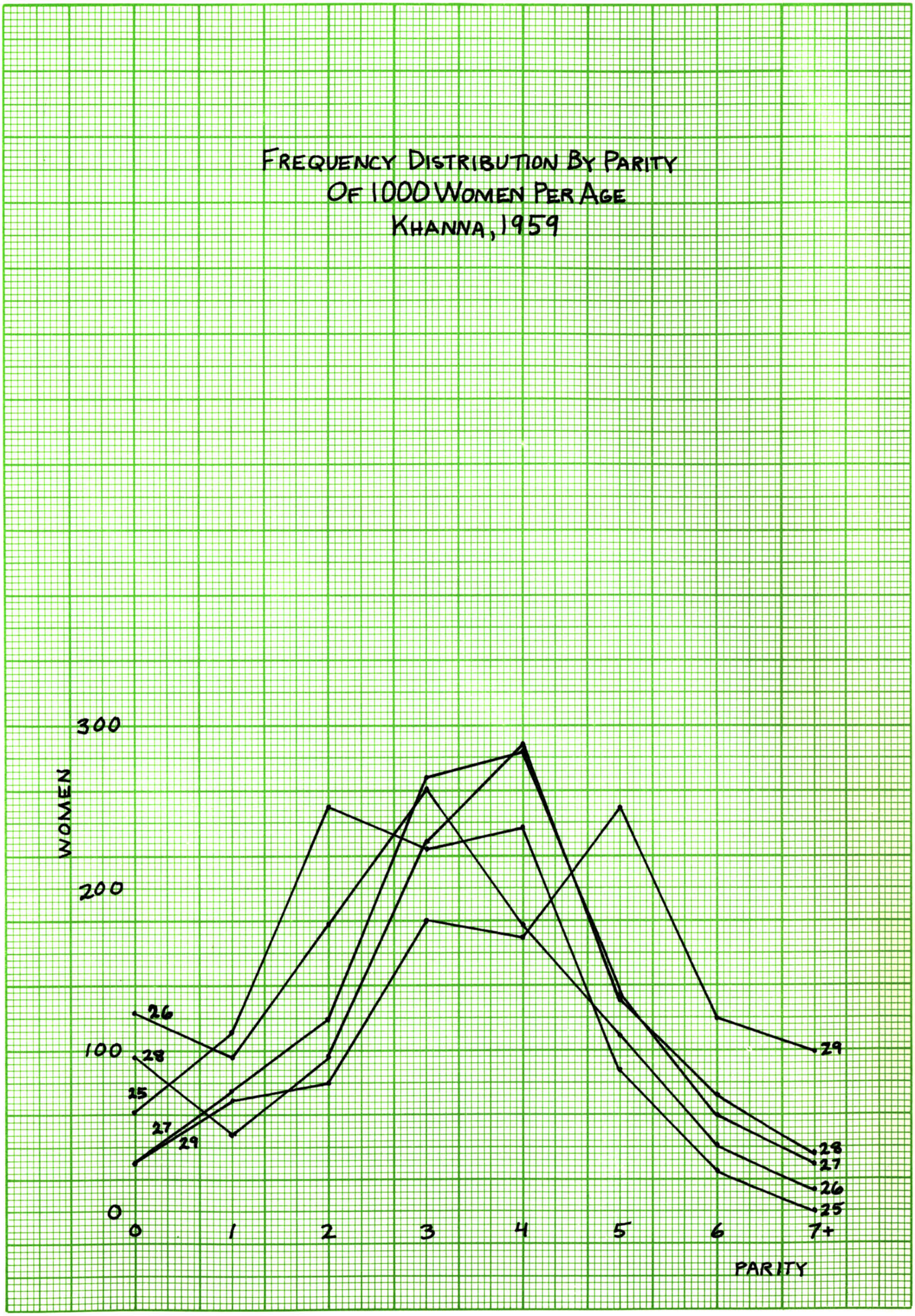
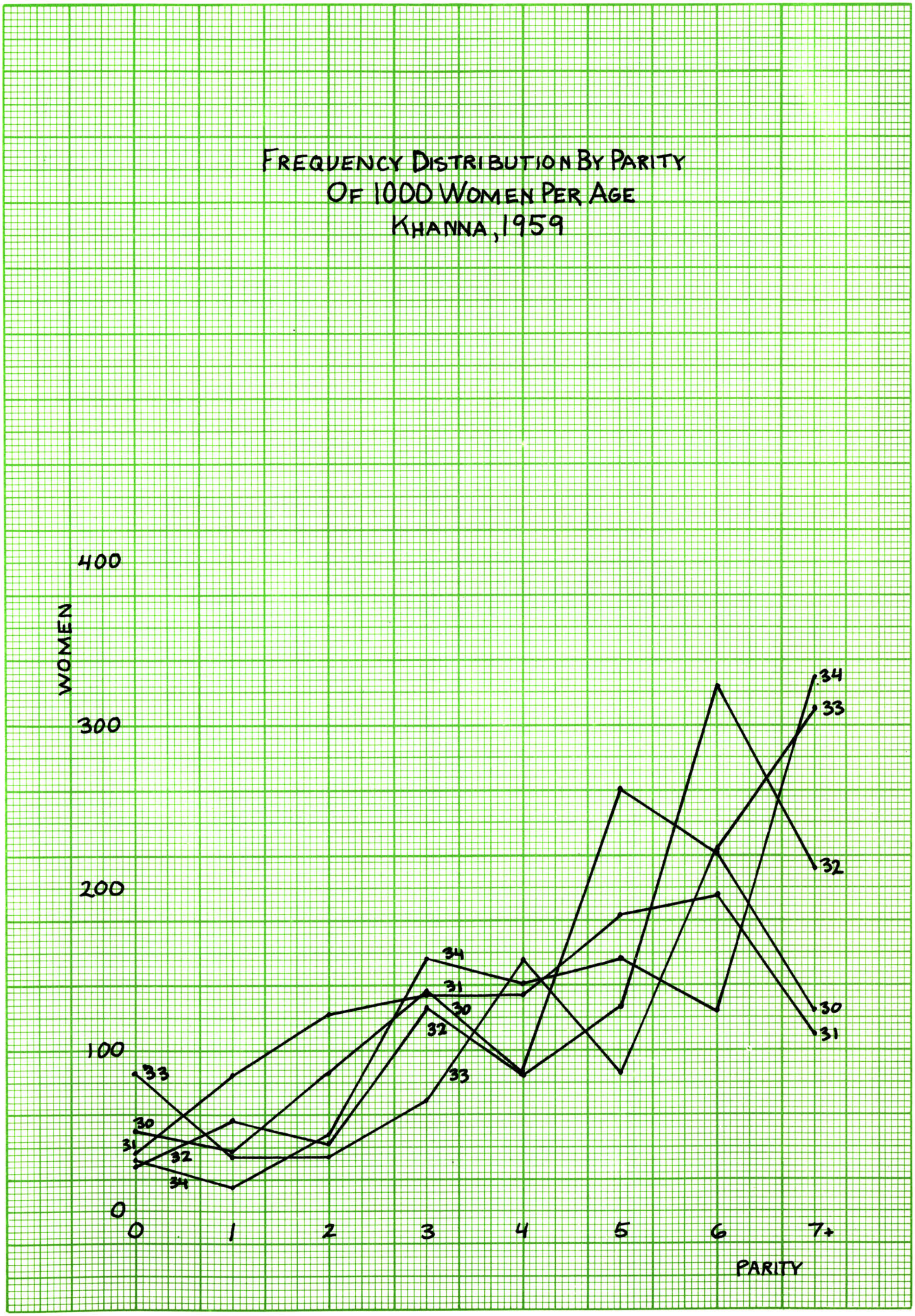
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
TO 777.8 AT PARITY 0
700
600
500
400
300
200
100
0
16
17
18
19
0
1
2
3
4
5
6
7+
PARITY

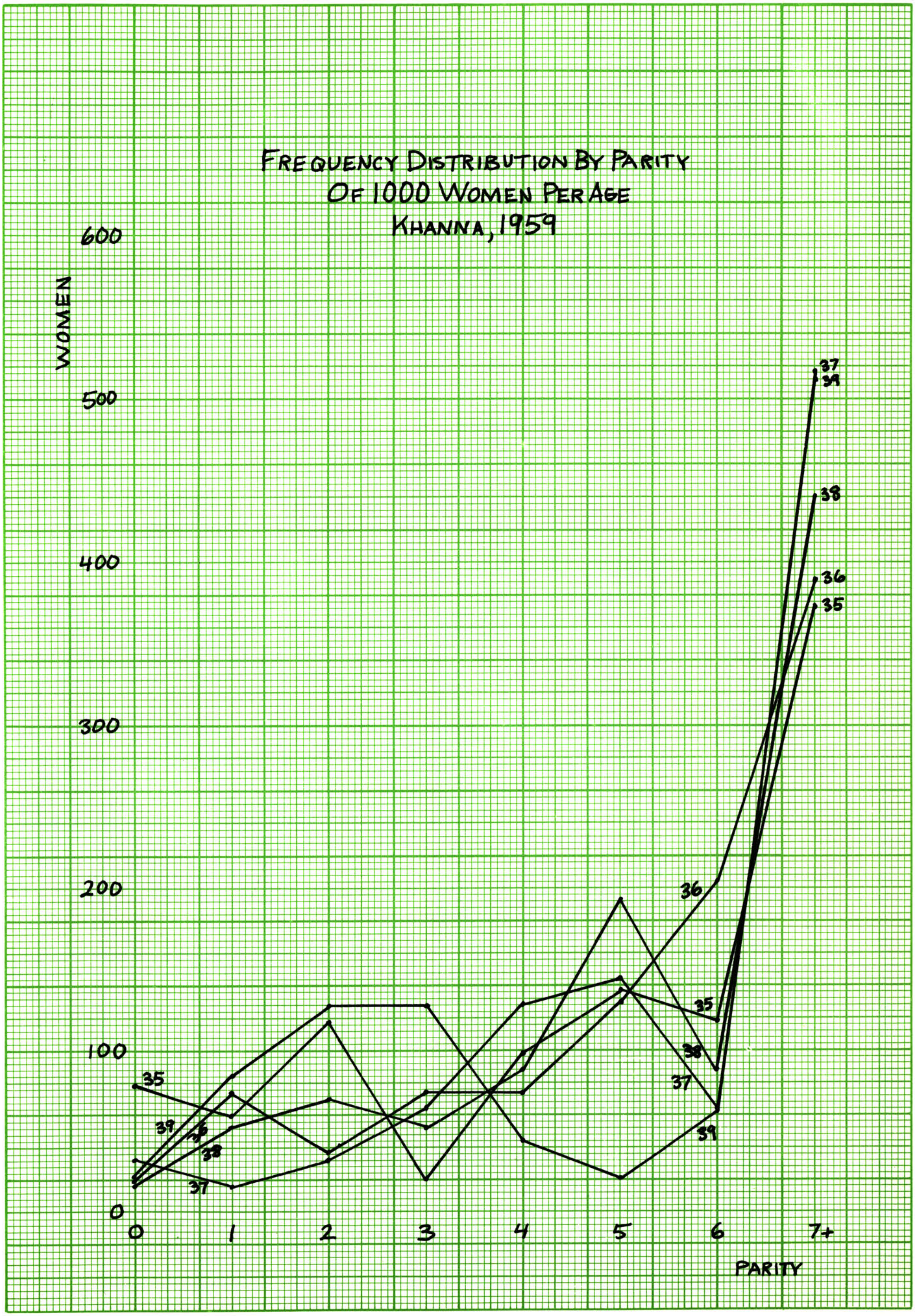


FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
500
400
300
200
100
0
20
21
22
23
24
0
1
2
3
4
5
6
7+
PARITY

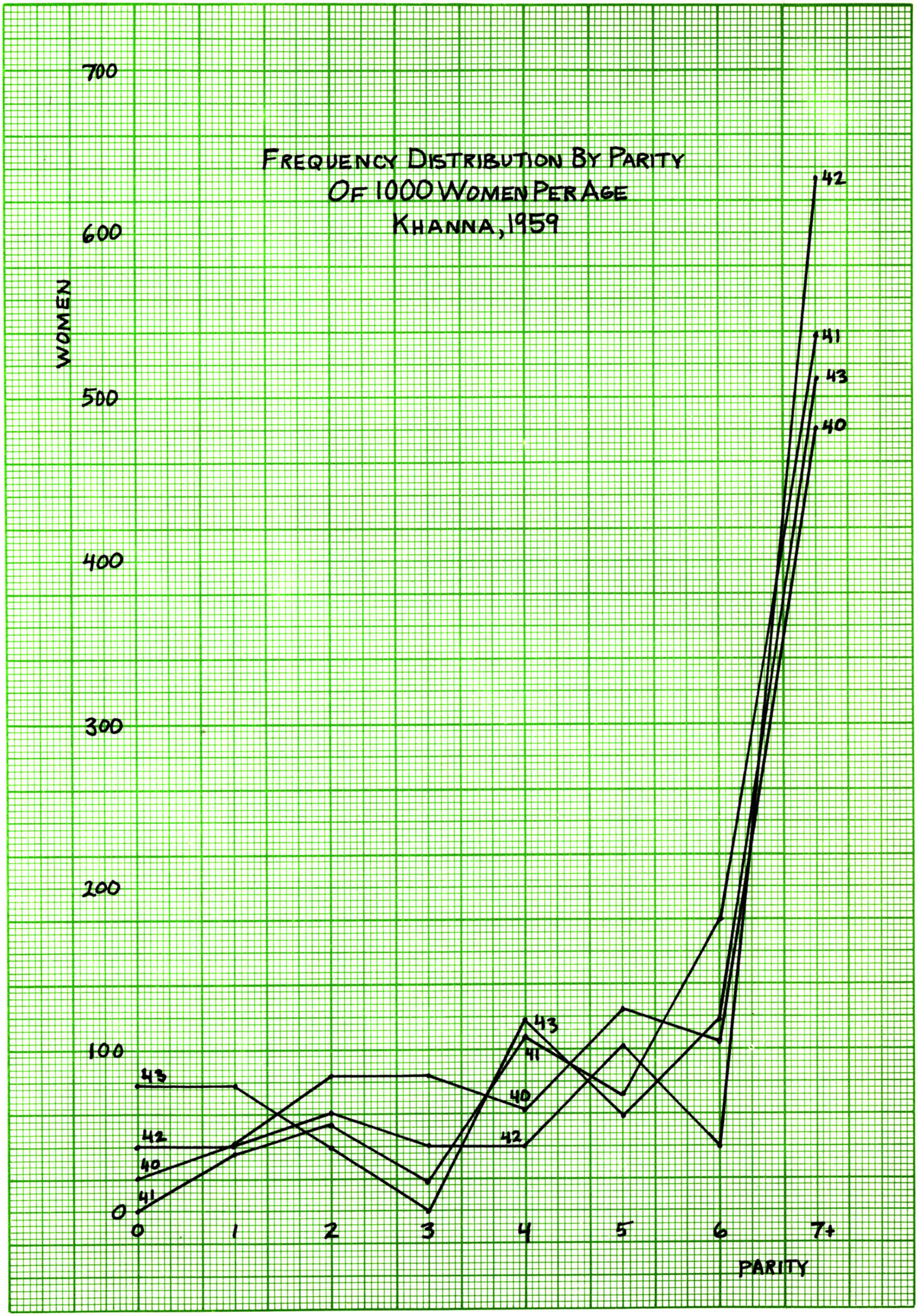


FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
300
200
100
0
0
1
2
3
4
5
6
7+
PARITY
26
28
25
27
29
29
28
27
26
25


FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
400
300
200
100
0
0
1
2
3
4
5
6
7+
PARITY
34
33
32
30
31
34
31
30
32
33
33
30
31
32
34



FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
600
500
400
300
200
100
0
0
1
2
3
4
5
6
7+
PARITY
35
39
36
38
37
36
35
38
37
39
37
39
38
36
35


FREQUENCY DISTRIBUTION BY PARITY
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
700
600
500
400
300
200
100
0
PARITY
0
1
2
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5
6
7+
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41
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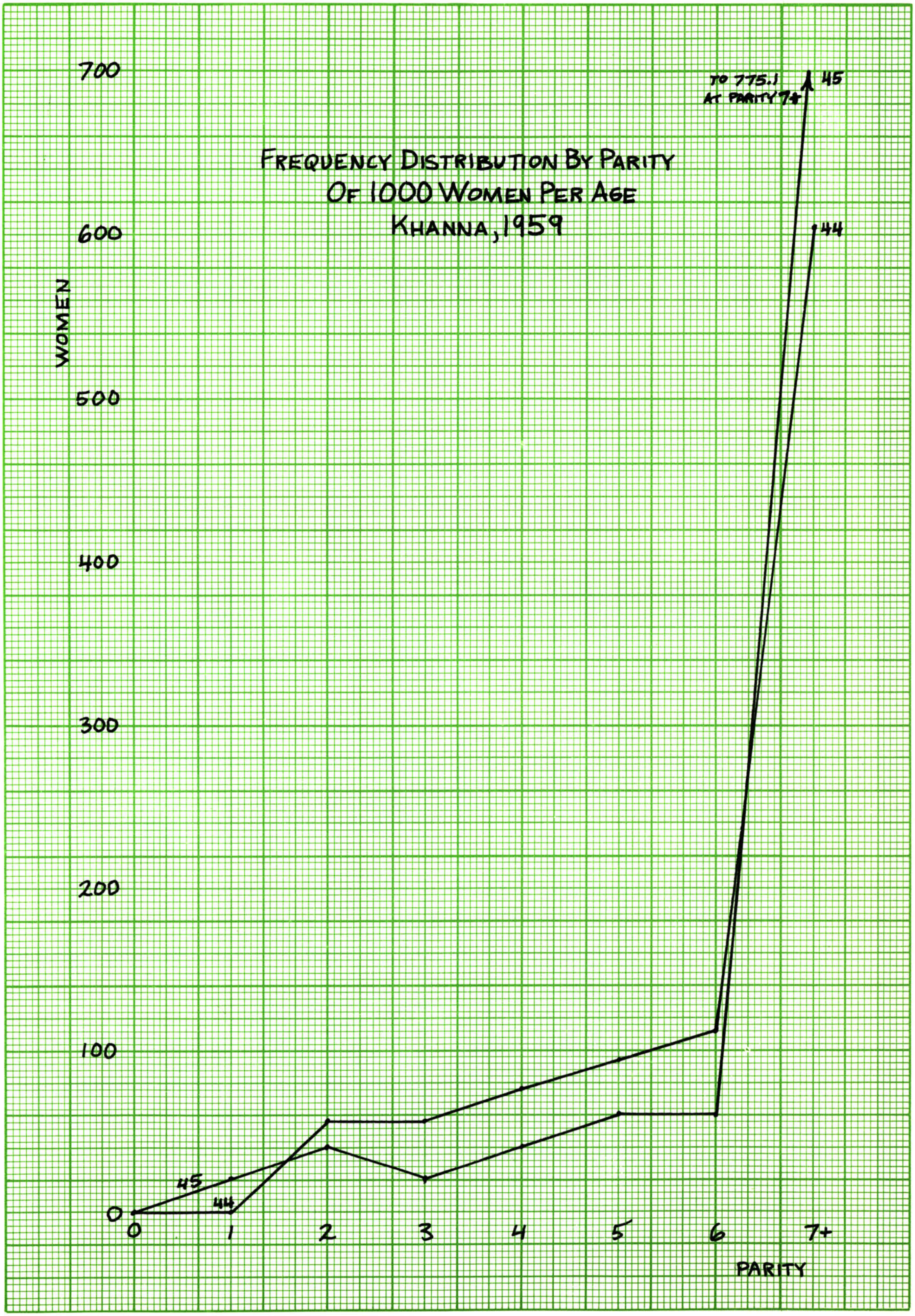
42
43
42
40
41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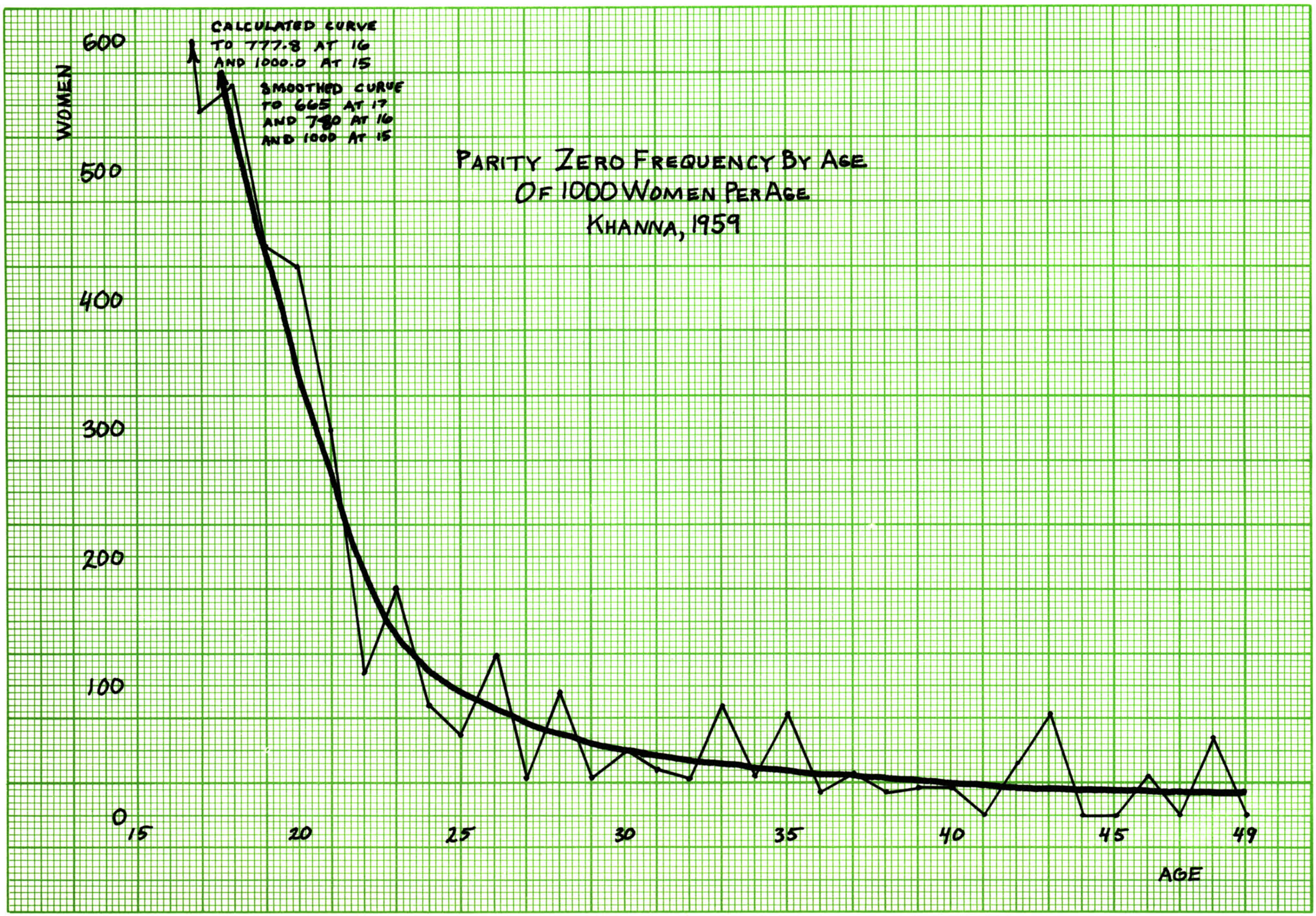
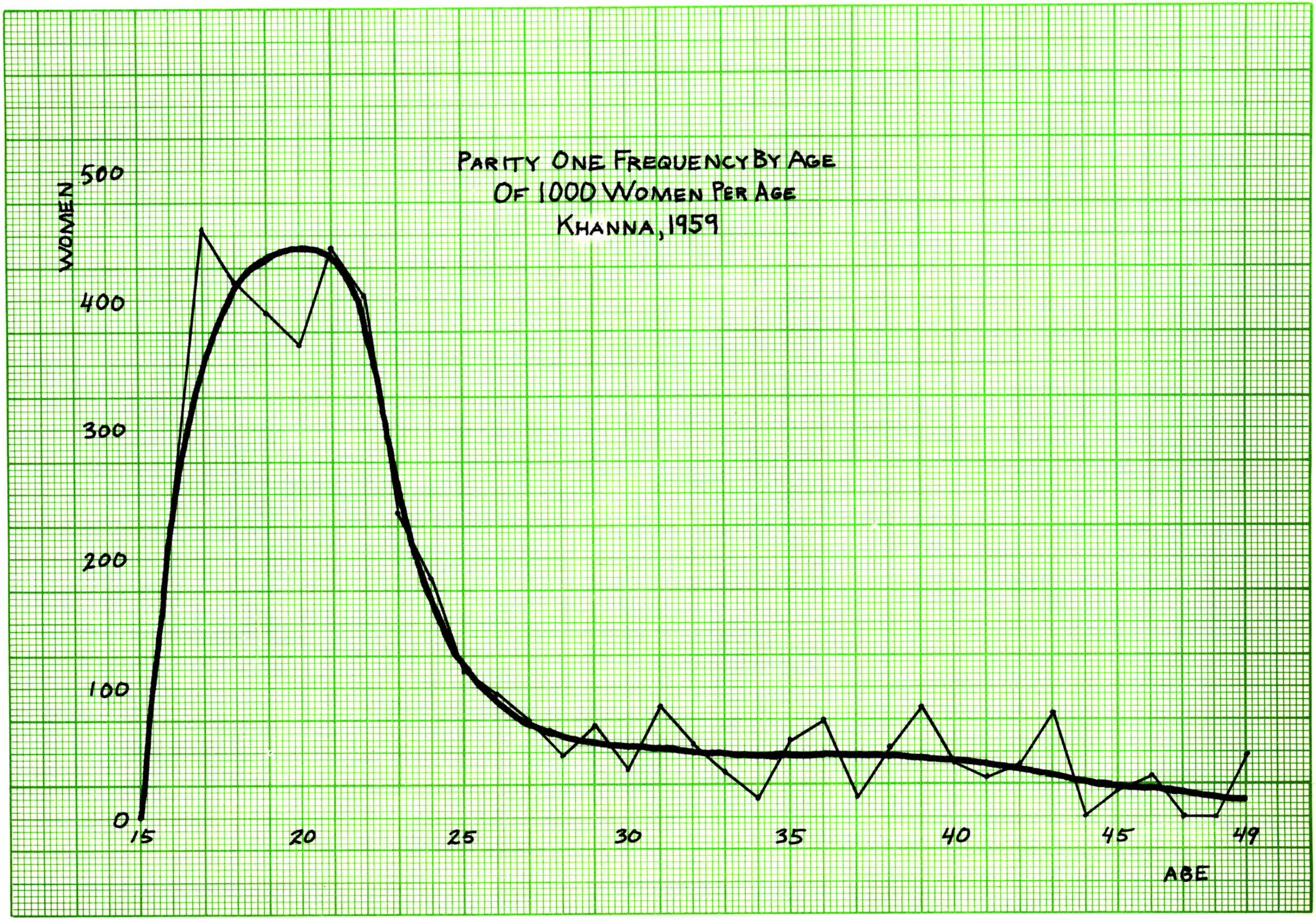
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
700
600
500
400
300
200
100
0
TO 775.1
AT PARITY 7+
45
44
45
44
0
1
2
3
4
5
6
7+
PARITY


PARITY ZERO FREQUENCY BY AGE
OF 1000 WOMEN PER AGE
KHANNA, 1959
CALCULATED CURVE
TO 777.8 AT 16
AND 1000.0 AT 15
SMOOTHED CURVE
TO 665 AT 17
AND 780 AT 16
AND 1000 AT 15
WOMEN
600
500
400
300
200
100
0
15
20
25
30
35
40
45
49
AGE


PARITY ONE FREQUENCY BY AGE
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
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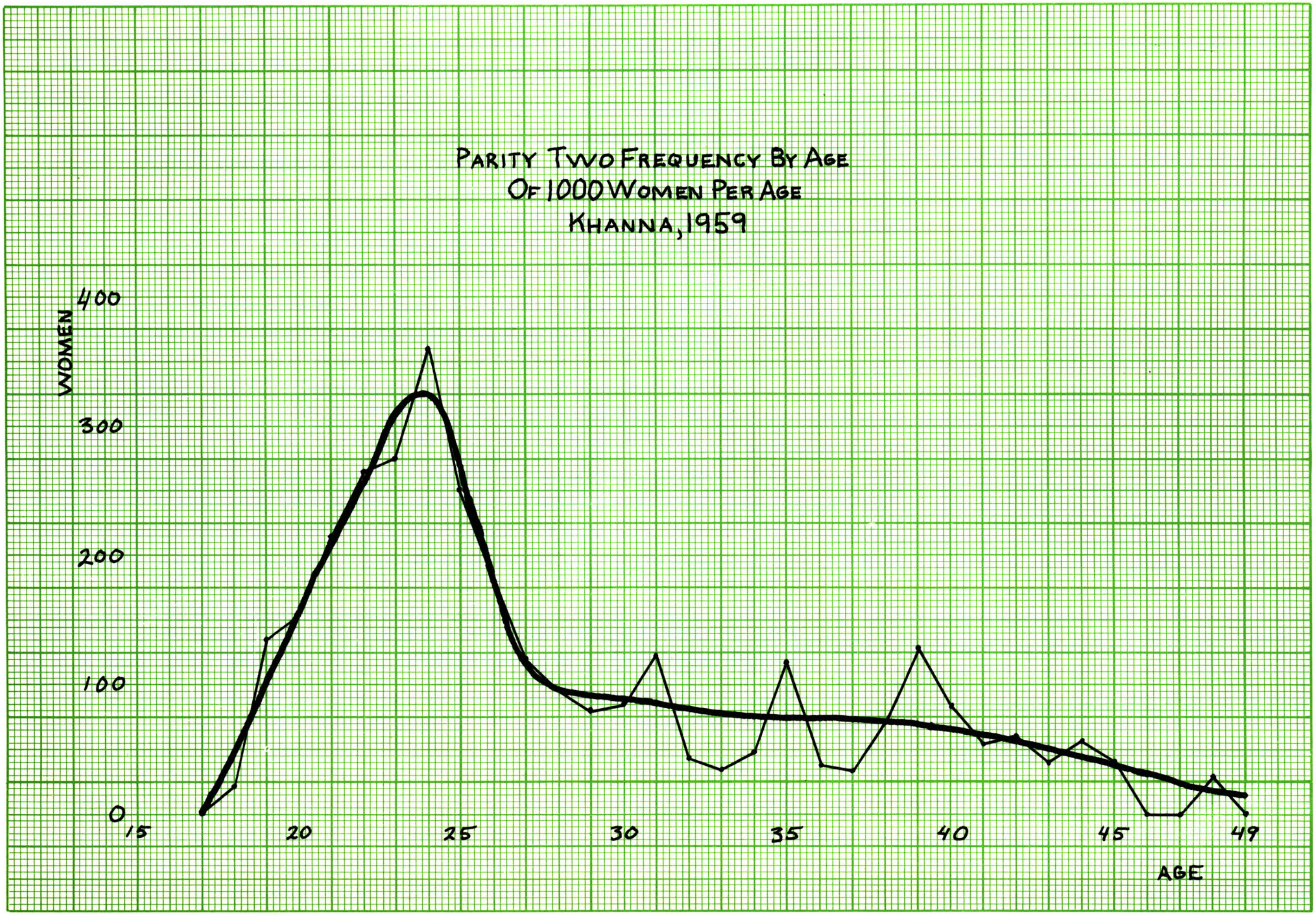
200
100
0
15
20
25
30
35
40
45
49
ABE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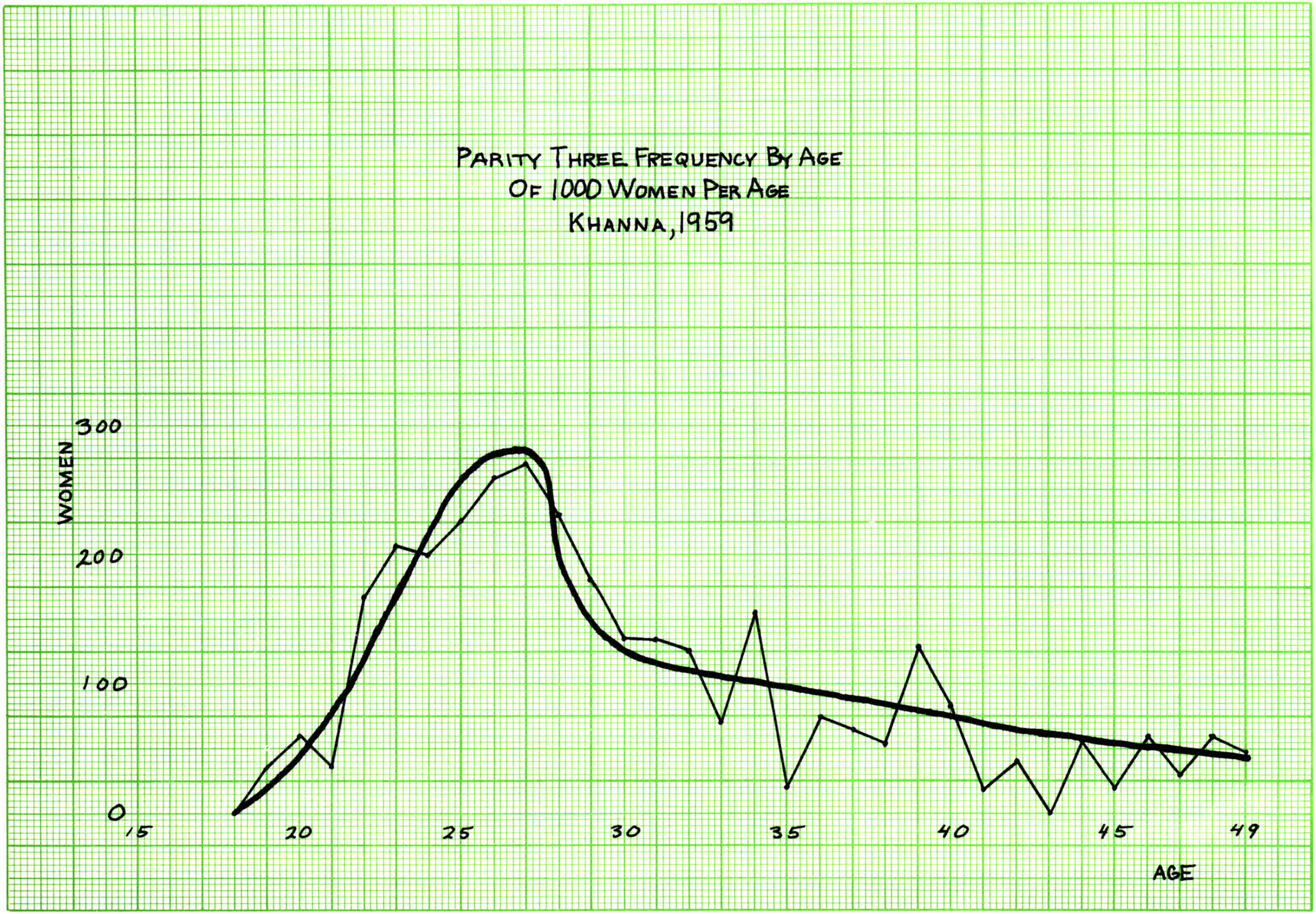
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
400
300
200
100
0
15
20
25
30
35
40
45
49
AGE


PARITY THREE FREQUENCY BY AGE
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
300
200
100
0
15
20
25
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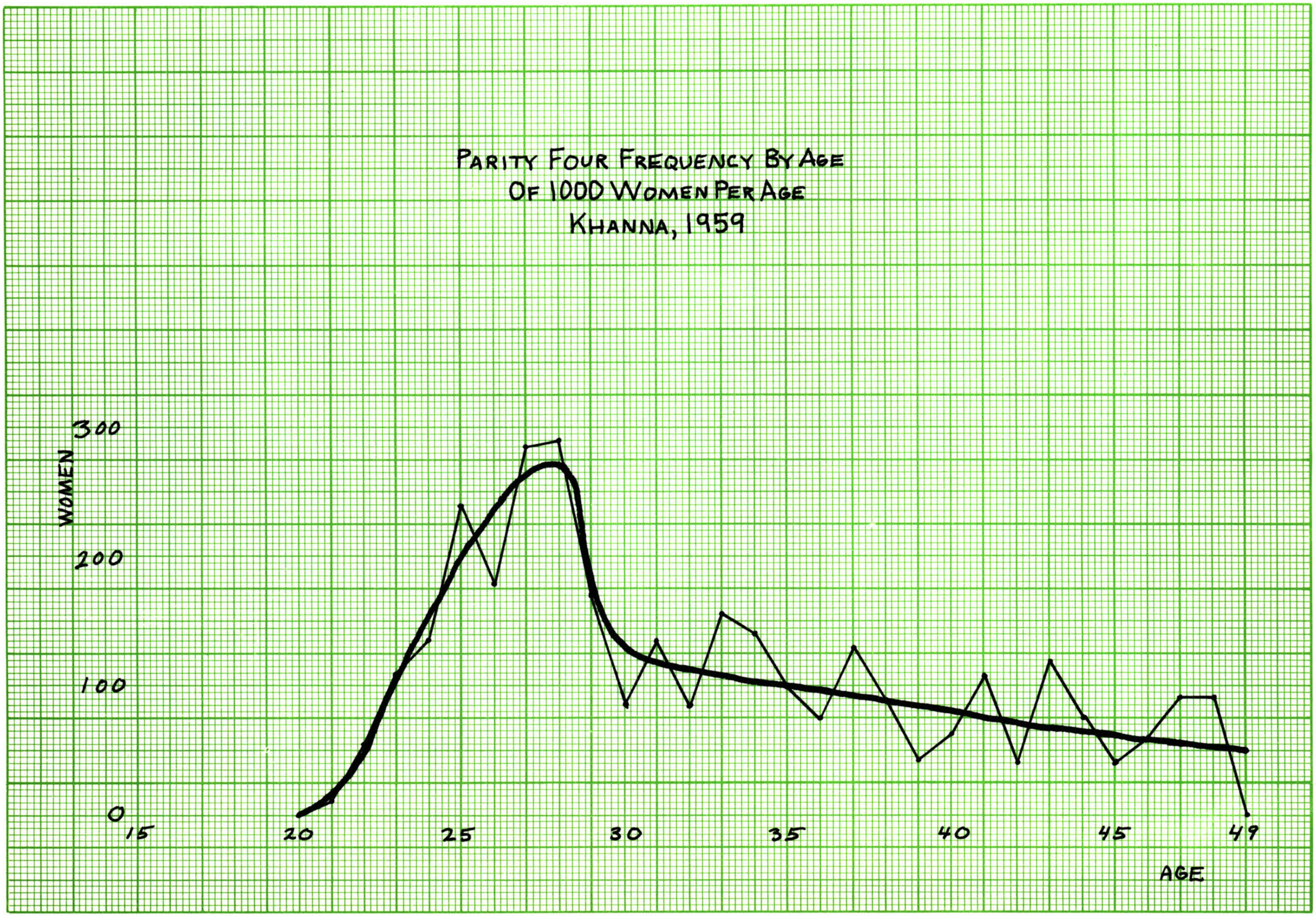
40
45
49
AGE

A 20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WOMEN
300
200
100
0
15
20
25
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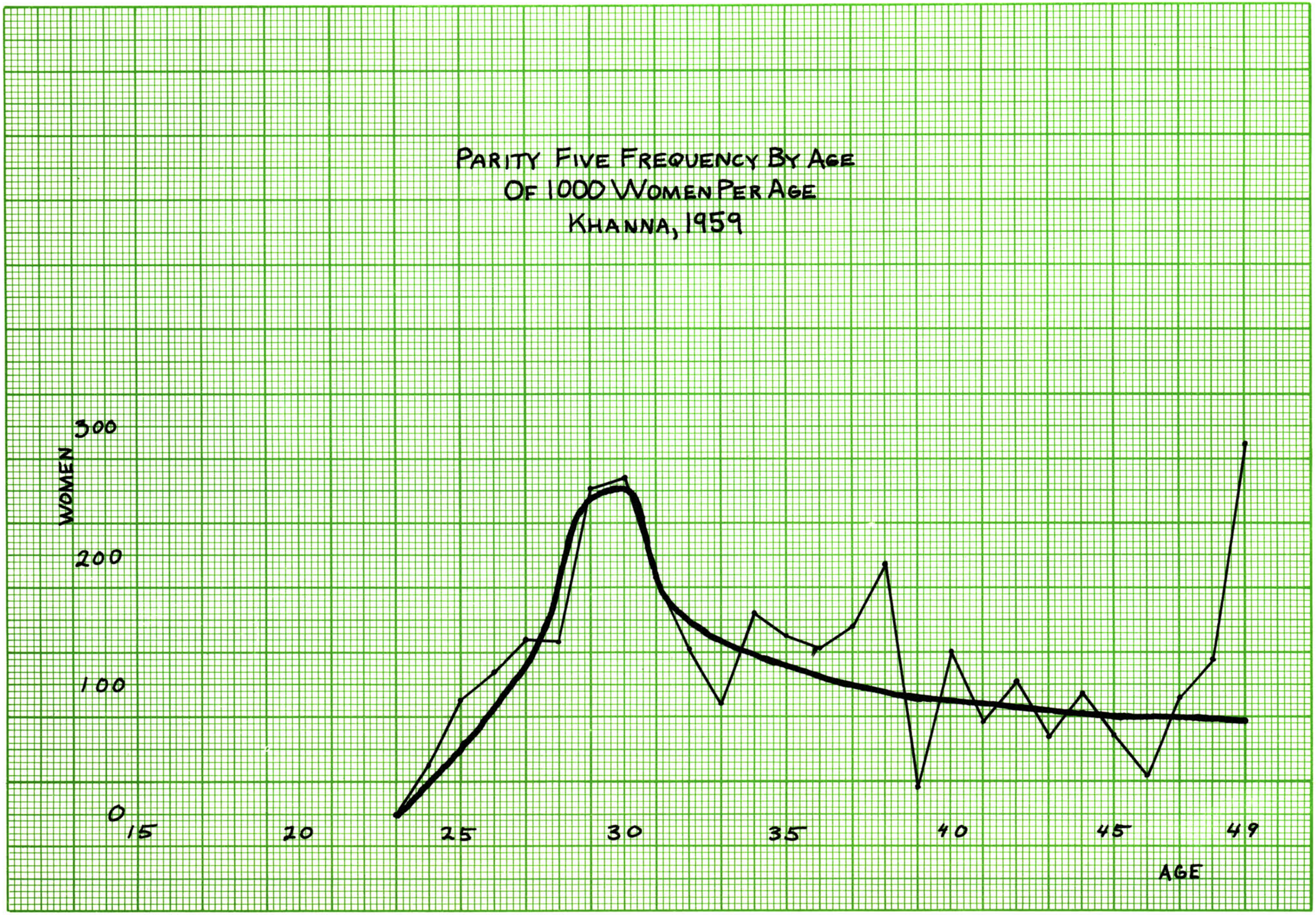
35
40
45
49
AGE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OF 1000 WOMEN PER AGE
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WOMEN
300
200
100
0
15
20
25
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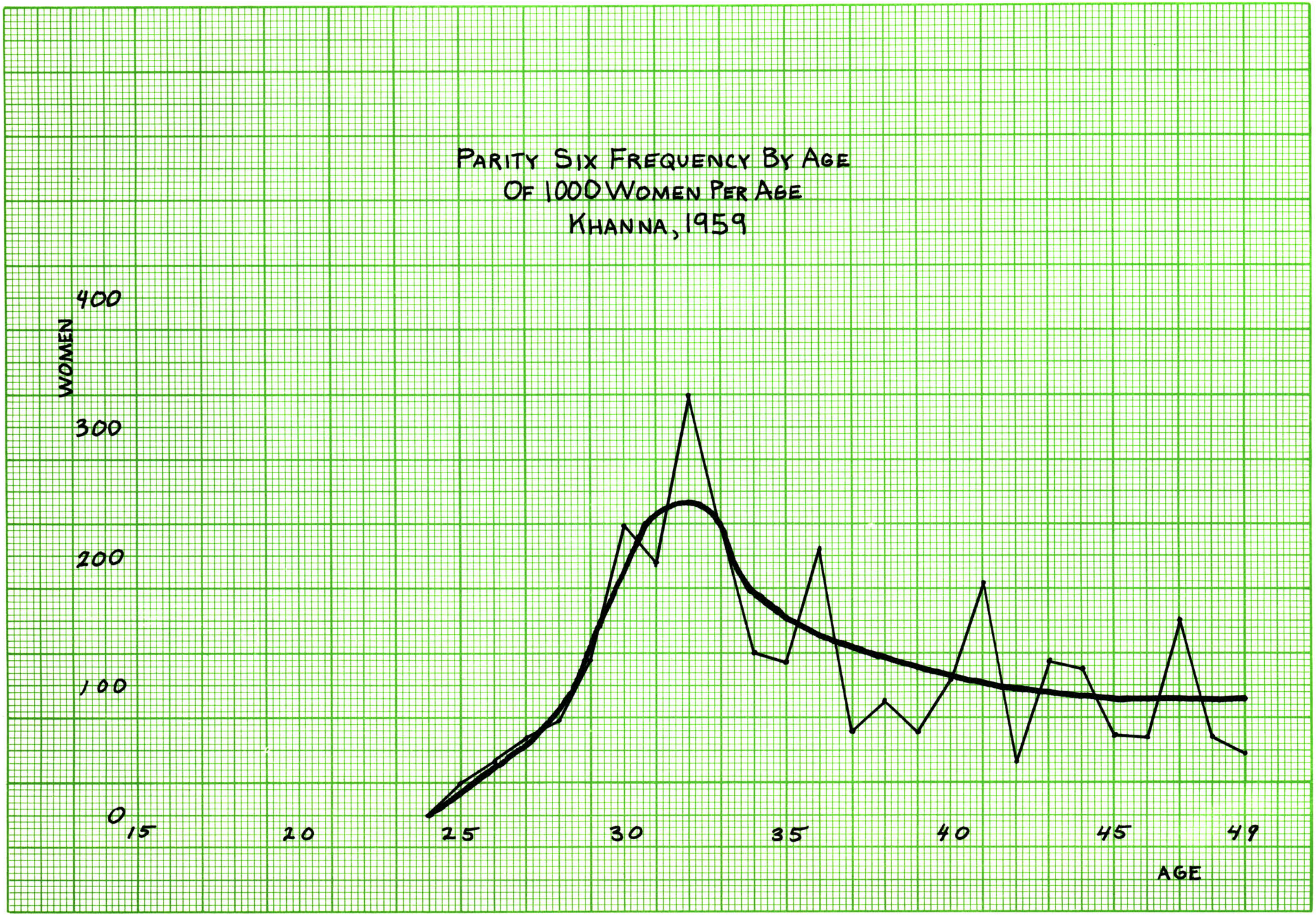
45
49
AGE

A 21

A22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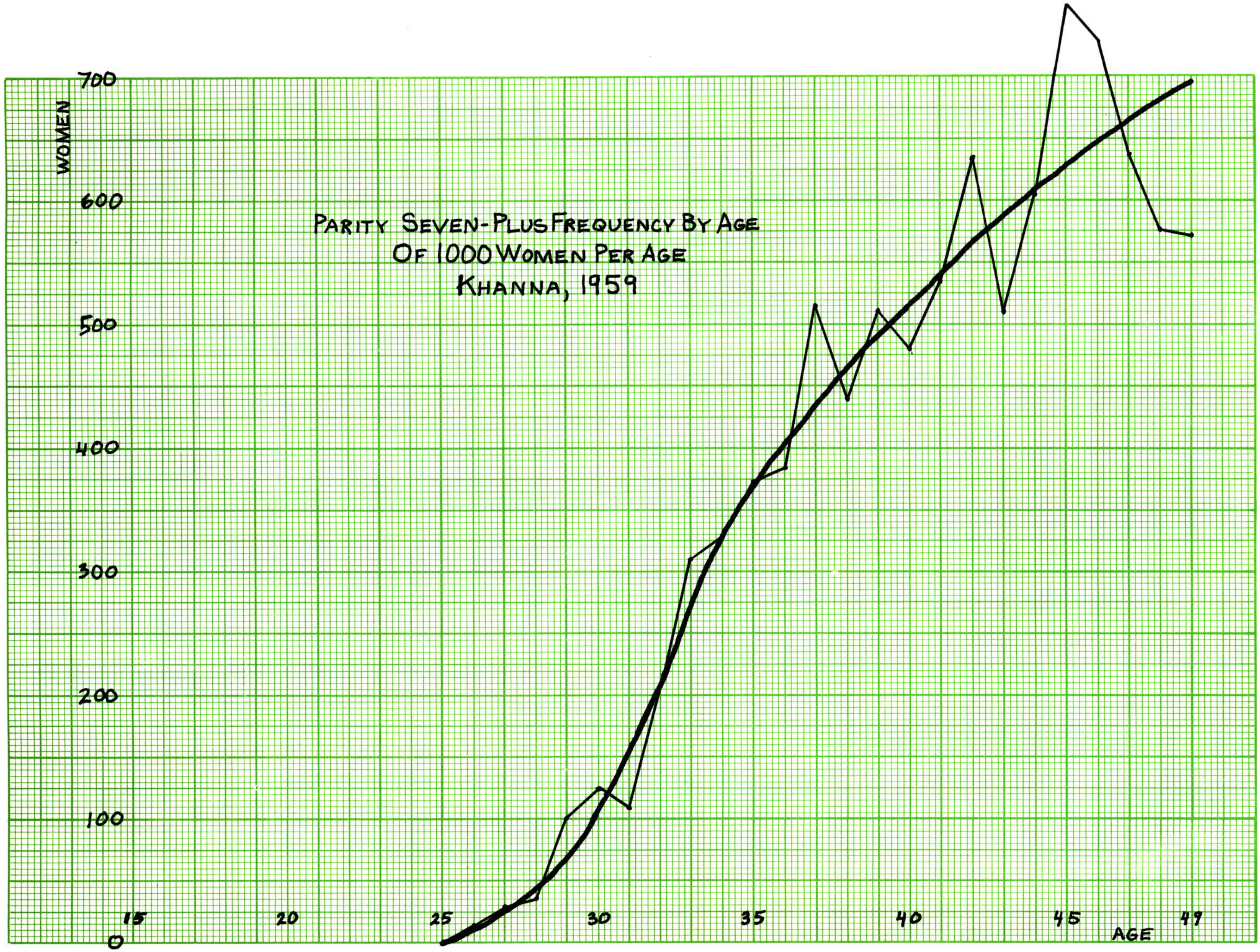
40
45
49
AGE



PARITY SEVEN-PLUS FREQUENCY BY AGE
OF 1000 WOMEN PER AGE
KHANNA, 1959
WOMEN
700
600
500
400
300
200
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0
15
20
25
30
35
40
45
49
AGE

A 23