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# Perceived slant of binocularly viewed large-scale surfaces: A common model from explicit and implicit measures

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It is known that the perceived slants of large distal surfaces, such as hills, are exaggerated and that the exaggeration increases with distance. In a series of two experiments, we parametrically investigated the effect of viewing distance and slant on perceived slant using a high-fidelity virtual environment. An explicit numerical estimation method and an implicit aspect-ratio approach were separately used to assess the perceived optical slant of simulated large-scale surfaces with different slants and viewing distances while gaze direction was fixed. The results showed that perceived optical slant increased logarithmically with viewing distance and the increase was proportionally greater for shallow slants. At each viewing distance, perceived optical slant could be approximately fit by linear functions of actual slant that were parallel across distances. These linear functions demonstrated a fairly constant gain of about 1.5 and an intercept that increased logarithmically with distance. A comprehensive three-parameter model based on the present data provides a good fit to a number of previous empirical observations measured in real environments.

Keywords: slant perception, exocentric distance perception, perceptual scale expansion, binocular disparity, virtual reality

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## Introduction

The perception of geographical slant (surface orientation relative to horizontal) is exaggerated for both uphill and downhill surfaces (Kammann, 1967; Proffitt, Bhalla, Gossweiler, & Midgett, 1995; Ross, 1974). Recently, it was found that the perception of downhill geographical slant differs with changes in viewing position that alter the direction of gaze to the surface, even when viewing distance is controlled for (Li & Durgin, 2009). Based on this finding, a geometric model was proposed to account for perception of downhill slant. In the model, perceived geographical slant was expressed as a function of perceived *gaze declination* (i.e., the perceived magnitude of the downward pitch of gaze), which was observed to be an exaggerated linear function of actual gaze declination, and perceived *optical slant* (perceived surface orientation relative to the direction of gaze, Sedgwick, 1986). The model not only fit the empirical data of downhill slant perception quite well but also predicted an exaggerated linear function between perceived and actual optical slants (from 5° to 50°); this prediction of the model was confirmed later by direct measurement (Durgin & Li, [submitted for publication](#); Durgin, Li, & Hajnal, 2010, Experiment 3).

There is reason to believe that large variations in viewing distance affect the perception of optical slant because farther portions of uphill slants appear steeper (Bridgeman & Hoover, 2008) whereas downhill slants

appear shallower when viewed from a far distance (Ross, 2006). The present study used parametric manipulation to model the effects of viewing distance on perceived optical slant using both traditional verbal estimation (Proffitt et al., 1995) and an aspect-ratio task that has been used to study exocentric distance perception (Loomis, Da Silva, Fujita, & Fukusima, 1992; Loomis & Philbeck, 1999; Philbeck, 2000). Whereas a variety of methods have been used to study perceived slant, some of these, such as haptic matching tasks (palm boards), have been shown to be biased measures (Durgin, Hajnal, Li, Tonge, & Stigliani, 2010). In contrast, verbal measures appear not to be intrinsically biased (Durgin, Li et al., 2010). For this reason, in the first experiment of the present study, verbal estimates of slant, an explicit measure of perceived slant, were used to develop a model of optical slant perception as a function of viewing distance. The model was then tested in [Experiment 2](#), using an aspect-ratio task to implicitly measure perceived slant.

The idea of treating the estimation of exocentric extents (relative to frontal extents) as an implicit measure of the perceived slant of the ground plane has precedent in the clever work of Ooi, Wu, and He (2006) and Wu, Ooi, and He (2004). Here we extend that idea to the more general case of optical slant perception by introducing a more general mathematical analysis. Theoretically, the ratio between a frontal interval and a perceptually matched interval that is extended along a surface at some tilt to gaze can be used to derive the local perceived surface

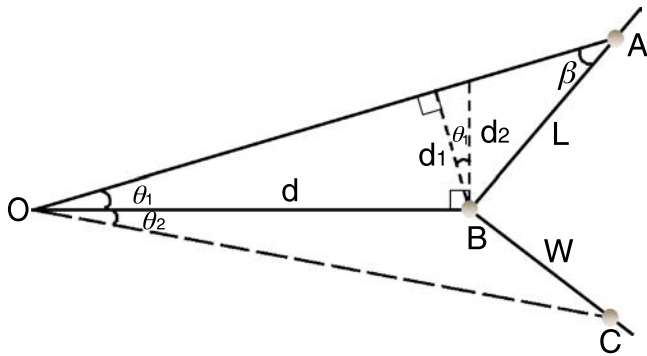


Figure 1. Illustration of the aspect-ratio task as an implicit measure of perceived optical slant. O is the station point. A, B, and C are targets on a slant, with B and C at the same height as O. The vector BC, which is perpendicular to OB and to AB, represents a horizontal interval that is frontal to the observer at O.

slant relative to the direction of gaze. As shown in Figure 1, balls A, B, and C are located on a slanted surface in front of the observer, O, with balls B and C at the same height as the station point O. (N.B. The horizontal vector BC is perpendicular both to AB and to BO). The slanted extent AB subtends a visual angle of  $\theta_1$  and the frontal extent BC subtends a visual angle of  $\theta_2$ . The optical slant at ball A is  $\beta$ .  $d_1$  is the frontal projected length of AB when gaze is directed at A.  $d_2$  is the frontal projected length of AB when gaze is directed at B. Because  $d_1$  equals  $d_2 \cos \theta_1$ , the aspect ratio ( $R$ ) between AB and BC (i.e.,  $L/W$ ) can be expressed as

$$R = \frac{d_2 \cos \theta_1}{W \sin \beta}. \tag{1}$$

If we assume that the perceived extents and visual angles (as expressed by adding a prime to the variables in Equation 1) follow the same equation, i.e.,

$$R' = \frac{d_2' \cos \theta_1'}{W' \sin \beta'}, \tag{2}$$

then the relation between perceived optical slant,  $\beta'$ , and actual optical slant,  $\beta$ , can be expressed by

$$R'/R = \frac{d_2' \cos \theta_1'}{W' \sin \beta'} / \frac{d_2 \cos \theta_1}{W \sin \beta}. \tag{3}$$

Since  $d_2$  and  $W$  are in the same fronto-parallel plane when gaze is directed to B, we assume that the perceived ratio between the two frontal extents is approximately equal to their actual ratio, i.e.,  $d_2'/W' \approx d_2/W$  (but see

Higashiyama, 1992; it may be appropriate to correct for the vertical–horizontal illusion, VHI, which is typically measured as being about 1.05). Thus, Equation 3 can be simplified as

$$\sin \beta' = \frac{R \cos \theta_1'}{R' \cos \theta_1} \cdot \sin \beta. \tag{4}$$

Although it has been suggested that  $\theta_1'$  may be scaled by a factor of 1.3 relative to  $\theta_1$  (Foley, Ribeiro-Filho, & Da Silva, 2004; see also Murray, Boyaci, & Kersten, 2006), when  $\theta_1$  is small,  $\cos \theta_1' / \cos \theta_1$  is still essentially 1.0. For example, even if a factor of 1.3 is assumed, when  $\theta_1$  is less than  $10^\circ$ ,  $\cos \theta_1' / \cos \theta_1$  would remain between 0.99 and 1.0. Thus, Equation 4 can be further simplified to

$$\beta' = \sin^{-1} \left( \frac{R}{R'} \cdot \sin \beta \right). \tag{5}$$

According to Equation 5, perceived optical slant  $\beta'$  can be deduced by measuring perceived aspect ratio  $R'$  in the aspect-ratio task. Whereas the aspect-ratio technique has been applied to the study of exocentric distances along the ground and interpreted in terms of geographical slant (e.g., Ooi et al., 2006), to our knowledge no published study has used it on slanted surfaces (but see Ooi & He, 2004). Although the trigonometry described by Equation 5 is similar to one of the equations published by Ooi et al. (in their Appendix A), the theoretical terms are entirely different. For example, our equation refers only to optical slant and includes no term for gaze declination; in contrast, the main equation of Ooi et al. includes terms for gaze declination and a term,  $\eta$ , referring to an imputed constant additive bias in geographical slant perception; even when a term for optical slant is substituted for the gaze declination term, the geographical slant error term,  $\eta$ , is retained. Thus, our equation was developed to express the perceived optical slant of any surface (e.g., a slanted surface floating in space), while the equation proposed by Ooi et al. concerns an imputed angular bias in the perception of a horizontal ground surface. These theoretical distinctions are important because our paper seeks to model perceived optical slant as a function of both distance and optical slant, rather than to propose a constant additive bias ( $\eta$ ) in geographical slant perception as Ooi et al. have done.

Loomis et al. (1992) used the aspect-ratio task to show that exocentric distance was increasingly compressed at farther distances. If we interpret their data using Equation 5, it suggests that the perceived optical slant was exaggerated by a factor of about 1.5 for their observers. This amount of exaggeration in perceived slant is consistent with recent studies of both large-scale and small-scale surfaces

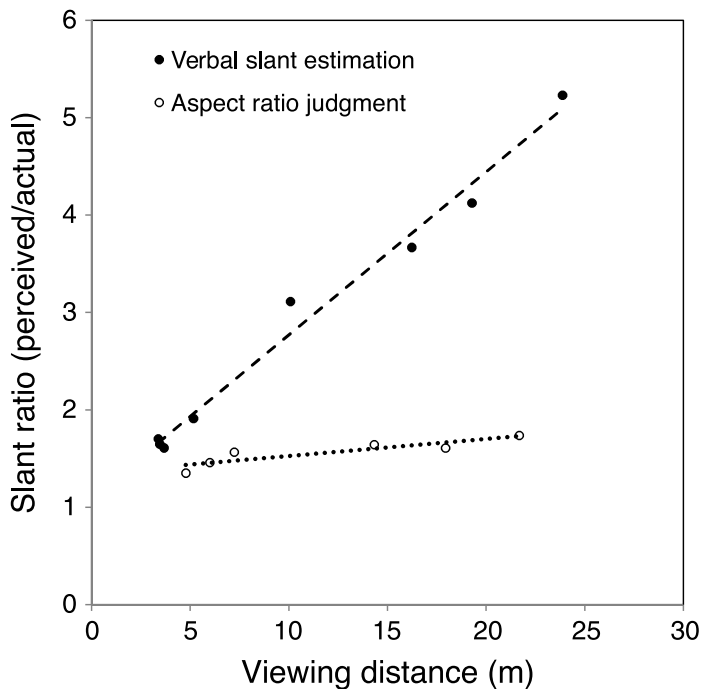


Figure 2. Slant overestimation ratios as functions of viewing distance. Solid circles show verbal report data of real hill slant perception (Proffitt et al., 1995; Experiment 1). Open circles represent aspect-ratio judgment data along the ground (Loomis & Philbeck, 1999, binocular condition with physical ratio of 1.5). Far slants in the verbal data are 10°, 6°, 5°, and 4° hills. Actual optical slants in the aspect-ratio judgment data are between 16° and 24° because the observers were elevated for the farther viewing distances.

(Durgin & Li, submitted for publication; Durgin, Li et al., 2010; Li & Durgin, 2009), which used both verbal and non-verbal measures of perceived slant. This seems to confirm that perceived aspect ratio is a good candidate for an implicit measure of perceived optical slant.

However, there is also reason to question whether the aspect-ratio technique measures the same things as slant tasks. When Loomis and Philbeck (1999) held optical slant constant (by elevating the observer for farther viewing distances along the ground), they reported that the aspect-ratio judgments were invariant with distance. In Figure 2 (open circles), we have plotted their aspect-ratio data (i.e., binocular condition with physical ratio of 1.5) interpreted as the ratio between (inferred) perceived optical slant and actual optical slant, which shows little effect of distance. In contrast, Bridgeman and Hoover (2008) have reported that perceived geographical slant increases logarithmically with distance.

It is not easy to directly compare these two results because Bridgeman and Hoover (2008) confounded viewing distance with direction of gaze. However, if we plot normative verbal slant data for outdoor surfaces with gaze forward (Proffitt et al., 1995; Experiment 1), these data also suggest that the exaggeration of perceived optical

slant (which is coincident with geographical slant when gaze is forward) increases with distance (Figure 2, solid circles). The divergent pattern between verbal and aspect-ratio measures might be taken to indicate that the aspect-ratio task is not a good implicit measure of perceived optical slant. However, because Loomis and Philbeck elevated their observers for far distances, the optical slants at far distances in these two studies were quite different. Thus, there may be interactions between slant and distance, which produce the divergent patterns of data in Figure 2.

In the present study, we parametrically studied the effects of slant and distance on perceived slant using both explicit slant measures (verbal report) and implicit slant measures (aspect-ratio judgments). Our study required the use of simulation (immersive virtual environments) to maintain experimental control across a large range of stimulus conditions. Previously, Li and Durgin (2009) used both virtual environments and real environments to show that gaze direction affected the perception of downhill geographical slant. The theoretical significance of the present work is that, in combination with our model of the effects of gaze declination (Li & Durgin, 2009), the present study may provide a fairly complete model of the full-cue perceptual experience of surface orientation in depth, while integrating a number of seemingly disparate findings and methods.

Although there is always concern about generalizability from virtual environments (Durgin & Li, 2010), we will show that the present results provide an excellent model of existing slant and aspect-ratio data collected in the real world (i.e., slant data in Proffitt et al., 1995, and aspect-ratio data in Kudoh, 2005 and Loomis & Philbeck, 1999), while providing a more complete picture of the relevant parameter space.

## Experiment 1

In this experiment, perceived optical slant was measured across parametric variations in simulated slant and viewing distance. Because perceived slant is sensitive to surface texture information (Cumming, Johnston, & Parker, 1993; Knill, 1998) and gaze direction (Li & Durgin, 2009), we controlled these parameters in this experiment by keeping them fixed across variations in viewing distance.

## Methods

The participants were twenty-three undergraduate students (12 females) fulfilling a course requirement. All participants had normal or corrected-to-normal vision and were able to correctly report the facing direction (up, down, left, or right) of cyclopean random-dot Es presented at the conclusion of the experiment.



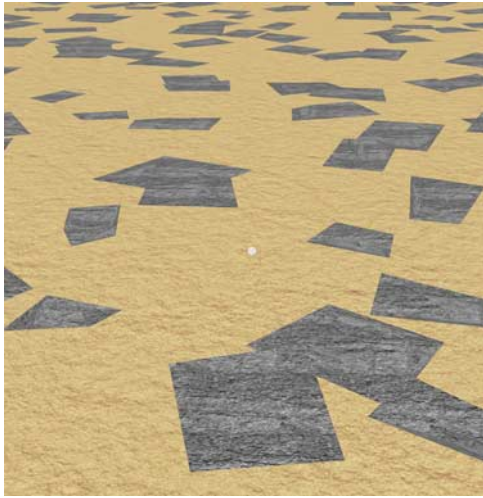


Figure 3. Sample visual stimulus presented in Experiment 1. The image is of a 24° slant at the 16-m viewing distance. Texture was scaled proportionally to the viewing distance so that retinal texture properties were identical at all viewing distances. The white sphere was at eye level and was also scaled with distance to maintain the same retinal size. Binocular disparity information locked the ball to the surface perceptually.

Prior to participation, the task of estimating optical slant was explained to participants with the help of a diagram depicting a side view of a person looking at a slanted surface. Once it was clear that the participant understood the angle to be given, they were fitted with the head-mounted display. There were five practice trials, and no feedback was given about their verbal estimates. The large-scale surfaces were simulated in an immersive virtual environment presented in an nVisor head-mounted display (HMD) with a nominal resolution of  $1280 \times 1024$  and 60-Hz refresh rate. Display images were pincushion-corrected, using a shader in a professional rendering system (Virtools 4.1) to predistort the image, and calibrated (Durgin & Li, 2010; see also Kuhl, Thompson, & Creem-Regehr, 2009). Each participant's head position and orientation were monitored by an optical tracking system (Vicon) and used to update the display of the HMD with a lag of less than 100 ms, with images appropriate to the location of each eye view based on measured interpupillary distance. (N.B. Our analysis assumes that the image planes in the two displays were parallel as per the manufacturer's specification.) The vertical field of view (FOV) subtended 34°. A simulated dark aperture restricted each eye's horizontal FOV to about 33°, with about 80% binocular overlap. The simulated aperture, a recent refinement added to our virtual environments, was used to eliminate a conflicting depth cue: Without it, the visual aperture formed by the physical frames of our HMD screens is binocularly specified to be infinitely far away even though it necessarily occludes near surfaces.

The surfaces were planes composed of a sand-like texture with irregular “flagstones” at irregular intervals (Figure 3). A white sphere ( $0.57^\circ$  visual angle) on the surface served as a fixation point, which was always presented at eye level. Five viewing distances to the fixation sphere (1, 2, 4, 8, and 16 m) were factorially combined with six surface orientations ( $6^\circ$ ,  $12^\circ$ ,  $18^\circ$ ,  $24^\circ$ ,  $30^\circ$ , and  $36^\circ$ ). These thirty experimental trials, presented in random order, followed five practice trials with a similar range of slants and viewing distances. The texture of the surface was scaled proportionally to viewing distance so that monocular slant information was similar at all distances. A distant texture of clouds was depicted in the sky.

Participants wore the HMD while sitting in a comfortable chair. They were asked to maintain their fixation at the white sphere during each trial and to give verbal estimates (in degrees) of the surface orientation relative to their gaze (i.e., optical slant). The experimenter typed in the number the participant reported and then started a new trial. On each trial, a different surface with randomly predetermined parameters was presented. A 1.5-s blank interval (black screen) separated trials. It took about 10 to 15 min for each participant to finish all 35 trials.

## Results and discussion

Mean numeric estimates of optical slant for each simulated slant are plotted as perceived slant ratios (estimated/actual slant) as a function of viewing distance in Figure 4. The mean perceived slant ratios increased

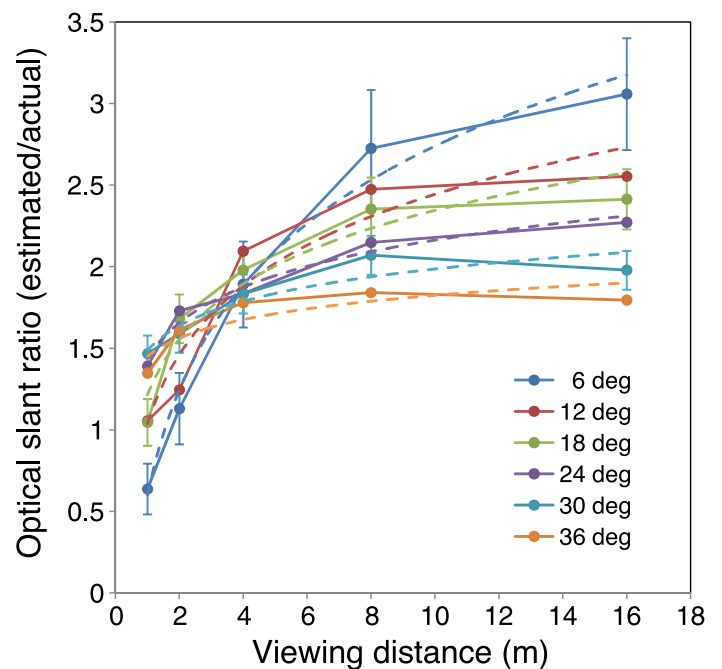


Figure 4. Slant estimates as ratios of simulated slant are plotted as a function of simulated viewing distance and slant (Experiment 1), with logarithmic fits (dashed lines). Error bars are standard errors of the means.

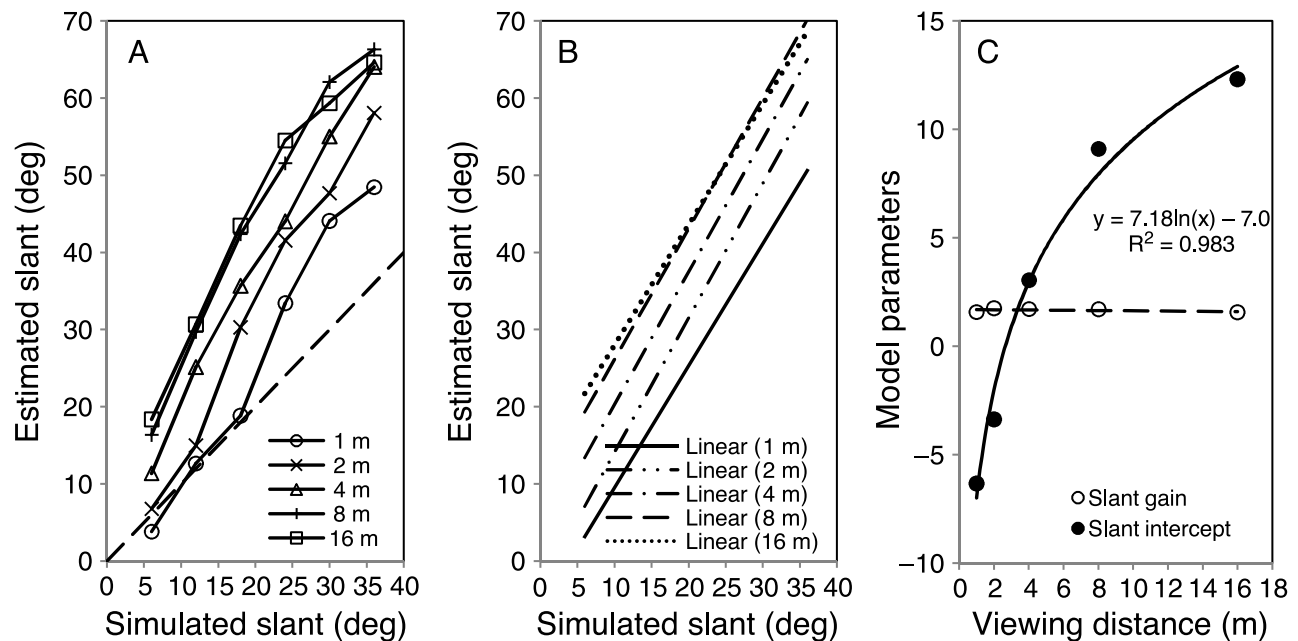


Figure 5. (A) Mean slant estimation data of Experiment 1 plotted as a function of viewing distance and simulated slant, (B) linear fits to those data at each viewing distance, and (C) linear fit parameters (slope and intercept) plotted as a function of viewing distance. The  $R$  squares of the linear fits are: 0.98, 0.99, 0.99, 0.98, and 0.96, respectively. A logarithmic fit is shown for the intercepts of the linear fits in (C).

logarithmically with viewing distance, and the increase was proportionally greater for smaller slants. The untransformed mean slant estimates are plotted in Figure 5A for each distance as a function of optical slant. Linear fits for each distance are plotted in Figure 5B. These plots show that the observed gain of perceived optical slant remained fairly constant across all viewing distances, while it was the intercept that changed with distance. The intercepts and gains from the fit lines in Figure 5B are plotted in Figure 5C as a function of viewing distance. This plot clarifies that it was the intercepts that increased approximately as a logarithmic function of viewing distance, while the gains remained constant. Because the slant gain was essentially constant with distance, the data were fit with a three-parameter model of perceived slant  $\beta_p$  as a function of simulated slant  $\beta$  and the log of viewing distance  $D$ :

$$\beta_p = k_1\beta + k_2\ln(D) + C. \quad (6)$$

Model parameters were computed using a mixed-effects model of the complete data set with subjects as a random effect. Best fit parameters were  $k_1 = 1.64$  (95% CI: 1.57 to 1.72,  $t = 44.4$ ,  $p < 0.0001$ ),  $k_2 = 6.96$  (95% CI: 6.17 to 7.77,  $t = 17.3$ ,  $p < 0.0001$ ), and  $C = -6.40$  (95% CI:  $-9.93$  to  $-2.91$ ,  $t = 3.07$ ,  $p = 0.0022$ ).<sup>1</sup>

The three-parameter model (Equation 6) was used to simulate the existing data of real-world studies (i.e., Loomis & Philbeck, 1999; Proffitt et al., 1995). To simulate the verbal data of Proffitt et al. (1995; Experiment 1), the

observer (with an eye height of 1.6 m) was assumed to be standing at the base of the hill and looking forward at the hill surface. For continuous variations in hill slant, Equation 6 was used to calculate the model ratio of estimated to actual optical slant for each combination of geographical slant and its associated viewing distance (blue line in Figure 6). To simulate the aspect-ratio data (Loomis & Philbeck, 1999, binocular condition with physical ratio of 1.5), predicted perceived optical slants were calculated using Equation 6 for the specific combinations of distance and optical slant that Loomis and Philbeck used. The resulting optical slant ratios are plotted as a function of viewing distance in Figure 6 (blue circles). The modeled data capture both patterns produced by slant estimation for hills (Proffitt et al., 1995) and by the aspect-ratio estimates collected by Loomis and Philbeck (1999). That is, both simulations are consistent with the trends of the respective empirical results (the black dashed lines). This suggests that (1) the apparent divergence between the effects of distance in the two prior studies was due to the different optical slants tested, and (2) the aspect-ratio task is indeed an implicit measure of perceived optical slant.

## Experiment 2

The fact that verbal slant estimates can be used to model implicit slant measures, such as the aspect-ratio

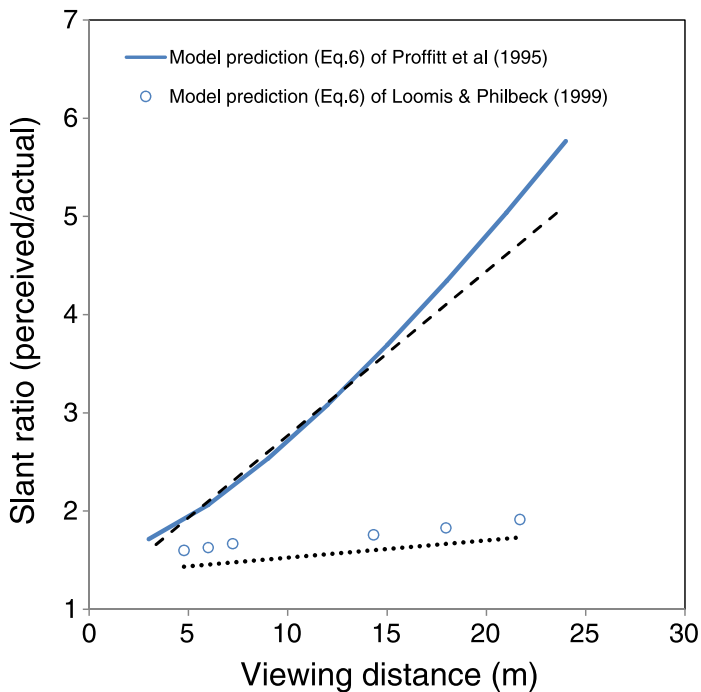


Figure 6. Slant overestimation ratios for model simulations of verbal slant estimation (Proffitt et al., 1995, blue curve) and aspect-ratio (Loomis & Philbeck, 1999, blue circles) data. Dashed lines are trend lines of corresponding empirical data (see Figure 2).

data of Loomis and Philbeck (1999), is consistent with the observation of Durgin, Li et al. (2010) that verbal numerical estimates of angles are not intrinsically biased measures. The verbal results in Experiment 1 indicate clearly how the apparently divergent patterns between the existing verbal slant data and aspect-ratio data shown in Figure 2 can be explained by the different optical slant ranges that were tested in the two paradigms. In Experiment 2, the aspect-ratio task was itself used as an implicit measure of perceived optical slant.

There are many possible variants of the aspect-ratio task. In one variant, participants were asked to give numerical estimates of the perceived ratio between the sagittal and frontal extents (e.g., Loomis et al., 1992; Loomis & Philbeck, 1999). This magnitude estimation version of aspect-ratio task is efficient but imprecise. In other variants, participants have been asked to adjust (or instruct the experimenter to adjust) the length of the sagittal (or frontal) extent until it perceptually matches the length of the frontal (or sagittal) extent (Beusmans, 1998; Wu et al., 2004). However, the method of adjustment is known to be biased by the initial length of the extent being adjusted (e.g., Purdy & Gibson, 1955).

An additional advantage of using virtual environments is that it is possible to employ more rigorous psychophysical methods efficiently. In Experiment 2, we used a staircase method to measure the simulated aspect ratio at which sagittal and frontal extents appeared equal. In this way, we could generate psychometric functions of the

perceived aspect ratio for each of 20 combinations of simulated slants and viewing distances.

## Methods

Twenty-one undergraduate students (11 females) participated in Experiment 2, fulfilling a course requirement. All had normal or corrected-to-normal vision and were able to correctly report the facing direction (up, down, left, or right) of cyclopean random-dot Es presented at the conclusion of the experiment. None had participated in Experiment 1.

Visual stimuli were presented in the same virtual environment as that used in Experiment 1. The surface parameters were similar to that used in Experiment 1, though the number of parameter values tested was reduced to accommodate the number of trials required to generate a psychometric function for each. Twenty conditions were measured for all observers: Five distances (1, 2, 4, 8, and 16 m) were factorially combined with four surface orientations ( $6^\circ$ ,  $12^\circ$ ,  $18^\circ$ , and  $24^\circ$ ).

On each trial, three identical white spheres were presented on the surface arranged in an L-shape, as illustrated in Figure 7. The participant's task was to compare the 3D distances of the two legs of the L along the ground and decide which was longer. The sphere at the corner of the L-shape was at eye level, with a fixed visual angle of  $0.57^\circ$  for all viewing distances. The texture of the surface and the length of the frontal extent of the L-shape were scaled proportionally to viewing distance. The length of the frontal extent of the L-shape subtended a constant visual angle of about  $7.1^\circ$ . The angular length of the sagittal extent of the L-shape was determined by the physical aspect ratio (sagittal/frontal) that was between 0.33 and 8.14.

A logarithmic up–down staircase procedure was used to simultaneously measure the PSEs between sagittal and frontal lengths for all the 20 distance–slant combinations. There were forty interleaved staircases. For each distance–slant combination, one staircase started with a physical aspect ratio of 0.33 (i.e.,  $1.15^{-8}$ ) and the other started with a physical aspect ratio of 8.14 (i.e.,  $1.15^{15}$ ). On each trial, a two-alternative forced-choice (2AFC) response was collected by means of key presses to indicate whether the sagittal extent appeared longer or shorter than the frontal extent. The value (i.e., the physical aspect ratio) of the next trial in that staircase was adjusted up or down by a variable multiplicative step size, depending on the response given and the number of “turns” in that staircase so far (e.g., Durgin, 1995; a “turn” is defined when two consecutive responses to the same “staircase” series differ).

Initial step size was by a ratio of  $1.15^8$ ; this value declined to  $1.15^4$  after the first turn, to  $1.15^2$  after the second turn, and to 1.15 after the third turn, where it remained thereafter. Five turns for each of the 40 staircases were normally required to finish the experiment.



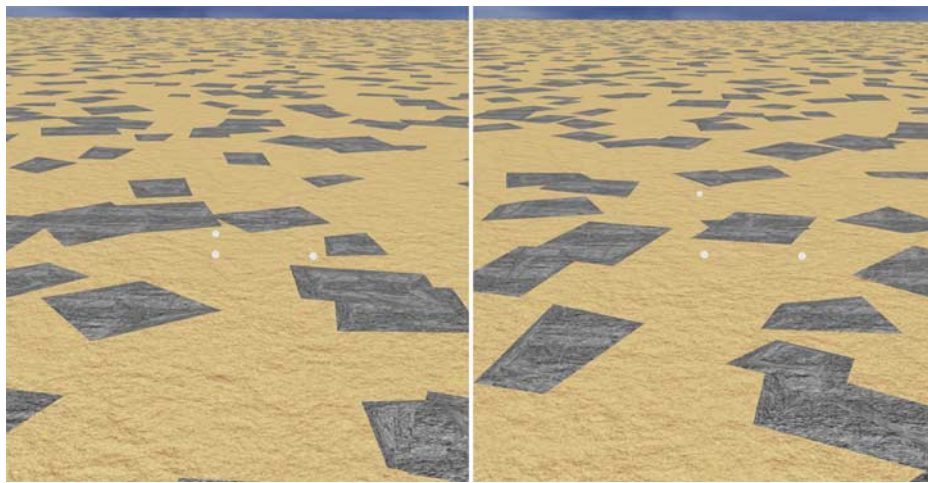


Figure 7. Sample images of visual stimuli used in [Experiment 2](#). Each image shows an 18° slant with sagittal ball separations judged as (left) nearer or (right) farther than the frontal extent. To avoid local shape cues to slant, the balls were not embedded in the surface nor were shadows used. Although in a single eye view it is unclear if the balls are in contact with the surface, the binocular view disambiguated that the balls were indeed on the plane of local surface elements.

Typically, about 300 trials were sufficient (which took about 25 min); the procedure also terminated if it reached 333 trials. The forty staircases were randomly interleaved with the relative probability of a staircase being selected on any given trial being proportional to the square of the number of “turns” remaining for that staircase. Thus, a staircase with 3 turns remaining was 9 times as likely to be selected as one with only 1 turn remaining. This rule served to roughly synchronize progress in the various staircases.

## Results and discussion

A logistic psychometric function was fit (with aspect ratio on a log scale) to each observer’s accumulated responses for each of the 20 distance–slant combinations, to calculate the PSEs and just noticeable differences (75%, JND). Because each psychometric function was fit to only about 16 data points, the JND estimates were used solely as a means of screening inattentive observers. The data of one participant were removed because of extremely high JNDs relative to those of other participants. The average Weber fraction was 7% overall. The aspect ratios (sagittal/frontal) at the mean PSEs (computed in log space) are shown in [Table 1](#) for each of the 20 distance–slant combinations.

Using [Equation 5](#), we computed the perceived slant based on these aspect ratios at the mean PSE and expressed these as a ratio of the actual surface slant. [Figure 8](#) shows the deduced optical slant ratios for each slant value as a function of viewing distance. Again, it is evident that slant increased logarithmically with viewing distance and that the increase was proportionally greater for smaller optical slants. Comparing [Figure 8](#) with [Figure 4](#), we can see that the pattern of the relationship between the

slant ratio and distance–slant is fairly similar for both numerical estimates and aspect-ratio measures. When the slant ratios are plotted for each distance as a function of simulated slant, the gain of the deduced perceived optical slant, as shown in [Figure 9](#), remained fairly constant, but the intercept increased as a logarithmic function of distance. Although [Figure 9A](#) shows evidence of systematic curvature, a linear approximation, such as the 3-parameter model we proposed in [Experiment 1](#) ([Equation 6](#)) can still be applied to describe the aspect-ratio data of [Experiment 2](#), with only the coefficients of the equation being changed. Model parameters were computed using a mixed-effects model of the complete data set, with parameter estimates as follows:  $k_1 = 1.40$  (95% CI: 1.34 to 1.46,  $t = 47.8$ ,  $p < 0.0001$ ),  $k_2 = 2.88$  (95% CI: 2.53 to 3.24,  $t = 16.6$ ,  $p < 0.0001$ ), and  $C = 3.27$  (95% CI: 1.67 to 4.81,  $t = 2.93$ ,  $p = 0.0035$ ),<sup>2</sup> i.e.,

$$\beta_p = 1.40\beta + 2.88 \ln(D) + 3.27. \quad (7)$$

When a correction of 1.05 was applied to the matched ratios to account for the VHI (i.e., the ratios were increased by a factor of 1.05 to compensate for the VHI), the model

Physical ratio	6°	12°	18°	24°
1 m	1.82 (0.10)	1.80 (0.12)	1.58 (0.07)	1.45 (0.06)
2 m	2.27 (0.15)	2.15 (0.12)	1.79 (0.09)	1.56 (0.06)
4 m	2.83 (0.19)	2.42 (0.13)	1.98 (0.08)	1.68 (0.06)
8 m	3.30 (0.26)	2.59 (0.13)	2.14 (0.11)	1.80 (0.08)
16 m	3.30 (0.28)	2.75 (0.15)	2.13 (0.10)	1.76 (0.06)

Table 1. Average physical aspect ratios of sagittal to frontal extents (with standard errors) necessary to make the lengths of the two extents appear equal in [Experiment 2](#).



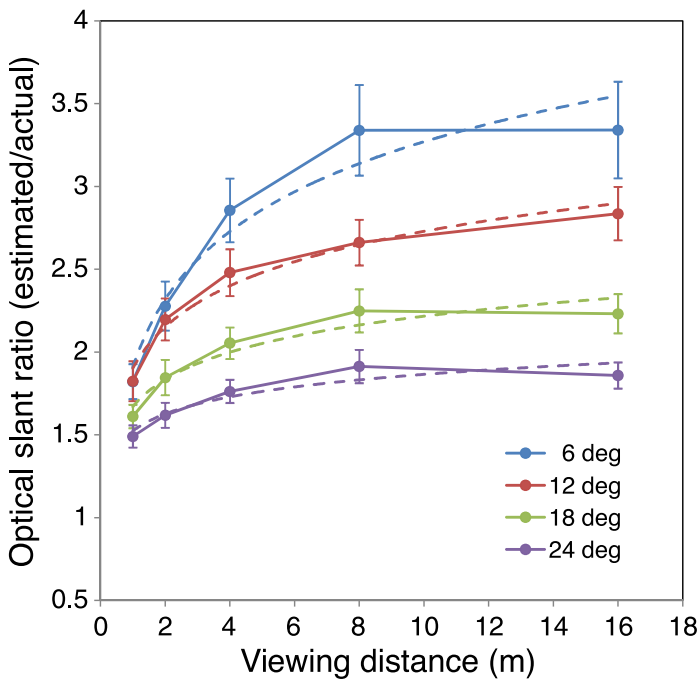


Figure 8. Proportional optical slant overestimation (deduced from the aspect-ratio data in Experiment 2) is shown as a function of simulated viewing distances and slants, with logarithmic fits (dashed lines). Error bars are standard errors of the means.

parameters computed using a mixed-effects model of the VHI-corrected data set were  $k_1 = 1.49$  (95% CI: 1.43 to 1.55,  $t = 47.5$ ,  $p < 0.0001$ ),  $k_2 = 3.07$  (95% CI: 2.67 to 3.42,  $t = 16.5$ ,  $p < 0.0001$ ), and  $C = 3.28$  (95% CI: 1.69 to 5.07,  $t = 2.75$ ,  $p = 0.0063$ ), i.e.,

$$\beta_p = 1.49\beta + 3.07\ln(D) + 3.28. \tag{8}$$

Equations 7 and 8 allow us once again to model the existing data of Loomis & Philbeck, 1999 and Proffitt et al., 1995, with the same assumptions and procedure used earlier for Figure 6. The modeled slant ratio data based on the aspect-ratio task are shown in Figure 10. The modeled data once again capture the divergent patterns between the verbal (Proffitt et al., 1995) and aspect ratio (Loomis & Philbeck, 1999) data collected in real world. Although there are clear differences in the estimates based on explicit verbal slant judgments and on implicit slant measures of extent comparison task, the basic patterns captured by both models are the same.<sup>3</sup> Thus, the aspect-ratio task of Experiment 2 provides important confirmation that (1) the apparent dissociation in the effect of distance on the perceived slant ratio between Loomis and Philbeck (1999) and Proffitt et al. (1995) is clearly due to the different optical slant ranges that were tested. Moreover, (2) the aspect-ratio technique can be used as an

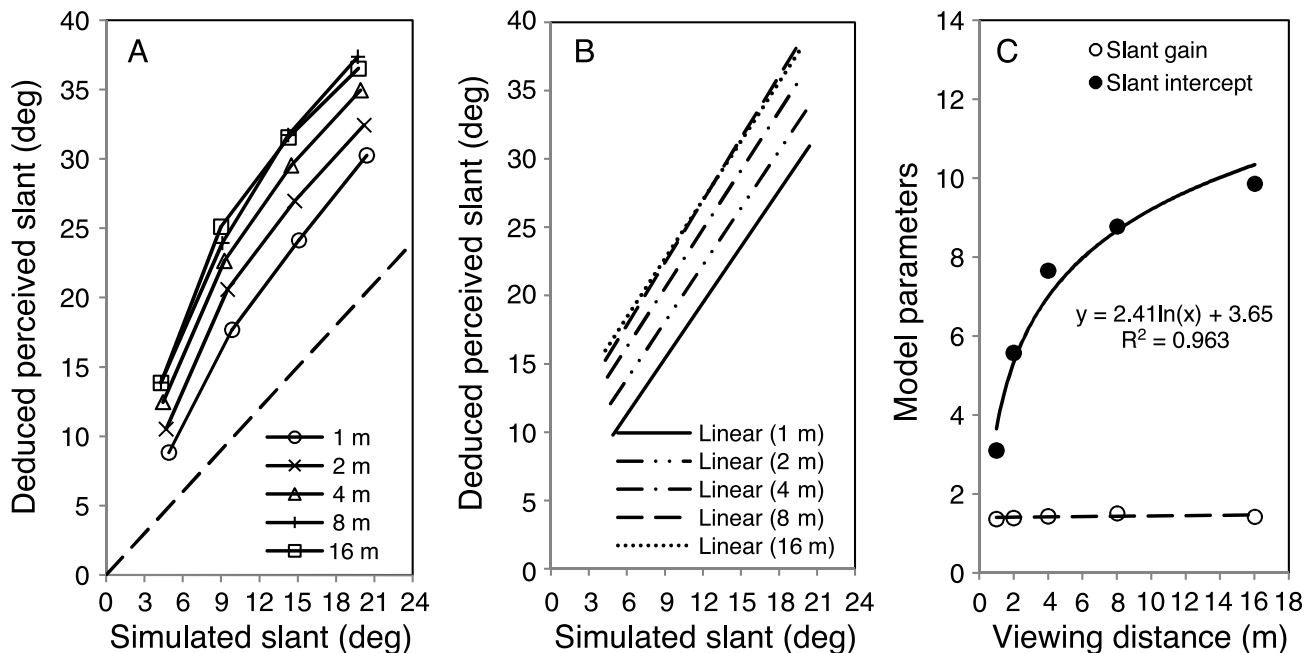


Figure 9. (A) Deduced slant data of Experiment 2 plotted as a function of viewing distance and simulated slant, (B) linear fits to those data at each viewing distance, and (C) linear fit parameters (slope and intercept) as a function of viewing distance. The  $R$  squares of the linear fits are: 0.99, 0.97, 0.97, 0.98, and 0.95, respectively. A logarithmic fit is shown for the intercepts of the linear fits in (C). Note that the optical slant on the abscissa in (A) is the optical slant at ball A (the far ball) rather than the geographical slant of the simulated surface.

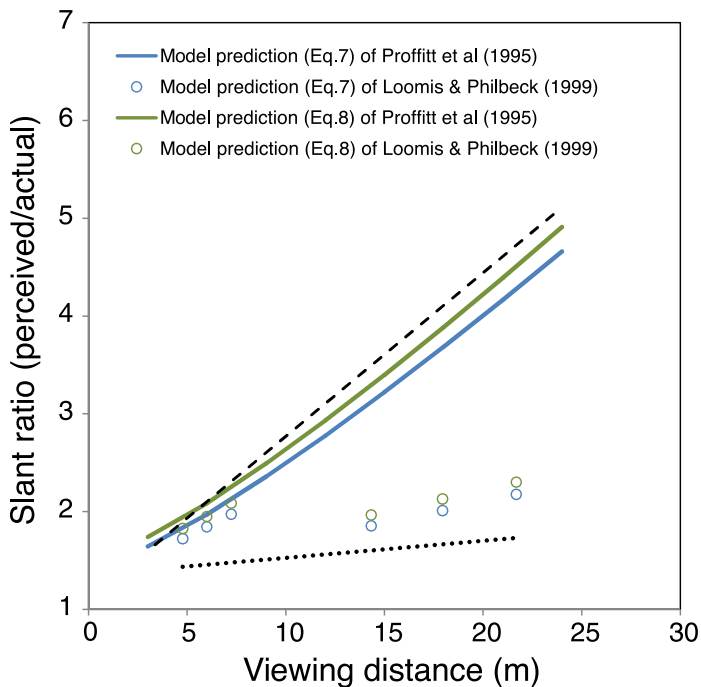


Figure 10. Modeled proportional slant overestimation as a function of viewing distance from the aspect-ratio data in Experiment 2. The green (VHI-corrected) and blue (uncorrected) curves are the model predictions for the viewing conditions of Proffitt et al. (1995; Experiment 1). The circles are the corresponding model predictions for the viewing conditions of Loomis and Philbeck (1999; binocular condition with physical ratio of 1.5). Dashed lines represent linear trend lines from the corresponding empirical data (see Figure 2).

implicit measure of perceived optical slant that provides a converging method of demonstrating this point. Because the aspect-ratio task may be susceptible to different forms of cognitive correction (e.g., Granrud, 2009) than the verbal slant estimation data, we regard the quantitative divergence between the two tasks as less important than the qualitative convergence.

## General discussion

We have used the advantages of virtual environments (excellent stimulus control) to study the perception of surface slant over a much broader parameter space than has previously been explored. We have done this by simulating large-scale surfaces of varying slant at a large range of viewing distances using carefully calibrated immersive binocular displays. We have used explicit angular magnitude estimation (Experiment 1) as well as an implicit slant task based on psychometric measurement of perceptual matches of sagittal and frontal extents on slanted surfaces (Experiment 2). This has allowed us to develop a more complete quantitative model of slant perception than

has previously been proposed, while testing it against existing data collected in real environments. Both methods support the same conclusions. There are three main features of the general form of the model we have developed.

First, even in very near space, we find that actual differences in slant are exaggerated in perception by a factor of about 1.5, and that this exaggeration of slant differences remains fairly constant over the range of distances we tested (1–16 m). As we will discuss below, this is consistent with the principles of scale expansion theory that we have developed elsewhere based on the study of slant perception for real surfaces (Durgin & Li, submitted for publication; Durgin, Li et al., 2010; Hajnal, Abdul-Malak, & Durgin, 2010; Li & Durgin, 2009).

Second, our data from both explicit and implicit methods support the claim that apparent slant increases logarithmically with distance (i.e., Bridgeman & Hoover, 2008), which was also discovered with real surfaces. Moreover, our model shows that this increase can be understood as a change in the intercept value of the perceived slant function, while the exaggerated slant gain remains constant over distance.

Third, the model provides a bridge between the investigation of errors in slant perception and the investigation of errors in distance perception. In this respect, our work builds on prior work that proposed that the ground might be perceived as slanted (Wu et al., 2004) and provides some corroborating evidence for the proposal that the ground plane might be perceived as a bowl (Gibson, 1950; Ooi et al., 2006). However, whereas these other models of ground plane bias have assumed that declination of gaze is perceived accurately and proposed that biases in exocentric distance perception are due to errors in the perceived slant of the ground (e.g., Ooi et al., 2006), Li and Durgin (2009; see also Durgin & Li, submitted for publication) have documented that perceived gaze declination is misperceived with a gain of 1.5. As we will show below, it is necessary to include this fact in a complete model of ground plane perception.

## Scale expansion theory

Durgin and Li (submitted for publication) have argued that some angular variables, such as perceived optical slant and perceived gaze declination, are coded on an exaggerated scale in order to increase the precision of their most commonly represented values for the sake of action. That is, replicating and extending an earlier report (Li & Durgin, 2009), they found that perceived gaze declination had a gain of 1.5 relative to actual gaze declination both when looking at real objects on ground surfaces and when looking at virtual objects suspended in space. Gaze declination was implicated in the perception of distance by Wallach and O’Leary (1982), who measured perceived size of objects presented upright on a ground surface that was viewed through a cylindrical

lens that reduced the perceived declination of gaze. Whereas Wallach and O’Leary attributed the use of gaze declination (or “slope of regard”) to a cue recruitment strategy (Haijiang, Saunders, Stone, & Backus, 2006), it is equally possible that monocular angular variables (such as gaze declination) play a primary role in the control of action.

Ooi, Wu, and He (2001; Ooi et al., 2006) have shown that, in the dark, walking and then gesturing to targets was accurate with respect to gaze declination even when it was inaccurate with respect to distance. Although Ooi et al. (2006) concluded that gaze declination was therefore perceived accurately, an alternative view is that action measures do not license such arguments. This is because calibrated actions, such as walking and gesturing should be uninformative about systematic biases in perceived gaze declination (e.g., Durgin, Hajnal et al., 2010; Durgin, Li et al., 2010). For example, if one wears prism glasses that offset gaze to one side, one’s reaching actions can become calibrated to the offset without there being any visual correction of the misperceived location (Harris, 1963, 1980). This shows that one’s perception can be biased, even though one’s actions are accurate. Similarly, the action of walking can become calibrated to distorted optic flow experience (Durgin et al., 2005; Rieser, Pick, Ashmead, & Garing, 1995). Presumably, it could become calibrated to biases in perceived gaze declination as well. Thus, the calibration of behavior with respect to gaze declination demonstrated by Ooi et al. does not answer the question of whether or not gaze declination is perceived correctly. Calibrated actions only require that any distortions in perception be stable and predictable, such as Durgin and Li (submitted for publication) have measured for gaze declination across a variety of conditions.

If gaze declination were misperceived while optical slant were accurately perceived, then conflict could arise in the perception of level ground surfaces. For example, if one felt one’s gaze was declined by 45°, when it was only declined by 30°, then accurate perception of the optical slant of the ground surface as only 30° might entail perceiving the ground as sloped downward by 15°. Instead, Durgin and Li (submitted for publication; Durgin, Li et al., 2010; Li & Durgin, 2009) have found that perceived optical slant of full-cue surfaces is misperceived with a similar gain (about 1.5) to that of perceived gaze declination. Thus, a person looking at a level ground plane will misperceive both their declination of gaze and the optical slant of the surface in a way that leaves the ground plane appearing approximately flat. Note that residual errors that might be expected could be adapted to as well, by frequent exposure to level surfaces, which would leave observers sensitive, in most contexts, to actual departures from level ground.

The models we have derived in the present study from explicit verbal slant estimation and from the implicit slant estimation task of matching sagittal and frontal extents were all based on situations where gaze was forward. Although small errors in perceived eye level might have affected geographical slant perception (e.g., O’Shea &

Ross, 2007; see also Matin & Li, 1992), such errors are irrelevant to optical slant. This is why we explicitly instructed participants to use optical slant estimates in [Experiment 1](#). The implicit slant task of [Experiment 2](#) is, by nature, a measure of optical rather than geographical slant. Thus, the present results provide a model of perceived optical slant that can be combined with a model of perceived gaze direction to form a more complete model of perceived surface orientation. Because our current model shows that perceived optical slant has a gain of about 1.5 at each viewing distance, it is consistent with the idea that perceptual exaggerations of optical slant and gaze declination are approximately balanced in near locomotor space.

According to scale expansion theory, the exaggerated gain of perceived gaze declination and the concomitant scale expansion of perceived optical slant serve to magnify the representations of these two variables for the sake of more precise action coding. Elsewhere, we have employed the analogy of the watchmaker’s magnifying glass (Durgin, Hajnal et al., 2010; Hajnal et al., 2010) to emphasize that perceptual exaggeration can clearly be advantageous for guiding action. Action can be calibrated to exaggerated perception so long as that perceptual experience is consistent over time.

## Differences from previous models of the ground plane

To understand why an overscaling of gaze declination may be said to complete the model, consider [Figure 11](#), which plots the deduced perceived shape of the ground plane for an observer with a standing eye height of 1.6 m under two different sets of assumptions.

The upper, red, convex line represents the expected perceived local slant along lines of sight to points on the ground between 1 and 10 m, assuming the local predicted slants of the model are integrated throughout. This plot is based on the model from our implicit measure of slant ([Experiment 2](#)), but it differs very little from that based on the verbal data. Thus, if we assume an integrated surface and we also assume that gaze declination is accurately perceived (i.e., the assumptions underlying the bowl model of Ooi et al., 2006), the surface shape predicted by our data is convex rather than concave and the surface is obviously too steep compared to our natural experience of the ground plane (which Ooi & He, 2007 have estimated to appear to be about 3°). Note that Ooi et al. have postulated a concave surface; however, their mathematical model presupposes a constant additive bias ( $\eta$ ) in geographical slant perception, which ought to imply a tilted planar surface.

The lower, blue, concave line shown in [Figure 11](#) plots the same integrated function under the assumption that gaze declination is misperceived by a factor of 1.5. Here the exaggeration of perceived optical slant captured by our model is offset by the exaggeration of perceived gaze declination (which is most pronounced in near space),

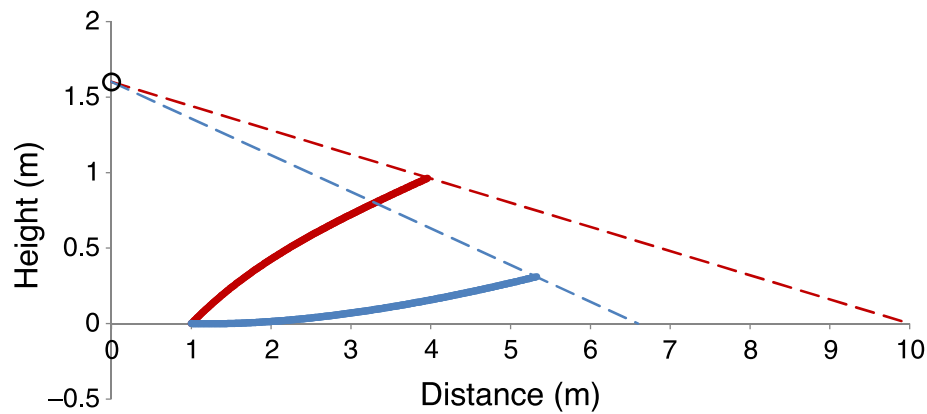


Figure 11. Possible deduced shapes of the ground plane from 1 to 10 m in front of an observer with 1.6-m eye height. Red convex line represents integration of slant error over distance with the assumption of an accurate perceived gaze gain. Blue concave line represents the same integrated function with the assumption of an exaggerated perceived gaze declination gain of 1.5.

producing the qualitative bowl-shape percept of the ground that has been postulated based on performance in the dark (e.g., Ooi et al., 2006). Thus, our findings may be said to provide evidence for their conjecture, with the important qualification that a systematic misperception of gaze declination must be included.

However, we must emphasize that we do not think that this lower line is an accurate representation of perceptual experience of the ground, and we reject one of the assumptions underlying its construction: It is not necessary to assume that locally perceived surface orientations get *integrated* into an increasingly elevated ground plane. Our model is currently silent on exactly how perceived distance should be understood along each theoretical ray of sight from the eye to the ground. In this sense, our model should be interpreted as accepting that there is a dissociation between perceived location (along the ground) and perceived slant (at each point). This distinction between location and shape has a long tradition (e.g., Loomis & Philbeck, 1999). Our theory captures the distinction in terms of differences between these two separate angular variables, gaze declination and optical slant. Optical slant is primarily a visual variable, the perception of which evidently varies with viewing distance, whereas gaze declination is primarily a proprioceptive variable, which is probably not so affected by viewing distance (but see O'Shea & Ross, 2007). Thus, evidence of dissociations between perceived egocentric distance and perceived exocentric distance as a function of distance (e.g., Loomis et al., 1992) is consistent with our theory (for an alternative view, see Wu, He, & Ooi, 2008).

Although we reject the idea that local surface slants must be integrated as depicted in Figure 11, the idea that surface integration is an important part of perceiving extended planar surfaces (Ooi et al., 2006) remains valid in an alternative sense: Continuous textures may allow for an accurate assessment of coplanarity that can override the errors in optical slant perception we have documented here. Thus, uniformly textured planar ground surfaces

tend to appear planar despite local misperceptions of optical slant that increase with distance. Breaks in texture may disrupt this (Wu, He, & Ooi, 2007). The issue of whether horizontal ground planes normally appear slanted or not may depend on how the question is framed. Implicit estimates of local optical slant (aspect-ratio tasks) have been interpreted by some as measures of perceived geographical slant (e.g., B. Wu et al.). Such an interpretation may not, in fact, correctly characterize the perceptual experience of planar surfaces, such as ground planes. At the very least, estimates of geographical slant based on optical slant measures (such as aspect-ratio tasks) should probably take perceived gaze declination into account, and there is good reason to believe that gaze declination is misperceived (e.g., Li & Durgin, 2009).

Although there is a superficial similarity between our analysis and that previously proposed by Ooi et al., 2006, the theoretical assumptions of the models are entirely distinct. Ours is a model of optical slant perception from which estimates of geographical slant can be derived when combined with models of perceived gaze declination. Our model is thus suited to explain both slant estimation data and aspect-ratio task data. Ooi et al. (see also Ooi & He, 2007; Wu et al., 2004) have proposed that errors in aspect-ratio tasks can be interpreted by supposing that the ground plane appears tilted up by a constant amount,  $\eta$ . However, their published estimates of  $\eta$  have varied from about  $3^\circ$  (Ooi & He) to about  $14^\circ$  (B. Wu et al.). Thus, their model has a free parameter ( $\eta$ ) that has varied by a factor of five from one context to another. Despite this, errors of even  $14^\circ$  are simply not large enough to account for the much larger errors in geographical slant perception reported by Proffitt et al. (1995), nor do they capture the effects of viewing distance that have been documented by Bridgeman and Hoover (2008). In contrast, our studies have consistently found evidence for an angular scaling factor of about 1.5 in perceived optical slant (Durgin & Li, submitted for publication; Durgin, Li et al., 2010; Li & Durgin, 2009).



Here we have further shown that the effect of distance can be modeled as a logarithmic change in the intercept, leaving the 1.5 scale expansion intact. Whereas our model produces similar predictions to that of Ooi et al., 2006, for aspect-ratio tasks along the ground plane, our model, unlike theirs, can be generalized to slanted surfaces at varying distances. This is because ours is a model of optical slant primarily. It has been empirically determined to require both a multiplicative bias and an approximately additive bias linked to distance. The multiplicative bias is of theoretical significance because it corresponds quantitatively with the multiplicative biases we have found for perceived gaze declination (Durgin & Li, submitted for publication; Li & Durgin, 2009) and for small-scale full-cue real surfaces (Durgin, Li et al., 2010). Similarly, the distance-linked bias has also been documented in the real world (Bridgeman & Hoover, 2008) and is here suggested to depend on binocular information. It is notable that both of these biases appear to be present in full-cue contexts (rather than being minimized under full-cue conditions as most theories suppose).

### A note on the role of binocular information in large-scale slant perception

Because we controlled monocular texture information across distance by scaling the simulated texture size proportional to distance, the effects of distance we have modeled appear to be due to the presence of binocular information, rather than a (non-existent) degradation of monocular cues with distance. It might be argued that texture size produced a conflict cue across our displays, but there was no such conflict within them: Texture size gradients were matched to binocularly specified slant and distance. Moreover, stereoscopic aspect-ratio judgments have been shown to be unaffected by familiar size scaling (Predebon, 1993). Allison, Gillam, and Vecellio (2009) have elegantly demonstrated the utility of stereoscopic information at far distances for specifying (compressed) depth. Allison, Gillam, and Palmisano (2009) have similarly studied the role of binocular information for perceiving the slant of the ground plane.

Unlike the typical rule of thumb, which states that retinal disparity is reduced roughly in proportion to the square of distance for an object of a given depth, the retinal gradient of disparities for a surface slanted in depth decreases roughly linearly with distance. This is because proportionally larger depths are subsumed by the same vertical retinal angle. This is illustrated in Figure 12 by plotting retinal disparities  $1^\circ$  above fixation for slanted surfaces of  $6^\circ$ – $24^\circ$  and for a fixed depth interval about fixation as distance increases from 1 to 16 m. The exponents of the power functions relating disparity to distance are  $-1$  for slanted surfaces, whereas the exponent for a fixed depth interval is  $-2$  (inverse square rule).

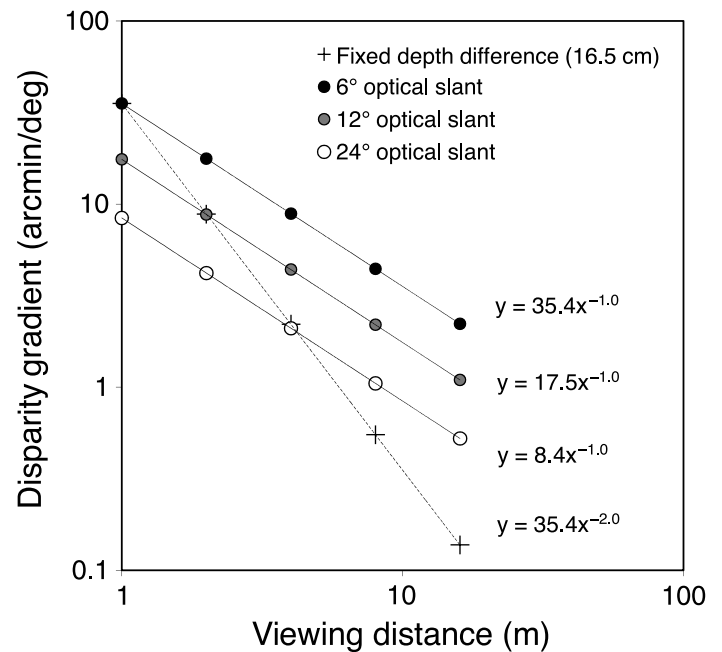


Figure 12. Theoretical retinal disparity gradients for slanted surfaces as a function of viewing distance. Disparities are expressed in arcmin, computed  $1^\circ$  above fixation (i.e., arcmin/ $^\circ$ ) for three optical slants (circles) and for a fixed depth interval of 16.5 cm (crosshairs). Power function fits are shown for each case.

Based on the close correspondence between the model derived from our displays and data collected in real environments, it seems likely that binocular information (retinal disparities and vergence information) may normally play an important role in producing effects of distance on the perceived slants of hills. We can draw this inference for the current displays because static monocular information did not differ as a function of viewing distance in our displays. We tentatively extend the inference to normal viewing condition based on the evidence that our data seem to align nicely with existing data collected in the real world.

### Fit to real-world slant and distance data

Proffitt et al. (1995) proposed that the perceived slants of hills followed a power function of actual slant. Here we have proposed an alternative model of hill perception, which requires taking both slant and viewing distance into account. Figure 13 compares the predicted values of our model for the hills tested by Proffitt et al. to the results reported by them. In order to estimate distance, we assume an observer with an eye height of 1.6 m, standing at the base of each hill with gaze forward, consistent with the conditions reported by Proffitt et al. It can be seen that the predictions derived from models of both our experiments closely replicate the verbal report observations of Proffitt et al. for the 8 specific hills they tested. The

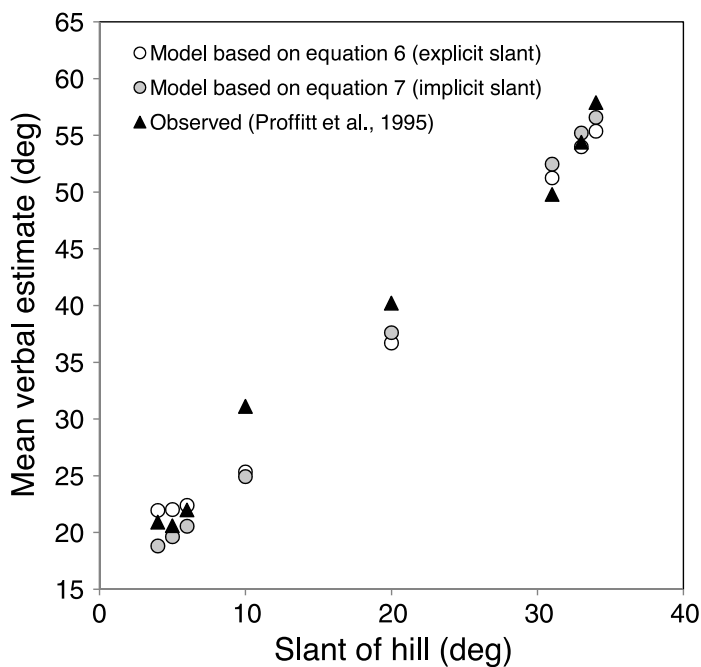


Figure 13. Predicted perceived geographical slant for observers standing at the base of each of the 8 hills tested by Proffitt et al. (1995), assuming a 1.6-m eye height. Model predictions of both the explicit slant task (Equation 6) and the implicit slant task (Equation 7) provide good approximations to the observed explicit slant estimates.

exception is their 10° hill, which appears, in other graphs as well, to have been something of an outlier (see Durgin, Hajnal et al., 2010; Figure 14). Thus, a model based on parametric investigation using virtual environments provides an excellent account of the classic measurements of Proffitt et al., which, in turn, serves to validate the current model.

Kudoh (2005) proposed that perceived aspect-ratio judgments maintained a relatively constant retinal depth/width ratio. More importantly, Kudoh tested a variety of depth, distance, and width parameters in an L-shaped task on the ground. Figure 14A replots Kudoh’s original exocentric extent data from this real-world aspect-ratio matching experiment. The graphical deviations from Kudoh’s prediction appear relatively random. However, by again assuming an eye height of 1.6 m, we can use Equation 7 (given that Kudoh did not correct for the VHI) to compute predicted values of perceived exocentric matches in Kudoh’s experiment based on his published viewing parameters and on our aspect-ratio measurements in Experiment 2. The predicted values are shown in Figure 14B. It is evident that our model captures a great deal of the detailed structure of Kudoh’s empirical observations. Note that this again provides excellent validation that our analysis of perceived optical slant for surfaces viewed with gaze forward generalizes to the perception of ground surfaces in the real world across an impressive range of real-world parameters.

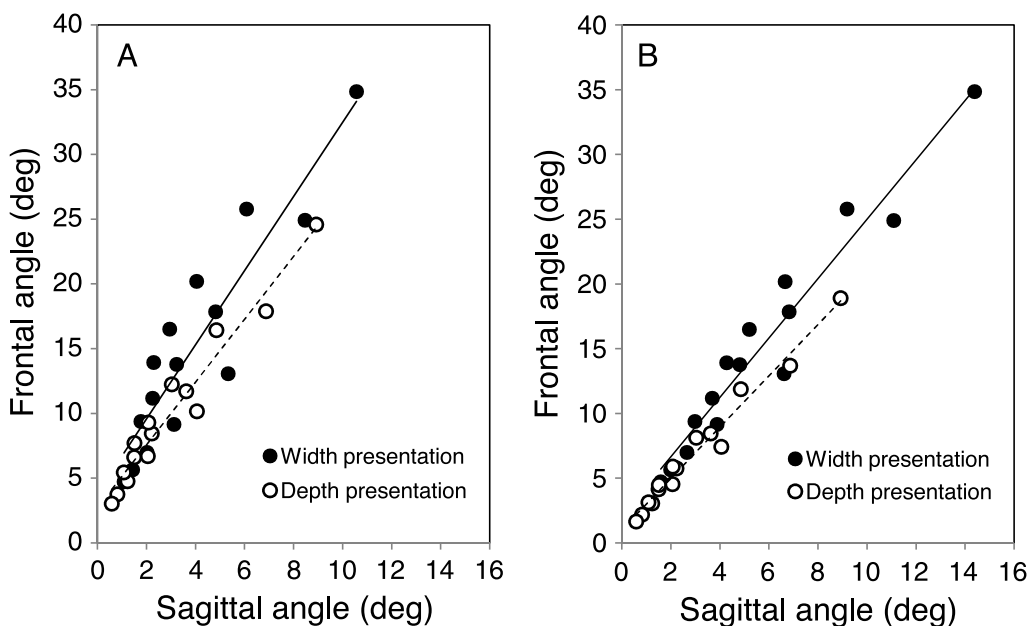


Figure 14. (A) Replot of Figure 3 in Kudoh (2005). The data points were calculated according to the reported aspect-ratio data, i.e., Figure 2 in Kudoh (2005), assuming a viewing eye height of 1.6 m. (B) Model prediction of Kudoh’s (2005) data, based on Equation 7, assuming an eye height of 1.6 m.

Although absolute distance perception may be poorly scaled in virtual environments (e.g., Loomis & Knapp, 2003; Thompson et al., 2004), the models of slant perception we have derived from investigations using virtual environments appear to provide excellent models of real-world data for both relative distance perception (aspect-ratio tasks) and geographical slant perception (estimates of hills). Because each of these tasks involves ratios and angles rather than absolute distance estimation, the results are consistent with the idea that the compression of perceived distance in virtual reality may nonetheless preserve similarity relationships (see Durgin & Li, 2010; Messing & Durgin, 2005). Elsewhere, we have shown that the perceived slants of near surfaces are fairly similar in virtual and real environments (Durgin & Li, submitted for publication; Durgin, Li et al., 2010).

## Conclusion

Other theorists have proposed the existence of biases in the perception of the ground plane (intrinsic bias: Ooi et al., 2006), optical slant (frontal tendency: Gibson, 1950), and distance (specific distance tendency: Gogel, 1969; Gogel & Tietz, 1973). Those studies were conducted under reduced-cue conditions. Our studies, though conducted using virtual environments, replicate findings collected under full-cue conditions in the real world with unrestricted views. By manipulating direction of gaze, Durgin, Li et al. (2010) showed that the exaggeration of perceived optical slant for full-cue, real surfaces in reach was not due to frontal tendency. Whereas all of the biases discussed above are evident in reduced-cue situations, our observation is that perceptual scale expansion of angular variables exists in full-cue viewing to which action has been calibrated. The failure of slant constancy with distance that we have also modeled here (see also Bridgeman & Hoover, 2008; Ross, 1974) is not due to reduced monocular visual information (because monocular slant information was constant across distance). Elsewhere, we have argued that failures of constancy are only problematic for action when they cannot be predicted. The model developed here for optical slant perception suggests that there are stable and predictable failures of constancy in the full-cue perception of slant. Such partial constancy failures may in fact be functional (Durgin, 2009; Durgin, Ruff, & Russell, in press).

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## Footnotes

<sup>1</sup>Because one reviewer was concerned that the logarithmic fit was caused by the inclusion of the 16-m distance, we conducted post-hoc analyses comparing mixed-effects models with linear and logarithmic distance terms for displays from 1 to 8 m. A model that included only the logarithmic term was no worse than one that included a linear distance term as well  $\chi^2(1) = 1.03$ ,  $p = 0.3092$ , whereas one that included only a linear term provided a reliably worse fit to the data than one that also included a logarithmic term,  $\chi^2(1) = 27.4$ ,  $p < 0.0001$ . Thus, these data strongly support the use of a logarithmic distance term. The model gain parameter for slope for distances of 1–8 m was 1.68 (95% CI: 1.60–1.76).

<sup>2</sup>Because one reviewer was concerned that the logarithmic fit was caused by the inclusion of the 16-m distance, we conducted post-hoc analyses comparing mixed-effects models with linear and logarithmic distance terms for displays from 1 to 8 m. A model that included only the logarithmic term was no worse than one that included a linear distance term as well  $\chi^2(1) = 1.18$ ,  $p = 0.2756$ , whereas one that included only a linear term provided a reliably worse fit to the data than one that also included a logarithmic term,  $\chi^2(1) = 29.1$ ,  $p < 0.0001$ . Thus, these data, like those of Experiment 1, strongly support the use of a logarithmic distance term in the model. The logarithmic distance model slant gain parameter for distances of 1–8 m was still 1.40 (95% CI: 1.34–1.46).

<sup>3</sup>We also fit a mixed-effects model to the combined data of Experiments 1 and 2 (not corrected for VHI), including task as an additional variable along with optical slant and log distance in our model. The effect of task was small ( $0.77^\circ$ ) and not reliable (95% CI:  $-2.97$ – $4.28$ ). The model parameters for the combined data were  $-3.94$  (intercept), 1.61 (optical slant gain), and 5.61 (gain with log distance). Excluding the data for slants greater than  $24^\circ$  in order to maintain the same range of slants for both tasks did not substantively alter the model, nor reveal any

reliable difference between the two tasks (95% CI:  $-2.37$ – $4.14$ ). It appears that a common model approximates the data from both tasks.

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