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Mentor-Participant Exchange In The Ask Dr. Math Service: Design And Implementation Considerations

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CHAPTER TEN

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The Math Forum and Ask Dr. Math

The Math Forum is an interactive digital library that consists of over 1.2 million pages. As such, it is both (a) a content site that has extensive archives and links to information and (b) an interactive site that promotes information exchange, discussion, and community building. The site contains several interactive services including Ask Dr. Math and its archives of frequently asked questions (FAQs).

Survey data indicate that Math Forum participants include a mix of teachers, students, parents, software developers, mathematicians, math educators, scientists, engineers, professionals, tradespeople (e.g., roofers, pipe fitters), and so forth (Renninger & Shumar; 2002; 2003). These participants vary in their levels of expertise and interest for mathematics, which means that a wide range of mathematics questions are submitted to Ask Dr. Math.

The Ask Dr. Math service provides participants with advice about how to find answers to the questions they pose. The goals of the service include helping participants who ask questions to think about underlying mathematical concepts, about how to rephrase a question in ways that make the

solution easier to find, and about how to locate resources that might be helpful. Ask Dr. Math is not designed to be a homework help service. Thus, as a general rule, participants do not simply receive answers to questions. If a participant asks, "How can I solve this problem?" the mentor is likely to respond, "What would you need to know to be able to see how to solve this problem—and others like it—on your own?"

The Ask Dr. Math service receives over 300 questions a day despite a submission process that is intended to help participants to find answers to their own questions instead of submitting them to the service. One full-time and two part-time staff members edit responses for the archives, train volunteer mentors, and facilitate discussion between mentors and between mentors and participants, in addition to responding to participant questions. There are approximately 400 registered volunteer mentors who help to respond to participant questions, although in any given month the number of volunteers answering questions is approximately 30 to 40.

Mentors for the service go through a training, or tenuring, process in which they work with already-tenured volunteer mentors and Math Forum staff members to respond to questions in ways that are consistent with site protocol. Site protocol involves engaging participants' questions with the assumption that the participants are asking questions in order to learn. Participants' questions are accepted at face value and mentors work with participants' interests and stated understandings. To enable the development of participants' mathematical thinking, mentors (and staff) use inquiry approaches to questioning, exploring, and modeling.

Mentors for Ask Dr. Math use an "office," or triage system, on the site that allows them to select questions that they want to answer. In addition to facilitating communication between mentors and participants, the office also supports communication among mentors. The office allows mentors to discuss questions and prospective answers via internal memos, ask for help from other mentors, and flag questions whose answers they do not know but they would be interested in learning. Interviews with volunteer mentors indicate that it is both the opportunity to stretch their own thinking about mathematics and the opportunity to help others that lead them to volunteer.

Because of rapid growth, or scaling, of questions submitted to this service, two questions surfaced for Math Forum staff members regarding mentor–participant exchanges. The first (Study 1) concerned the impact of changes in

the Ask Dr. Math service on mentor–participant exchanges. Three types of changes were instituted in the service. These included training volunteer mentors, developing a triage system for submitted questions, and posting directions that explain mentors' expectations that participants will use the site archives and FAQs to try to answer their questions for themselves.

The second (Study 2) addressed differences in the length of mentor–participant exchanges after changes to the service. Were single and sustained exchanges different in the mathematical thinking that they were fostering? A related question was whether mentors should be encouraged to engage participants in shorter exchanges as a means to address more of the questions that the service receives.

Studies of Mentor–Participant Exchange

General Methods

Mentor–participant exchanges were drawn for study from the public Ask Dr. Math archive in a fixed random order to meet the specifications of Study 1 (sustained exchanges, Time 1 and Time 2) and Study 2 (single and sustained exchanges, Time 2).

Analyses of mentor–participant exchanges were conducted using a coding scheme developed in work with a separate, randomly drawn sample of exchanges in the Ask Dr. Math archives.

In accordance with findings of multilevel analyses undertaken by Packer and Scott (1992), Lave and Wenger (1991), Rogoff (1995, 1998), Valsiner (1998), and Wertsch and Toma (1995), as well as of discussions and studies of tutoring (Hogan & Pressley, 1997), student learning (e.g., Bransford, Brown, & Cocking, 1999), and mathematical thinking (Renninger, Farra, & Feldman-Riordan, 2000; Schoenfeld, 1992), the coding scheme consists of three levels of analysis:

1. *Individual engagement*, or that which the mentor and the participant each bring to their work with the Ask Dr. Math service (i.e., interest, risk-taking behavior, and resourcefulness)

2. *Interaction between the mentor and the participant* (i.e., the ability of the mentor and the participant to connect to the problem/task in question, their ability to generate and adjust use of strategies to meet the problem/task demand, and the amount of support required to accurately

address the problem/task demands)

3. *The norms or cultural context of the mentor and of the participant* (i.e., norms and positioning).

Two people trained in use of this coding scheme consensually coded all mentor–participant exchanges. Disagreements were resolved by a third person who was also trained in use of the coding scheme. Each exchange was coded for both the mentor and for the participant. Sustained exchanges differed in length and ranged from 5 to 18 turns, where a turn consisted of a participant–mentor–participant exchange. To allow comparison of sustained exchanges (Study 1), as well as single and sustained exchanges (Study 2), the total score for each indicator (e.g., interest, risk taking, resourcefulness for the individual level of analysis) in each exchange was divided by the number of turns in that exchange.

Descriptive findings for both Study 1 and Study 2 were similar. The school level of the participant, the difficulty level of the question posed, and the type of question (mathematical concepts, skill development and practice, or application and real world) did not differ for participants at Time 1 or Time 2 (Study 1) or for participants who engaged in single or sustained exchanges (Study 2). Representative of the Ask Dr. Math archive, most of the exchanges drawn for study came from high school–age students. The level of difficulty of the questions for that age group was typically intermediate to hard. In addition, the most frequent type of question posed involved skill development and practice.[1]

Study 1: Mentor–Participant Exchange at Time 1 and Time 2

Study 1 focused on mentor–participant exchanges before (Time 1) and after (Time 2) refinement of service design. A total of 40 (19 at Time 1; 21 at Time 2) sustained exchanges were studied, each of which consisted of 5 or more turns. Findings from Study 1 are presented in Tables 10–1 and 10–2.

Inspection of Tables 10–1 and 10–2 suggest that at both Time 1 and Time 2, mentors and participants were seriously engaged. Mentors took steps in their responses to dissipate their own power in the interaction by telling jokes, adding smiley faces, and so forth. That mentors were more likely to engage in reciprocal exchange at Time 2 than they had been at

1. No analysis by gender was undertaken because mentors for this service are typically male and the gender of participants is often not known.

TABLE 10-1.
Study 1: Mentor–Participant Exchanges at Time 1 and Time 2, Mentors

	Equally Likely	More Likely At Time 1	More Likely At Time 2
Individual Engagement			
Push discussion of the questions	X		
Provide further information to clarify/modify the question or response	X		
Create situations in which the participant must engage to learn	X		
Raise other questions, express uncertainty, or ask for input	X		
Stay on topic; give additional information		$M = 0.07$	$M = 0.01$ $t(39) = 2.96, p < 0.004$
Pool efforts to answer questions		$M = 0.18$	$M = 0.04$ $t(39) = 3.48, p < 0.001$
Take a risk in discussion	X		
Mentor–Participant Interaction			
Connect the mathematics being discussed to the real world	X		
Provide resources or background information	X		
Explain mathematical concepts	X		
Describe decision making	X		
Emphasize the importance of problem solving (as opposed to getting the correct answer)	X		
Focus mathematical thinking through the use of tools such as models, examples, and scaffolding		$M = 0.37$	$M = 0.72$ $t(39) = 3.79, p < 0.000$
Make connections between the question and related topics		$M = 0.28$	$M = 0.04$ $t(39) = 3.82, p < 0.000$
Cultural Norms and Behaviors			
Use Ask Dr. Math conventions of greeting and encouraging response	X		
Take action to dissipate power dynamic	X		
Emphasize reciprocity in exchange		$M = 1.43$	$M = 0.57$ $t(39) = 4.70, p < 0.000$

TABLE 10–2.
Study 1: Mentor–Participant Exchanges at Time 1 and Time 2, Participants

	Equally Likely	More Likely At Time 1	More Likely At Time 2
Individual Engagement			
Explain what is understood about the question	X		
Clarify or modify their question		$M = 0.21$	$M = 0.54$ $t(39) = 4.11, p < 0.000$
Work to figure out an answer	X		
Take the chance of being wrong	X		
Stay on topic; give additional information	X		
Are resourceful about getting information; use other parts of the Math Forum to find answers	X		
Take a risk in discussion	X		
Mentor–Participant Interaction			
Connect the mathematics being discussed to the real world	X		
Make connections to related topics		$M = 0.15$	$M = 0.03$ $t(39) = 2.86, p < 0.006$
Make use of other resources and background information	X		
Explain mathematical concepts	X		
Describe decision making		$M = 0.19$	$M = 0.06$ $t(39) = 2.33, p < 0.023$
Cultural Norms and Behaviors			
Use netiquette and social cues	X		
Emphasize reciprocity in exchange		$M = 0.50$	$M = 0.82$ $t(39) = 2.27, p < 0.03$

Time 1, however, suggests that training may have heightened awareness of their need to be sensitive to power dynamics in their responses.

From Table 10–1 it appears that changes introduced to refine the Ask Dr. Math service shifted the way in which mentors supported partici-

pants' mathematical thinking. In fact, both providing additional information and pooling resources *and* providing models, examples, and scaffolding are theoretically consistent with the literature on the development of mathematical thinking (e.g., De Corte, Verschaffel, & Op't Eynde, 2000; Ginsberg, Klein, & Starkey, 1998; Math Forum BRAP Group, 2000; Schoenfeld, 1992).

The design of the Ask Dr. Math service requires participants to provide explanations of the questions they submit. Responses from the mentors are intended to scaffold participants' abilities to more fully articulate and help themselves determine answers. This aspect of the service did not change.

Participants at Time 1 and Time 2 were equally likely to explain what they understood about the question in posing it, work to figure out the answer to the question, take the chance of being wrong in order to figure out an answer, use resources including the Math Forum to answer their questions, and take a risk in conversation so that they could learn (e.g., "I am 100% sure this polynomial exists.").

Participants at Time 2, however, were more likely to be focused on clarifying the question they posed than they were to be trying to make connections between the question and prior work or explaining their decision making. This finding suggests that there were some differences in the kinds of opportunities for reflection that were available at Time 1 and Time 2. The process of describing connections to prior work and explaining decision making are aspects of reflection that may be steps in the process of being able to ask a better question. An obvious and intriguing question for future investigation is whether the present emphases of the Ask Dr. Math service enhance critical aspects of participant reflection or eliminate aspects of reflection that are necessary for participant learning. This more nuanced understanding of reflection is not well understood. Findings such as those of the present study, however, suggest that the on-line environment may be an ideal context in which to pursue research that addresses the relation between participant reflection and learning.

Study of the various strengths and needs of learners to be taught and/or encouraged to be reflective at different points in their lives (e.g., in certain grade levels or grade bands, in relation to expertise) also would be helpful for addressing ways in which mentors might adjust responses to enhance participant learning (see discussion in Renninger, 1998). Finally, study of the

way in which Ask Dr. Math responses, archives of Ask Dr. Math exchanges, and the FAQs are used by participants could be useful for determining how much support participants need from mentors to answer questions. Such information would also be useful for purposes of indexing.

Study 2: Study of Single and Sustained Exchanges

In Study 2, the 21 sustained exchanges that were evaluated at Time 2 in Study 1 were studied in relation to 22 single exchanges.

Findings from Study 2 are presented in Tables 10–3 and 10–4. As depicted in Table 10–3, it appears that mentors of single and sustained exchanges were equally likely to push the participant to think about his or her question and to provide additional information to support this thinking.

Only in sustained exchanges, however, did mentors provide additional information to clarify or modify the question being discussed, raise other questions, pool efforts to answer questions, or take risks in the discussion. There are at least three ways to think about these findings. These findings may suggest that

1. Sustained exchanges afford both mentors and participants the time and space to work through questions together. In fact, from Table 10–4, it seems that it was only in sustained exchanges that participants were likely to indicate what they understood about the questions they posed, clarify or modify their questions, or use other resources such as those available on the Math Forum site to help themselves answer the questions.

2. Mentors and participants in single exchanges felt that they had addressed the questions adequately and that no further exchange was necessary.

3. Both the mentors and the participants continued to think about and talk about the questions and the mentors' responses outside of the context of the Ask Dr. Math service.

Definitive information about the likelihood of these possibilities is not available.

Findings from this study do suggest that mentors of single exchanges were more likely than mentors of sustained exchanges to explain mathematical concepts and to focus participants' mathematical thinking through the use of models, examples, and scaffolding. Mentors of single exchanges were also more likely to emphasize the reciprocal nature of the exchange than were mentors of sustained exchanges. Despite the fact that mentors of both types of exchanges had similar training, it appears that the responses of the mentors of single exchanges were more succinct, informational, and even

TABLE 10-3.
Study 2: Single and Sustained Exchanges, Mentors

	Equally Likely	More Likely In	
		Single	Sustained
Individual Engagement			
Push discussion of the question	X		
Provide further information to clarify/modify the question or response		$M = 0$	$M = 0.57$
Create situations in which the participant must engage to learn	X		
Raise other questions, express uncertainty, or ask for input		$M = 0$	$M = 0.08$
Stay on topic; give additional information	X		
Pool efforts to answer questions		$M = 0$	$M = 0.08$
Take a risk in discussion		$M = 0$	$M = 0.08$
Mentor–Participant Interaction			
Connect the mathematics being discussed to the real world	X		
Provide resources or background information	X		
Explain mathematical concepts		$M = 0.41$	$M = 0.16$ $t(42) = 2.21, p < 0.03$
Describe decision making	X		
Emphasize the importance of problem solving (as opposed to getting the correct answer)	X		
Focus mathematical thinking through the use of models, examples, and scaffolding		$M = 0.86$	$M = 0.57$ $t(42) = 2.95, p < 0.005$
Make connections between the question and related topics		$M = 0$	$M = 0.16$
Cultural Norms and Behaviors			
Use Ask Dr. Math conventions of greeting and encouraging response	X		
Take action to dissipate power dynamic to create a more equal footing with the participant	X		
Emphasize reciprocity in exchange		$M = 2.14$	$M = 0.71$ $t(42) = 12.40, p < 0.000$

TABLE 10-4.
Study 2: Single and Sustained Exchanges, Participants

	Equally Likely	More Likely In	
		Single	Sustained
Individual Engagement			
Explain what is understood about the question		$M = 0$	$M = 0.15$
Clarify or modify their question		$M = 0$	$M = 0.43$
Work to figure out an answer	X		
Take the chance of being wrong	X		
Stay on topic; give additional information	X		
Are resourceful about getting information; use other parts of the Math Forum to find answers		$M = 0$	$M = 0.27$
Take a risk in discussion	X		
Mentor–Participant Interaction			
Connect the mathematics being discussed to the real world	X		
Make connections to related topics		$M = 0$	$M = 0.07$
Make use of other resources and background information		$M = 0.41$	$M = 0.16$ $t(42) = 2.17, p < 0.04$
Explain mathematical concepts	X		
Describe decision making		$M = 0$	$M = 0.11$
Cultural Norms and Behaviors			
Use netiquette and social cues	X		
Emphasize reciprocity in exchange		$M = 1.18$	$M = 0.46$ $t(42) = 5.18, p < 0.000$

in tone than mentors of sustained exchanges. Interestingly, post hoc review of mentor–participant exchanges in the Ask Dr. Math archives indicates that some mentors engage only in single exchanges whereas others use a combination of single and sustained exchanges.

It may be that participants who received more succinct explanations from mentors had their needs met and had no further need to engage in dis-

cussion. It also may be that it is in the less-directive, sustained exchange that breakthroughs in a person's understanding are realized and that it is the process of engaging in discussion that facilitates this shift. Support for this possibility can be found in literature that has addressed elementary school–age students' work with mathematics concepts (e.g., Ball, 1993; Cobb & Bauersfeld, 1995; Lampert, 1986; the Math Forum BRAP Group, 2000). Further study is needed to determine the impact of clear explanations about terminology and concepts on the development of participants' mathematical thinking.

Mentors of both single and sustained exchanges were likely to connect the mathematics about which they were talking with real-world applications, provide resources or background information as explanation, describe their decision making, and underscore the importance of the process of problem solving.

Furthermore, participants in both single and sustained exchanges were equally likely to work to figure out an answer, take the chance of being wrong as they explained what they did understand, stay focused on the question they originally posed provide additional information that furthered discussion, and take a risk in the discussion to assert their perspective, even if, for example, the mentor had suggested another approach.

As suggested in discussion of Study 1, the mathematical thinking of participants appears to be supported through mentor–participant exchanges in the Ask Dr. Math service. It also appears that differences that exist are the result of differences in individual engagement.

In fact, findings from the present study further suggest the usefulness of recognizing and appreciating individual differences among mentors and participants. It may be that emphasis should not be placed on single over sustained exchange but instead on the possibility that there are learning-style differences among both mentors and participants or developmental differences in their readiness for one versus another type of exchange. Such differences could also suggest that some participants are more predisposed/or ready to engage learning in single exchanges and others in sustained exchanges. If either of these possibilities is the case, a match in type of mentor and participant might be optimal for maximizing learning from work with Ask Dr. Math (e.g., Gustafsson & Undheim, 1996; Renninger 1998).

Such interpretation might also suggest recognition that the conventions

of inviting a participant to write back if he or she has further questions is not sufficient for some participants if encouraging sustained exchanges is the goal. Mentors might be encouraged to send further thoughts about issues on to participants if these occur to them as ways to provide follow-up and contribute to participants' abilities to further consolidate their understandings of the topics they were discussing.

It does seem that an on-line service such as Ask Dr. Math is uniquely positioned to work with and adjust instruction in relation to the individual differences of participants precisely because it is on-line, involves the mentor and the participant in writing, and, as such, involves mentors in reflecting on their exchanges with participants. It may not make sense for an on-line service to identify one mode of facilitation (e.g., encouraging sustained exchanges) that can and will apply to everyone. In fact, these data suggest that some learners will benefit more from sustained exchanges, and others, from the clear explanations of the single exchanges.

Conclusion

Findings from the studies reported here suggest that the on-line environment may be a good place to deepen investigations of student reflection; for tutoring or mentoring that meets students' goals, interests, and needs; and so forth. By design, the service and its mentors focus on participant questions and expect that it is the student who needs to answer his or her own question and who, in the course of doing so, will experience a shift in the relationship. Because exchanges in Ask Dr. Math are written exchanges, they require reflection. Given that these design principles are constants for the service, its mentors are then positioned to focus their efforts on facilitating reciprocal relationships with participants and positioning themselves to understand participants' interests and goals (e.g., Pea, 1993; Sansone, Sachau, & Weir, 1989). Findings from the present study address aspects of this process. Further study must be directed to the content of these exchanges and the match between responses and the types of questions that are posed.

Findings from the present studies beg for further consideration of the impact and the connections between the kind of problem, level of mathematical thinking stimulated, and amount of improved understanding that emerges from work with the service. In other words, how, when, and with whom

should single and/or sustained exchanges be undertaken? Understanding more about individual differences in how participants make use of responses from mentors also would be very useful. Are participants working with the responses in the context of classrooms, with other people, or by themselves?

Postscript

Changes have been made to the Math Forum's Ask Dr. Math service since the studies reported here were conducted. These include changes intended to make it easier for mentors and participants to engage in sustained exchanges and changes in the archiving of exchanges. To make it easier for mentors to engage participants in sustained exchanges, participants are now provided with a uniform resource locator (URL) where they can go to ask follow-up questions. Because staff members are copied on these questions, they then can notify mentors via e-mail when follow-up messages arrive.

The service also switched from an e-mail–based to a Web-based submission process, which cut down on the amount of spam in the triage listing and made it possible for mentors looking through the triage system to alert other mentors to responses from participants that did not come through the usual follow-up procedure. Mentors looking at the listing of responses now have their names highlighted, which makes it easier for them to find follow-up responses or memos specifically directed to them. Ask Dr. Math staff members report that these features are getting a lot of use and that they appear to have impacted both the frequency and the turnaround time of sustained exchanges. Sustained exchanges now often occur within a single day rather than over several days, as they had in the past.

In addition, the Math Forum has now instituted a process of identifying particular questions (and responses) for archiving on the basis of whether they filled a gap in the archives. The staff members changed the archiving procedures because the archive had begun to collect a lot of repeated questions and because editing, indexing, and making sure each question and response is searchable takes staff time. Changes in the way in which archiving is handled affect the ability to select exchanges from the archives for further study that are representative of all exchanges in the service. This shift in the archiving also may mean that the site inadvertently is supporting single responses and succinct explanations, because these now tend to be archived more frequently than sustained exchanges.

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REFERENCES

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal*, 93(4), 373–397.

Bransford, J. S., Brown, A. L., & Cocking, R. R. (1999). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academy Press.

Cobb, P., & Bauersfeld, H. (Eds.) (1995). *The emergence of mathematical meaning: Interaction in classroom cultures*. Mahwah, NJ: Lawrence Erlbaum.

De Corte, E., Verschaffel, L., & Op't Eynde, P. (2000). Self-regulation: A characteristic and a goal of mathematics education. In M. Boekaert, P. R. Pintrich, & M. Zeidner (Eds.), *Handbook of self-regulation* (pp. 687–726). New York: Academic Press.

Hogan, K., & Pressley, M. (Eds.) (1997). *Scaffolding student learning: Instructional approaches and issues*. Cambridge, MA: Brookline Books.

Ginsberg, H. P., Klein, A., & Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In I. E. Sigel & K. A. Renninger (Vol. Eds.), *Child psychology and practice* (Vol. 4), in W. Damon (Gen. Ed.), *Handbook of child psychology* (5th ed., pp. 401–476). New York: John Wiley & Sons.

Gustafsson, J-E., & Undheim, J. O. (1996). Individual difference in cognitive functions. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 186–242). New York: MacMillan.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. *Cognition and Instruction*, 3(4), 305–342.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. New York: Cambridge University Press.

Math Forum BRAP Group (2000). *Encouraging mathematical thinking: Discourse around a rich problem*. Retrieved July 21, 2003, from http://www.mathforum.org/brap/wrap/

Packer, M. J., & Scott, B. (1992). The hermeneutic investigation of peer relations. In L. T. Winegar & J. Valsiner (Ed.), *Children's development within social context: Vol. 2. Research and methodology* (pp. 75–111). Hillsdale, NJ: Lawrence Erlbaum.

Pea, R.D. (1993). Learning scientific concepts through material and social activities: Conversational analysis meets conceptual change. *Educational Psychologist,* 28(3), 265–277.

Renninger, K. A. (1998). Developmental psychology and instruction: Issues from and for practice. In I. E. Sigel & K. A. Renninger (Vol. Eds.), *Child psychology in practice* (Vol. 4), in W. Damon (Gen. Ed.), *Handbook of child psychology* (5th edition, pp. 211–274). New York, NY: John Wiley & Sons.

Renninger, K. A., Farra, L., & Feldman-Riordan, C. (2000). The impact of the Math Forum's Problems of the Week on students' mathematical thinking. *Proceedings of ICLS 2000.* Mahwah, NJ: Lawrence Erlbaum. Available at http://www.mathforum.org/articles/rennin2_2000.html

Renninger, K. A., & Shumar, W. (2002). Community building with and for teachers at the Math Forum. In K. A. Renninger & W. Shumar (Eds.), *Building virtual communities: Learning and change in cyberspace* (pp. 60–95). New York: Cambridge University Press.

Renninger, K. A., & Shumar, W. (2003). The centrality of culture and community to participant learning at and with the Math Forum. In S. Barab, R. Kling, & J. Gray (Eds.), *Designing for virtual communities in the service of learning.* New York: Cambridge University Press.

Rogoff, B. (1995). Observing sociocultural activity on three planes: Participatory appropriation, guided participation, and apprenticeship. In J. V. Wertsch, P. del Rio, & A. Alvarez (Eds.), *Sociocultural studies of mind* (pp. 139–164). New York: Cambridge University Press.

Rogoff, B. (1998). Cognition as a collaborative process. In D. Kuhn & R. S. Siegler (Vol. Eds.) *Cognition, perception, and language* (Vol. 2), in W. Damon (Gen. Ed.), *Handbook of child psychology* (5th ed., pp. 679–744). New York: John Wiley & Sons.

Sansone, C., Sachau, D. A., & Weir, C. (1989). Effects of instruction on intrinsic interest: The importance of context. *Journal of Personality and Social Psychology* 57(5), 819–829.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 334–370). New York: Macmillan.

Valsiner, J. (1998). The development of the concept of development: Historical and epistemological perspectives. In R. Lerner (Vol. Ed.), *Theoretical models of human development* (Vol. 1), in W. Damon (Gen. Ed.), *Handbook of child psychology* (5th ed., pp. 189–232). New York: John Wiley & Sons.

Wertsch, J. V., & Toma, C. (1995). Discourse and learning in the classroom: A sociocultural approach. In L. P. Steffe & J. Gale (Eds.), *Constructivism in education.* Hillsdale, NJ: Lawrence Erlbaum.