

# A HIGHER-ORDER FINE-GRAINED LOGIC FOR INTENSIONAL SEMANTICS

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*Abstract.* This paper describes a higher-order logic with fine-grained intensionality (FIL). Unlike traditional Montogovian type theory, intensionality is treated as basic, rather than derived through possible worlds. This allows for fine-grained intensionality without impossible worlds. Possible worlds and modalities are defined algebraically. The proof theory for FIL is given as a set of tableau rules, and an algebraic model theory is specified. The proof theory is shown to be sound relative to this model theory. FIL avoids many of the problems created by classical course grained intensional logics that have been used in formal and computational semantics.<sup>1</sup>

## 1. Introduction

It has frequently been noted that the characterization of intensions as functions from indices to denotations, as in Montague (1974), yields a semantics which is not sufficiently fine-grained. For example, logically equivalent expressions are cointensional and so intersubstitutable in all contexts, including the complements of propositional attitude predicates. The view that expressions are cointensional just in case they have the same denotation across indices has been dominant in formal semantics at least since Carnap (1947).

An alternative view, which we refer to as *hyperintensionalism*, posits propositions as independent intensional entities, and takes truth to be a derived relational property. In the past twenty years a variety of hyperintensionalist theories have been proposed, including Thomason (1980), situation semantics (Barwise and Perry 1983, Barwise and Etchemendy 1990, and Seligman and Moss 1997), Landman (1986), property theory (Chierchia and Turner 1988, Turner 1987 and Turner 1992), Muskens (1995), and Lappin and Pollard (1999). With the exception of Turner (1992), these theories have focused on the interpretative structures while remaining inexplicit or programmatic about the logic.

We depart from this tradition by constructing an explicit proof theory for a fine-grained logic and then defining a class of models in which the logic is sound. The logic is broadly similar to Church (1940)'s simple theory of types (SST), augmented with a coextensionality predicate. It differs crucially from more familiar elaborations of SST (such as Henkin (1950) and Gallin (1975)) in having models that falsify the axiom of propositional extensionality.

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## 2. Fine-Grained Intensional Logic

Fine-Grained Intensional Logic (FIL), like SST, incorporates the typed  $\lambda$ -calculus into a classical logic.<sup>2</sup>

### 2.1 The Set of Types

We define the set of types in our fine-grained intensional logic FIL as follows.

#### 2.1.1 Basic Types

1.  $i$  (*individuals concepts*)
2.  $p$  (*propositions*)

#### 2.1.2 Exponential Types

If  $A, B$  are types, then  $A^B$  is a type.

This is the type system of Church (1940). Unlike Gallin (1975)'s Ty2, there is no additional type  $s$  for possible worlds. For each type  $A$  there is (i) a (possibly non-empty) denumerable set of non-logical constants of type  $A$  and (ii) a denumerably infinite set of variables of type  $A$ . The constants serve as logical translations of words in a natural language whose semantic structure is being represented. As usual, in higher-order logic, connective and quantifier symbols are syntactic sugar for logical constants.

We define the set  $E_A$  of expressions of type  $A$  as follows.

1. Every variable of type  $A$  is in  $E_A$ .
2. Every constant of type  $A$  is in  $E_A$ .
3. If  $\alpha \in E_A$  and  $u$  is a variable in  $E_B$ , then  $\lambda u \alpha \in E_{A^B}$ .
4. If  $\alpha \in E_{B^A}$  and  $\beta \in E_A$ , then  $\alpha(\beta) \in E_B$ .
5. If  $\alpha, \beta \in E_A$ , then  $\alpha = \beta \in E_p$ .
6. if  $\alpha, \beta \in E_A$ , then  $\alpha \cong \beta \in E_p$ .
7.  $\top$  and  $\perp \in E_p$ .
8. If  $\phi, \psi \in E_p$ , then so are
  - (a)  $\neg\phi$
  - (b)  $\phi \vee \psi$
  - (c)  $\phi \wedge \psi$
  - (d)  $\phi \rightarrow \psi$

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<sup>2</sup>In Fox et al. (2002) we present an extended first-order Property Theory with Curry typing (PTCT) as an alternative implementation of the fine-grained intensionalist approach to natural language semantics that we propose here. We are in the process of exploring the correspondence relations that hold between FIL and PTCT.

$$(e) \phi \leftrightarrow \psi$$

9. If  $\phi \in E_p$  and  $u$  is a variable in  $E_A$ , then  $\forall u\phi$  and  $\exists u\phi \in E_p$ .

10. If  $\phi \in E_p$ , then  $\Box\phi$  and  $\Diamond\phi \in E_p$ .

## 2.2 A Proof Theory for FIL

We formulate our proof theory for FIL as a set of tableau rules. We follow Fitting (2000), and Fitting and Mendelsohn (1999) in using prefixed rules, where a prefix  $\sigma$  is a positive integer that corresponds to a possible world.<sup>3</sup> A prefixed formula  $\sigma\phi$  is interpreted as  $\phi$  is true at  $\sigma$  ( $\phi \in w$ ,  $w$  the world corresponding to  $\sigma$ ).

A path  $P$  of a tree is *closed* iff (i)  $P$  contains both a formula  $\phi$  and its negation  $\neg\phi$ , or (ii)  $P$  contains  $\perp$ .  $P$  is *open* iff it is not closed. A proof of  $\phi$  is a tree  $T$  with  $1\neg\phi$  at the root of  $T$  and all the branches of  $T$  are closed.

The symbol  $*$  following a premis indicates that it has been used in the course of a proof and is no longer available for further application of tableau rules. When a rule requires more than one premis, the premises are separated by commas. In addition to tree construction rules, the following tableau rules contain three path constraints, the conditions for Non-Identity, and for Equivalence (1) and (2). We suppress type subscripts in the rules where the type of an expression is obvious, or not relevant to the rule. In the interests of space, for the connectives we only give the rules for conjunction, implication, and double negation.

### Top

For any prefix  $\sigma$ ,

$$\begin{array}{c} \sigma\neg\top \\ | \\ \sigma\perp \end{array}$$

### Conjunction

For any prefix  $\sigma$ ,

$$\begin{array}{c} \sigma(s \wedge t)^* \\ | \\ \sigma s \\ \sigma t \end{array}$$

### Negated Conjunction

For any prefix  $\sigma$ ,

$$\begin{array}{c} \sigma\neg(s \wedge t)^* \\ \swarrow \quad \searrow \\ \sigma\neg s \quad \sigma\neg t \end{array}$$

### Implication

For any prefix  $\sigma$ ,

$$\begin{array}{c} \sigma(s \rightarrow t)^* \\ \swarrow \quad \searrow \\ \sigma\neg s \quad \sigma t \end{array}$$

### Negated Implication

For any prefix  $\sigma$ ,

$$\begin{array}{c} \sigma\neg(s \rightarrow t)^* \\ | \\ \sigma s \\ \sigma\neg t \end{array}$$

### Boolean Negation

For any prefix  $\sigma$ ,

$$\begin{array}{c} \sigma\neg\neg s^* \\ | \\ \sigma s \end{array}$$

### Negated Quantification

For any prefix  $\sigma$ ,

$$\begin{array}{c} \sigma\neg Qu\phi^* \\ | \\ \sigma Q'u\neg\phi \end{array} \quad \begin{array}{l} \text{where } Q' \text{ is the dual} \\ \text{of } Q \end{array}$$

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<sup>3</sup>In our model theory for FIL we define a possible world as an ultrafilter of the boolean pre-lattice of propositions.

## Universal Quantification

$\sigma\forall u\varphi$  where (i)  $\alpha$  is grounded, and  
 $\sigma\varphi[\alpha/u]$  (ii)  $\alpha$  occurs on the path (or  $\alpha$  is a new constant in the path)<sup>4</sup>

## Identity

For any prefix  $\sigma$ ,

$$\sigma\alpha = \beta, \sigma\varphi$$

$$\sigma\varphi[\alpha/\beta]$$

## $\alpha$ -reduction

For any prefix  $\sigma$ ,

$\sigma\phi$   
 $\sigma\phi'$

where  $\phi'$  is obtained from  $\phi$  by substituting variables of the appropriate type that appear in the path for corresponding bound variables in  $\phi$

## $\eta$ -reduction<sup>5</sup>

For any prefix  $\sigma$ ,

$$\sigma\dots\lambda u\phi(u)\dots$$

$$\sigma\dots\phi\dots$$

## Equivalence (1)

For any prefix  $\sigma$ ,

$$\sigma\neg(\alpha \cong \alpha)$$

$$\sigma\perp$$

## Equivalence (3)

For any prefix  $\sigma$ ,

$$\sigma(\alpha \cong \beta), \sigma(\beta \cong \gamma)$$

$$\sigma(\alpha \cong \gamma)$$

## Co-extensionality (1)

For any prefix  $\sigma$ ,

$\sigma(\phi \cong \psi)$  where  $\phi, \psi \in B^A$ ,  
 $\sigma(\phi(\alpha) \cong \psi(\alpha))$   $\alpha \in A$ , and  $\alpha$  is a grounded term that appears in the path

## Existential Quantification

$\sigma\exists u\varphi^*$  where  $\beta$  is a new constant that does not occur on the path and  
 $\sigma\varphi[\beta/u]$   $\beta, u \in A$

## Non-Identity

For any prefix  $\sigma$ ,

$$\sigma\neg(\alpha = \alpha)$$

$$\perp$$

## $\beta$ -reduction

For any prefix  $\sigma$ ,

$$\sigma\dots(\lambda u\phi)(\alpha)\dots$$

$$\sigma\dots\phi[\alpha/u]\dots$$

## Equivalence (2)

For any prefix  $\sigma$ ,

$$\sigma(\alpha \cong \beta), \sigma\neg(\beta \cong \alpha)$$

$$\sigma\perp$$

## Equivalence (4)

For any prefix  $\sigma$ ,

$$\sigma(s \cong t)^*$$

$$\sigma s \quad \sigma\neg s$$

$$\sigma t \quad \sigma\neg t$$

## Co-extensionality (2)

For any prefix  $\sigma$ ,

$\sigma\neg(\phi \cong \psi)^*$   $\phi, \psi \in B^A$ ,  $\beta \in A$ ,  
 $\sigma\neg(\phi(\beta) \cong \psi(\beta))$  and  $\beta$  is a new constant that does not appear in the path

<sup>4</sup>A term is grounded in the sense of Fitting (2000) iff it is a closed term (constant, closed  $\lambda$  term, or closed formula) with a prefix  $\sigma$ .

<sup>5</sup> $\eta$ -reduction permits us to prove EXT of Church (1940):  $\vdash \forall f, g(\forall u(f(u) = g(u)) \rightarrow f = g)$ . As the antecedent of EXT requires intensional identity of  $f(u)$  and  $g(u)$  for every  $u$ , it does not compromise the intensionality of FIL. Specifically, it does not entail  $\forall u, v \in A(u \cong v \rightarrow u = v)$ . We are grateful to Paul Gilmore for pointing this out to us.

## Negated Modality

For any prefix  $\sigma$ ,

$$\begin{array}{l} \sigma \neg M \phi^* \\ | \\ \sigma M' \neg \phi \end{array} \quad \begin{array}{l} \text{where } M \text{ and } M' \text{ are} \\ \text{modal operators, and} \\ M' \text{ is the dual of } M \end{array}$$

## Modality $\Box$ S5

$$\begin{array}{l} \sigma \Box s \\ | \\ \sigma' s \end{array} \quad \begin{array}{l} \text{where } \sigma' \text{ is a prefix interger that} \\ \text{appears in the path} \end{array}$$

## Modality $\Diamond$ S5

$$\begin{array}{l} \sigma \Diamond s^* \\ | \\ \sigma' s \end{array} \quad \begin{array}{l} \text{where } \sigma' \text{ is a new prefix interger} \\ \text{that does not appear in the path} \end{array}$$

Tableau proof methods for other modal logics can be defined by modifying and adding to the rules for the modal operators, as in Fitting and Mendelsohn (1999).

### 2.3 Distinguishing Between Equivalence and Identity

The relation  $=$  corresponds to identity of intension, while  $\cong$  is interpreted as extensional equivalence of entities of the same type.<sup>6</sup> The tableau rules allow us to prove **IDENT** but not **EXTEN**.

**IDENT:**  $\forall u, v \in A (u = v \rightarrow u \cong v)$

**EXTEN:**  $\forall u, v \in A (u \cong v \rightarrow u = v)$

The proof for **IDENT** is as follows.

$$\begin{array}{l} 1 \neg \forall u, v \in A (u = v \rightarrow u \cong v)^* \\ 1 \exists u, v \in A \neg (u = v \rightarrow u \cong v)^* \text{ (Negated Quantification)} \\ 1 \neg (a = b \rightarrow a \cong b)^* \text{ (Existential Quantification)} \\ 1 a = b \\ 1 \neg (a \cong b) \text{ (Negated Implication)} \\ 1 \neg (a \cong a) \text{ (Identity)} \\ 1 \perp \text{ (Equivalence (1))} \end{array}$$

The tree is closed. The following tableau provides a counterexample to **EXTEN**.

$$\begin{array}{l} 1 \neg \forall u, v \in A (u \cong v \rightarrow u = v)^* \\ 1 \exists u, v \in A \neg (u \cong v \rightarrow u = v)^* \text{ (Negated Quantification)} \\ 1 \neg (a \cong b \rightarrow a = b)^* \text{ (Existential Quantification)} \\ 1 a \cong b \\ 1 \neg (a = b) \text{ (Negated Implication)} \end{array}$$

The tree is finished and open.

The proof theory for FIL entails that two expressions can be logically equivalent but not cointensional, and so distinct propositions can imply each other.

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<sup>6</sup>Within the framework of program specification theory, Maibaum (1997) discusses the use of a weak non-logical equality predicate to express the equivalence/congruence of possibly distinct expressions within a theory. Gilmore (2001) constructs an intensional simple theory of types (ITT) in which an intensional ( $=$ ) and an extensional ( $=_e$ ) identity predicate are defined. His proposal differs from that of Fox and Lappin (2001) and the current version of FIL in that (i) his extensional identity predicate is not type general, but is only defined for propositions and predicates, and (ii) for us identity and equivalence are primitive, whereas Gilmore defines them in terms of substitution and bi-implication.

### 3. A Semantics for FIL

#### 3.1 Intensional Models

Before defining our class of models, we first review the notions of a frame and a boolean prelattice, in terms of which these models will be specified. We take a (*Henkin*) *frame* to be a type-indexed family of sets  $S = \langle S_A \rangle$  such that  $S_{B^A}$  is a (possibly proper) subset of the set of functions from  $S_A$  to  $S_B$ . We take a *boolean prelattice* to be a set  $B$  with a preorder (a relation that is transitive, reflexive, but not necessarily antisymmetric)  $\sqsubseteq$ , two nullary operations  $T$  and  $F$ , one unary operation  $'$  (written postfix), and four binary operations  $\sqcap$ ,  $\sqcup$ ,  $\Rightarrow$ ,  $\Leftrightarrow$ , subject to the following conditions (here  $\equiv$  is the relation on  $B$  such that  $a \equiv b$  iff  $a \sqsubseteq b$  and  $b \sqsubseteq a$ ).

1.  $T$  is a greatest element.
2.  $F$  is a least element.
3.  $\sqcap$  is a greatest lower bound operation.
4.  $\sqcup$  is a least upper bound operation.
5.  $\Rightarrow$  is a relative pseudocomplement operation, i.e. For all  $a, b, c \in B$ ,  $c \sqsubseteq a \Rightarrow b$  iff  $c \sqcap a \sqsubseteq b$ .
6. For all  $a, b \in B$ ,  $(a \Leftrightarrow b) \equiv ((a \Rightarrow b) \sqcap (b \Leftarrow a))$ .
7. For all  $a \in B$ ,  $a' \equiv a \Rightarrow F$ .
8. For all  $a \in B$ ,  $a'' \equiv a$ .

This boolean prelattice is like a boolean algebra except that antisymmetry does not hold: two distinct elements can be greater than or equal to each other.

We can now define the class of intensional structures into which FIL is to be interpreted. An *intensional structure*  $\mathbf{S}$  consists of a frame  $S = \langle S_A \rangle$ , where  $S_p$  is a boolean prelattice, together with enough additional structure to interpret all FIL logical constants. The preorder  $\sqsubseteq$  is called *entailment*. It is a theorem that  $a \sqsubseteq b$  iff  $b$  belongs to every ultrafilter that  $a$  belongs to. More generally, we say that a subset  $X$  of  $S_p$  *entails*  $b \in S_p$  if every ultrafilter containing  $X$  as a subset has  $b$  as a member. The specified operations of  $S_p$  already provide interpretations for the boolean logical constants, and the entailment preorder models logical consequence. However we must add enough operations to interpret (at each type  $A$ ) the equality and coextensionality predicates as well as the universal and existential quantifiers. The additional operations that we require are as follows.

**Equality:**  $=_A$  is interpreted as an (*intensional*) *identity* operation  $id_A$  of type  $(p^A)^A$  such that, for all  $a, b \in S_A$ ,  $id(a, a) \equiv T$  and  $id(a, b) \equiv F$ , whenever  $a \neq b$ .<sup>7</sup>

**Universal:**  $\forall_A$  is interpreted as a *type-restricted universal quantification* operation  $\prod_A$  of type  $p^{p^A}$  such that  $\{f(a) | a \in S_A\}$  entails and has as a greatest lower bound  $\prod_A(f)$ .

**Existential:**  $\exists_A$  is interpreted as a *type-restricted existential quantification* operation  $\coprod_A$  of type  $p^{p^A}$ , such that,  $\coprod_A(f)$  is the least upper bound of  $\{f(a) | a \in S_A\}$ .

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<sup>7</sup>In this setting,  $=$  ( $\neq$ ) is a meta-theoretic notion of identity (non-identity).

**Coextensionality:**  $\cong_A$  is interpreted as an *extensional identity* operation  $extid_A$  of type  $(p^A)^A$  such that, for all  $a, b \in S_A$ ,  $extid(a, a) \equiv T$ ,  $extid(a, b) \equiv extid(b, a)$ , and  $(extid(a, b) \sqcap extid(b, c)) \sqsubseteq extid(a, c)$ . Also, if  $A = C^B$ , then for all  $f, g \in S_A$ ,  $extid(f, g) \equiv \prod_B(h)$ , where  $h$  is the function that maps each  $B$ -intension  $b$  to the proposition  $extid(f(b), g(b))$ . Moreover,  $extid_p \equiv \Leftrightarrow$ .

Thus universal and existential quantification are treated as infinitary conjunction (greatest lower bound) and disjunction (least upper bound), respectively.

An *intensional model* of FIL, is an intensional structure  $\mathbf{S}$ , together with a function  $Int$  which assigns to each nonlogical constant a type-appropriate intension in  $S$ , and to each logical constant the obvious operation in  $S$ . If  $g$  is an assignment of variables to type-appropriate intensions in  $S$ , then  $Int$  extends uniquely to an interpretation  $Int_g$  of all FIL terms if we require that  $Int_g(x) = g(x)$  for every variable  $x$ , and that application and term abstraction receive their customary interpretations; i.e.  $Int_g(\gamma(\alpha)) = (Int_g(\gamma))(Int_g(\alpha))$ , and  $Int_g(\lambda x \in_A \alpha)$  is the function that maps each  $a \in S_A$  to  $Int_{g(x/a)}(\alpha)$ . Clearly the value assigned by  $Int_g$  to a closed term depends only on  $Int$  (not on  $g$ ), and interpretation respects primitive equality in the sense that lambda-equivalent terms and terms differing only alphabetically in their bound variables are assigned the same intensions.

To summarize, an intensional model assigns intensions to terms in such a way that logical constants are interpreted as designated operations, term application and abstraction are interpreted in the standard way, and lambda and alphabetic variant-equivalent terms receive the same intensions. Formulas (terms of type  $p$ ) have as their intensions *propositions* (elements of the boolean prelattice  $S_p$ ), terms of type  $i$  have as their intensions *individual concepts* (elements of  $S_i$ ), and  $A$ -predicates (terms of type  $p^A$ ) have as their intensions  *$A$ -properties* (propositional functions with domain  $S_A$ ). Crucially, sentences which are provably coextensional but not provably equal ( $\lambda$ -interconvertible or alphabetic variant) are interpreted as distinct but equivalent (mutually entailing) propositions.

### 3.2 Constructing Worlds and Denotations Algebraically

Let us hold  $Int$  and  $S$  fixed. Generalizing Kripke (1959)—and Jonsson and Tarski (1951)—we define a (*possible*) *world* (we have no impossible worlds) to be a maximal consistent sets of propositions, i.e. an ultrafilter of the boolean prelattice  $S_p$ . These are in one-to-one correspondence with the *valuations* on  $S_p$ , i.e. the boolean homomorphisms from  $S_p$  to the two-element boolean algebra  $2$  (or, equivalently, the characteristic functions of ultrafilters). The valuation  $Val_w$  corresponding to the ultrafilter  $w$  is its characteristic function.

Again following Kripke, we define a *modal intensional model*  $M = \langle \mathbf{S}, Int, W, R \rangle$ , where  $W$  is the set of worlds, and  $R$  is an accessibility relation on the elements of  $W$  (Kripke 1959). For the present version of FIL we assume that  $R$  is an equivalence relation in order to sustain S5.

Given a world  $w$  and two intensions  $s, r$  of the same type, we say  $s$  and  $r$  are *w-coextensional* just in case the proposition  $extid(s, r)$  is in  $w$ . It is easy to see that *w-coextensionality* is an equivalence relation at each type. We define the type-indexed family  $D_w = \langle D_{w,A} \rangle$  of *denotations* at  $w$  by type recursion as follows.

1.  $D_{w,p} = 2$  (the truth values).
2.  $D_{w,i} =$  the quotient of  $S_i$  by the  $w$ -coextensionality relation (i.e. the individuals are the  $w$ -coextensionality equivalence classes of individual concepts).

3.  $D_{w,BA}$  = the set of functions from  $S_A$  to  $D_B$

For any type  $A$ , the denotations of type  $p^A$  are functions from  $S_A$  to 2, i.e. characteristic functions of sets of  $A$ -intensions.

We can extend the valuation  $Val_w$  associated with  $w$  to all of  $S$  (i.e. to intensions of all types) by type recursion.

1.  $Val_w(s) = t$  if  $s \in w$ ,  $f$  otherwise (for  $s$  a proposition).
2.  $Val_w(s) =$  the  $w$ -coextensionality equivalence class of  $s$ , for  $s$  an individual concept.
3.  $Val_w(s) =$  the function from  $S_A$  to  $D_B$  that maps each  $a \in S_A$  to  $Val_w(s(a))$  (for  $s \in S_{BA}$ .)

We can prove Theorems 1 and 2 by type induction.

**Theorem 1** *Two intensions are  $w$ -coextensional iff they have the same value at  $w$ .*

Given a world  $w$  and a variable assignment  $g$ , we define the *denotation* of a term  $\alpha$  at  $w$ , relative to  $g$ , written  $\|\alpha\|^{M,w,g}$ , to be  $Val_w(Int_g(\alpha))$ . If  $\alpha$  is closed, the subscript  $g$  can be omitted. We then have

**Theorem 2** *For any term  $\alpha$ ,  $\|\alpha\|^{M,w,g}$  is*

1.  $Val_w(Int(\alpha))$ , if  $\alpha$  is a constant.
2.  $Val_w(g(\alpha))$ , if  $\alpha$  a variable.
3.  $\|\gamma\|^{M,w,g}(Int_g(\beta))$ , if  $\alpha$  is  $\gamma(\beta)$ .
4. the function  $f$  with domain  $S_A$  such that for any  $a \in S_A$ ,  $f(a) = \|\beta\|^{M,w,g(x/a)}$ , if  $\alpha$  is  $\lambda x_{\in A}\beta$ .
5.  $t$  if  $\alpha$  is  $\top$ .
6.  $f$  if  $\alpha$  is  $\perp$ .
7.  $t$  iff  $\|\phi\|^{M,w,g} = f$ , if  $\alpha$  is  $\neg\phi$ .
8.  $t$  iff  $\|\phi\|^{M,w,g} = \|\psi\|^{M,w,g} = t$ , if  $\alpha$  is  $\phi \wedge \psi$ .
9.  $t$  iff  $\|\phi\|^{M,w,g} = t$  or  $\|\psi\|^{M,w,g} = t$ , if  $\alpha$  is  $\phi \vee \psi$ .
10.  $t$  iff  $\|\phi\|^{M,w,g} = f$  or  $\|\psi\|^{M,w,g} = t$ , if  $\alpha$  is  $\phi \rightarrow \psi$ .
11.  $t$  iff  $\|\phi\|^{M,w,g} = \|\psi\|^{M,w,g}$ , if  $\alpha$  is  $\phi \leftrightarrow \psi$ .
12.  $t$  iff  $\|\phi\|^{M,w,g(x/a)} = t$  for all  $a \in S_A$ , if  $\alpha$  is  $\forall x_{\in A}\phi$ .
13.  $t$  iff  $\|\phi\|^{M,w,g(x/a)} = t$  for some  $a \in S_A$ , if  $\alpha$  is  $\exists x_{\in A}\phi$ .
14.  $t$  iff  $\|\beta\|^{M,w,g} = \|\gamma\|^{M,w,g}$ , if  $\alpha$  is  $\beta \cong \gamma$ .
15.  $t$  iff  $Int_g(\beta) = Int_g(\gamma)$ , if  $\alpha$  is  $\beta = \gamma$ . (So the truth-value of an equality is independent of the choice of world.)



In addition to the semantic clauses of Theorem 2 we adopt the following conditions on  $Val_w$ .

**Modal Operator Conditions:** For  $\phi \in \mathcal{P}$ ,  $\|\phi\|^{M,w,g}$  is

16.  $\mathfrak{t}$  iff  $\|\psi\|^{M,w_i,g} = t$  for all  $w_i \in W$  such that  $wRw_i$ , if  $\phi$  is  $\Box\psi$ .
17.  $\mathfrak{t}$  iff  $\|\psi\|^{M,w_i,g} = t$  for some  $w_i \in W$  such that  $wRw_i$ , if  $\phi$  is  $\Diamond\psi$ .

It is not difficult to show that Theorem 3 holds.

**Theorem 3** *If there is a proof of a sentence  $s$  of FIL, then  $s$  is valid relative to the set of models defined for FIL.*

For reasons of space, we will limit ourselves to an outline of the proof. First it is necessary to prove a lemma stating that if there is a model  $M$ , a world  $w_1$ , and a valuation  $g$  such that for the root  $1$   $s$  of a finished tree  $T$ ,  $\|s\|^{M,w_1,g} = t$ , then there is an open path  $P$  in  $T$  in which for every full sentence  $\sigma r$  in  $P$ ,  $\|r\|^{M,w_\sigma,g} = t$ . To establish this lemma, we need to prove the downward correctness of the tableau rules and constraints. This is done by showing that each rule and constraint preserves truth, given the clauses of Theorem 2 and the Modal Operator Conditions of our model theory.

We then prove the lemma through induction on application of the tableau rules and constraints to the sentences in the open path of a tree. The base of the induction is the case in which the tree consists of just one sentence to which no rules or constraints apply. For the inductive step we observe that any expansion  $P'$  of an open path  $P$  is obtained by a tableau rule to a sentence of  $P$ . Downward correctness of the rules insures that every sentence  $\sigma r$  of  $P'$  is true relative to  $\|\cdot\|^{M,w_\sigma,g}$ , and the sentences of  $P$  satisfy the tableau constraints. Hence, at any point in the construction of a finished tree  $T$  whose root is a sentence  $1$   $s$  such that  $\|s\|^{M,w_1,g} = t$ , there is an open path  $P$  in  $T$  in which for every sentence  $\sigma r$   $\|r\|^{M,w_\sigma,g} = t$ . This result holds for both finite and infinite trees (trees with infinite paths). This establishes the lemma.

Given the lemma, it follows that if there is a proof for  $s$ , then there is no model  $M$ , no world  $w$ , and no valuation  $g$  such that  $\|¬s\|^{M,w,g} = t$ . Therefore, Theorem 3 holds.

## 4. Conclusion

We have presented a higher-order fine-grained intensional logic for the semantic representation of natural languages. The logic contains type general predicates for intensional identity and extensional equality. The proof procedure permits us to prove that identity of intension entails identity of extension, but that the converse does not hold. We have constructed an algebraic semantics for our logic relative to which our tableau proof procedure is sound. Unlike alternative hyperintensionalist frameworks that have been proposed, we can distinguish among equivalent propositions without resorting to impossible worlds to sustain the distinction. We treat modality separately from intensions. On our approach intensions are taken as primary, and truth, denotation, and modality are defined in terms of them.

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