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Abstract

In recent times, rapid advances in the field of information technology have enabled firms selling information products (for example, in the form of a CD) to provide subscription services using remote servers. Making the product vs. service decision involves a trade-off. Online subscription offerings are perceived to be of lower quality because of data security and network reliability issues but can reduce or even eliminate the credible commitment (of prices) problem typically faced by firms selling durable products. We model an infinite horizon game between two symmetric firms that choose the design architecture i.e. product or service (and consequent contracting scheme: selling or subscription) and then set prices to optimize profits. The perceived quality loss enables firms to differentiate based on design architecture/contracting scheme and make positive profits. Further, the firm that sells a product makes higher profit than the firm that offers a subscription service. However, as the discount factor increases, the presence of a seller leads to a credible commitment problem for both firms leading to a reduction in prices. The results offer an explanation for the persistence of firms selling products rather than services. Further, a firm offering a subscription service may be subject to price pressure due to the price commitment issues of a competing seller. We also extend our insights to other settings: one involving asymmetric firms and the other involving different discount rates for product quality and producer / consumer surplus.

Keywords: pricing; competitive strategy; forward-looking customers; online subscription services;

1 Introduction

Renting and leasing of goods and services is widely prevalent in the business world. It can be observed across many industries: real estate, cars, machinery, books etc. There are many pros and cons of renting and these vary from product to product. As a result, the rental model is more prevalent in the case of some product categories as opposed to others. Broadly, renting turns out to be useful under the following conditions: 1) The customer cannot make the large payments associated with buying and would rather pay a smaller per period price. To some extent, this problem can be ameliorated in situations where appropriate financing schemes are available. 2) The customer requires flexibility of use and needs the product only over a limited time horizon. For example, renting a car when on a trip to a new city or renting a house for a few months. 3) The customer does not have adequate information about the quality of the product or his needs and would like to try the product for a limited duration before making a purchase decision. 4) The customer anticipates declining selling prices and consequently does not buy a product when it is first introduced. The customer then rents till the point of purchase. This last characteristic may not be the driving force behind renting in product categories such as automobiles but is of particular relevance in the context of information goods. In this paper, we focus on this last characteristic of rental models in the context of the information goods industry.

In recent times, information goods that were once sold as products can now be offered in the form of subscription services. The services range from the delivery of software-as-a-service to online gaming and also include information services such as newspapers and industrial databases. This transition to subscription services has been enabled by significant recent advances in information and communication technologies. In this context, customers may prefer renting for another reason: information goods such as software are frequently upgraded and a rental contract typically allows for free upgrades. Customers who would not like to incur huge upgrade transition costs would then rent rather than buy. In this paper, we do not consider upgrades though other researchers have looked at this issue. For example, Bala and Carr(2009) evaluate the impact of customer upgrade costs on the firm's upgrade pricing policy. Sankaranarayanan(2007) analyzes Microsoft's insurance policy that protects against frequent upgrades.

Many information goods are development-intensive, a term defined by Krishnan and Zhu (2006). A firm that produces these information goods incurs an upfront fixed cost of developing the product that depends on the quality level of the product but incurs negligible marginal costs per copy of product sold (any installation costs are assumed to be passed on to the users). Unlike many other durable goods such as automobiles, information goods typically do not physically deteriorate, yet may have much less value in the future than the present (newspapers, for example). Transaction costs of renting could be significant for a firm offering certain kinds of durable goods. This typically involves the administrative cost that goes into managing the rental service. In fact, in the automobile industry, this function is significant enough to be completely taken over by third party rental agencies such as Hertz and Enterprise. Such transaction costs are low for information goods, unless the firm offers non-linear pricing schemes. Offering non-linear pricing schemes may involve a cost of monitoring usage, which may be significant when compared to the low variable costs of production (Sundararajan, 2004). For a treatment of a competitive setting where firms offer usage-based pricing, we refer the readers to Bala & Carr (2006). In the current paper, we assume that the firms offer only fixed prices but may differ in the length of their contracts. In particular, we assume that selling a product is a contract of infinite length (or at least till the end of the horizon under consideration) while offering a subscription service terminates the contract at the end of a time period.

The industrial motivation for the specific problem addressed here is visible in the CRM (customer relationship management) software world. The largest CRM software vendor in the world is Siebel (now acquired by Oracle). It is widely regarded as the dominant player in the industry despite the entry of ERP (enterprise resource planning) vendors such as SAP. These ERP vendors hope to create their own CRM systems and retrofit their current ERP systems with this new offering. Yet, Siebel is the only large firm with specific core competence in this area. In the year 2000, Salesforce.com entered this market with a mission to deliver account management, sales pipeline, and CRM software via the web. The company promotes its unique selling point as low installation costs and a low per month charge per customer. This is low compared to the large installation cost for Siebel and significantly higher one time per customer fee. As a result, Salesforce.com is doing well in a market niche, small to medium sized companies. Further, Siebel has long grappled with the idea of offering its own subscription service, and in fact it does offer one currently. This paper seeks to model situations such as these with the aim to provide insights into the nature of prices and market segments across time in the competitive market for such products and services.

From the customer's perspective, using an online subscription service reduces the installation costs and consequently the price paid, but increases the level of vulnerability of the service due to network disruptions and poor data security[1]. We model this by incorporating a perceived quality loss for a product that is offered as an online subscription service as opposed to one that is sold outright. Thus, if a monopoly firm has the choice to either sell a product or offer a subscription service, Coase conjecture might apply but subscription may not always dominate selling due to perceived quality loss. We can easily verify this using our model setup. The objective of this paper is to evaluate a situation where symmetric firms pick their design architecture and hence consequent contracting strategies and then compete in the market.

The rest of the paper is structured as follows: Section 2 looks at the related literature in this area. Section 3 elaborates on the setup of the base model and the equilibrium results. Section 4 provides a numerical analysis of these results. Section 5 present a couple of extensions to the base model. Section 6 concludes the analysis and discusses directions for future research. Throughout the paper, we use the terms renting, leasing and subscription interchangeably and this should be clear from the context.

2 Related Literature

The earliest work on the economic benefits of renting / leasing is by Engelbourg(1966) who conducts an empirical study to determine conditions under which leasing is prominent and finds that leasing is widely used in industries where stability of income, price discrimination and market power are important issues. Flath(1980) constructs an analytical model and concludes that the ubiquity of leasing is due to the fact that it economizes upon the costs of detecting, assuring, and maintaining quality, costs of search, and costs of risk bearing. Miller and Upton (1976) analyze the risk and tax related motivation for leasing.

The renting/leasing decision also relates to the realm of the durable goods monopolist literature in economics, primarily as a tool to price discriminate appropriately. The earliest work in this area is by Coase(1972) who postulates that a monopolist selling a durable good to rational consumers can-

[1] www.howstuffworks.com/asp.htm

not capture monopoly profits. Consumers will look ahead, anticipate a decreasing price trajectory and hence postpone their purchase till price equals marginal cost. Stokey(1981) models this process over an infinite horizon for a non-depreciating durable good and confirms the results. Bulow(1982) uses a two-period model with a second hand market for a durable good and shows that while a monopolist renter can capture monopoly profits in each time period, a monopolist seller produces goods less durable then either competitive firms or monopolist renters. However, DeGraba(1994) modifies a key assumption in Bulow(1982) to show that it is impossible to come up with a lease short enough to completely solve the time inconsistency problem as postulated by Coase.

While the literature seems firm that pure renting dominates pure selling, it is less clear on the subject of concurrent renting and selling. While the Coase conjecture seems to hold for the case of complete information, price trajectories are affected by consumer and producer experiences. Consumers have expectations about future prices and product quality and these estimates could be incorrect. This topic is analyzed by Shapiro(1983) for different cases in order to obtain optimal price paths. Producers are affected by the learning curve which in turn affects production costs. This was first studied by Stokey(1979) who found that decreasing production costs could support a decreasing price trajectory. Balachander and Srinivasan(1988) extend this to the case where consumers have expectations for both prices and the level of experiential learning for the producer. A conclusion that we derive from this is that while leasing dominates selling in the case of complete information, this need not be the case when consumers cannot evaluate product quality beforehand. Bucovetsky and Chilton(1986) analyze concurrent renting and selling in a durable goods monopoly under the threat of entry and find that there is an optimal preentry mix. Desai and Purohit(1998) find that different combinations of leasing and selling are possible depending on the rates at which leased and sold goods depreciate. They base their model on the market for new and used cars.

As evidenced by the past literature, one of the major objectives pursued by researchers after the initial results on durable goods markets has been to test major results such as the Coase conjecture in specific industry and market settings. Each industry and market setting brings with it a unique set of assumptions that deviate from the baseline assumptions of the earlier models. Balachander and Srinivasan(1988) and Desai and Purohit(1998) are examples of such work. Research efforts have also gone into analyzing the lease/sell decision in the context of competition. Bucovetsky and Chilton(1986) and Desai and Purohit(1999) are examples of such work. Along similar lines, our work has a dual objective. It considers the information goods industry, particularly that part of the industry where products can now be offered as online subscription services using the current state of information technology, and models business outcomes for this industry in the context of competition.

The economics of online services differs from other durable products in many ways. First, the marginal costs of production are negligible, but there might be a significant fixed cost of developing the product. Second, although there is no physical depreciation of these services, they do depreciate in value in a way similar to other products. Third, the high rate of innovation in the information goods sector also ensures that there are no major used goods markets. This phenomenon is further accentuated for some service categories such as software services in which there might be strict licensing requirements. Finally, as mentioned before, offering an information good as an online service might cause a loss in service quality due to issues related to data security and network reliability. Our paper incorporates these key elements of information goods in a competitive model. We specify circumstances under which firms might be better off using traditional selling models as opposed to subscription services. Thus, this paper also adds to the literature on the economics of durable goods by testing the generalizability of the Coase conjecture to a competitive setting.

3 The Base Model

We model all cases in the paper as sequential decision processes with the producers and consumers making rational decisions at each stage. We also assume complete information; that is the consumers are aware of (or can anticipate) the producers pricing and product quality decisions in future stages and this fact is known to the producer. Both future utilities and prices are discounted in the calculation of present value of consumer surplus. The discount factor is assumed common to both producer and consumers. The discount factor is also assumed common to utilities and prices in this base model. In Section 5.2, we analyze a case where this assumption is relaxed. This assumes that utilities are readily converted to dollar values. The analysis then proceeds by first constructing demand functions for the product based on product attributes and prices that are chosen by the firm and on assumptions about customer heterogeneity and behavior. These functions are then incorporated into an optimization program for each firm that generates best response prices. These best response prices are then solved for equilibrium prices.

Customer willingness to pay (WTP) for the product / service, denoted by θ, is uniformly distributed over the interval [0,1]. For a customer of type θ, the function $\theta\cdot U$ is the monetary value or utility that the customer would ascribe to owning a product / service of quality U over an infinite horizon. This implies that a customer of type θ would be willing to purchase a product / service if it would provide surplus that is greater than the sales / rental price; i.e. $\theta\cdot U - p > 0$. A customer selecting among multiple (and mutually exclusive) options will select the one that provides the highest surplus. Let δ be the discount factor. Consequently, the utility derived by a customer in exactly one period of a infinite horizon problem is $(1-\delta)\cdot U$.

3.1 Structure of the game

In a particular time period, we define the state of the system to be the highest type customer who has not bought the product prior to that period. Customers with a higher type are not active participants in the market since they have already bought the product. From here on, we speak only of strategies for those customers who are active in a given period (and their type is necessarily less than the state of the system). A customer's strategy is the nature of the contract (buy, rent, do nothing) that he selects. The payoff to the customer resulting from this strategy is the surplus corresponding to each contract. The objective of the customer is to pick a strategy that maximizes this surplus. A demand curve can be constructed based on this choice.

A firm's strategy is a vector of functions that maps the state of the system (defined on the interval [0,1]) in each time period into a price (also defined on the interval [0,1]) for that period:

$$\begin{aligned} \text{Strategyset } S &= \{f_1, f_2, ...f_\infty\} : \text{statespace } \rightarrow \text{pricespace} \\ &= \{f_1, f_2, ...f_\infty\} : [0,1] \rightarrow [0,1] \end{aligned}$$

When the function in period t as a function of state σ_t is of the form:

$$f(\sigma_t) = \mu_t \cdot \sigma_t$$

where μ_t is a constant between 0 and 1[2], the strategy is said to be linear. When the elements μ_1 to

[2] Restricting this constant between 0 and 1 ensures that customers who purchase in period t receive positive surplus.

μ_∞ are such that $\mu_1 = \mu_2 = ... = \mu_\infty$, this strategy is said to be stationary. We use the subgame perfection equilibrium concept and all subgame equilibria are defined with respect to the firms for given demand curves.

3.2 Duopoly Setting

Consider a symmetric duopoly: both firms sell a product of similar quality. Further, when each of these firms offers a subscription service, the loss in product quality is the same for both firms. Consider a two stage game where firms choose a product architecture and hence the consequent contracting strategy (sell or lease) in the first stage and then compete in the market using prices in the second stage. We first observe the second stage outcome of this game assuming that firms have made the choice of contracting strategies. This results in three possible outcomes: 1) both firms sell their product 2) both firms offer a subscription service or 3) one firms sells its product while the other firm offers a subscription service. Note that from these three outcomes, only the third outcome involves asymmetric strategies adopted by the firms. First, we observe that the first two outcomes involve Bertrand competition and each firm's equilibrium price is zero[3]. Consequently, we only describe the price equilibrium for the third asymmetric outcome.

Let period t represent a point in time such that $t-1$ periods have already elapsed. We first describe the consumers' decision problem. The value of the seller's product over the infinite horizon is U_s and the similar value for the renter is U_r. The corresponding prices in period t are p_{st} and p_{rt}. The state of the system in period t is the highest type of customer who has not bought the product at the beginning of period t and is denoted by σ_t. From the definition of σ_t as the highest type customer who has not bought prior to period t, it follows immediately that σ_t is weakly decreasing as time elapses:

$$\sigma_t \geq \sigma_{t+1} \geq \sigma_{t+2} \geq ..... \text{ for every } t$$

The value function of customer type in each period is the maximum surplus for the customer starting from that period over the infinite horizon. Hence, the value function $V_\theta(\sigma_t)$ for a customer

[3] The proof of this result is straightforward and available with the authors.

of type θ at state σ_t can be stated as a recursive function as follows:

$$V_\theta(\sigma_t) = Max \left\{ \begin{array}{lr} \text{Buy:} & \theta U_s - p_{s,t}) \\ \text{Rent:} & \theta(1-\delta)U_r - p_{r,t} + \delta V_\theta(\sigma_{t+1}) \\ \text{Do nothing:} & \delta V_\theta(\sigma_{t+1}) \end{array} \right\} \tag{1}$$

The first line of equation (1) indicates that if the customer buys the product in period t, then the customer exits the market and has no more choices to make in future periods. The second line indicates that if the customer chooses the subscription option, then the customer is free to choose any of the three options in the next period. The customer who chooses the "do nothing" option in period t simply postpones the decision to the next period but with a discount factor applied to utilities and prices. We set $U_s = 1$ and $Ur = r \leq 1$. Thus, r is the new perceived quality (after quality loss due to the subscription model). A lower r indicates a greater perceived quality loss due to subscription. Using the customer value function in equation (1) and the fact that $U_s \geq U_r$, we state a lemma characterizing the equilibrium states of the game:

Lemma 1 *Given optimal pricing by both firms, σ_{t+1} is the threshold customer type that is indifferent between buying in period t versus renting in the period t and buying in period $t+1$:*

$$\sigma_{t+1} = \frac{p_{s,t} - p_{r,t} - \delta p_{s,t+1}}{(1-\delta)(1-r)}$$

The result in lemma 1 immediately leads to the calculation of the demand curve in each period. If σ_t is the highest customer type that has not bought the product in period t and σ_{t+1} is the highest customer type that has not bought the product in period $t+1$, then the demand in period t is simply a summation of all customer types located between these thresholds. Given that customer type θ is uniformly distributed, it follows that this demand is equal to the difference between the two thresholds. Thus, the demand in each period for the seller ($q_{s,t}$ in period t for the seller) is simply the difference between successive states. Customers who do not buy in period t may rent provided the one-period consumer surplus obtained by them through renting is positive. Given that the single period utility through a subscription service is $(1-\delta)\,r$ and the price paid is $p_{r,t}$, a customer θ who does not buy will rent only when $\theta \cdot (1-\delta)\,r - p_{r,t} \geq 0$. Given that, in any period t, only customer types below σ_{t+1} have not bought the product, we can once again invoke the uniform distribution of customer type to calculate subscription / rental demand in period t. Thus, the demand in each period for the renter ($q_{r,t}$ in period t for the renter) depends on the successive state and the per-period rental price offered in period t. Both demand functions are stated below.

$$q_{s,t} = \sigma_t - \sigma_{t+1} \tag{2}$$

$$q_{r,t} = \sigma_{t+1} - \frac{p_{r,t}}{(1-\delta)\,r} \tag{3}$$

Let $\pi_{s,t}$ be the total discounted profit of the seller starting in period t over the infinite horizon. Let $V_s(\sigma_t)$ be the value function for the seller at state σ_t. The Bellman equation for the seller is:

$$V_s(\sigma_t) = Max_{p_{s,t}}\left(\pi_{s,t} = p_{s,t}(\sigma_t - \sigma_{t+1}) + \delta V_s(\sigma_{t+1})\right) \tag{4}$$

and note that σ_{t+1} is a function of $p_{s,t}$.

Similarly, let $\pi_{r,t}$ be the total discounted profit of the firm offering the subscription service starting in period t over the infinite horizon. Let $V_r(\sigma_t)$ be the value function for the renter at state t. The Bellman equation for the renter is:

$$V_r(\sigma_t) = Max_{p_{r,t}}\left(\pi_{r,t} = p_{r,t}\left(\sigma_{t+1} - \frac{p_{r,t}}{(1-\delta)r}\right) + \delta V_r(\sigma_{t+1})\right) \tag{5}$$

We look for an equilibrium by restricting attention to linear and stationary strategies[4]. Similar to the monopoly model of selling versus renting in Tirole(1988), we define the following functions:

$$p_{s,t} = \mu_s \cdot \sigma_t \tag{6}$$

$$p_{r,t} = \mu_r \cdot \sigma_t \tag{7}$$

$$\sigma_{t+1} = \lambda(p_{s,t} - p_{r,t}) \tag{8}$$

where λ, μ_r, and μ_s are constants with $0 < \mu_r,\ \mu_s \leq 1$ and $\lambda > 0$.

Proposition 1 *Restricting attention to linear and stationary strategies, the best response functions for the firm offering a subscription service and the firm selling a product, μ_r and μ_s respectively, are:*

$$\mu_r^* = \frac{(1-\delta)r\mu_s}{1-\delta+\delta\mu_s}$$

$$\mu_s^* = \frac{\sqrt{(1-\delta)(1-r)^2 + \delta\mu_r(1-r)} - (1-\delta)(1-r)}{\delta}$$

[4] We restrict attention to linear strategies because the demand in each period is a linear function of prices. Stationary strategies are chosen because they are easy to implement if a stationary equilibrium indeed exists.

with $0 < \mu_r \leq 1$, $0 < \mu_s \leq 1$. *Given these response functions, there exists a unique subgame perfect equilibrium in pure strategies.*

Proof in the appendix

Given these response functions, there exists a unique subgame perfect equilibrium in pure strategies. Given the nature of the equilibrium pricing policy and a state σ_t, the equilibrium profit in period t for both the seller and the renter can be written out. This is accomplished by setting $p_{s,t}$, $p_{r,t}$ and σ_{t+1} in terms of equilibrium values μ_s^e, μ_r^e, and λ^e using proposition 1. Define the equilibrium profits for the seller and renter in period t to be $\pi_{s,t}^e$ and $\pi_{r,t}^e$. The expressions for the profits are:

$$\pi_{s,t}^e = \left(\frac{\mu_s^e(1-\lambda^e(\mu_s^e-\mu_r^e))}{1-\delta\lambda^{e2}(\mu_s^e-\mu_r^e)^2}\right)\sigma_t^2 \tag{9}$$

$$\pi_{r,t}^e = \left(\frac{\mu_r^e}{(1-\delta)r}\right)\left(\frac{(1-\delta)r\lambda^e(\mu_s^e-\mu_r^e)-\mu_r^e}{1-\delta\lambda^{e2}(\mu_s^e-\mu_r^e)^2}\right)\sigma_t^2 \tag{10}$$

4 Numerical Results

The previous section analyzes the equilibrium outcomes of duopoly price competition between differentiated products over an infinite horizon. However, the equilibrium prices and the state of the system are not available in closed form. As a result, we can analyze the comparative statics at equilibrium only numerically. Figure 1 charts the states of the system across time at equilibrium prices for $r = 0.5$. The state of the system is exponentially decreasing and asymptotically approaches zero.

Figure 2 does the same for equilibrium prices and reveals similar results. However, the seller's price is always higher than the renter's price since the seller offers a higher value product over a longer time horizon. In figure 3, we chart the profit of both firms in the duopoly as the level of differentiation decreases (r increases).

In the seller-renter duopoly, consumers postpone their commitment to a high value seller by renting a lower value product from a renter for a limited number of time periods. They eventually switch to the seller but at a substantially lower price. However, this does not answer the question as to

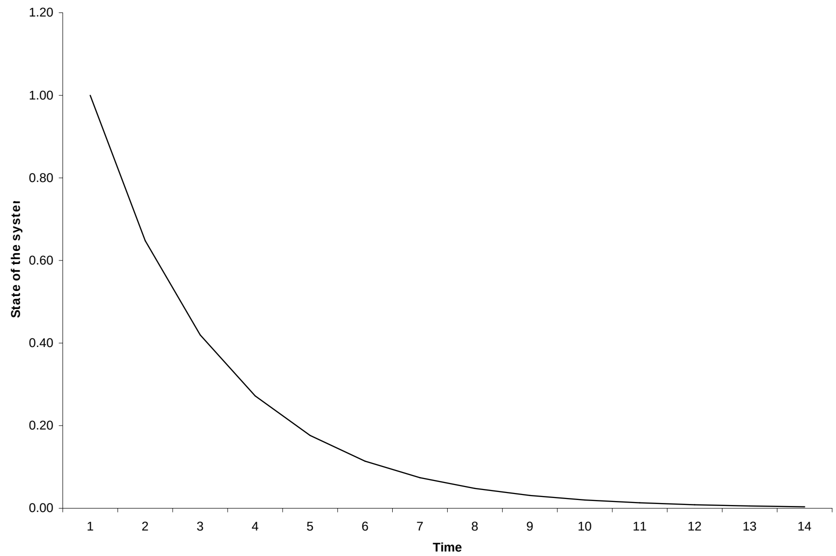

Figure 1: States of the system across time

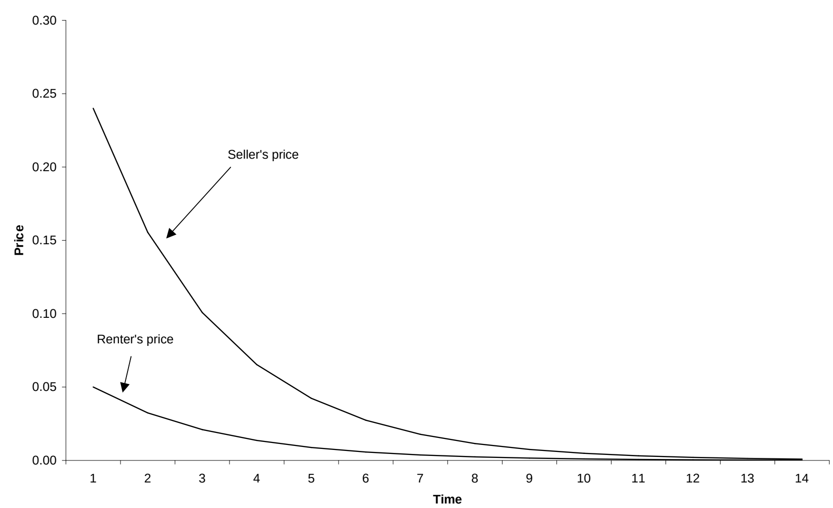

Figure 2: Equilibrium prices across time

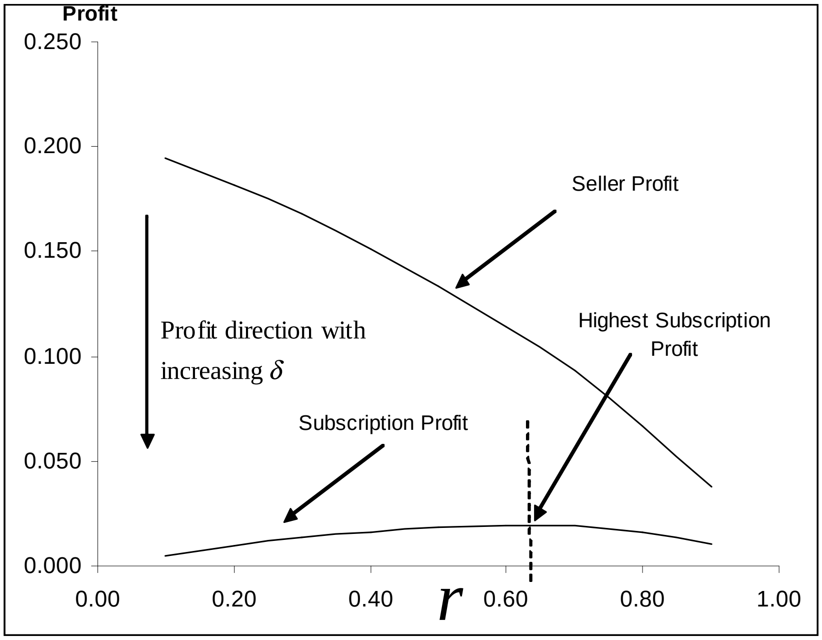

Figure 3: Profits in a seller-renter duopoly

how the level of differentiation (the parameter r) affects individual firm profit. Through figure 3, we capture the effects of differentiation on firm profits (we fix $\delta = 0.5$ for this purpose). At low values of r (high differentiation), the subscription service incorporates little value in its product and hence makes low profits. As the value of r increases (differentiation decreases), the value of the subscription service increases and hence profit increases. However, as the differentiation continues to decrease (r increases), the effects of price competition take over and both firms experience a significant decline in profits. This implies that there is an optimal level of differentiation for the subscription service. Since the two firms are symmetric and differentiation is caused primarily by loss in quality of the subscription service, this implies that there is some sort of "optimal loss" for the subscription service. This result is similar in spirit to the differentiated price competition result in Moorthy(1988). This provides guidance on making investments in enhancing quality for the subscription service. It might be worth making such investments only if the loss level brings quality below the optimal point of differentiation. When the loss level is low and the effective service quality is above the optimal point of differentiation, the firm providing the subscription service might make further efforts to reduce service quality by perhaps disabling some features of the original product.

We also seek to analyze the change in firm profits as a function of discount factor δ. At $\delta = 0$, both settings are alike with no regard for future time periods. As the discount factor increases, the seller is hit by the time inconsistency problem and in the duopoly, this also affects the competing renter. Hence the profit for each firm in the duopoly involving a seller and a renter decreases with an increase in the discount factor and approaches zero at $\delta = 1$. This illustrates the generalization of the Coase conjecture from a monopoly to a duopoly setting involving competition between a higher value seller and a lower value renter.

5 Extensions to the base model

In this section, we discuss the impact of relaxing some of the assumptions underlying the base model from the previous sections. In particular, we examine two extensions: first, we examine the case of firms that are asymmetric in their base quality offering; second, we analyze a case where the discount rate used for product quality and producer / consumer surplus are different.

5.1 Asymmetric firms

Suppose that the two firms vary in the quality of the base product. The firm that offers a product of higher quality is labelled H while the other firm is labelled L. We normalize the high quality product to 1 and denote the quality of the low end product by $\triangle < 1$. Further, we assume that the perceived quality loss when the product is offered as a subscription service is the same in percentage terms for both firms. As in the base model, offering a subscription service requires us to adjust the quality of the base product (for both firms) by a multiplicative factor r. However, since the firms are differentiated ex-ante, the exact level of differentiation will vary from the base model depending on the exact product / subscription service strategy. We denote different strategy combinations as follows: (X, Y) denotes a strategy combination where the firm with a higher base product quality offers X while the firm with lower base product quality offers Y. In our model, S denotes a product selling strategy and R denotes a subscription service or rental strategy. As is commonly observed in practice, we will restrict attention to the equilibrium where the firm with higher base quality adopts a product selling strategy and the firm with lower base quality offers a subscription service. Profit for a firm at equilibrium prices given a particular product / subscription strategy combination

(X, Y) is denoted by $\pi^T(X, Y)$ where T represents firm type (whether H or L). With the notation in place, we are in a position to write the conditions required for an equilibrium where firm H adopts a product selling strategy while firm L adopts a subscription service strategy.

$$\pi^H(S, R) \geq \pi^H(R, R)$$

$$\pi^L(S, R) \geq \pi^L(S, S)$$

The conditions listed above ensure that neither firm has the incentive to unilaterally deviate from its strategy in the (S, R) combination. The profit function for each firm at equilibrium depends on the level of differentiation. In the (S, R) combination, the level of differentiation is $1 - r \cdot \triangle$ whereas it is $\gamma \cdot (1 - \triangle)$ in the (R, R) combination and $1 - \triangle$ in the (S, S) combination. For asymmetric firms, $\triangle < 1$ and hence $\pi^H(R, R)$ and $\pi^L(S, S)$ are both strictly positive. This is unlike the symmetric firm case where both are equal to zero. Similarly, a lower $\triangle$ implies higher differentiation in the (S, R) combination thereby increasing $\pi^H(S, R)$ but this may either increase or decrease $\pi^L(S, R)$. Consequently, it is unclear based on differentiation alone whether an (S, R) equilibrium is more or less likely. However, tracking the movement of equilibrium profit with respect to δ provides greater insight. In the symmetric firm case, we know that (S, R) is an equilibrium for all values of $\delta < 1$ and provides equal (= zero) profit with respect to (R, R) and (S, S) at $\delta = 1$. For asymmetric firms, we know that profits of both firms under (S, R) equals zero at $\delta = 1$ (the analysis is the same as in the base model except that the lower quality product is adjusted by a factor $\triangle$). However, (R, R) provides positive profits[5] at $\delta = 1$. This implies that (S, R) is not an equilibrium at $\delta = 1$ for $\triangle < 1$. It can be shown that there exists a $\delta^* \in [0, 1)$ such that (S, R) is an equilibrium only for $\delta < \delta^*$. The exact value of δ^* depends on r and $\triangle$[6] with $\delta^* = 1$ at $\triangle = 1$. Thus, when the asymmetric firm case is compared with the symmetric case, the region over δ where (S, R) is an equilibrium decreases as firm asymmetry increases.

5.2 Different discount factor for quality and producer / consumer surplus

In many contexts, product or service quality and monetary values may be discounted differently. In the base model, we assigned a common discount factor δ to both quality and monetary values. In this subsection, we relax this assumption with respect to the base model. We assume that in

[5] An analysis of this case is straightforward and is available with the authors.

[6] The exact analysis is available from the authors.

any period t, current period utility for a customer is calculated from lifetime value of the product using a discount factor $d < 1$ and future period surplus is discounted using the same discount factor as before, δ. Since we are using δ as the discount factor for monetary values, we apply the same discount factor to producer profit in future periods. This alters the the expression for the state of the system as compared to that stated in Lemma 1:

$$\sigma_{t+1} = \frac{p_{s,t} - p_{r,t} - \delta p_{s,t+1}}{(1-\delta) - (1-d)(1-r)}$$

This leads to a change in the best response functions for both firms as compared to those stated in Proposition 1. The new expressions are:

$$\mu_r^* = \frac{1}{2}\left[\frac{(1-d)\cdot r\cdot \mu_s}{1-\delta(1-\mu_s)}\right]$$

$$\mu_s^* = \frac{\sqrt{(1-\delta)(1-r-\delta+d\cdot r)[1-(1-d)r-\delta(1-\mu_r)]} - (1-\delta)^2 + r(1-d)(1-\delta)}{(1-\delta)\delta}$$

Setting $d = \delta$ in the above best response functions gives us the same equations as in Proposition 1. In addition, the above best response functions have the same properties for any $d < 1$ as the corresponding ones in the base model and lead to a unique stationary equilibrium in pure linear strategies. While the exact values of the equilibrium states of the system and the prices may change when $d \neq \delta$, the basic insights remain the same. Thus, the insights from our base model are robust to differences in discount rate between product / service quality and producer / consumer surplus.

6 Discussion

The notion of offering online subscription services has become very common in the information goods industry. In some industries such as software services, this is merely a rebirth of a model that was widely prevalent during the era of mainframes. Many information goods can be classified as durable goods, and this leads one to conclude that the ability to offer subscription services can only be good for the firm. This conclusion follows from a basic application of the Coase conjecture. However, an often observed fact with online subscription services is the loss in quality due to issues with data security and network reliability. The objective of our work was to understand how this loss in quality combined with competition might affect our original intuition.

To examine this closely, we build an infinite horizon competitive model. When two symmetric firms compete in the market, quality loss may offer an avenue for differentiation. This occurs particularly when one firm decides to sell while the other decides to rent. This differentiation benefits both firms but ensures higher profits for the firm that sells as compared to the firm that rents, since the firm that offers a subscription has lower quality. However, if the loss in service quality is too high for the subscription firm, it might lead to much lower profits. For instance, when quality deteriorates to zero, the firm no longer makes a profit as customers can derive no value from the subscription service. Thus, a firm that chooses to offer a subscription service in the presence of a competitor that sells its product must be aware of an optimal level of quality loss that maximizes its profit. This acts as a guide towards making investments in improving quality. If quality loss is not high, there may not be much value in making such investments as they are only likely to hurt the firm by minimizing differentiation in the context of competition. On the other hand, if the loss level is high, then such investments would bring the firm closer to an optimal level of differentiation. We also discuss the robustness of these results to other settings, particularly the case of asymmetric firms and a case where the discount factor used for product / service quality is different from the discount factor used for producer / consumer surplus. We find that the main insights remain the same although the specific values of equilibrium prices may change.

Going back to the base model, one question to ask would be: since the subscription service makes lower profit than the product seller, which of the two symmetric firms would occupy the position of the subscription service? A possible answer is that the first mover might occupy a seller's position while a late entrant would offer the subscription service. Our model of competition only addresses the outcome of competition in a market over a snapshot in time where such decisions have already occurred in the past. Future work might consider evaluating the outcome of such firm entry dynamics.

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7 Appendix

Proof of Lemma 1 From the definition of σ_t as the highest type customer who has not bought prior to period t, it follows immediately that σ_t is weakly decreasing as time elapses:

$$\sigma_t \geq \sigma_{t+1} \geq \sigma_{t+2} \geq ..... \text{ for every } t$$

Next, we show that in every period, for given renter's prices, there exists a threshold that splits the market into two segments: one segment of customer types higher than the threshold that has bought the product prior to the period, and a segment of customer types lower than the threshold that has not bought the product. This is shown by contradiction. Suppose the threshold does not exist. Then there exist customers θ_1 and θ_2 such that $\theta_1 < \theta_2$ and in some time period, customer of type θ_1 has already bought the product and θ_2 has not. This contradicts the fact that $V_\theta(t)$ that is monotonically increasing in θ[7]. Hence the threshold must exist. Since this threshold exists, and using the definition of the state of the system, this threshold in period t must be σ_t. Furthermore, this implies that if $\sigma_t = \sigma_{t+1}$ for some t, the seller experiences zero demand in period t. At optimal pricing, at each period t, given the renter's prices, the seller will never experience zero demand. This is again shown by contradiction. Assume that the seller experiences zero demand in period t. Now suppose that the seller applies a new price in period t: $p_{s,t}^{new} - p_{r,t} = p_{s,t+1} - p_{r,t+1}$. This ensures that some of the original demand in period $t+1$ occurs in period t without affecting the demand (and hence profit) from period $t+2$ onwards.

$$q_{s,t}^{new} + q_{s,t+1}^{new} = q_{s,t+1}^{old}$$

$$q_{s,v}^{new} = q_{s,v}^{old} \text{ for all } v > t+1$$

where $q_{s,t}^{old}$ and $q_{s,t}^{new}$ represent the demands in period t for the seller before and after the change in prices.

Since the original prices have the firm earning zero revenue at time t, the new price sequence would give the firm strictly higher discounted profits. In other words, under optimal pricing, no period experiences zero demand for the seller, which further implies:

$$\sigma_t > \sigma_{t+1} > \sigma_{t+2} > .... \text{ for every } t$$

[7] Proof of the fact that $V_\theta(t)$ that is monotonically increasing in θ is straightforward and is available with the authors.

Given the existence of the threshold that splits the market into customers who have already bought and those who have not, all $\theta \in (\sigma_{t+1}, \sigma_t)$ buys in period t and all $\theta \in (\sigma_{t+2}, \sigma_{t+1})$ buys in period $t-1$. Setting $U_s = 1$ and $U_r = r < 1$ in equation (1), the value function for the customer, a similar argument for the renter shows that all $\theta \in \left(\left(\frac{1}{1-\delta}\right)\frac{p_{r,t}}{r}, \sigma_{t+1}\right)$ rent the product in period t and this segment is never zero at optimal renters prices. So σ_{t+1} is the indifference point between two market segments: buying in period t versus renting in period t and buying in period $t-1$. We express this condition using equation (1):

$$\sigma_{t+1} - p_{s,t} = \sigma_{t+1}(1-\delta)r - p_{r,t} + \delta(\sigma_{t+1} - p_{s,t+1})$$

which simplifies to:

$$\sigma_{t+1} = \left(\frac{1}{(1-\delta)(1-r)}\right)(p_{s,t} - p_{r,t} - \delta p_{s,t-1}) \tag{11}$$

as required.

Proof of Proposition 1 We substitute for the state of the system in period $t+1$ using equation (8) in the value functions of the two firms given by equations (4) and (5):

$$V_s(\sigma_t) = Max_{p_{s,t}}\left(\pi_{s,t} = p_{s,t}(\sigma_t - \lambda(p_{s,t} - p_{r,t})) + \delta V_s(\lambda(p_{s,t} - p_{r,t}))\right) \tag{12}$$

$$V_r(\sigma_t) = Max_{p_{r,t}}\left(\pi_{r,t} = p_{r,t}\left(\lambda(p_{s,t} - p_{r,t}) - \frac{p_{r,t}}{(1-\delta)r}\right) + \delta V_r(\lambda(p_{s,t} - p_{r,t}))\right) \tag{13}$$

Addressing the seller's problem first, the profit from period t onwards over the infinite horizon (denoted by $\pi_{s,t}$) is:

$$\pi_{s,t} = p_{s,t}(\sigma_t - \lambda(p_{s,t} - p_{r,t})) + \delta V_s(\lambda(p_{s,t} - p_{r,t})) \tag{14}$$

For any given renter's price $p_{r,t}$, differentiate the profit function with respect to $p_{s,t}$ to give:

$$\frac{\partial \pi_{s,t}}{\partial p_{s,t}} = \sigma_t + \lambda p_{r,t} - 2\lambda p_{s,t} + \delta\lambda V_s'(\lambda(p_{s,t} - p_{r,t})) \tag{15}$$

$$\frac{\partial^2 \pi_{s,t}}{\partial p_{s,t}^2} = -2\lambda + \delta\lambda^2 V_s''(\lambda(p_{s,t} - p_{r,t})) \tag{16}$$

Differentiating equation (12) with respect to t and invoking the envelope theorem:

$$V_s'(\sigma_t) = p_{s,t} \tag{17}$$

Using the linearity of $p_{s,t}$ as a function of σ_t from equation (6) and substituting in equation (17):

$$V_s'(\sigma_t) = \mu_s \sigma_t \tag{18}$$

and

$$V_s''(\sigma_t) = \mu_s \tag{19}$$

Using equations (18) and (19) in equations (15) and (16) respectively.

$$\frac{\partial \pi_{s,t}}{\partial p_{s,t}} = \sigma_t + \lambda p_{r,t} - 2\lambda p_{s,t} + \delta\lambda^2\mu_s(p_{s,t} - p_{r,t}) \tag{20}$$

$$\frac{\partial^2 \pi_{s,t}}{\partial p_{s,t}^2} = -2\lambda + \delta\lambda^2\mu_s \tag{21}$$

Using equations (6) and (8) in the indifference equation from lemma 1 to get the identity:

$$\lambda = \frac{1}{(1-\delta)(1-r) + \delta\mu_s} \tag{22}$$

which on rearrangement gives:

$$\delta\lambda\mu_s = 1 - (1-\delta)(1-r)\lambda \tag{23}$$

Using equation (23) in equations (20) and (21), setting the first derivative to zero and further simplification:

$$\mu_s^* = \frac{\sqrt{(1-\delta)(1-r)^2 + \delta\mu_r(1-r)} - (1-\delta)(1-r)}{\delta} \tag{24}$$

$$\frac{\partial^2 \pi_{s,t}}{\partial p_{s,t}^2} = -\lambda(1 + (1-\delta)(1-r)\lambda) \tag{25}$$

Equation (25) shows the concavity of the profit function in $p_{s,t}$ for price as a linear, stationary function. Equation (24) is the first order condition that provides the best response price for the seller.

We shift attention to the renter's problem. The profit from period t onwards over the infinite horizon (denoted by $\pi_{r,t}$) is:

$$\pi_{r,t} = p_{r,t}\left(\lambda(p_{s,t} - p_{r,t}) - \frac{p_{r,t}}{(1-\delta)r}\right) + \delta V_r(\lambda(p_{s,t} - p_{r,t})) \tag{26}$$

Knowing the structure of V_r, it is clear that $V_r(\lambda(p_{s,t} - p_{r,t}))$ is not a function of $p_{r,t}$. Consequently, taking derivatives in equation (26):

$$\frac{\partial \pi_{r,t}}{\partial p_{r,t}} = \lambda p_{s,t} - 2\lambda p_{r,t} - \frac{2p_{r,t}}{(1-\delta)r} \tag{27}$$

$$\frac{\partial^2 \pi_{r,t}}{\partial p_{r,t}^2} = -2\lambda - \frac{2}{(1-\delta)r} < 0$$

The negative second derivative guarantees concavity and setting the first derivative in equation (27) to zero thus provides the best response renters price:

$$\mu_r^* = \frac{1}{2}\left(\frac{(1-\delta)r\mu_s}{1-\delta+\delta\mu_s}\right)$$

Rearranging this equation:

$$\mu_s = \frac{2(1-\delta)\mu_r^*}{(1-\delta)r - 2\delta\mu_r^*} \tag{28}$$

Differentiating equation (28) with respect to μ_r :

$$\frac{d\mu_s}{d\mu_r^*} = \frac{2(1-\delta)^2 r}{((1-\delta)r - 2\delta\mu_r^*)^2} \text{ and } \frac{d^2\mu_s}{d\mu_r^{*2}} = \frac{8\delta(1-\delta)^2 r}{((1-\delta)r - 2\delta\mu_r^*)^3} \tag{29}$$

Using equation (28), the second derivative can be rewritten as:

$$\frac{d^2\mu_s}{d\mu_r^{*2}} = \frac{\delta r}{(1-\delta)} \cdot \left(\frac{\mu_s}{\mu_r^*}\right)^2 > 0$$

Thus, by inspection, μ_s is a strictly convex increasing function of μ_r^* :

For the seller:

$$\mu_s^* = \frac{\sqrt{(1-\delta)(1-r)^2 + \delta\mu_r(1-r)} - (1-\delta)(1-r)}{\delta} \tag{30}$$

Evaluating the slope of the best response function:

$$\frac{d\mu_s^*}{d\mu_r} = \frac{1-r}{2\sqrt{(1-\delta)(1-r)^2 + \delta\mu_r(1-r)}} \tag{31}$$

and

$$\frac{d^2\mu_s^*}{d\mu_r^2} = -\frac{\delta(1-r)^2}{4\left((1-\delta)(1-r)^2 + \delta\mu_r(1-r)\right)^{\frac{3}{2}}}$$

By inspection, μ_s is a strictly concave increasing function of μ_r.

Next, we state a lemma (proof available from the authors) that aids in proving uniqueness of equilibrium.

Lemma 2 *If f (strictly convex) and g (strictly concave) are increasing functions and for $m, n \in R$ $(n > m)$:*

$f(m) < g(n)$ and $f(n) > g(m)$

Then f and g intersect exactly once in the interval (m, n)

Using equations (28) and (30), set f and g as follows:

$$f(\mu_r) = \frac{2(1-\delta)\mu_r}{(1-\delta)r - 2\delta\mu_r}$$

$$g(\mu_r) = \frac{\sqrt{(1-\delta)(1-r)^2 + \delta\mu_r(1-r)} - (1-\delta)(1-r)}{\delta}$$

Also, set $m = 0$ and $n = \frac{(1-\delta)r}{2}$, and this gives:

$$f(0) = 0 < \sqrt{1-\delta}\left(\frac{1-\sqrt{1-\delta}}{\delta}\right)(1-r) = g(0)$$

$$f\left(\frac{(1-\delta)r}{2}\right) = 1 > \frac{\sqrt{(1-\delta)(1-r)^2 + \frac{\delta(1-\delta)r(1-r)}{2}} - (1-\delta)(1-r)}{\delta} = g\left(\frac{(1-\delta)r}{2}\right)$$

Using the lemma, f and g have a unique intersection and this provides a unique equilibrium in pure strategies that are linear and stationary.