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THE MODELS OF ECONOMIC CHOICE THEORY: A PARADIGM FOR NON-ECONOMISTS

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Introduction

The ultimate goal of social science is to explain individual and group behavior within given institutional constraints. In practice this means developing models which effectively describe and predict human behavior. Recent experience has shown a particular approach to modeling individual behavior to be especially useful. The approach in question has been developed by economists and consists of using the analytical structure of utility theory to focus attention on the determinants of individual choice and then analyzing the responsiveness of individual choices to changes in these determinants. The success of model building in this format is evidenced by the fact that a major portion of microeconomic theory is now known as choice theory.

Choice theoretic problems are naturally partitioned into two classes: Problems in which the consequences of alternative courses of action (choices) are deterministic and problems in which choices are associated with stochastic consequences. The characteristic of choice theory problems that differentiates them from typical decision theory problems is the fact that the models of choice theory are qualitative. The abundance of qualitative models in the social sciences in general and in economics in particular is due primarily to a lack of knowledge about the form of the functions involved in the various models and the consequent unwillingness on the part of theorists to specify any but their most general properties (e.g. convexity or concavity). Although the explanatory power of qualitative models is a far cry from that of the models of classical physics, many interesting questions may be addressed.

It is the purpose of this paper to provide a fairly detailed analysis of a specific model of individual choice with the intention of exposing the apparatus of choice theory to non-economists. Formally, the model to be presented is representative of a broad class of models in economics and assumes optimizing behavior on the part of the individual agent. In the present context, "optimizing behavior" is taken to mean that the agent chooses from among alternative courses of action in such a manner that the particular alternative(s) chosen leaves the agent as "well off" as possible. Questions of primary interest include the response of an agent's choices to changes in policy parameters and other underlying "conditions."

In what follows we will be studying a particular kind of criminal behavior. Specifically, we model the ignition decision problem confronting an arsonist. This choice reflects our desire to illustrate the scope of the analytical framework utilized and at the same time to provide an interesting example of an activity where the stochastic nature of consequences is an essential ingredient in the decision problem.

For choice problems with stochastic consequences, economists have come to rely heavily upon the expected utility theorem of von Neumann and Morgenstern.[1] In a nutshell, the expected utility theorem consists of a set of axioms concerning the structure of an individual's preferences, which if satisfied, imply that choices among alternative uncertain courses of action are made *as if* the individual were maximizing expected utility. This is not

[1]The classical reference to this theorem is von Neumann and Morgenstern [1944]. A more recent proof may be found in Arrow [1965], Chapter I.

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the place for a discussion of these axioms.[2] It suffices to say that they are quite general and imply the individual's preferences are *representable;* that is, there exists a function which assigns a real number to the consequence of each possible action in such a way as to reflect the individual's preferences. Economists call such a function a *utility function.* The expected utility theorem provides a powerful analytical framework for modeling choice problems under uncertainty, a framework utilized in this paper.

In the first section of the paper our analysis is focused on the case where punishment for arson is by fine only. The sensitivity of incendiary activity to several policy changes is investigated. In the following section, prison sentences are added to fines as a possible punishment and the effects of the same policy changes are explored. The effect of including both sentences and fines as penalties is to further complicate penalty options, thereby reducing the number of unambiguous results obtainable. The final section of the paper contains a brief methodological discussion on modeling human behavior.

A Choice Theoretic Model of Arson

The question at hand is how to best represent the decision problem confronting an arsonist. An economist is likely to consider the arsonist's problem as a time allocation problem with uncertain consequences. In particular, how much time will be allocated to the planning and execution of ignitions if the arsonist acts in his "best interest," as he sees it.[3] And this is precisely the tack taken in this paper. Ignitions are viewed as the outcome of a time allocation decision with uncertain consequences. Outcomes are uncertain since once a fire is started the individual may be apprehended and subjected to a fine, a prison term, or both. Because traditional policy prescriptions designed to deter arson (or for that matter any type of criminal behavior) invariably include increases in prosecution and penalties, we will be interested in determining whether such prescriptions follow from our model of individual choice.[4]

[2]Discussions of this theorem and alternative sets of axioms which lead to the maximization of expected utility may be found in the previous two references and in Marschak [1950], Herstein and Milnor [1953] and Marschak and Radner [1972]. An easy to read discussion of human choice theory may be found in McFadden [1974].

[3]Arson may be committed either for "fun" or for economic gain (e.g. insurance payoffs). So as to give a unified interpretation throughout, we assume the individual's motives are non-economic. The fact that an arsonist's *motives* are non-economic does not mean he or she experiences no economic *repercussions* from arson. Indeed, if apprehended and convicted, the fine or prison sentence or both and the earnings and opportunities foregone from "loss of name" and/or time in prison, may constitute a severe economic loss. But at the same time, the fire may have been started for "fun"—a non-economic motive.

[4]For a detailed methodological discussion on modeling criminal behavior, see Block and Heineke [1975].

The central concept in an analysis of time allocation under uncertainty is the individual's von Neumann-Morgenstern (N-M) utility function. This function contains all the needed information pertaining to the individual's evaluation of the various "states of the world," and provides a ranking of states in terms of their relative worth as perceived by the individual. The N-M utility function is denoted as $U(W, L)$, where W represents the individual's wealth and L his time allocation to arson related activities (incendiary activity).[5] We assume that the utility function is differentiable, that the individual prefers more wealth to less, $U_W > 0$ and that planning and starting fires is a desirable activity in the eyes of an arsonist, $U_L > 0$ (see f.n. 3).

The following definitions will be used:

$D(L)$ $\equiv$ the damage caused by a fire. Damage is a function of the amount of time an incendiarist spends planning and executing ignitions and is assumed to be a differentiable function of L. It seems reasonable to assume that the more time spent the higher will be the damages. That is, $D'(L) > 0$. Obviously, $D(0) \equiv 0$.

p $\equiv$ the individual's estimate of the probability of apprehension if a fire is started.[6]

W^o $\equiv$ the individual's "initial" wealth.

F $\equiv$ the fine as a multiple of fire damages, $F > 0$.

W $\equiv$ $W^o - FD(L)$, the individual's wealth if apprehended after starting a fire causing damage $D(L)$.

According to the expected utility theorem, the time allocation decision will be made by the individual *as if* it were the solution to

$$(1)\quad \max_{L} \left\{(1-p)\,U(W^o, L) + pU(W^o - FD(L), L)\right\}$$

subject to the conditions that $W \geq 0$ and $0 \leq L \leq \hat{L}$, where $\hat{L}$ is the maximum amount of time which could be allocated

[5]For a general analysis of the allocation of time under uncertainty, see Block and Heineke [1973].

[6]It is often reasonable to assume $p = p(L)$ with $p'(L) > 0$. Although many of the results reported below would remain unchanged under this specification, so as not to needlessly complicate what is essentially an expository paper, I have assumed p to be independent of L. It should also be noted that p may not be the "true" probability of apprehension. All that is needed to model the arsonist's decision problem is his *subjective* evaluation of his chances of being caught—our parameter p. An interesting question which is outside the scope of this paper, involves this relation between p and the "true" value of p, say, P. This relationship is especially important in law enforcement since the control variable for a law enforcement agency is P and not p. See Block and Lind [1975] for a discussion of this topic.

to arson and arson-related activity. The expression within the brackets { } in (1) is the expected utility the individual derives from L "hours" of arson related activity, given an initial wealth of W^o. The arsonist's decision problem in this setting is to choose L such that this expression is maximized.[7] We will designate L^* as the solution to (1).[8]

The necessary condition for a relative maxima in L is then[9]

$$(2) \quad H_L = (1-p)U_L^1 + p\,[U_W^2\,(-FD') + U_L^2\,] \leqslant 0.$$

where $H \equiv (1-p)\,U\,(W^o, L) + pU\,(W^o - FD(L), L)$, $U^1 \equiv U\,(W^o, L)$ and $U^2 \equiv U\,(W^o - FD(L), L)$. This inequality is the fundamental concept in our analysis since it determines the time the individual will spend at arson related activities. Consequently, most of the rest of the paper may be thought of as a series of operations designed to ferret out the information implicit in (2).

Let's begin by considering policies designed to deter arson. Intuition and the "conventional wisdom" lead one to suspect that increasing p or F or both should result in decreases in incendiarism. The implicit reasoning behind such a conclusion is simple: Increases in either of these policy variables has the effect of increasing the expected costs of starting fires and hence should tend to deter incendiarism. Unfortunately, such reasoning is not valid in general as examination of inequality (2) indicates. Since F and p enter both $U_W{}^2$ and $U_L{}^2$ it will not be possible to determine the response of L to changes in p or F without further analysis. In any event, it will generally not be desirable to adopt policies which completely deter incendiarism (if such policies exist). The reason for this is that investigating the origins of fires, apprehending those who start them, and then convicting them in a court of law is a resource consuming process. It will be desirable to plow resources into these activities only as long as increases in expenditure yield even greater decreases in losses. Typically, the optimal policy will be to tolerate some arson and, in this case, the marginal response of L to policy changes is of interest. To explore this question in more depth, we consider the responsiveness of incendiary activity to changes in wealth, the severity of punishment, and the level of enforcement.

[7]Since any value chosen for L induces a probability density function on W, the decision problem may be viewed as one of choosing among probability densities such that expected utility is maximum.

[8]We assume the "upper bound" constraint on L is non-binding, i.e., $0 \leqslant L^* < \hat{L}$, since it seems highly unlikely that "all" of the individual's time would be allocated to L, given the nature of the activity.

[9]As usual, we adopt the practice of indicating partial derivatives of a function with subscripts. So for example $H_L \equiv \partial H/\partial L$, $H_{LL} \equiv \partial^2 H/\partial L^2$ and $U_{LW} \equiv \partial^2 U/\partial L \partial W$.

The Method of Analysis

The question now arises as to how one goes about calculating the responsiveness of incendiary activity to these various parameter changes. Clearly we are not interested in the responsiveness of just any old value of L. Instead our interest lies in how L^*, the optimal value of L, responds to changes in wealth, the severity of punishment, and the level of enforcement. Now if a continuously differentiable function ϕ exists such that

$$(3) \quad L^* = \phi\,(p, F, W^o),$$

then the answers to these questions may be interpreted as being given by the signs of $\partial L^*/\partial W^o$, $\partial L^*/\partial F$ and $\partial L^*/\partial p$, respectively.[10]

For the important case where H has a "regular," "internal" relative maxima at L^* such a function ϕ does indeed exist and is unique.[11] This follows from the hypothesis of the implicit function theorem which, in the case at hand, requires only that the Jacobian associated with equation (2) not vanish at L^*. For a regular, internal maxima this is assured.[12]

Economists would call (3) the *reduced form* of the model given in equation (1). In general, reduced form equations are the "solutions" of a model in which all endogenous variables have been expressed as a function of the model's parameters. In this context, the purpose of the present analysis is to sign the partial derivatives of the reduced form equation. In addition, it should be pointed out that an analysis of the type we have sketched does not require explicit solution for L^*, since merely knowing that the function ϕ exists allows us to solve for the reduced form derivatives.

[10]In equation (3) we have represented only parameters which appear explicitly in our model, the implicit parameters in the utility function and D (L) being subsumed into ϕ. If we were willing to furthur specify U or D, say for example as to the class to which one or both of these functions belong, then the parameters associated with these classes of functions would also appear explicitly in (3).

[11]We use the words "regular" and "internal" to mean $H_{LL} < 0$ at L^* and $L^* > 0$, respectively. "Regularity" of the maximum insures $H_{LL} < 0$ and not, e.g., $H_{LL} = 0$ and $H_{LLLL} < 0$, while "internal" solutions restrict our attention to analyzing only the behavior of individuals who actually spend some time at arson and related activities, i.e., $L^* \neq 0$. We maintain these assumptions throughout. Notice also that the fact that ϕ is unique is crucial. If not, one would have a different derivative for each possible function ϕ which would greatly complicate the analysis.

[12]The Jacobian associated with (2) at L^*, is of course merely H_{LL} evaluated at L^*. Hence, as long as the maxima is "internal" and "regular," $H_{LL} < 0$ at L^* and the Jacobian does not vanish.

Incendiary Activity and Wealth

The first question to be examined is the response of incendiary activity to changes in the level of wealth, W^o. As we have seen, this may be accomplished for "internal" solutions by differentiating equation (2) with respect to W^o. In which case

$$(4)\ \partial L^*/\partial W^o = [pFD'U^2_{WW} - (1-p)\,U^1_{LW} - pU^2_{LW}\ ]/H_{LL}$$

Although the denominator of (4), H_{LL}, must be negative for a regular, interior relative maxima and p, F and D′ are each positive, signing (4) will require, at a minimum, some assumption about the signs of U^2_{WW} and EU_{LW}.[13]

The function U_{LW} measures the sensitivity of U_L to changes in wealth, where U_L represents the "enjoyment" the arsonist derives from planning and executing ignitions (at the margin). The question to be answered at this point is what effect small changes in wealth have upon the marginal psychic rewards to arson. It would seem to be acceptable as a first approximation to assume that these psychic rewards are invariant in wealth. The sign of $\partial L^*/\partial W^o$ then depends upon U_{WW}. In models in which the consequences of actions are uncertain, the sign of U_{WW} has special significance–the import of which is the subject of the following digression.

Digression on Sign [U_{WW}]

In economic theory the sign of U_{WW} is a measure of the agent's behavior toward risk. To explore this concept briefly consider a N-M utility function with only one argument, wealth, say V (W). As we have noted previously, an obvious restriction on V is $V_W > 0$. But what interpretation is to be given to the rate of change of V_W? The answer to this question depends upon the agent's preferences among prospects possessing varying degrees of risk. To see this consider an agent with initial wealth W^o who is offered a bet which involves winning or losing an amount h with probabilities p and 1-p respectively. The agent will be willing to accept the bet for values of p sufficiently large (certainly for p=1) and will refuse it if p is small (certainly for p=0). Now consider the special case where p = .5 (a fair bet). The choice is then between a certain wealth of W^o and a random wealth taking on values of W^o-h and W^o+h with probability .5 each. A *risk averse* individual is defined as one who prefers the certain income. In terms of the expected utility hypothesis, the utility associated with the certain wealth W^o is strictly greater than the expected utility associated with the fair gamble, i.e.

$$(5)\quad V(W^o) > (\tfrac{1}{2})\,V(W^o-h) + (\tfrac{1}{2})\,V(W^o+h)$$

or

$$(6)\quad V(W^o) - V(W^o - h) > V(W^o + h) - V(W^o)$$

The utility differences corresponding to equal changes in wealth are decreasing as wealth increases. Thus the utility function of a risk averter is characterized by the condition that V_W is strictly decreasing in wealth, i.e. $V_{WW} < 0$. Similar reasoning leads to the definitions of agents who are *risk neutral,* $V_{WW} = 0$ and agents with a preference for risk, $V_{WW} > 0$. If the utility function contains several arguments the agent is said, for example, to display *conditional risk aversion* if the second partial derivative with respect to the wealth argument is negative. (The agent's aversion to risk is conditional upon given values of the other arguments in the utility function.) Finally, we note that meaningful results using the expected utility theorem require that the individual's utility function be bounded from above and below and that boundedness implies the individual must be predominantly a risk averter.[14] In addition, since both risk neutrality and risk preference seem to be at odds with most observed behavior, analysts usually assume that individuals are risk averse, an assumption we shall accept.[15]

We are now in a position to sign equation (4). The assumptions that the agent is risk averse and that $U_{LW} = 0$ imply[16]

$$(4')\quad \partial L^*/\partial W^o > 0$$

If fines are the only form of punishment, increases in the level of wealth lead to increased incendiary activity! This conclusion obviously assumes that "tastes" do not change with wealth. For example, if increased wealth were automatically accompanied by an "emotional maturity" that made arson repugnant, then inequality (4′) would not hold. But, since there is little evidence that wealth induces such character transformations, we conclude that as long as incendiary activity is punishable *only* by fine, transfer payments of any kind will exacerbate the problem of incendiary ignitions. As is implicit in the last statement, punishment which includes prison sentences *may* alter this conclusion, a point returned to below.

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[13]The symbol E represents the expectation operator.

[14]See Arrow [1965, p. 93] for a discussion of this point.

[15]Note that risk aversion in wealth, $U_{WW} < 0$, in no way prevents the arsonist from being a risk taker in L, $U_{LL} > 0$. For a discussion of behavior toward risk in arguments of U other than wealth, see Block and Lind [1975].

[16]The two assumptions, $U_{LW} = 0$ and $U_{WW} < 0$, will be retained throughout.

Incendiary Activity and Fines

We next examine the response of incendiary activity to changes in the severity of the punishment. In this section, the "severity of punishment" is given by the magnitude of the fine. The response of the arsonist's time allocation to changes in the magnitude of the fine is given by

$$(6)\ \partial L^*/\partial F = (pD'U_W^2)/H_{LL} + Dp(U_{LW}^2 - FD'U_{WW}^2)/H_{LL}$$

Under the assumptions adopted, both terms in this sum are negative and

$$(6')\ \partial L^*/\partial F < 0.$$

Increasing the severity of monetary punishments has a deterrent effect on incendiary activity.

It should be kept in mind that this result does not imply that it will be possible to reduce incendiarism to zero or to any other arbitrary level by increasing the fine sufficiently. The reason for this lies in the limit on the magnitude of fines which is imposed by the non-negativity constraint on wealth. Even if W^o is interpreted as the discounted life-time earnings of the individual, the same problem may appear. Of course, the "more fun" it is to start fires (the larger is EU_L), the more difficult it will be to deter such activity via fines or any other means.

Incendiary Activity and Enforcement

We now investigate the response of incendiary activity to changes in the level of enforcement. The enforcement variable is p, the probability of apprehension once a fire is started and, like F, is a policy variable in the model. Differentiation of equation (2) yields

$$(7)\ \partial L^*/\partial p = (U_L^1 - U_L^2 + FD'U_W^2)\ /\ H_{LL}$$

It can be shown that

$$(7')\ \partial L^*/\partial p < 0$$

independent of any assumption about the individual's preferences other than behavior in accordance with the axioms of the expected utility theorem.[17] In particular, no assumption about U_{LW} is needed, nor is any assumption needed about the individual's behavior toward risk.

Enforcement Versus Fines: The Relative Effectiveness

It has been shown that increases in either the probability of apprehension or the severity of the fine will have a deterrent effect on incendiary activity. The obvious extension of the analysis is to ask, "which is most effective?" That is, would a one-percent increase in p or a one-percent increase in FD (by increasing F) have a greater impact in reducing incendiarism? Remarkably, even at the present level of generality, this question can be answered.

To see this, consider a simultaneous change in p and FD which leaves the expected punishment unchanged, i.e. $d(pFD) = 0$. Increasing p and decreasing F so that expected punishment is unchanged is a formal method of determining which of the two policy variables has the larger deterrent capability. The requirement on the expected fine is then

$$d(pFD) = pdFD + FdpD = 0$$

so that

$$dF/dp = -(F/p)$$

is the condition that insures that expected punishment is unchanged when p and F are both varied. Calculation of $\partial L^*/\partial p$, from (3), when this condition holds yields:

$$(8)\ \left.\frac{\partial L^*}{\partial p}\right|_{d(pFD)=0} = \frac{\partial L^*}{\partial p} - \frac{\partial L^*}{\partial F}\left(\frac{F}{p}\right)$$

The terms $\partial L^*/\partial F$ and $\partial L^*/\partial p$ were derived and signed above in equations (6′) and (7′). Since each is negative, $\partial L^*/\partial p$ and $-\partial L^*/\partial F\ (F/p)$ are of opposite sign. If the sign of equation (8) is determinable, then one of these two opposing effects dominate—which one depends upon the sign.

Using equations (6) and (7), equation (8) may be rewritten as

$$(8')\ \left.\frac{\partial L^*}{\partial p}\right|_{d(pFD)=0} = [U_L^1 - U_L^2 + DF(FD'U_{WW}^2 - U_{LW}^2)]/H_{LL}$$

and hence

$$(8'')\ \left.\frac{\partial L^*}{\partial p}\right|_{d(pFD)=0} > 0$$

Percentage increases in the probability of apprehension will deter incendiarism less than will equal percentage increases in fines.[18] This important result holds for risk averse individuals for which $U_{LW} \geqslant 0$ and in penalty systems which are based on fines only.

The results shown as inequalities (4′), (6′), (7′) and (8″) rest upon several assumptions about individual prefer-

[17] To see this, note that for internal solutions, the first order conditions may be written as $U_L^1 = p[U_L^1 - U_L^2 + FD'U_W^2]$. Since $U_L^1 > 0$, the right hand side of this expression is positive. But the right hand side is the numerator of (7) and hence $\partial L^*/\partial p < 0$.

[18] That is, the second term in equation (8) dominates the first.

ences and underscore an important, although somewhat pedantic point: Policy recommendations in general, and policy recommendations designed to deter arson in particular, rest upon assumptions about the preferences of individuals. For example, in a penalty system based on fines only, a sufficient condition for the recommendation that the probability of apprehension be increased as a means of deterring arsonists, is merely that individuals act in accordance to the axioms of the expected utility theorem. No other preference information is necessary. On the other hand, deducing the deterrent capabilities of fines requires additional preference restrictions. For example, for risk averse individuals, unambiguous deterrence via fines requires $U_{LW} \geqslant 0$.

Prison Sentences and Fines as Punishment for Incendiarism

Since the wealth of an individual imposes limits upon the monetary penalty which can be assessed, one might be tempted to conjecture that a system of penalties incorporating both fines and prison sentences would be a more effective deterrent. This question is briefly explored in this section as an extension of the previous model. It is assumed that both fines and prison sentences are possible penalties if an arsonist is apprehended.

The individual's utility function is now U (W, L, S) where S represents the time length of a prison sentence. Obviously, $U_S < 0$. The expected utility associated with L "hours" on incendiary activity is

$$(9) \quad (1\text{-}p)\, U(W^o, L, 0) + pU(W^o - FD(L), L, S)$$

The individual will choose the amount of time to spend at incendiary activity *as if* (9) were being maximized. The level of incendiary activity is determined by

$$(10) \quad (1\text{-}p)\, U_L^1 + p\,[U_W^2\,(\text{-}FD') + U_L^2 + U_S^2\, f'] \leqslant 0$$

where sentence length is assumed to be an increasing function of L, i.e. $S = f(L)$ and $f'(L) > 0$.[19] As before, the reduced form equation associated with (10) is a unique, continuously differentiable function of p, F and W^o.

Calling the solution to (10), L^o, the three "policy" derivatives from above are now repeated in this more general context:[20]

$$(11) \quad \partial L^o/\partial W^o = [pFD'U_{WW}^2 - pf'U_{SW}^2 - E(U_{LW})]/H_{LL}$$

$$(12) \quad \partial L^o/\partial F = [pD'U_W^2\,]/H_{LL} + Dp\,[U_{LW}^2 + f'U_{SW}^2 - FD'U_{WW}^2\,]/H_{LL}$$

$$(13) \quad \partial L^o/\partial p = [U_L^1 - U_L^2 + FD'U_W^2 - f'U_S^2\,]/H_{LL}$$

The only unsigned term in these expressions is U_{SW} for which there would seem to be an obvious choice, viz. $U_{SW} < 0$–increasing the length of a prison sentences "hurts" more the wealthier one is. Or, equivalently, increases in sentence length become more disagreeable the more that is given up.

Inspection of equations (11) and (12) reveals that without further preference information the effects on incendiary activity of changes in wealth levels and the effects of changes in the amount of the fine are inherently ambiguous when penalties are a mixture of fines and prison sentences. As we saw, in a penalty system based upon fines only increased wealth and increased fines have incentive and disincentive effects respectively on incendiary activity. But if the penalty system is a mixture of fines and sentences, the effects of the same wealth and fine changes are qualitatively ambiguous. Of course, increases in, say, fines *may* deter incendiary activity but at this level of generality it is impossible to say for sure.

Although $\partial L^o/\partial W^o$ is qualitatively ambiguous, equation (11) does reveal one interesting conclusion that can be drawn about policies directed toward changing wealth levels: If incendiarism is punishable only by prison sentences ($F \equiv 0$) then

$$(11') \quad \partial L^o/\partial W^o < 0$$

Increases in affluence unambiguously deter incendiary activity. Wealth increases have incentive effects in a fine-only penalty structure and disincentive effects in a sentence-only penalty structure. This result not only explains the ambiguity of the mixed fine-sentence penalty structure, but also has interesting policy implications.

The only remaining policy variable in the model is the enforcement variable, p. It was shown above that increases in the probability of apprehension had deterrent effects on incendiary activity if punishment was by fine. This result required no preference information other than behavior in accordance with the expected utility theorem. According to equation (13), the same conclusion may be drawn about increases in the probability of apprehension in a mixed fines-sentences penalty structure! That is,[21]

$$(13') \quad \partial L^o/\partial p < 0$$

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[19]Superscripts on functions indicate the point where the function has been evaluated.

[20]Again, only "regular," "internal" solutions to (10) are considered in what follows.

[21]The proof of this statement is identical to that presented for inequality (7').

This remarkable result requires only that arsonists be expected utility maximizers. So, whether penalties are fines or a mix of fines and prison terms, increases in the probability of apprehension will unambiguously deter arson.

Some Concluding Remarks on Modeling Human Behavior

In this final section we attempt to pinpoint the difference between the approach to modeling taken here and what might be called the "Forrester (World Dynamics) Approach." Our approach has been to construct a model using rather fundamental behavioral hypotheses (e.g. individuals maximize expected utility, are risk averse, etc.). The basic model was then restricted to the extent possible using whatever theoretical and *a priori* information one has (e.g. $U_{WW} < 0$ and $U_W > 0$). In general, the purpose of an analysis such as ours is to utilize the behavioral hypotheses and available restrictions on the model to deduce as much information as possible about reduced form equations.[22] The motivation for this approach is evident: The fundamental explanatory equations of the model (the reduced form) are directly linked to explicit underlying propositions about individual behavior. Consequently, one is able to use the derived properties of reduced form equations confident of their implications concerning the underlying preferences of individuals.

On the other hand, one can *assume* the existence of reduced form equations without ever bothering to build a model from which the equations follow.[23] In reference to the above model, one could very credibly *assume* that the level of incendiary activity is a function of the probability of arrest, the severity of punishment and the individual's wealth position. Superficially one then has equation (3) above, without all the theoretical niceties, a circumstance particularly appealing to those in a hurry to "solve" real world problems. Of course, since this equation has not been derived from a model, one is forced to make *ad hoc* assumptions about its properties. And at this point serious problems emerge. Since one has no model, one has no way of knowing what implications any assumed properties of the reduced form carry for underlying preferences. In general, one is fortunate if any given set of *ad hoc* assumptions about the reduced form of a model is not mutually contradictory in reference to a reasonable class of models.

To illustrate several of these arguments, let's return once again to the model analyzed in this paper. If one were to begin an analysis by assuming the existence of equation (3), then *ad hoc* considerations would undoubtedly call for assigning negative signs to each of the derivatives $\partial L/\partial p$, $\partial L/\partial F$ and $\partial L/\partial W^o$. But, as we saw above, this follows in general *only* for $\partial L/\partial p$. Without further restrictions on U the other two derivatives are unsigned. And after the model has been restricted in what would seem to be the most reasonable manner, we found that $\partial L/\partial F$ is unambiguously negative only when penalties consist of fines alone. If both fines and sentences are admitted as penalties, $\partial L/\partial F$ is inherently ambiguous. In addition, we found that sign $[\partial L/\partial W^o]$ depended even more critically on the penalty specification. If fines were the only type of punishment, then incendiary activity increases in wealth ($\partial L/\partial W^o > 0$),[24] while if the only punishment is a prison term, incendiary activity decreases in wealth ($\partial L/\partial W^o < 0$). But if both types of punishment are possible, sign $[\partial L/\partial W^o]$ is indeterminant.

In summary, our point here is not that this derivative or that derivative has a particular sign, but instead that the properties of reduced form equations rest ultimately upon the structure of the underlying model and are in general much too subtle to be specified in an *ad hoc* manner.

Summary and Conclusions

In this paper we have attempted to introduce the reader to an interesting class of models which appear in economic theory. These models are qualitative in nature and are based upon the optimizing behavior of the individual agent. Obviously, the robustness of such models varies inversely with the number of preference restrictions (restrictions on U) used in the analysis. The fewer the restrictions the wider is the class of agents to which the model is applicable and accordingly the more confident one is in derived results. For example, the fact that $\partial L/\partial p < 0$ whether penalties are fines or a mix of fines and sentences and independent of all preference restrictions except behavior consistent with the axioms of the expected utility theorem, provides a fairly strong foundation upon which to base policy. At the same time, one could not recommend increasing fines as a deterrent to arson with the same degree of confidence, although the assumptions upon which this result rests (in the fines-only penalty system) seem to be quite reasonable.

Ultimately, this model, like all others in science, is interesting only if the deduced properties of the reduced form are empirically verifiable. Here the problem of verification amounts to estimating the reduced form via regression analysis or some other estimation technique, and then calculating whether the partial derivatives of the estimated equation are statistically significant and of the

[22]In this paper, attention was focused primarily on determination of but one property of the reduced form—the sign of partial derivatives.

[23]For example, see Rabow [1974] in which the signs of partial derivatives and the functional form of what are essentially reduced form equations have been specified. Other examples are plentiful. The "model" in Forrester's *World Dynamics* [1971] is primarily a collection of aggregated reduced form equations which have been assigned *ad hoc* properties. For a detailed analysis of the Forrester model, see Nordhaus [1974].

[24]Notice that under the postulated circumstances, if one unwittingly made the seemingly plausible assumption that $\partial L/\partial W^o < 0$, one ends up with a contradiction. That is, in this situation $\partial L/\partial W^o < 0$ is inconsistent with behavior according to the expected utility theorem. (Of course, $\partial L/\partial W^o < 0$ will be consistent with behavior under *some* hypothesis.)

correct sign. We hasten to add that the process of estimation will force the investigator to make an assumption about the class of functions to which (3) belongs. Care must be exercised to insure the class chosen is consistent with the underlying model.

As is no doubt obvious by now, both the strength and weakness of utility analysis stem from the generality of the approach. As the number of arguments in U increases so does the difficulty of obtaining unambiguous results. Successful application of this technique depends heavily upon ingenuity in transforming plausible behavioral hypotheses into meaningful restrictions on utility functions.

BIBLIOGRAPHY

1. Arrow, K. J., *Essays in the Theory of Risk Bearing,* Jrjö Johansson Foundation, Helsinki, 1965.

2. Block, M. K. and Heineke, J. M., "The Allocation of Effort Under Uncertainty: The Case of Risk Averse Behavior," *Journal of Political Economy,* vol. 81, no. 2, (March/April 1973), pp. 376-85.

3. Block, M. K. and Heineke, J. M., "A Labor Theoretic Analysis of the Criminal Choice," *American Economic Review,* June, 1975.

4. Block, M. K. and Lind, R., "A Choice Theoretic Analysis for Crimes Punishable by Imprisonment," *Journal of Legal Studies,* forthcoming.

5. Forrester, J., *World Dynamics,* Wright-Allen Press, Cambridge, Mass., 1971.

6. Herstein, I. N. and Milnor, J., "An Axiomatic Approach to Measurable Utility," *Econometrica,* vol. 21, 1953, pp. 291-97.

7. Marschak, J., "Rational Behavior, Uncertain Prospects, and Measurable Utility," *Econometrica,* vol. 18, 1950, pp. 111-41.

8. Marschak, J. and Radner, R., *Economic Theory of Teams,* Yale University Press, New Haven, Conn., 1972.

9. McFadden, D., "Conditional Logic Analysis of Qualitative Choice Theory," Chapter 4 in *Frontiers in Economics,* Edited by P. Zurembka, Academic Press, New York, 1974.

10. Nordhaus, W. D., "World Dynamics: Measurement Without Data," *Economic Journal,* vol. 83, no. 332, December 1973, pp. 1184-1200.

11. Rabow, Gerald, "A Qualitative Theory of Criminal Justice," *IEEE Transactions on Systems, Man and Cybernetics,* vol. SMC4, no. 3, May 1974, pp. 303-03.

12. von Neumann, J. and Morganstern, O., *Theory of Games and Economic Behavior,* Princeton University Press, Princeton, New Jersey, first edition, 1944.