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Technical and allocative efficiency: preliminary ideas toward discrimination between the hypotheses

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TECHNICAL AND ALLOCATIVE EFFICIENCY: PRELIMINARY IDEAS TOWARD DISCRIMINATION BETWEEN THE HYPOTHESES

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INTRODUCTION

Two levels of efficiency lie behind the supply and demand equations of neoclassical economic theory. First, firms are assumed to be technically efficient, in that maximum output is obtained from any given mix of inputs. Second, firms are assumed to be allocatively (or price) efficient, in that input and output mixes are chosen such that profits are maximum.

Although it has often been argued that firms must be "efficient" in a competitive economy, only a very limited amount of work has been directed to measuring the extent of any inefficiencies. In this paper we provide a framework for such measurements with a special emphasis on decomposing observed inefficiencies into technical and allocative components.

THE PROBLEM

We consider a firm producing n+s outputs. In the period of interest a decision must be made as to the appropriate production level of n outputs, while the s remaining output levels are assumed to have been determined in an earlier production period, determined by an outside agency or in any case are exogenous as far as current period decisions are concerned. The n outputs are termed variable outputs and denoted y_i, i = 1, 2, . . ., n; the s remaining outputs are fixed outputs and denoted q_j, j = 1, 2, . . ., s. Firm outputs are produced with m variable inputs, v_k, k = 1, 2, . . ., m, and ℓ fixed inputs, q_j, j = s+1, s+2, . . ., s+ℓ.

In order to introduce the notion of technical efficiency we write the firm's production function as f (y, v, q) – ϵ, where y and v are n and m dimensional vectors of variable outputs and inputs, q is an s+ℓ dimensional vector of fixed outputs and inputs and ϵ is a non-negative stochastic disturbance reflecting the fact that a firm's output must lie on or below its production frontier. As the dispersion of ϵ approaches zero the stochastic production model collapses into the traditional deterministic frontier model.[1] So if, for example, one specifies a two parameter distribution for ϵ the hypotheses of technically efficient production may be treated by testing whether the estimates of μ_ϵ and σ_ϵ^2 are significantly different from zero.

Although the stochastic production frontier appears to be a useful means of modeling technical inefficiency, any attempt to estimate the model would run into difficulties. These difficulties arise due to the fact that the data set to be used may also reflect inefficiencies in variable output and/or input decisions, i.e., given the production technology, input and/or output decisions may not be consistent with profit maximization. In other words, the firm may have errored either in its choice of input levels or its output mix decision or both. We term these errors *price inefficiency* errors. It is of considerable interest to specify a model of production in which it is possible to econometrically identify the relative magnitude of the two sources of economic inefficiency. Although several other authors have studied the price and technical efficiency problem (see Lau and Yotopoulos [1977], Yotopoulos and Lau [1973] and Schmidt and Lovell [1977]), our approach is more general in that, (i) we allow price inefficiency to result not only from erroneous input decisions but also from erroneous output mix decisions; (ii) very weak assumptions are made about the nature of the deviation from the price efficient input and output mixes; and (iii) the analysis requires few restrictions as to the class of functional forms which may be used to represent the firm's

[1] A production frontier of the sort we have specified has been estimated by Aigner, Lovell and Schmidt (1977) and by Schmidt and Lovell (1977) in a linear model with one output.

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production frontier.[2]

To motivate our approach we formally write out the firm's decision problem. Recall that the firm's problem is to choose the levels of n variable outputs and m variable inputs given fixed values for s predetermined (or exogenous) outputs and ℓ fixed inputs. We assume the firm makes these decisions with the goal of maximizing profits. The problem is then,

$$(1)\quad \max_{y,v,\lambda} \Pi(y, v, \lambda; \epsilon, q) = \max_{y,v,\lambda} \{\Sigma P_i y_i - \Sigma w_j v_j - \lambda [f(y, v, q) - \epsilon]\}$$

where P_i and w_j are the given unit prices of variable outputs and inputs, respectively, and λ is a Lagrange multiplier. First order conditions for an internal maxima are

$$P_i - \lambda\, \partial f/\partial y_i = 0 \qquad i = 1, 2, \ldots, n$$

$$(2)\quad w_j - \lambda \partial f/\partial v_j = 0 \qquad j = 1, 2, \ldots, m$$

$$f(y, v, q) - \epsilon = 0$$

Under appropriate concavity conditions on f (·), the unique solution to equations (2) is

$$y_i = \phi_i (P, w, q, \epsilon) \quad , \quad i = 1, 2, \ldots, n$$

$$(3)\quad v_j = \psi_j (P, w, q, \epsilon) \quad , \quad j = 1, 2, \ldots, m$$

$$\lambda = \psi_{m+1} (P, w, q, \epsilon)$$

Output supply and input demand equations are seen to depend upon output and input prices, the level of fixed inputs and outputs and the distribution of ϵ which determines the extent of any technical inefficiency. Of course equations (3) as they stand cannot be used to measure price inefficiency as they were derived under the hypothesis that the "correct" profit maximizing input and output decisions were taken. An appealing means of introducing the possibility of price inefficiency into firm decisions is to rewrite equations (2) as

$$\lambda\, \partial f/\partial y_i - g_i (P_i) = 0 \qquad i = 1, 2, \ldots, n$$

$$(2')\quad \lambda \partial f/\partial v_j - h_j (w_j) = 0 \qquad j = 1, 2, \ldots, m$$

$$f(y, v, \lambda, q) - \epsilon = 0$$

where the functions g_i (·) and h_j (·) are analytic functions of P_i and w_j and are determined by parameters $a_i = \{a_{i1}, a_{i2}, \ldots, a_{i\sigma}\}$ and $b_j = \{b_{j1}, b_{j2}, \ldots, b_{j\Sigma}\}$ respectively. If g_i (·) and h_j (·) are identity functions for all i and j then firm decisions are price efficient. If not, output supply and input demand functions (3) become

$$y_i = \tilde{\phi}_i (P, w, q, \epsilon, a, b) \quad , \quad i = 1, 2, \ldots, n$$

$$(3')\quad v_j = \tilde{\psi}_j (P, w, q, \epsilon, a, b) \quad , \quad j = 1, 2, \ldots, m$$

$$\lambda = \tilde{\psi}_{m+1} (P, w, q, \epsilon, a, b)$$

where $a = (a_1, a_2, \ldots, a_n)$ and $b = (b_1, b_2, \ldots, b_m)$ represent the $n\sigma$ and $m\Sigma$ parameters of the functions g_i (·) and h_j (·).

Testing the hypotheses of technical and price efficiency could proceed by first estimating equations (3′) as they stand; then reestimating with the distribution of ϵ degenerate at zero; then reestimating again with g_i (·) and h_j (·) as identity functions; and finally reestimating with both of these conditions holding, i.e., estimating equation (3). One could begin by testing the hypotheses that firm decisions are neither technically or allocatively efficient against the hypothesis that decisions are technically efficient but price inefficient, against the hypothesis that decisions are technically inefficient but price efficient and finally against the hypothesis that decisions are both technically and price efficient. Whatever the outcome of the test, one could then proceed to estimate equations (3) and (3′) conditional on the outcome. Since the models are "nested," asymptotic likelihood ratio tests could be used to distinguish between the structures.

THE VARIABLE PROFIT FUNCTION

To actually undertake the estimation and testing regime we have described, one must make assumptions either directly or indirectly about the functional form of ϕ_i, $\tilde{\phi}_i$ and ψ_j, $\tilde{\psi}_j$. One way of proceeding would be to specify a functional form for the production function, f (·), and functions g_i and h_j, and derive explicit solutions for equations (3) and (3′). The difficulty with this procedure lies in the fact that it will generally not be possible to obtain explicit solutions to these equations unless the functional specifications for f (·), g_i and h_j are very simple.[3] This accounts for the fact that the majority of econometric studies of firm or household decisions adopt *ad hoc* functional specifications for reduced form equations (our equations (3) and (3′)). Such a procedure is generally undesirable in that, unless care is taken, the resulting equations will not be consistent with the behavioral hypothesis generating them; i.e., it will not be possible to integrate *ad hoc* reduced form equations and obtain the underlying objective function.

Fortunately one need not explicitly solve first order conditions to obtain equations (3) or (3′), nor is it necessary to make *ad hoc* functional specifications if the theorems of modern duality theory are applicable. In this case one need only go to the dual structure and perform the appropriate differentiation to obtain the model's reduced form.

[2]The chosen functional forms need only be capable of satisfying certain regularity conditions needed for the duality between variable profit functions and transformation functions.

[3]For example, see Schmidt and Lovell [1977] who explicitly solve the firm's cost minimization problem to derive input demands and the cost function from a log-linear production function.

For the case at hand define the variable profit function[4] associated with both price and technical efficiency as

$$(4)\quad \max_{y,v,\lambda} \Pi_1^*(y,v,\lambda;q) \equiv \Pi_1(y^o,v^o,\lambda^o) \equiv \Pi_1^*(P,w,q)$$

where y^o and v^o are profit maximizing vectors of output supply and input demand functions and λ^o is the profit maximizing value of λ.

Differentiation of (4) with respect to P and w yields the n variable output supply functions ϕ and the m input demand functions ψ, equation (3). Differentiation of Π_1^* with respect to elements of q yields the shadow prices of the fixed outputs and inputs.[5] Formally

$$\frac{\partial \Pi_1^*}{\partial P_i} = \phi_i(P,w,q) \qquad i = 1,2,\ldots,n$$

$$(4')\quad \frac{\partial \Pi_1^*}{\partial w_j} = \psi_j(P,w,q) \qquad j = 1,2,\ldots,m$$

$$\frac{\partial \Pi_1^*}{\partial q_t} = \Lambda_t(P,w,q) \qquad t = 1,2,\ldots,s+\ell$$

where $\Lambda_t(\cdot)$ is the shadow price of the t^{th} predetermined output $t = 1, 2, \ldots, s$ or t^{th} fixed input $t = s+1, s+2, \ldots, s+\ell$. Since the direct maximization problem need not be explicitly solved the investigator is free to choose the functional form of Π_1^* in sufficient generality so as to leave the properties of supply and demand equations unrestricted vis a vis the measurements of interest.

In terms of the sequence of tests outlined above, four sets of output supply, input demand and shadow price equations are of interest. These are; the case in which price efficient decisions are made given a technically efficient production frontier, system (4′); the case of price efficient but technically inefficient decisions; the case of price inefficient decisions given a technically efficient production frontier; and the system in which both technical and price inefficiency reign.

A short discussion of these cases is probably in order. We view these phenomena, to the extent they occur, as being the consequence of less than perfect information about the actual production structure and/or less than perfect information about input and output market conditions. For example, due to the complexity and interdependence of production processes certain technically inefficient processes may not have yet been "weeded out." And given the firm's perception of its production possibilities, be it the frontier or in the interior," conditional" price inefficiency may arise when managements' forecasts of output and input prices are incorrect. Presumably this occurs quite easily when there are lags between production decisions and purchases of inputs and/or sales of output. On the other hand, price inefficiency could arise if marginal production costs or marginal productivity functions are known with less than certainty, whatever the firm's perception of its production frontier. In either case first order conditions will not hold and equations (2′) are applicable either as they stand or with $\epsilon \equiv 0$.

In this framework, the variable profit equation and resulting supply and demand equations of interest are

$$(5)\quad \max_{y,v,\lambda} \Pi_2^*(y,v,\lambda;\epsilon,q) \equiv \Pi_2^*(y^o,v^o,\tau^o) \equiv \Pi_2^*(P,w,\epsilon,q)$$

$$\frac{\partial \Pi_2^*}{\partial P_i} = \phi_i(P,w,\epsilon,q) \qquad , i = 1,2,\ldots,n$$

$$(5')\quad \frac{\partial \Pi_2^*}{\partial w_j} = \psi_j(P,w,\epsilon,q) \qquad , j = 1,2,\ldots,m$$

$$\frac{\partial \Pi_2^*}{\partial q_t} = \Lambda_t(P,w,\epsilon,q) \qquad , t = 1,2,\ldots,s+\ell$$

for the case of technically inefficient production.

The case of price inefficient decisions are obtained by substituting $g_i(P_i)$ and $h_j(w_j)$ for P_i and w_j in either equations (4′) and (5′) depending upon whether production is technically efficient or inefficient. As noted above, one could test the efficiency hypothesis by estimating these four sets of equations.

THE TRANSLOG VARIABLE PROFIT FUNCTION

It is now time to choose a functional specification for the variable profit function. For most purposes a second order approximation to the variable profit function will provide a sufficiently general framework for estimation of the equation systems of interest. We illustrate using a transcendental logarithmic variable profit function. The translog model of technically inefficient production is then

$$(6)\quad \ln \Pi_2^*(P,w,\epsilon,q) = a_0 + \sum_1^n a_i \ln P_i + \sum_1^m b_j \ln w_j + \sum_1^{s+\ell} c_k \ln q_k + d \ln \epsilon$$
$$+ \frac{1}{2}\sum_1^n\sum_1^n \alpha_{ij} \ln P_i \ln P_j + \frac{1}{2}\sum_1^m\sum_1^m \beta_{ij} \ln w_i \ln w_j + \frac{1}{2}\sum_1^{s+\ell}\sum_1^{s+\ell} \gamma_{ij}$$
$$\ln q_i \ln q_j + \delta(\ln\epsilon)^2 + \sum_1^n\sum_1^m \alpha'_{ij} \ln P_i \ln w_j$$
$$+ \sum_1^n\sum_1^{s+\ell} \beta'_{ij} \ln P_i \ln q_j + \ln\epsilon \sum_1^n \gamma'_i \ln P_i$$
$$+ \sum_1^m\sum_1^{s+\ell} \delta'_{ij} \ln w_i \ln q_j + \ln\epsilon \sum_1^m \tau_j \ln w_j$$

[4]See Diewert [1974] for an interesting discussion of variable profit functions and an overview of duality results with an emphasis toward application.

[5]See Diewert [1974: 139-140].

$$+ \ln\epsilon \sum_{1}^{s+\ell} \rho_i \ln q_i$$

Although supply and demand functions associated with this profit function are non-linear in the parameters of (6), the value share equations $P_i y_i / \Pi_2^*$ and $w_j v_j / \Pi_2^*$ are linear in the parameters. ($P_i y_i / \Pi_2^* = \partial \ln \Pi_2^* / \partial \ln P_i$ and $w_j v_j / \Pi_2^* = \partial \ln \Pi_2^* / \partial \ln w_j$.) For this reason our exposition is in terms of the supply and demand "share" equations.

$$(6') \quad \frac{P_i y_i}{\Pi_2^*} = a_i + \sum_{1}^{n} \alpha_{ij} \ln P_j + \sum_{1}^{m} \alpha'_{ij} \ln w_j + \sum_{1}^{s+\ell} \beta_{ij} \ln q_j + \gamma'_i \ln \epsilon \qquad i = 1, 2, \ldots, n$$

$$\frac{w_j v_j}{\Pi_2^*} = b_j + \sum_{1}^{m} \beta_{ij} \ln w_i + \sum_{1}^{n} \alpha'_{ij} \ln P_i + \sum_{1}^{s+\ell} \delta'_{ij} \ln q_j + \tau_j \ln \epsilon \qquad j = 1, 2, \ldots, m$$

Up to this point the only stochastic component in our model is the one-sided term ϵ which arises if technically inefficient decisions are taken. For estimation purposes we append to equations (6)–(6′) classical disturbance terms v_1, v_{2i}, $i = 1, 2, \ldots, n$, and v_{3j}, $j = 1, 2, \ldots, m$, respectively, which capture random variation in these equations due either to factors exogeneous to the firm or as a result of the fact that the translog variable profit function provides only an approximation to the "true" underlying production structure. In addition, we assume that the "onesided" disturbance ϵ is of the form $\epsilon = e^u$ and that the density functions for v_1, v_{2i} and v_{3j} may be adequately approximated with normal density functions. The stochastic components of equations (6)–(6′) are then composed of two components, the traditional components v_1, v_{2i} and v_{3j} which account for exogenous randomness or approximation error and u which accounts for production inside the frontier. The error structure for equations (6)–(6′) is then

$$(7) \quad (d + \sum_{1}^{n} \gamma'_i \ln P_i + \sum_{1}^{m} \tau_j \ln w_j + \sum_{1}^{s+\ell} \rho_i \ln q_i)\, u + \delta u^2 + v_1, \qquad \text{for equation (6)}$$

$$(7') \quad \begin{array}{ll} \gamma'_i u + v_{2i} & , i = 1, 2, \ldots, n \\ \tau_j u + v_{3j} & , j = 1, 2, \ldots, m \end{array} \qquad \text{for equation (6')}$$

The first n disturbances in (7′) are associated with output supply functions. Since u is non-negative the notion of technical inefficiency implies $\gamma_i < 0$, for all i, and hence output supply functions will be bounded from above by traditional stochastic supply frontiers. An analogous argument indicates $\tau_j > 0$, all j, and hence input demand functions will be bounded from below by traditional stochastic input demand frontiers. Therefore persistent decrements in output supplies and persistent excesses in factor demands are due to technical inefficiency and equal $\gamma'_i u$ in the i^{th} supply function and $\gamma_j u$ in the j^{th} input demand function.

A final task prior to estimation is to impose the restrictions implied by profit maximization on the translog variable profit function, equation (6), and hence on the resulting supply and demand equations, equations (6′). First we require $\alpha_{ij} = \alpha_{ji}$, $\beta_{ij} = \beta_{ji}$ and $\gamma_{ij} = \gamma_{ji}$. In addition, the variable profit function is homogeneous of degree one in variable output and input prices.[6] Π_2^* (P, w, ϵ, q) is homogenous of degree one in P and w if

$$\sum_{i=1}^{n} a_i + \sum_{j=1}^{m} b_j = 1$$

$$\sum_{i=1}^{n} \beta'_{ik} + \sum_{j=1}^{m} \delta'_{jk} = 0 \qquad , k = 1, 2, \ldots, s+\ell,$$

$$(8) \quad \frac{1}{2}\left(\sum_{\ell=1}^{n} \alpha_{i\ell} + \sum_{\ell=1}^{n} \alpha_{\ell i} \right) + \sum_{j=1}^{m} \alpha'_{ij} = 0 \ , i = 1, 2, \ldots, n,$$

$$\frac{1}{2}\left(\sum_{h=1}^{m} \beta_{hj} + \sum_{h=1}^{m} \beta_{jh} \right) + \sum_{i=1}^{n} \alpha'_{ij} = 0 \ , j = 1, 2, \ldots, m.$$

$$\sum_{i=1}^{n} \gamma'_i + \sum_{j=1}^{m} \tau_j = 0$$

Identifying the Components of the Residual Variance

The other question of interest here concerns the relative importance of the two sources of random error. Recall that the non-negative disturbance u reflects the fact that each firm's output must be on or below its production frontier. Any deviation from the frontier is the result of factors under the firm's control. The disturbances v_1, v_{2i} and v_{3j} reflect the fact that the frontier itself may vary across firms or within a firm over time. As we noted above, such variation arises from exogenous shocks, both favorable and unfavorable, and the fact that the translog variable profit function only approximates the underlying production structure and consequent variable profit function.

For convenience we repeat equations (7)–(7′), which define the error structure of our model, as

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[6] See Diewert [1974].

$$(d + \sum_1^n \gamma_i' \ln P_i + \sum_1^m \tau_j \ln w_j + \sum_1^{s+\ell} \rho_i \ln q_i)\, u + \delta u^2 + v_1$$

$$(9) \quad \gamma_i' u + v_{2i} \qquad , i = 1, 2, \ldots, n$$

$$\tau_j u + v_{3j} \qquad , j = 1, 2, \ldots, m.$$

If we assume that errors from the two sources of random variation are independent, it is straightforward to obtain estimates of the variances of u, v_{2i} and v_{3j} and hence to get an idea of their relative importance.

One method of isolating the two sources of random error is to estimate the profit function and m+n−1 of the supply and demand functions using a systems approach such as SUR. Then calculate the second and third moments of the residuals for each estimated output supply and input demand function. These sample moments are consistent estimators of

$$E(\gamma_i' u + v_{2i})^2 \equiv \mu_{2i}^2, i = 1, 2, \ldots, n, \; E(\tau_j u + v_{3j})^2 \equiv \eta_{3j}^2, j = 1, 2, \ldots, m,$$

$$E(\gamma_i' u + v_{2i})^3 \equiv \mu_{2i}^3, i = 1, 2, \ldots, n, \text{ and } E(\tau_j u + v_{3j})^3 \equiv \eta_{3j},$$

j = 1, 2, . . ., m. It is again straightforward to show

$$(10) \quad \begin{aligned} \mu_{2i}^2 &= (\gamma_i')^2 \sigma_u^2 + \sigma_{2i}^2 \\ \mu_{2i}^3 &= \gamma_i' \sqrt{2/\Pi}\sigma_u [2(\gamma_i')^2 \sigma_u^2 + 3\sigma_{2i}^2] \qquad , i = 1, 2, \ldots, n \end{aligned}$$

$$(10') \quad \begin{aligned} \eta_{3j}^2 &= \tau_j \sigma_u^2 + \sigma_{3j}^2 \\ \eta_{3j}^3 &= \tau_j \sqrt{2/\Pi}\sigma_u [2\tau_j^2 \sigma_u^2 + 3\sigma_{3j}^2] \qquad , j = 1, 2, \ldots, m \end{aligned}$$

Equations (10) and (10′) are m+n pairs of equations each pair in two variables σ_u^2 and either σ_{2i}^2 or σ_{3j}^2. Hence by replacing theoretical moments (10) and (10′) with sample moments and solving each pair of equations, one can derive consistent estimates of σ_u^2 and σ_{2i}^2 and σ_{3j}^2.[7] Our estimates of σ_u are given by the roots of

$$(11) \quad \begin{aligned} -\sqrt{2/\pi}\,(\gamma_i')^3 \sigma_u^3 - 3\gamma_i' \hat{\mu}_{2i}^2 \sigma_u - \hat{\mu}_{2i}^3 &= 0, \quad i = 1, 2, \ldots, n \\ -\sqrt{2/\pi}\,\tau_j^3 \sigma_u^3 - 3\tau_j \hat{\eta}_{3j}^2 \sigma_u - \hat{\eta}_{3j}^3 &= 0, \quad j = 1, 2, \ldots, m \end{aligned}$$

[7]See Schmidt and Lovell [1977] for more detail. An alternative approach would be to use maximum likelihood methods to estimate σ_u^2 and the variance-covariance matrix of v_{2i} and v_{3j}. The major difficulty with this approach is solving the necessary conditions for the maximum.

where a "hat" denotes a parameter estimate. Our estimates of σ_{2i}^2 and σ_{3j}^2 are given by

$$(14) \quad \begin{aligned} \hat{\sigma}_{2i}^2 &= \hat{\mu}_{2i}^2 - (\gamma_i')^2 \hat{\sigma}_u^2, \qquad i = 1, 2, \ldots, n \\ \hat{\sigma}_{3j}^2 &= \hat{\eta}_{3j}^2 - \tau_j^2 \hat{\sigma}_u^2, \qquad j = 1, 2, \ldots, m. \end{aligned}$$

Notice that although the mean of the conglomerate disturbance in each equation is non-zero, only the consistency of the intercept terms in the estimated versions of (6) and (6′) will be affected. All other parameters will be consistently estimated. Consistent estimates of intercepts may be obtained by subtracting estimated means of $\gamma_i' u + v_{2i}$ and $\tau_j u + v_{3j}$ from the estimated intercepts.

CONCLUDING REMARKS

We have presented a framework for decomposing observed firm inefficiency into its technical and allocative components. Our specification is considerably more general than that of previous work. The next task at hand is the empirical implementation of the model.

REFERENCES

1. Aigner, D., C. A. K. Lovell and P. Schmidt, "Formulation and Estimation of Stochastic Frontier Production Function Models," *Journal of Econometrics,* 6 (1977), 21–37.

2. Diewert, W. E., "Application of Duality Theory" in M. D. Intriligator and D. A. Kendrick (eds.), *Frontiers of Quantitative Economics,* Vol. II, North-Holland, 1974.

3. Lau, L. J., and P. A. Yotopoulos, "A Test for Relative Efficiency and Application to American Agriculture," *American Economic Review,* 61 (1971), 94-109.

4. Schmidt, P., and C. A. K. Lovell, "Estimating Technical and Allocative Inefficiency Relative to Stochastic Production and Cost Frontiers," University of North Carolina at Chapel Hill.

5. Yotopoulos, P. A., and L. J. Lau, "A Test for Relative Economic Efficiency: Some Further Results," *American Economic Review,* 63 (1973), 214-223.