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Optimal Pricing Policy in the Presence of Experience Effects*

We use a general model to analyze the optimal intertemporal pricing policy for a monopolist when current and past output play a role in determining future cost and/or demand conditions through "experience" in production and/or in consumption. As would be expected, the optimal price path depends on the manner in which experience affects demand and cost functions. Three special cases are scrutinized: (1) learning by doing in which production costs are *scaled* downward over time; (2) learning by doing in which production costs are *translated* downward over time; and (3) the case of demand satiation. For these cases, the optimal price paths are shown to be, respectively, decreasing, increasing, and nonmonotonic.

I. Introduction

In this paper we analyze the problem of optimal *intertemporal* pricing for a monopolist when current (and past) output affect future cost and/or demand conditions through "experience" in production and/or in consumption. Learning by doing, the experience curve, contagion, habit formation, bandwagon, and snob effects are all examples of terminologies used to describe such situations. We call these "experience effects" for convenience and explore profit-maximizing pricing behavior when such effects exist.[1]

In the traditional profit-maximization model, a firm chooses the price-output combination so as to maximize short-run (i.e., current) profits. The familiar $MC = MR$ equality (given that price is higher than average variable cost and a nonbinding capacity constraint) is a necessary condition for price takers as well as price makers. This assumes that the current pricing decision has no bearing on the future, so that long-run profit is maximized by a series of short-run maximizing

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1. Here we used the word "experience" to encapsulate all situations where the present depends on the integral or sum of past values of decision variables.

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decisions. When this independence condition does not hold, however, it is not possible to maximize long-run profits by using such myopic decision rules. For example, a firm may set a price lower than that dictated by the short-run optimum in order to increase current sales, if this larger sale leads to lower costs in the future. The phenomenon of cost falling with cumulative production is often referred to as "learning by doing" or the "experience curve" phenomenon. In this case, a price lower than the myopic price may seem to be intuitive. However, a series of questions immediately comes to mind for which neither intuition nor the traditional marginal conditions provide easy solution. Among these questions are the following: Should price be continuously lowered throughout the planning horizon? If so, at what rate? In other words, what is the optimal price path? Are there situations when price goes up, goes down, or fluctuates? Does it vary continuously or exhibit jumps? Obviously the answers to these questions depend on specifics: In particular, exactly how do current output and sales affect future costs and/or demand?

These questions have been touched upon in the literature, mostly in the context of experience curves and the market penetration (or contagion) model. The former deals with the widely observed phenomenon of falling cost due to experience (learning) in production. Such cost reductions are not a result of economies of scale, but rather of endogenously induced technological change.[2] In this case, production costs will be a function not only of current output but also of past production (cumulative output). The latter phenomenon deals with situations such as diffusion of innovations in which market demand for a durable good increases at first and decreases eventually as the product becomes more widely accepted and the market becomes saturated. Demand, therefore, depends not only on price but also on past purchases (cumulative sales).

Bass (1969, 1980) focused on the question of diffusion and adoption rates of consumer durables. Assumed in his model (1980) are: (1) a marginal cost function which decreases with total cumulative output, and (2) a constant price elasticity of demand with changing market size. In addition, Bass postulates that (3) firms set prices according to short-run profit-maximization principles. This latter assumption combined with (1), results in a monotonically decreasing price over time. The model discussed by Bass is fundamentally different from the present model which focuses on the intertemporal or long-run optimizing behavior of a monopolist in the presence of experience effects.

Robinson and Lakhani (1975) have adopted a model similar to that of Bass to analyze the question of dynamic pricing strategy. Again de-

2. The primary distinction between technological change induced by learning and the more familiar technological change of, say, growth theory, is the endogenous and exogenous origins, respectively, of changes.

mand and cost vary with past sales. In a discretized example involving specific parameterizations of demand and cost functions, the optimal price path is calculated numerically and is shown to be quite different from that given by short-run profit maximization.

Dolan and Jeuland (1981) and Jeuland and Dolan (1982) have analyzed optimal pricing strategies when experience effects exist on both cost and demand sides of the market. In these papers, however, the authors assume that unit production costs are independent of the production rate and follow Bass in adopting specific parametric representation of demand, thus limiting the scope of their conclusions.

Spence (1981) explores some of the implications for competition in an industry where firm costs decline over time due to "learning." He focuses on the case of no discounting and, like each of the authors referenced, adopts a specialized functional form for firm cost functions. We show below that such cost specifications constrain the optimal price trajectory to be monotonically falling over time. This will not be the case in general.

It is the purpose of this paper to build and analyze a general model which encompasses various experience situations including the models discussed above. The next section presents this model. The third section deals in more detail with two cases of learning by doing in which learning influences cost in two distinct manners: In one case, the optimal price falls and, in the other, increases throughout the entire planning period. The satiation process in demand is analyzed in a similar manner in Section IV. The fifth section contains an example in which the optimal price path exhibits jump discontinuities, even though all the data are continuous. A brief summary and conclusions are followed by the Appendix which addresses the question of the existence of optimal price paths.

II. The General Model

We denote by x the (cumulative) quantity sold of commodity X in the past and p the price of X (which can be interpreted as the reservation price). Then,

$$\dot{x} = q(x,p), \tag{1}$$

where $\dot{x} = dx/dt$, current output, and $q(x,p)$ is the demand function. In addition, we have

$$c(x,q), \tag{2}$$

a given production cost function. Both q and c are assumed to be twice continuously differentiable, with the partial derivative $q_p < 0$ everywhere, and $q > 0$ for all nonnegative prices.

The problem the firm faces is to choose the optimal time path for p in such a way as to maximize the present value of the stream of profit. That is

$$\max_{p(t)} \int_0^T e^{-\delta t}\{pq(x,p) - c[x,q(x,p)]\}dt, \tag{3}$$

where $p(\cdot)$ is piecewise continuous and assumes values in $(0,\infty)$, and x satisfies (1) and $x(0) = x_0$, the cumulative output at $t = 0$. We shall assume that an optimal solution (x,p) exists (the existence question is addressed in the Appendix).

We proceed by applying the maximum principle to this optimal control problem (see, e.g., Intriligator 1971). The Hamiltonian $H(t,x,\lambda,p)$, where λ is the adjoint variable, is given by $\lambda q(x,p) + e^{-\delta t}\{pq(x,p) - c[x,q(x,p)]\}$. The adjoint equation in this case is

$$\dot{\lambda} = e^{-\delta t}(c_x + c_q q_x - pq_x) - \lambda q_x. \tag{4}$$

The fact that $\mathrm{H}(t,x,\lambda,\cdot)$ achieves a maximum over $(0,\infty)$ at p gives $H_p = 0$; in this case

$$\lambda q_p + e^{-\delta t}(q + pq_p - c_q q_p) = 0. \tag{5}$$

We solve (5) for λ to obtain

$$\lambda = e^{-\delta t}(c_q - p - q/q_p). \tag{6}$$

Substitution of (6) into (4) gives

$$\dot{\lambda} = e^{-\delta t}(c_x + qq_x/q_p). \tag{7}$$

The maximum principle also yields the transversality condition

$$\lambda(T) = 0. \tag{8}$$

For later reference we note the relation $H_{pp} \leqslant 0$ along the optimal trajectory (x,p). Therefore

$$c_{qq}q_p^2 - 2q_p + qq_{pp}/q_p \geqslant 0. \tag{9}$$

It is common practice to interpret λ as a shadow price, and then use equations such as (4) and (5) as the basis for policy prescriptions. However, it is possible to eliminate λ from these equations and to define a function ϕ which appreciably enhances the interpretability of solutions. We set

$$\phi(x,p) = c_q[x,q(x,p)] - p - q(x,p)/q_p(x,p). \tag{10}$$

In view of (6), along the optimal path $[x(t),p(t)]$, we have $\phi[x(t),p(t)] = e^{\delta t}\lambda$, the "spot shadow price," a function of t which is continuous. By using (6) and (7), we then calculate

$$\frac{d}{dt}\phi[x(t),p(t)] = \dot{\phi} = \delta\phi + qq_x/q_p + c_x. \tag{11}$$

Let us note that, in light of (8), one has $\phi(T) = 0$. Further (9) implies

$$\phi_p q_p \geqslant 0 \text{ at } [x(t), p(t)] \tag{12}$$

Solving the ordinary differential equation (11) for $\phi(t)$, we obtain

$$\phi(t) = -\int_t^T (c_x + qq_x/q_p)\, e^{-\delta(\tau - t)} d\tau.$$

When this is expressed in terms of the original variables, we obtain the following theorem.

THEOREM: The following relationship holds along the optimal price path:

$$p(t) + q/q_p = c_q + \int_t^T (c_x + qq_x/q_p)\, e^{-\delta(\tau - t)} d\tau. \tag{13}$$

Notice that $p(t) + q/q_p$ is the usual "short-run" marginal revenue (MR), the immediate benefit, from selling one more unit, and c_q is the usual "short-run" marginal cost (MC) of producing one more unit. The integral term is the present value of the "long-run effects" of the current and past actions. It is obvious from (13) that when $c_x \neq 0$ and/or $q_x \neq 0$ (i.e., when experience effects are present), the usual short-run marginal cost pricing rule no longer yields an optimal policy. There must be a "wedge" between MR and MC (the value of the integral in [13]). As an example, consider the case in which $c_x < 0$ (i.e., learning by doing) and $q_x = 0$. Since the integral is necessarily negative in this case, we have $MC > MR$ along the optimal path for all time except at T. The wedge between MC and MR represents the present value of future cost reductions due to an increase in cumulative output (experience). In this sense, (13) can be considered as the dynamic analogue of the short-run (static) marginal condition. Of course one may interpret (13) as a condition which equates "full long-run MC" to "full long-run MR"; the theorem simply tells one how these full marginal values are to be calculated.

If p is differentiable, we derive from (10) the following relation:

$$\dot{p}\phi_p = \dot{\phi} - q\phi_x. \tag{14}$$

But $\dot{\phi} = \delta\phi + qq_x/q_p + c_x$ from (11). Substituting this into (14), we get

$$\dot{p}\phi_p = \delta\phi + qq_x/q_p + c_x - q\phi_x. \tag{15}$$

Recall that ϕ is the wedge between MC and MR, or $\phi(t) = MC(t) - MR(t)$. Furthermore, we have from (10): $\phi_x = \partial MC/\partial x - \partial MR/\partial x$. Since $q = \dot{x}$, we can now rewrite (14) as

$$\dot{p}\phi_p = \dot{\phi} - \left(\frac{\partial MC}{\partial x} - \frac{\partial MR}{\partial x}\right)\dot{x},$$

or equivalently

$$\phi_p dp = d\phi - \left(\frac{\partial MC}{\partial x} - \frac{\partial MR}{\partial x}\right)dx. \tag{16}$$

As long as $\phi_p \neq 0$, we have dp determined by the difference between $d\phi$ and $[\cdot\,]dx$. The $d\phi$ represents how the wedge is changing, and $[\cdot\,]dx$ represents how MC and MR are affected by the cumulative output. In other words, the direction of change in the optimal price depends on the two (possibly opposing) effects: (1) how fast the spot shadow price of x changes, and (2) how MC and MR are affected by x. One might interpret these effects as long-run (potential) and short-run (realized) effects. Equation (16) proves to be extremely useful in interpreting our results. In particular, diagrammatic analyses will be presented in the next two sections to sort out the two effects and to analyze how the optimal price changes.

As we show in Section V, $p(t)$ may not be continuous, let alone differentiable everywhere. There is, however, a simple additional hypothesis that will guarantee continuity of $p(t)$.

PROPOSITION 1: Suppose that q and c satisfy globally the inequality

$$c_{qq}q_p^2 - 2q_p + qq_{pp}/q_p > 0. \tag{17}$$

Then the optimal price path $p(t)$ is continuous and differentiable on $[0,T]$.

PROOF: If (17) holds, then $H_{pp} < 0$ everywhere. Thus, the equation $H_p[t,x(t),\lambda(t),p] = 0$ uniquely defines $p = p(t)$, and differentiability of $p(t)$ follows as a consequence of the implicit function theorem.

REMARK: The inequality in the statement of the proposition is satisfied, for example, if $q(x,p) = f(x)g(p)$, in which $f > 0$, $g \geqslant 0$, $g' < 0$, g is concave and, in addition, c is convex in q. Examples include exponential functions for $g(p)$.

III. Learning by Doing

We shall now study two special cases of the general model in which we impose additional structure, and in so doing arrive at certain global information regarding the behavior of the optimal price path. Both cases are instances of learning by doing, that is, experience has the beneficial effect of lowering the cost function ($c_x < 0$). Yet, as we shall see, the specific way in which costs are lowered is important in determining the price profile: in one case the optimal price is an increasing function of time and, in the other, a decreasing function of time.

The Case of Scaling in c.

We assume that in this subsection $c(x,q)$ has the form

$$c_0 + m(x)h(q), \tag{18}$$

where $m(x) > 0$, $m'(x) < 0$, $h(0) = 0$, $h(q) > 0$, $h'(q) > 0$, and h is convex. Thus, the production cost curve is scaled downward as x increases.[3] We assume that demand is unaffected by experience: $q_x = 0$. Note that for a range of parameter values, a differentiable optimal price path $p(t)$ is known to exist (see Appendix). For present purposes, however, we merely assume that such exists.

PROPOSITION 2: In this case, the optimal price path is a decreasing function of time.

PROOF: The equation (11) becomes in this case

$$\dot{\phi} = \delta\phi + m'(x)h(q) < \delta\phi.$$

Now if ϕ were ever negative or 0, then the solution to this differential equation would necessarily be negative beyond that point. But $\phi(T) = 0$ from (8), so we conclude that $\phi(t) > 0$ for all $t < T$. Equation (15) becomes in this case

$$\dot{p}\phi_p = \delta\phi + c_x - qc_{qx} \tag{19}$$

since, for this problem, $\phi_x = c_{qx}$. We now claim that $c_x - qc_{qx}$ is nonnegative; it then follows from (19) that $\dot{p}$ is negative, since the right-hand side is positive and $\phi_p \leqslant 0$ (from [12]). To prove this claim, observe that $c_x - qc_{qx}$ reduces to

$$m'(x)[h(q) - qh'(q)],$$

so it suffices to show $h(q) - qh'(q) \leqslant 0$. But from the convexity of h, we know

$$h(q) = h(q) - h(0) \leqslant qh'(q).$$

Q.E.D.

We now turn our attention to a diagrammatic exposition of this special case. Since $q_x = 0$, equation (16) becomes $\phi_p dp = d\phi - \partial MC/\partial x\, dx$, which is strictly positive for $t < T$. This implies that

$$\left|\frac{\partial MC}{\partial x}\right| dx + d\phi > 0.$$

The case when $d\phi$ is negative is shown in figure 1. Comparing two points of time t and $t + \Delta$, we see the MC curve shifting from MC_t to $MC_{t+\Delta}$. The wedge between MC and MR required for dynamic profit maximization changes from ϕ_t to $\phi_{t+\Delta}$ (with $\phi_{t+\Delta} < \phi_t$). But if this reduction in the wedge is smaller than the reduction in MC (i.e., ΔMC in the diagram), then the optimal output will increase from q_t to $q_{t+\Delta}$, hence price falls. If $d\phi$ is positive, then it is clear that the two effects reinforce each other, resulting in a price reduction.

In this case, even though the long-run potential benefit from added

3. A case treated by Bass and others which falls in this category is that in which marginal cost is constant (i.e., h linear in q) and $m(x) = x^{-\gamma}$.

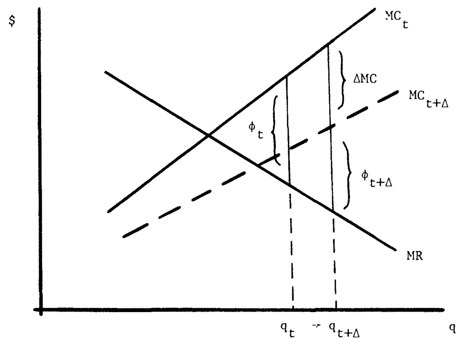

FIG. 1

experience may be decreasing as the end of the planning period is approached, this is outweighed by the (realized) reduction in marginal cost. Thus, the firm decides to lower price continuously.

The Case of Translation in c

We assume now that $c(x,q)$ has the form

$$c_0 + s(x) + r(q), \tag{20}$$

where $r' > 0$, $s' < 0$, and s is convex on $(0,\infty)$. Thus, in this case, experience translates (shifts) the cost curve downward. We continue to assume $q_x = 0$. Again, the proposition in the Appendix implies the existence of a smooth optimal price path for at least a subclass of such problems.

PROPOSITION 3: In this case, the optimal price path is an increasing function of time.

PROOF: If we substitute our particular c into ϕ, we see that $\phi_x = 0$ in this case, and (15) now becomes

$$\dot{p}\phi_p = c_x + \delta\phi = s'[x(t)] - \delta \int_t^T s'[x(\tau)]e^{-\delta(\tau-t)}\, d\tau.$$

Our task reduces to proving that the right-hand side of this equation is negative (for $\phi_p \leqslant 0$ from [12]), which we proceed to do.

Since s is convex, s' is nondecreasing, so:

$$\begin{aligned} s'[x(t)] - \delta \int_t^T s'[x(\tau)]e^{-\delta(\tau-t)}\, d\tau \\ &\leqslant s'[x(t)] - \delta s'[x(t)]\int_t^T e^{-\delta(\tau-t)}d\tau \\ &= s'[x(t)]\left\{1 - \delta\int_t^T e^{-\delta(\tau-t)}\delta\tau\right\} \\ &= s'[x(t)]e^{-\delta(T-t)} < 0. \end{aligned}$$

Q.E.D.

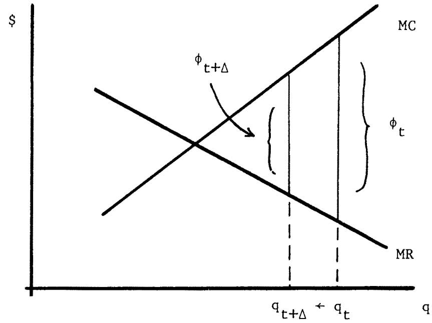

FIG. 2

It is a simple matter to show (see fig. 2) why the optimal price increases. Since the cost function is additively separable, we have $\partial MC/\partial x = 0$. Thus, (16) becomes $\phi_p dp = d\phi$. But $\dot{\phi} = \delta\phi + c_x = \delta\phi + s' = \dot{p}\phi_p$, which has been shown to be strictly negative above. Thus, $d\phi$ or $\dot{\phi} < 0$. As the wedge between MC and MR becomes smaller (since future benefits from a cost reduction become smaller), the optimal quantity falls, hence the optimal price increases.

This is a case in which experience reduces total cost, without affecting marginal cost. On the other hand, "fixed cost" is reduced; for example, organization of the production process may benefit from experience. However, actual production of each unit may not; since MC in terms of, say, additional materials and labor time required might be exactly the same. In such a case, larger sales (by pricing lower) are warranted in view of future cost reductions. But the gains from such policies decrease continuously over time: hence, a continuous price increase.

In summary, the two cases studied in this section both deal with learning by doing, where production cost is reduced by experience, yet they are dramatically dissimilar with respect to optimal pricing policy. It has been shown that this is due to the nature of the cost reduction: The marginal cost function is lowered in the first case, but not in the second.

IV. Demand Experience

When we examine demand cases analogous to those of Section III, we find that, in contrast to the case of learning, the behavior of the price path is not generally monotonic. We shall illustrate this in the case in which $q(x,p)$ is of the form

$$q(x,p) = \sigma(x)\rho(p), \tag{21}$$

and $c(x,q) = c_0 q$. This formulation was used by Robinson and Lakhani in an attempt to model the case of consumer durables.[4]

We assume that $\sigma(x)$ and $\rho(p)$ are positive, with $\rho' < 0$, that for x in $(0,x_0)$, $\sigma'(x) = 0$, and that for x in $[x_0,\infty)$, $\sigma'(x) < 0$. Thus, x_0 can be viewed as a threshhold beyond which consumer satiation begins.

PROPOSITION 4: In this case, for sufficiently large horizons (T), the optimal price path exhibits at least one period of increase and at least one period of decrease.

PROOF: In this case (11) becomes $\dot{\phi} = \delta\phi + \sigma'\rho^2/\rho' \geqslant \delta\phi$ and it follows as in Proposition 2 that $\phi \leqslant 0$. We suppose T is large enough so that the threshold x_0 is breached; thus, $\dot{\phi} > \delta\phi$ for t beyond a certain point. It then follows that ϕ is strictly negative for $t < T$.

We turn to (15), which now transmutes to $\dot{p}\phi_p = \delta\phi + \rho^2\sigma'/\rho'$. Initially (when $x < x_0$) the right-hand side is negative, so $\dot{p} > 0$. For t near T, $\phi(t)$ is almost zero, since $\phi(T) = 0$, so the right-hand side must be positive; that is, $\dot{p} < 0$. Q.E.D.

To supplement the mathematical derivation of the property of $\dot{p}$, we shall now give a heuristic exposition of this case using MR and MC curves. Equation (16) becomes $\phi_p dp = d\phi + \partial MR/\partial x \; dx$.

For x in $(0,x_0)$, we have $\sigma'(x) = 0$, thus $\dot{\phi} = \delta\phi < 0$ (since $\phi < 0$) and $\partial MR/\partial x = 0$. Thus, in this case, the (negative) wedge between MC and MR is growing larger in magnitude, resulting in price increases. This can be interpreted as growing spot shadow prices as the satiation point is approached; that is, the negative benefit is felt more strongly as we get close (see fig. 3).

However, once the satiation point ($x > x_0$) is reached, two effects again begin to interact. When (16) is positive, we have $d\phi + \partial MR/\partial x \; dx > 0$.

Assume that consumer satiation decreases both demand and MR (see fig. 4). Since $\phi(T) = 0$ and $\phi < 0, \dot{\phi} > 0$ sufficiently near T ($d\phi > 0$). Thus, we have

$$\left| \frac{\partial MR}{\partial x} dx \right| < d\phi \text{ or } \Delta\phi > \Delta MR.$$

The wedge is shrinking fast enough to compensate for the reduction in MR. In this case we see an increasing q and, hence, a decreasing p.

V. Example

The purpose of this section is to give a specific example in which the unique optimal price path is a discontinuous function of time. Besides

4. In their formulation, demand for the durable at any point of time is a function of x and p, but is independent of pricing history. We feel this is not an entirely realistic model for durables. Since consumers "typically" buy only one unit, current demand should depend on which segments of population (differentiated by their reservation prices) have bought in the past.

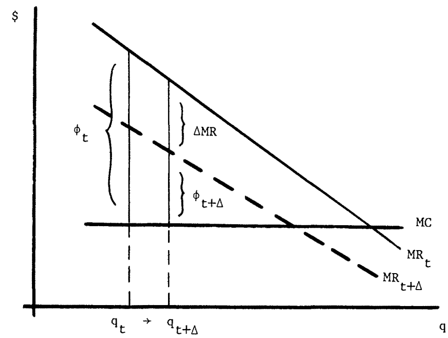

FIG. 3

showing that our attention to this issue in Section II was not unwarranted, the example demonstrates, perhaps counterintuitively, that sudden large price fluctuations are not necessarily a sign of mismanagement—they may, in fact, be unavoidable in the optimal price path.

We take for $q(x,p)$ the function $e^{-p/100}$, and we choose [0,1] as the planning period, with $x_0 = 0$. It follows that x and q lie in [0,1] at all times. Let $\alpha < \beta < \gamma < 1/3$ be three positive numbers, and let $f(q)$ be any continuously differentiable function which satisfies the following conditions:

i) $f(q) \geqslant 0$ for $q \in [0,1]$; $f(q) = 0$ if and only if $q = \beta$ or γ.
ii) $|f'(q)| \leqslant 1$ for $q \in [0,1/3]$.
iii) $f'(q) \geqslant 1$ for $q \in [1/3,3/4]$.
iv) $f'(q) > 104$ for $q \in [3/4,1]$.

We use the cost function defined by $c(x,q) = -100q \ln q + f(q) + g(x,q)$, where $g(x,q) = [\max\{(x - \alpha)(q - \beta)^2, (\alpha - x)(q - \gamma)^2\}]^2$. The function g is continuously differentiable, as is $q \ln q$ for $q > 0$. Further, conditions i–iv above were specified to imply $c_q > 0$. (It is a simple matter to verify this on each of the intervals [0,1/3], [1/3,3/4] and [3/4,1]

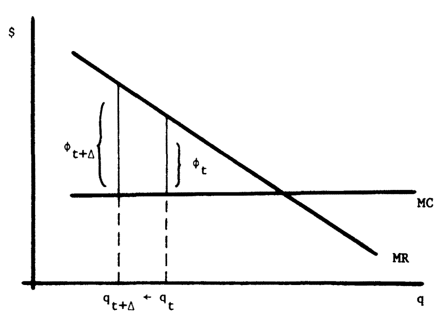

FIG. 4

if one observes the inequality $|g_q| \leqslant 4$.) Thus, c is not unreasonable in the role of a cost function.

For this case, the optimal pricing problem reduces to minimizing

$$-\int_0^1 e^{-\delta t}[pq - c(x,q)]dt$$

$$= -\int_0^1 e^{-\delta t}[-100q \ln q - c(x,q)]dt$$

$$= \int_0^1 e^{-\delta t}[f(\dot{x}) + g(x,\dot{x})]dt$$

$$= \int_0^1 e^{-\delta t} h(x,\dot{x})dt,$$

where we have defined $h = f + g$. Note that h is nonnegative by construction, and can only equal zero when both f and g are zero. This can only happen in two ways: if $q = \gamma$ and $x \leqslant \alpha$, or if $q = \beta$ and $x \geqslant \alpha$. It follows that there is a unique policy which makes $h = 0$ everywhere (and consequently is optimal): Use $q = \dot{x} = \gamma$ until $x = \alpha$, then switch to $q = \dot{x} = \beta$. In terms of price, this means that price is piecewise constant with an upward jump at time $t = \alpha/\gamma$.

VI. Summary and Conclusions

We have analyzed the optimal intertemporal pricing policy for a monopolist in the presence of experience effects. The solution to this "optimal control problem" is completely determined mathematically by the differential equations and boundary conditions of Section II. The dynamic analogue of the short-run marginal condition (Theorem, Sec. II) was shown to be

$$MR = MC + \int_t^T (c_x + qq_x/q_p)e^{-\delta(\tau - t)}d\tau, \tag{22}$$

where we have referred to the integral term as the wedge between MC and MR required for long-run optimality.

It is clear from our analysis that short-run profit-maximizing decision rules will not, in general, lead to maximum long-run profit when experience effects are present. Although the phenomenon of temporally falling prices is often observed in markets where experience effects are alleged to exist, it was shown that this is not the only scenario possible. The optimal price path depends on precisely how experience effects influence demand and cost functions. Three special cases were scrutinized in which the mathematical solution was supplemented by diagrammatic analysis.

In the first case of learning by doing, we found that the optimal price falls throughout the period. This result is due to the nature of the cost reduction: Cost is scaled downward and hence the MC curve falls over time. Intuitively we may say that the change in the long-run effect is outweighed by the change in the short-run effect. If MC falls fast enough, the firm should lower price continuously.

When cost is translated downward due to experience, on the other hand, the optimal price increases throughout the period. Translation implies an MC function which is independent of x. Thus, the short-run effect is absent, and the change in the long-run effect determines the price path. Since the long-run benefit from experience declines over time, the incentive to sell large quantities also diminishes: hence, an increasing price path.

A similar analysis was made of the case of experience effects on the demand side. If cumulative scales have negative impact on future demand (i.e., eventual satiation), then the price path will exhibit both periods of increase and decrease.

Appendix
Existence of Optimal Price Paths

Direct application of general existence theorems from optimal control or the calculus of variations is difficult in the case of the problem treated in this article, for these would mandate the presence of constraints such as $p_0 \leqslant p \leqslant p_1$, which we do not impose. To show, however, that existence is not a hopeless feature of the problem, we prove by ad hoc means the following result for a subclass of problems.

Let $q(x,p)$ have the form $f(p) = \alpha e^{-\beta p}$ for $\alpha < 1$.

PROPOSITION 5: Suppose that for some $m > 0$, the cost function c satisfies: (a) $c_{qq} \geqslant -1/(\alpha\beta)$; (b) $c_x \geqslant -m$; (c) $c_q \geqslant me^{\delta T}/\delta - 1/\beta$. Then there is an optimal price path $p(t)$ for the problem of Section II, with $0 < p(t) < \infty$, and p is continuous and differentiable.

PROOF: The problem may be recast as that of minimizing

$$\int_0^T e^{-\delta t}\,[c(x,\dot{x}) - f^{-1}(\dot{x})\dot{x}]dt$$

$$= \int_0^T e^{-\delta t}\,[c(x,\dot{x}) + \dot{x}\,\ln(\dot{x}/\alpha)/\beta]dt, \tag{23}$$

where $\dot{x} = q$ lies in $(0,\infty)$. Now, the function $v \rightarrow c(x,v) + v\,\ln(v/\alpha)/\beta$ is convex as a consequence of (a) (its second derivative is positive on $[0,\alpha]$), so that, from standard results, it can be shown that there is a solution $x(t)$ to minimizing (23) subject to $0 \leqslant \dot{x} \leqslant \alpha$. It remains to show that $\dot{x}$ is never equal to 0 or α. We will show this by invoking the necessary conditions of Clarke (1976) for this "generalized problem of Bolza." These assert the existence of a function λ satisfying:

$$\dot{\lambda} = e^{-\delta t}c_x \text{ for all } t,\ \lambda(T) = 0, \tag{24}$$

and the inequalities

$$\lambda(t) \leqslant e^{-\delta t}\left\{c_q + \frac{d}{dv}\,[v\,\ln(v/\alpha)/\beta]_{v=\dot{x}}\right\} \text{ if } 0 \leqslant \dot{x} < \alpha$$

$$\lambda(t) \geqslant e^{-\delta t}\left\{c_q + \frac{d}{dv}\,[v\,\ln(v/\alpha)/\beta]_{v=\dot{x}}\right\} \text{ if } \dot{x} = \alpha.$$

Now suppose in fact that $\dot{x}$ had the temerity to equal α somewhere. Then by the preceding $\lambda(\tau) \geqslant e^{-\delta\tau}\{c_q(x,\alpha) + 1/\beta\}$ for some τ, so by (b) we deduce $\lambda(\tau) \geqslant m/\delta$. On the other hand, (24) together with (c) yields $\lambda(t) \leqslant m(1 - e^{-\delta T})/\delta$ for all t, which is a contradiction. Thus, $\dot{x}$ is never α.

That $\dot{x}$ can never be 0 follows from the first general inequality for λ, together with the observation $d/dv\ [v\ \ln(v/\alpha)/\beta]_{v=0} = -\infty$.

As for the smoothness of p, it follows from applying proposition 1, Section II. Q.E.D.

References

Bass, Frank M. 1969. A new product growth model for consumer durables. *Management Science* 15 (January): 215–27.

Bass, Frank M. 1980. The relationship between diffusion rates, experience curves, and demand elasticities for consumer durable technological innovations. *Journal of Business* 53, no. 3 (July): 51–67.

Clarke, Frank H. 1976. The generalized problem of Bolza. *SIAM Journal of Control and Optimization* 14, no. 4 (July): 682–99.

Dolan, Robert J., and Jeuland, Abel P. 1981. Experience curves and dynamic demand models: implications for optimal pricing strategies. *Journal of Marketing* 45, no. 1 (Winter): 52–62.

Intriligator, Michael D. 1971. *Mathematical Optimization and Economic Theory*. Englewood Cliffs, N.J.: Prentice-Hall.

Jeuland, Abel P., and Dolan, Robert J. 1982. An aspect of new product planning: dynamic pricing. In A. Zolters (ed.), *TIMS Studies in the Management Sciences, Special Issue on Marketing Planning Models*. Amsterdam: North-Holland, in press.

Robinson, Bruce, and Lakhani, Chet. 1975. Dynamic price models for new product planning. *Management Science* 21, no. 10 (June): 1113–22.

Spence, Michael A. 1981. The learning curve and competition. *Bell Journal of Economics* 12, no. 1 (Spring): 49–70.