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Law Enforcement Agencies as Multiproduct Firms: An Econometric Investigation of Production Costs

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LAW ENFORCEMENT AGENCIES AS MULTI-PRODUCT FIRMS: AN ECONOMETRIC INVESTIGATION OF PRODUCTION COSTS*

by

M.N. DARROUGH AND J.M. HEINEKE**

I. INTRODUCTION

In this paper we study the relationship between costs, input prices and activity levels in a sample of approximately thirty medium sized city police departments for the years 1968, 69, 71, and 73. Our interest lies in determining the functional structure of law enforcement production technology.

Since efficient allocation of resources to activities requires knowledge of relative incremental costs for the activities involved, we are particularly interested in determining marginal cost functions for, and rates of transformation between the various outputs. We adopt a quite general functional specification which permits testing the hypothesis on the underlying technology. In a more general context we model and estimate the structure of production for a multiple output — multiple input firm in a manner which places few restrictions on first and second order parameters of the underlying structure.

One question which arises immediately in any discussion of cost or production functions associated with law enforcement agencies concerns the appropriate measure of “output”. Clearly police departments produce multiple outputs (services) for a community, ranging from directing traffic, quieting family squabbles, and providing emergency first aid, to preventing crimes and solving existing crimes. In this study we view police output as being of essentially two types: (1) general service activities as epitomized by the traffic control and emergency first aid care functions of police departments; and (2) activities directed to solving existing crimes. Strictly speaking, “solving existing crimes” may be an intermediate output with deterrence or prevention of criminal activity being the final product. But due to the difficulty of measuring crime prevention we use the number of “solutions” by type of crime as output measures.¹

In the past few years a number of authors have addressed the problem of determining the structure of production in law enforcement agencies. Since under certain rather mild regularity conditions there exists a duality between cost and production functions, either the cost function or the production function may be used to characterize the technological structure of a firm. The studies of Chapman, Hirsch and Sonenblum [2], Ehrlich [10, 11], Votey and Phillips [13, 19], and Wilson and Boland [21] all proceed by estimating production functions while Popp and Sebold [14] and Walzer [20] estimate cost functions. It is of some interest to briefly review the findings of these authors.

Chapman, Hirsch and Sonenblum estimate a rather traditional production function, at least from a theoretical point of view. All police outputs are collapsed into one aggregate, which is then regressed on input use levels. They find strongly increasing returns to scale — often a two to four percent output response to a one percent change in input usage.

Ehrlich also uses an aggregate solution rate as the output measure, which he regresses on per capita expenditures on police, the aggregate offense rate and a series of exogenous (“environmental”) variables. The expenditure variable is an index of overall input use levels while the aggregate offense rate is included to measure the effects of “crowding” or capacity constraints on output. This is a substantial departure from a neoclassical approach in which the shape of the production function itself will reflect diminishing returns as capacity is pressed. But it is a specification that has been widely adopted by those who have followed Ehrlich. (For example, see Vandaele [18], Phillips and Votey [13], or Votey and Phillips [19].) Using per capita expenditures to measure the scale of output, Ehrlich finds that a one percent increase in expenditures per capita leads to much less than a one percent increase in the solution rate.

We should point out that two different arguments have been used for including the offense level in police agency production functions. In addition to the argument based upon police resource capacities, some authors have justified inclusion of the offense level in the production function using what is essentially a “fisheries argument”. Viz., that the total number of fish in the ocean is a determinant of the number caught. So if the number of offenses is high, then *ceteris paribus*, it should be easier to obtain a solution that if there are but few offenses. Obviously, the argument goes, if there are no offenses there can be no solutions. But this is really not the question. The question is whether in the neighborhood of observed solution levels, changes in the total number of offenses would change solution levels.

Whichever rationale is used, the neoclassical production function is modified and written as $y = f(v_1, v_2, \dots, v_m, 0)$, where y is the number of solutions, v_i is the level

appropriateness of this specification is to assume that 0 does not belong in the production function and then estimate the function $y/0 = f(v_1, v_2, \dots, v_m) 0^\gamma$, where $y/0$ is the solution *rate*. If γ is significantly different from minus unity, the offense level probably influences solution levels. If not, one has some evidence that the production function for solutions is independent of the level of offenses.

Phillips and Votey [13] report three estimates of the production function $y/0 = \alpha v^\beta 0^\gamma$. Using their reported parameter estimates and standard errors, one cannot reject the hypothesis that $\gamma = -1$ in any one of the estimated equations at the .05 level. In addition, Ehrlich's [11] estimate of γ is -.908 which again is not significantly different from minus unity. We conclude, at least tentatively, that the production of solutions does not depend upon offenses and do not consider the matter further in this study.

Similar to the work of Votey and Phillips [19], Wilson and Boland study the production of solutions to several property crimes. But instead of input levels as determinants of solutions, they utilize a "capacity" variable as well as variables meant to account for productivity differences between departments. Here the authors cannot address the question of scale economies due to the fact that only a subset of all outputs are included in these studies.

Finally, both Popp and Sebold, and Walzer estimate cost functions and attempt to measure scale economies. The former use population size in the police jurisdiction as their measure of "scale" along with a large number of demographic and environmental variables to estimate the per capita costs of police service. The authors find diseconomies of scale throughout the entire range of population sizes. Of course the population variable provides a considerably different concept of scale than economists are accustomed to considering. In fact, Walzer has argued that population size is a poor measure of scale for several reasons — the most important being a tendency on the part of police administrators to determine manpower needs as a proportion of population size. In such a case there is obviously a strong bias toward constant returns to scale.

In his study, Walzer recognizes that offenses cleared, accidents investigated, etc., all make up the output of a police department. But instead of estimating a multiple output cost function, he creates an "index of police service" by collapsing all outputs into one.² The estimated cost function contains the offense rate as an argument in addition to measures of input prices, *input usage* and several variables meant to pick up externally determined differences in productivity. Using the service index to measure output, Walzer finds evidence of economies of scale, although they seem to be rather slight. Interestingly enough, he also finds that input costs are not significantly related to overall production costs.

II. OUTLINE OF THE PAPER

A number of strong hypotheses concerning the production structure of law enforcement agencies have been implicitly maintained in the studies sketched above. First, the arguments entering cost and production functions have for the most part differed considerably from what one would expect from classical production theory. In addition, in the one case where input costs do enter the cost function ([20]), linear homogeneity in input costs has not been imposed on the estimated cost function. One possible explanation for these deviations from classical theory is that classical theory, and cost minimizing behavior in particular, is deemed not capable of explaining observed choices in public agencies. While this is a plausible hypothesis, it should be tested rather than maintained.³

Second, each of the estimated production functions upon which we have reported is either linear or linear logarithmic. Such functions may be viewed as first order approximations to an arbitrary production function. It is well known that first order approximations severely restrict admissible patterns of substitution among inputs and admissible rates of transformation among outputs as well as having other undesirable empirical implications.⁴ In addition, linear logarithmic functions do not permit scale economies to vary with output. We noted above that each of the production studies surveyed included the offense rate or level as an argument. A possible explanation for this inclusion might be based upon the restrictiveness of the chosen functional forms and a consequent attempt on the part of the authors to provide output responses which do vary with the scale of operation, in functions which do not naturally possess this property. For these reasons and others, we adopt a second order approximation to the underlying cost and production structure thereby leaving the various elasticity measures of common interest free to be determined by the data.⁵

Third, the Chapman, Hirsch and Sonenblum, Walzer and Ehrlich studies all utilize a single output aggregate. If the results of such aggregate studies are to be used for decision purposes, it is desirable that the aggregate measure be a consistent index over all police outputs. In what follows we estimate a multiple output cost function and test whether the various subsets of outputs may be consistently aggregated into single categories.

Fourth, the Wilson and Boland, Votey and Phillips, and Vandaele studies each implicitly maintain the hypothesis of nonjoint outputs by estimating separate production functions for different types of solutions. Again, instead of maintaining this hypothesis we estimate a multiple output function and then test the nonjointness hypothesis.

To summarize, in this study we characterize the structure of production in a

the existence of consistent aggregate indices of police output, for nonjointness of output, and for consistency of our estimated equations with the optimizing behavior of classical theory. In addition, we calculate (1) marginal and average cost functions for solutions to the property crimes of burglary, robbery, larceny and motor vehicle theft, and for solution to crimes against the person; (2) marginal rates of transformation between these activities; and (3) an estimate of scale economies based upon the response of total cost to simultaneous variation in all police outputs.

III. MOTIVATION OF AGENCIES

In this section we provide a framework within which the structure of law enforcement production technology could be estimated. The model is essentially a value maximization model and implies that input decisions are reached in cost minimizing manner. We assume that police administrators, either implicitly or explicitly, assign "seriousness" weights to crimes by type and use these weights along with the costs of solving crimes by type to determine the solution mix. This might be termed a "bounty hunter"⁶ model of police decision making since resources are allocated to solutions by type as if police remuneration were proportional to the "value" of solved crimes and assumes that police decision makers are primarily interested in solutions and not deterrence. We believe that on a day to day basis a strong argument can be made that police administrators are primarily concerned with solutions and not deterrence and that for property crimes average values stolen are likely to be reasonable approximations to the weights used in allocating resources to solving property crimes.

Using P_i to represent the value to police of a solution of a crime of type i , the police agency's decision problem is

$$(1) \quad \max \sum_{i=1}^n P_i y_i - C(y, w),^7$$

which provides the familiar system

$$(2) \quad P_i - \partial C / \partial y_i = 0 \quad i = 1, 2, \dots, n.$$

These may be used as a basis for estimating $C(y, w)$. Equation (2) instructs police administrators to allocate resources to solving crimes of type i until the marginal cost of a type i solution is equal to the assigned weight, P_i . Note that if $C(y, w)$ is approximated with a polynomial in y and w , equations (2) alone may not be sufficient to determine the cost function. This can be remedied by including $C(y, w)$ itself in the system to be estimated. In which case

$$(3) \quad \begin{aligned} P_i - \partial C / \partial y_i &= 0, & i &= 1, 2, \dots, n \\ C - C(y, w) &= 0 \end{aligned}$$

becomes the system of interest. In the circumstances we have outlined it is reasonable to assume that values P_i are determined jointly by the activities of police and offenders in earlier periods — i.e., P_i are predetermined. Assuming that input costs are exogenous, equations (3) determine the n endogenous solution levels as functions of exogenous and predetermined variables.

One problem in implementing this system in an econometric context is obvious: The weights to be given the various types of solution are at best difficult to obtain. But as we have indicated above, in the case of property crimes average values stolen probably provide reasonable approximations to the seriousness of these crimes in the eyes of the police. No such convenient measure is available for the case of “crimes against the person”, e.g., homicide, rape, and assault.

One method of dealing with this problem is to assume the property crime solutions are separable⁸ from all other police activities. This is equivalent to assuming that marginal rates of transformation (MRT) between solutions to all pairs of property crimes be invariant to the level of nonproperty crime solutions, and to the level of other police services provided, and to input prices. In this case, it can be shown that there exists functions C^* and f such that the cost function may be written as

$$(4) \quad C = C^*(f(y_1, \dots, y_p, w), Y_{p+1}, \dots, y_n, w)$$

where y_1, \dots, y_p represent solutions to crimes against property and y_{p+1}, \dots, y_n represent solutions to crimes against the person and the service activities performed by police. Equations (3) with this modification are estimated below for the case of four property crimes, burglary, robbery, motor vehicle theft and larceny, an aggregate of crimes against the person and an aggregate police service indicator.⁹

IV. THE TRANSLOG MODEL

From an econometric point of view equation system (3) is only of limited interest until a specific functional form has been assigned to the cost function $C^*(y, w)$. The primary concern in choosing a functional form for C^* is that the chosen class of functions be capable of approximating the unknown cost function to the desired degree of accuracy. In wide-spread use in the literature in the past few years are the class of so called “flexible” functional forms which includes the generalized Leontief function, the generalized Cobb-Douglas function, the transcendental logarithmic function and many hybrids.¹⁰ These functions may all be viewed as

particular place no restrictions on elasticities of substitution between inputs or elasticities of transformation between outputs and allow returns to scale to vary with the level of output. We have chosen to approximate $C^*(y, w)$ with the translog function due primarily to the fact that most past studies of law enforcement agency production technology have adopted linear logarithmic functions which are special cases of the translog function.

The translog cost function may be written as

$$(5) \quad \ln C(y, w) = a_0 + \sum_1^n \alpha_i \ln y_i + \sum_{n+1}^{n+m} b_i \ln w_i + 1/2 \sum_1^n \sum_1^n \alpha_{ij} \ln y_i \ln y_j \\ + 1/2 \sum_{n+1}^{n+m} \sum_{n+1}^{n+m} \beta_{ij} \ln w_i \ln w_j + \sum_{n+1}^{n+m} \sum_1^n \gamma_{ij} \ln y_i \ln w_j.$$

It can be shown that second order parameters of this function must be symmetric if supply functions are to be well behaved, i.e., $\alpha_{ij} = \alpha_{ji}$ and $\beta_{ij} = \beta_{ji}$, for all i and j and $\gamma_{ij} = \gamma_{ji}$ for $i = 1, \dots, n$ and $j = n+1, \dots, n+m$. Our maintained hypothesis of separability (see equation (4)) between property crime solutions and all other activities of the police agency implies the following restrictions on equation (5):

$$(6) \quad \alpha_{ij} = 0, \quad i = 1, 2, \dots, p, j = p+1, p+2, \dots, n.^{11}$$

In general, hypotheses concerning the nature of production technology impose certain restrictions on the values of the parameters of the empirical cost function. In particular, the hypothesis of *linear homogeneity* of $C(y, w)$ in input prices, which is an implication of cost minimizing behavior, imposes the following restrictions on the translog cost function:

$$(7) \quad \sum_{n+1}^{n+m} b_i = 1, \quad \sum_j \beta_{ij} = \sum_i \beta_{ij} = \sum_j \gamma_{ij} = 0.$$

If these restrictions are imposed, then proportional increases in input prices lead to equi-proportional increases in production costs. The hypothesis of *constant returns to scale* implies

$$(8) \quad \sum_i \alpha_i = 1, \quad \sum_j \alpha_{ij} = \sum_i \alpha_{ij} = \sum_i \gamma_{ij} = 0.$$

Another hypothesis of considerable interest is that of *nonjointness* of outputs. If outputs are nonjoint one may estimate a separate cost function for each output. In terms of the translog cost function a test for nonjointness of outputs may be based

$$(9) \quad \alpha_{ij} = -a_i a_j \quad i, j = 1, 2, \dots, n, i \neq j.$$

These restrictions and others on the production technology of law enforcement agencies are tested below.¹²

V. THE ECONOMETRIC MODEL

In this section we specify the model to be estimated econometrically. We had available for this study information on annual police budgets for the years 1968, 1969, 1971, and 1973 for a sample of approximately thirty medium size cities;¹³ the average wages of officers by rank, the number of crimes of type i cleared by arrest ("clearances") and the average value stolen for each of the property crimes in the FBI index. The police budget and wage information was gathered by the Kansas City Police Department and circulated for use by participating cities under the title of the *Annual General Administrative Survey*. The data on clearances and average values stolen are from unpublished sources at the FBI. We have used clearances by arrest for the seven FBI "index crimes" as our measures of "solutions". In particular, we have called burglary clearances (solutions), y_1 , robbery clearances, y_2 , motor vehicle theft clearances, y_3 , and larceny clearances, y_4 . We have used the aggregate number of homicide, rape and assault clearances to represent solutions to crimes against the person and have labeled this output, y_5 . Finally, a very large component of the output of all law enforcement agencies are the rather mundane but important service functions — directing traffic, investigating accidents, breaking up fights, providing emergency first aid, etc. We group all such service functions together as y_6 . The question is what to use to measure these activities. We have adopted the hypothesis that the quantity of services of the type we have been discussing is proportional to the size of the city (population) in which the agency is located.

We had available wage information on eight grades of police officers from patrolman to chief. As one might expect, these wage series are highly collinear. To test for the existence of a Hicksian price index, we computed correlation coefficients between the wages of the various ranks and found very high coefficients. For example, the correlation between wages of patrolmen and a weighted average of the wages of all other ranks is .955. Unfortunately, there does not appear to be a way of testing whether a sample correlation is significantly different from one since the distribution of this statistic is degenerate at that point. But with correlations this high it appears safe to assume the conditions for Hicks' aggregation are fulfilled and hence we use a weighted average of all police wages as an aggregate measure of unit labor costs, denoted w .¹⁴

The translog cost function of (5) above may now be written as a function of six outputs and one input as follows

$$(10) \quad \ln C^*(y, w) = a_0 + \sum_{i=1}^6 a_i \ln y_i + b \ln w + 1/2 \sum_{i=1}^6 \sum_{j=1}^6 \alpha_{ij} \ln y_i \ln y_j \\ + 1/2 \beta \ln w^2 + \sum_{i=1}^6 \gamma_i \ln w \ln y_i$$

where $\alpha_{15} = \alpha_{16} = \alpha_{25} = \alpha_{26} = \alpha_{35} = \alpha_{36} = \alpha_{45} = \alpha_{46} = 0$ due to the imposed separability of property crime solutions from all other police activities.

Given the hypothesis of separability between property crime solutions and all other police activities there are a total of eleven possible groupings of property crime solutions which might be considered for indexing.¹⁵ Our question here is not whether an index exists in any of these cases, because an index can always be found, but whether a *consistent* index exists.¹⁶ It is important to keep in mind that the existence of a separable group of outputs does not in general imply existence of a consistent index for the group.

For the translog cost function, it is convenient to express equations (3) in the following "value share" form:

$$(11) \quad P_i y_i / C^* = a_i + \sum_{j=1}^6 \alpha_{ij} \ln y_j + \gamma_i \ln w, \quad i = 1, 2, 3, 4$$

$$\ln C^* = a_0 + \sum_{i=1}^6 a_i \ln y_i + b \ln w + 1/2 \sum_{i=1}^6 \sum_{j=1}^6 \alpha_{ij} \ln y_i \ln y_j + 1/2 \beta \ln w^2 + \sum_{i=1}^6 \gamma_i \ln w \ln y_i$$

where $\alpha_{ij} = 0$, for $i = 1, 2, 3, 4, j = 5, 6$ and $\alpha_{ij} = \alpha_{ji}$, for all i and j . (The first four equations here give the value of y_i solutions to property crime i as a proportion of total police expenditures.) In order to provide a stochastic framework for equations (11), we append classical additive disturbances to each of the five equations in the model. These disturbances arise either as a result of random error in the maximizing behavior of police administrators, or as a result of the fact that the translog function provides only an approximation of the "true" underlying production structure. We assume that noncontemporaneous disturbances are uncorrelated both within and across equations. We make no other assumptions about the distribution of disturbances other than they be uncorrelated with right hand variables in each equation.¹⁷

VI. EMPIRICAL RESULTS

We have fitted the five equations of system (11) under the stochastic specification

in the system. Since no assumption has been made concerning the distribution of disturbances, our estimation procedure may be thought of as multiequation, nonlinear least squares. In the computations we used the Gauss-Newton method to locate minima. The results of all estimations that are conditional upon cost minimization are presented in Table 1.

The estimates reported in column two contain no restrictions other than symmetry and homogeneity of C^* in input prices and entails estimating twenty parameters. Given the primarily cross section nature of the data, the model fits quite well with R^2 figures of .72 for the cost function and .35, .13, .30, and .27 for the value of solution equations $p_i y_i / C^*$, $i = 1, 2, 3, 4$, respectively.

In columns three, four, and five are reported parameter estimates for the cases of nonjoint outputs, linear logarithmic costs and constant returns to scale, each conditional on the cost minimization hypothesis. The restrictions of nonjointness (Column three) reduce the number of parameters which must be directly estimated to thirteen. (See equations (9).) The linear logarithmic cost function (Column four) was estimated primarily to contrast our functional form with that implied by the linear logarithmic production functions which have been estimated in the majority of earlier papers. The total number of parameters to be estimated is now reduced to seven.

Our tests of the various hypotheses are based upon the test statistic

$$(12) \quad \lambda = \max L^R / \max L^{\bar{R}}$$

where $\max L^R$ is the maximum value of the likelihood function for the model with restrictions R and $\max L^{\bar{R}}$ is the maximum value of the likelihood function without restriction. Minus twice the logarithm of λ is asymptotically distributed as chisquared with number of degrees of freedom equal to the number of restrictions imposed. Logarithms of the likelihood function ($\ln L$) are given in Table 1 for each of the model specifications. Throughout we choose a critical region based upon the .01 level of significance.

We now report the results of statistical tests performed on the estimated models. The model was first estimated in unrestricted form conditional only upon the maintained hypothesis of functional separability of property crime solutions from other police activities.¹⁸ We have contrasted this specification with the model implied by cost minimization (i.e., the model with symmetry and linear homogeneity of input prices imposed). Our interest here, of course, lies in determining whether our sample is consistent with cost minimization (or more generally with value maximization). Without the optimization hypothesis the "share" equations of system (11) above are interpreted as the average (observed)

TABLE 1
Parameter Estimates for Four Cost Models*
(Conditional Upon Cost Minimization)

Parameter	Symmetry & Homogeneity	Symmetry & Homogeneity & Non-joint Outputs	Symmetry & Homogeneity & Linear Logarithmic Costs	Symmetry & Homogeneity & Constant Returns to Scale
a_0	-114.974 (33.466)	5.150 (4.201)	-4.469 (1.092)	-.5043 (1.307)
a_1	-.0606 (.0166)	-.0794 (.0075)	.0292 (.0016)	.0171 (.0021)
a_2	-.0068 (.0039)	-.0053 (.0011)	.0065 (.0003)	.0088 (.0007)
a_3	-.0724 (.0216)	-.0771 (.0086)	.0459 (.0026)	.0653 (.0031)
a_4	-.0327 (.0100)	-.0376 (.0035)	.0198 (.0009)	.0102 (.0013)
a_5	-1.944 (1.210)	-.4885 (1.136)	.2448 (.0376)	.4338 (.3991)
a_6	19.080 (5.291)	.1020 (.0722)	.9113 (.0902)	.4646 (.3992)
b	1	1	1	1
α_{11}	.0246 (.0020)	.0235 (.0016)		.0214 (.0015)
α_{22}	.0033 (.0004)	.0031 (.0004)		.0032 (.0004)
α_{33}	.0293 (.0024)	.0298 (.0023)		.0199 (.0020)
α_{44}	.0126 (.0009)	.0127 (.0008)		.0117 (.0007)
α_{55}	.0191 (.0524)	-.0028 (.0535)		.0313 (.0607)
α_{66}	-1.461 (.4214)	.0308 (.0468)		.0313 (.0607)
α_{12}	-.0020 (.0006)	-.00042 (.00009)		-.0027 (.0005)
α_{13}	-.0064 (.0017)	-.00613 (.00075)		-.0123 (.0015)
α_{14}	-.0048 (.0010)	-.00299 (.00034)		-.0063 (.0008)
α_{23}	-.0003 (.0005)	-.00041 (.00009)		-.0013 (.0004)
α_{24}	.0013 (.0004)	-.00020 (.00004)		.0008 (.0003)
α_{34}	-.0028 (.0010)	-.00290 (.00037)		-.0062 (.0008)
α_{56}	.1501 (.0997)	.04984 (.09617)		-.0313 (.0607)
$\ln L$	1633.58	1620.66	1483.57	1600.13

value of property crime solutions as functions of solution activity levels and as a proportion of total agency budgets.

Symmetry and linear homogeneity in input prices impose fifteen restrictions in addition to those imposed by the functional separability of property crime solutions from all other police activity. The $\chi^2_{.01}$ critical value is 30.58. Minus twice the logarithm of the likelihood ratio falls far into the critical region of the test. Our sample appears to be inconsistent with cost minimizing (and consequently value maximizing) behavior on the part of police decision makers. Nonetheless, as we have indicated earlier, we will proceed with all other tests conditional upon the cost minimization hypothesis. We have chosen this approach due first to the fact that the translog cost function is but an approximation to the true cost structure and hence in this sense our test result is only approximate; second, due to the fact that our maintained hypothesis of functional separability of property crime solutions from all other police activity is a strong assumption and could possibly have distorted the result of our test; third, due to the fact that average values transferred may not accurately reflect departmental evaluation of outputs; and fourth, due to the fact that we are uncertain as to the power of the likelihood ratio test in finite samples. Finally we note that past studies indicate that tests of the symmetry restriction tend to be very difficult to pass. Any one of these factors could cause us to reject the structure implied by cost minimization when in fact it is true. Of course, still another and perhaps more fundamental reason for proceeding conditional upon cost minimization is the lack of interpretation and the shallowness of explanation one must be content with once the behavioral hypotheses underlying equation systems are abandoned.

Conditional upon cost minimization we next test the validity of the hypothesis of nonjoint outputs — a hypothesis which has been maintained in a great many past studies. The likelihood ratio statistic indicates that we must reject the hypothesis. We conclude that one may not go about estimating separate production functions or separate cost functions for each of the outputs of police agencies. The interaction between outputs must be accounted for if one is to adequately characterize the structure of cost and production in this “industry”.

Contrasting the linear logarithmic structure implied by these data with our more general model (columns four and two), we find that the linear logarithmic specification is overwhelmingly rejected. The loss in explanatory power resulting from adopting the Cobb-Douglass functional form for C^* is obviously substantial.

We next test the hypothesis of constant returns to scale. Linear homogeneity in outputs imposes seven additional restrictions on the model. The value of the test statistic is 66.90 and hence these data lend no support to the constant returns

We have estimated the models associated with each of the ten possible output aggregates. As above, we test these hypotheses (aggregation by homothetic separability) conditional on the validity of the cost minimization hypothesis via likelihood ratio test. All potential aggregates are definitely rejected, although the aggregate (y_1, y_2, y_4) is only marginally rejected. Our sample does not support the existence of a category function for any aggregate.

Even if the sufficient conditions for consistent aggregation of outputs via homothetic separability are not met, another possibility for consistent aggregation remains — the value of P may be perfectly correlated. We have calculated the correlation matrix for P to check for the possibility of a Hicksian aggregate. The correlations are $r_{12} = .065$, $r_{13} = .065$, $r_{14} = .901$, $r_{23} = .197$, $r_{24} = .014$ and $r_{34} = 0.026$. (Of course, such calculations permit testing only pairwise groupings of outputs in the first step.) The question here is whether .901 is significantly different from 1.0. It is not possible to test this proposition as the distribution of the sample correlation coefficient is degenerate at 1.0. However, .901 seems distant enough from 1.0 to conclude that y_1 and y_4 may not be treated as a single output. We conclude that the data in our sample does not support the existence of a consistent output aggregate. Hence future efforts directed to estimating cost and production functions for law enforcement should attempt to determine the appropriateness of summing police outputs into aggregates prior to using such measures.²⁰

VII. MARGINAL COSTS, RATES OF TRANSFORMATION AND RETURNS TO SCALE

The marginal cost function for activity i is given by $\partial C^*/\partial y_i = (\partial \ln C^*/\partial \ln y_i)(C^*/y_i)$ and may be calculated using the formula

$$(13) \quad \partial C^*/\partial y_i = (a_i + \sum_{j=1}^6 \alpha_{ij} \ln y_j + \gamma_i \ln w) (e^{\ln C^*} / y_i) \quad i = 1, 2, \dots, 5.$$

Equation (13) will be valid for each of the crime solving outputs, y_1, y_2, \dots, y_5 but not for y_6 . Recall that the sixth output was an aggregate of the "non-crime solving" services provided by police. Since we have postulated only that the production of this output is proportional to population size, it will be possible to determine $\partial C^*/\partial y_6$ only up to this factor of proportionality.

The rate of transformation of output i for output j gives the number of solutions to crimes of type i which must be foregone for an additional solution to a crime of type j ; given fixed levels of all other outputs. Formally, the rate of transformation between outputs i and j may be written as $-\partial y_i/\partial y_j = (\partial C^*/\partial y_j)/(\partial C^*/\partial y_i)$, $i, j = 1, 2, \dots, 5, i \neq j$, and may be calculated using the formula

$$(14) \quad -\frac{\partial y_i}{\partial y_j} = \frac{(a_j + \frac{6}{I} \sum \alpha_{jk} \ln y_k + \gamma_j \ln w) y_i}{(a_i + \frac{6}{I} \sum \alpha_{ik} \ln y_k + \gamma_i \ln w) y_j} \quad i, j = 1, 2, \dots, 5, i \neq j.$$

As with marginal cost functions, it will not be possible to obtain transformation rates between output six and other outputs.

Traditional measures of scale economies (or diseconomies) are predicated on the single output firm and must be modified for use here. We measure scale economies as the inverse of the percentage response of costs to a small equal percentage change in all outputs. That is, if

$$(15) \quad \epsilon \equiv dC^*/C^* = \sum \frac{6}{I} (\partial \ln C^* / \partial \ln y_i) (dq/q),$$

where dq/q is the percentage change in outputs, then $1/\epsilon$ is the usual measure of economies of scale.²¹ ϵ measures the percentage response of costs to an equal percentage change in all solutions and in the service output.²²

Defining average cost functions for the various outputs presents something of a problem in the case of multiple output production structures. We have calculated the average cost of solutions of type i by evaluating

$$(16) \quad [C^*(\bar{y}_1, \dots, \bar{y}_i, \dots, \bar{y}_6, \bar{w}) - C^*(\bar{y}_1, \dots, \min y_i, \dots, \bar{y}_6, \bar{w})] / (\bar{y}_i - \min y_i)$$

where an overbar indicates a sample mean and $\min y_i$ is the minimum sample value of y_i .²³ This approach holds input prices and all outputs, except y_i , constant and yields the average value of the increment in costs over the region between the minimum value of y_i and the mean of y_i .

In Table 2 we have evaluated the cost responsiveness function, ϵ , marginal cost functions, average cost functions, and marginal rates of transformation between outputs at sample means.

We find that estimated marginal costs are lowest for solving larcenies at \$353, followed by those for robbery at \$584, burglary at \$786, motor vehicle solutions at \$3,065, and solutions to crimes against the person at \$7,569.²⁴

Rates of transformation between outputs at sample means range from .11 between motor vehicle theft solutions and larceny solutions to twenty-one between larceny solutions and solutions to crimes against the person. Hence, the estimated cost function predicts that on average it will be necessary to forego between eight and nine larceny solutions to solve one additional motor vehicle theft (at the mean) and approximately twenty-one larceny solutions to solve an additional crime against

TABLE 2

Marginal and Average Costs of Outputs, Rates of Transformation and Cost Responsiveness Function, ϵ , at Sample Means*

MC_1	\$ 786.70	AC_1	\$ 662.80	MRT_{12}	.743	MRT_{24}	.605
MC_2	\$ 584.89	AC_2	\$ 720.34	MRT_{13}	3.82	MRT_{25}	12.94
MC_3	\$ 3,065.29	AC_3	\$ 3,367.77	MRT_{14}	.449	MRT_{34}	.115
MC_4	\$ 353.94	AC_4	\$ 332.87	MRT_{15}	9.62	MRT_{35}	2.47
MC_5	\$ 7,569.61	AC_5	\$ 13,097.40	MRT_{23}	5.24	MRT_{45}	21.39
						ϵ	1.083

*Standard errors were calculated for ϵ and for marginal cost functions. Each was highly significant at the .01 level.

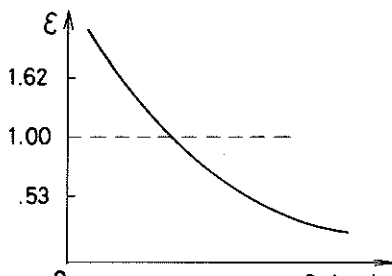
Similar interpretations hold for the other transformation rates.

Unit costs of clearing larcenies are \$332, followed by burglary at \$662, robbery at \$720, motor vehicle solutions at \$3,367, and solutions to crimes against the person at \$13,097. Comparing marginal cost estimates with associated average costs, indicates that marginal costs of solving robberies, auto thefts and crimes against the person are below average costs and hence unit costs are falling (at the sample mean) for these activities. Marginal costs are greater than average costs for solving burglaries and larcenies, indicating rising unit costs (at the sample mean) for these activities.

We have estimated the value of ϵ to be 1.083, which turns out to be not significantly different from unity. But as Fig. 1 indicates, scale economies vary greatly

FIGURE 1

Cost Responsiveness: All Police Activities



over the sample with decreasing, then constant, then increasing returns to scale as output levels increase. Sample values of ϵ range from 1.62 to .53. To the extent that small cities have low solution levels, it appears that "large" cities have technological advantages in the provision of police services.

In interpreting this finding one should keep in mind that the cities in our sample range in size from approximately one third million to only a little over one million. Therefore, one should not conclude that very large American cities experience increasing returns to scale in the provision of police services, since scale diseconomies may appear as city size continues to increase.²⁵

VIII. SUMMARY AND CONCLUSIONS

In this paper we have adopted the economic model of an optimizing firm as a framework for characterizing the production structure of a sample of medium sized U.S. law enforcement agencies. Unlike previous studies we have begun with a second order approximation to an arbitrary multi-output-multi-input production possibilities function. This rather general functional specification has permitted us to test a number of hypotheses which have been implicitly maintained in earlier work. Of particular interest are the findings that, at least in our sample, the decisions of police administrators seem to be inconsistent with cost minimization. We also rejected the hypothesis of nonjoint production. We also found that our sample did not support the existence of a consistent index for any one of the possible sub-aggregates of outputs. These findings make explicit that the usual aggregation of all police outputs into one measure or estimation of separate production (cost) functions is accompanied by a loss of information. In addition, we strongly rejected the hypothesis of constant returns to scale and found that scale economies varied considerably with activity levels — which in turn pointed up the inappropriateness of maintaining a Cobb-Douglas production structure in studies of law enforcement production technology. All tests of functional structure were performed conditional on cost minimizing behavior, although the same results are forthcoming in tests against the unrestricted model.

Finally, we calculated returns to scale, marginal costs, average costs and marginal rates of transformation at the sample mean. As always much work remains to be done. Among the more challenging and potentially promising tasks is to disaggregate the "crimes against the person" output and to incorporate these variables directly into the decision problem underlying estimation. Initial work in this area seems to indicate that unit costs for clearing homicides are an order of magnitude greater than that of any other police activity.

NOTES

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¹ See Chapman, Hirsch, and Sonenblum [2] for an attempt to measure crime prevention as an output of police agencies. In addition, see Shoup [15, pp. 115—18] and the references there cited.

² The weights used are average times spent on each type of activity.

³ This hypothesis is explicit in Wilson and Boland, [21, p. 8,] who state, "In our view, police departments do not behave in accordance with the economic model of the firm".

⁴ For example, linear logarithmic production functions imply input expenditure shares which are independent of the level of expenditure, while linear production functions imply perfect input substitutability and consequently rule out internal solutions to the cost minimization problem.

⁵ In the Popp and Sebold, and Walzer studies the production cost function is specified to be quadratic in the scale argument although all other second order parameters are restricted to be zero.

⁶ M.K. Block has suggested this terminology which is particularly descriptive of the model.

⁷ Of course, there is a constraint on the decision problem which we have not taken into account: Viz., that $C(y,w) \leq A$, where A is the agency's budget for the period.

⁸ See Darrough and Heineke [6] for a detailed discussion on theoretical background.

⁹ An alternative approach to estimating the production structure of law enforcement agencies would be to assume that police take as given a vector of outputs which is minimally acceptable to the community and provide at least that level of service at minimum cost. Among other reasons for choosing the value maximization framework over the cost minimization framework is that the former explicitly addresses the output mix problem rather assuming that this decision is exogenous. See Darrough and Heineke [6] for more detail.

¹⁰ See Diewert [7, 8, 9], and Christensen, Jorgensen, and Lau [3, 4, 5].

¹¹ An alternative means of imposing separability on the translog cost function exists, but is not pursued here. See Darrough and Heineke [6] for more detail.

¹² See Darrough and Heineke [6] and the accompanying references for further discussion of these restrictions.

¹³ The largest city in our sample is Houston, Texas, (1,230,000), the smallest is Birmingham, Alabama (300,000). Mean population over the sample is 561,000.

¹⁴ Cost and wage series have been deflated using an index based upon BLS Intermediate Family Budget data. (See B.L.S. Bulletins No. 1570—7 and the *Monthly Labor Review*).

¹⁵ These groups are: (y_1, y_2) , (y_1, y_3) , (y_1, y_4) , (y_2, y_3) , (y_2, y_4) , (y_3, y_4) , (y_1, y_2, y_3) , (y_1, y_2, y_4) , (y_1, y_3, y_4) , (y_2, y_3, y_4) , and (y_1, y_2, y_3, y_4) .

¹⁶ An example of such a question is whether it is possible to aggregate burglary, robbery, and larceny solutions into a composite category such as "non automobile theft" solutions so that the aggregate index

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consistent index may be derived by Hicks' aggregation theorem (Hicks [12]) or a homothetically separable production structure.

¹⁷ The latter is in fact a rather strong assumption which may be eliminated by using a set of instrumental variables to generate "predicted" values of y_i , say \hat{y}_i , and then replacing y_i with \hat{y}_i when estimating system (11).

¹⁸ The value of $\ln L$ is 1696.65.

¹⁹ According to Table 2, returns to scale at the sample mean are $1/\epsilon = .923$ which turns out to be not significantly different from one. Obviously this does not imply constant returns to scale throughout the relevant output region.

²⁰ The hypotheses of nonjoint production, linear logarithmic costs, constant returns to scale, and functional separability of the several aggregates were also tested against the unrestricted model. In each case these hypotheses were rejected.

²¹ E.g., if $dq/q = 1$, and $\epsilon < 1$ at y^* , then the production function exhibits increasing returns to scale at the output $mix y^*$, etc.

²² The proportionality between population size and y_6 causes no problem in calculating returns to scale since the percentage change in y_6 is equal to the percentage change in population size.

²³ $C^*(y, w) \equiv e^{InC^*(y, w)}$

²⁴ Of course, the model insures that "on average" marginal costs are equal to values stolen. Notice that this does not imply that marginal costs evaluated at the mean are equal "on average" to values transferred. More importantly, our interest in this study is primarily in the structure of law enforcement production technology and hence not local properties of marginal and average cost functions.

²⁵ In the past few years there has been considerable discussion concerning the share of the total police budget going to non-crime solving activities. All parties seem to agree that the share is high and has been increasing. For example, unpublished studies by the Vera Institute of Justice and the Cincinnati Institute of Justice indicate that police officers spend only about 15 to 20 percent of their time in crime solving activities. To provide additional information on this point, we have calculated $AC_6(\bar{y}, \bar{w}) \cdot \bar{y}_6 / C(\bar{y}, \bar{w})$ to measure the budget share of activity six—non-crime solving activities. (This calculation assumes that unit costs of these police services are approximately constant up to y_6 . See equation (16) above.) We find that the budget share of non-crime solving activities is slightly more than 80 percent at the sample mean — a result strikingly consistent with the studies mentioned.

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Summary: *Law Enforcement Agencies as Multi-Product Firms: An Econometric Investigation of Production Costs.* — In this paper we have adopted the economic mode of an optimizing firm as a framework for characterizing the production structure of a sample of medium sized U.S. law enforcement agencies. A translog function is used to test hypotheses as to the nature of agency behavior as well as hypotheses concerning the characteristics of the underlying production function. Our empirical tests have rejected cost minimizing behavior, nonjoint production, the existence of sub-aggregates of outputs, and constant returns to scale. We also calculated returns to scale, marginal costs, average costs, and marginal rates of transformation at sample means.

Résumé: *Les organismes chargés du respect des lois étudiés en tant que firmes à production multiple: une investigation économétrique des coûts de production.* Nous utilisons dans cet article le modèle économique de comportement d'optimisation de l'entreprise afin de caractériser la structure de production d'un échantillon d'organismes, de taille moyenne, chargés du respect des lois aux Etats-Unis. Une fonction "translog" est utilisée afin de tester les hypothèses quant à la nature du comportement des organismes ainsi que les hypothèses relatives aux caractéristiques de la fonction de production sous-jacente. No tests

l'existence de sous-agrégats d'outputs et de rendements d'échelle constants. Nous avons aussi calculé les rendements d'échelles, les coûts marginaux, les coûts moyens et les taux marginaux de transformation à la moyenne échantillon.

Zusammenfassung: *Polizeibehörden als Multi-Produkt-Unternehmen: Eine ökonometrische Untersuchung der Produktionskosten.* – In diesem Artikel wird das ökonomische Optimierungsmodell eines Unternehmens als Rahmen übernommen, um die Produktionsstruktur einer Stichprobe von Polizeibehörden mittlerer Größe zu charakterisieren. Es wird eine "translog" Funktion verwendet, um sowohl Verhaltenshypothesen als auch die Charakteristika der zugrundeliegenden Produktionsfunktionen dieser Behörden zu untersuchen. Unsere empirischen Untersuchungen wiesen kein Kostenminimierungsverhalten, keine gemeinsame Produktion, keine Unteraggregate der Produktion und keine konstanten Skalenerträge auf. Ebenso wurden Skalenerträge, Grenzkosten, Durchschnittskosten und Grenzraten der Transformation auf der Grundlage von Durchschnittswerten berechnet.