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## A THEORY OF HOUSEHOLD BEHAVIOR UNDER UNCERTAINTY\*

*M.K. Block and J.M. Heineke*

In this paper we are concerned with the effects of uncertainty on household decisions. In particular we are interested in the response of a single economic agent's factor allocations to "changes in the amount of uncertainty" with which the agent's beliefs regarding his income are held. For problems of choice under uncertainty the recent work of Arrow [1], Sandmo [9, 10, 11], Leland [5, 6], Stiglitz [12], and others clearly testifies to the power of the expected utility hypothesis as an analytical framework. In what follows two models are examined in which the agent is assumed to be confronting a two period planning horizon and making a joint labor-saving supply decision in the expected utility sense. Although Sandmo [10] and Leland [5] have investigated the effects of uncertainty on the savings decision and Block and Heineke [4] have investigated the effects of uncertainty on the labor decision, to our knowledge no general treatment of the effects of uncertainty on the joint labor-savings decision has been done.<sup>1</sup> It will be shown that the assumptions underlying the Sandmo-Leland analysis are not sufficient to generate their savings results in the more general household model. Additional assumptions are presented that are sufficient not only for the Sandmo-Leland savings results, but which also provide the requisite preference information for an analysis of the labor decision.

Our analysis is divided into three cases: The case of (1) "pure income uncertainty"; (2) wage uncertainty; and (3) savings rate uncertainty.<sup>2</sup> In each case attention is focused upon the response of the factor supply decisions of a risk averse agent to pure dispersion changes.

The following definitions are used.

$C_i$  : the agent's consumption level in period  $i$ ,  $i = 1, 2$ .

$L$  : the amount of labor supplied.

$w$  : the wage rate.

$Y_i$  : total income in period  $i$ ,  $i = 1, 2$ .

$Y_i^0$  : autonomous income in period  $i$ ,  $i = 1, 2$ .

$S$  : the amount of  $Y_1$  saved,  $S \equiv Y_1 - C_1$

$x$  : the rate of return earned on savings

$U(C_1, C_2, L)$  : the agent's cardinal utility indicator, which is assumed to be continuous and to possess continuous first, second and third derivatives.

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<sup>1</sup>A special case of the joint labor savings decision is investigated in Block and Heineke [3].

<sup>2</sup>The formal difference between "pure income uncertainty" and uncertainty in the return to a factor lies in the manner in which uncertainty affects the decision variables, which in turn is a crucial determinant of the qualitative properties of the models.

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As is well known, a general analysis of the response of an agent's decisions to changes in the "amount of uncertainty" in a system requires more detailed information about preference orderings than the agent's simple behavior toward risk (risk aversion or preference). An appealing method of providing additional information is to postulate plausible hypotheses concerning the agent's behavior toward risk as the various decision variables change. For example, a risk averse individual may become relatively less risk averse as his income increases. This hypothesis, Arrow-Pratt decreasing absolute risk aversion, has been widely utilized and has yielded many interesting results.<sup>3</sup>

Designating  $R^i = -U_{ii}/U_i$ <sup>4</sup> as the coefficient of absolute risk aversion in period  $i$  we assume:

(i)  $\partial R^2/\partial C_2 < 0$ ,

(ii)  $\partial R^2/\partial C_1 > 0$ ,

(iii)  $\partial R^2/\partial L = 0$ .

In (i) we assume the individual displays Arrow-Pratt decreasing absolute risk aversion in period 2; (ii) is Sandmo's [9,10] assumption on the intertemporal ranking of consumption bundles; assumption (iii) states that risk aversion is invariant with respect to the labor supply decision and intuitively means that the odds demanded by the agent for taking a risky action are not affected by how much he works.<sup>5</sup>

### Pure Income Uncertainty

We begin by examining the case where the individual's income is not uniquely determined by his factor allocation but contains a stochastic component which is independent of the labor-savings decision. The wage rate and the return earned by savings are both taken to be known. Under these conditions the agent's consumptions in the two periods are

(1)  $C_1 = Y_1 - S$  and

(2)  $C_2 = (1+x)S + Y_2^0$

where  $Y_1 = Y_1^0 + wL$ .<sup>6</sup> In this section we take  $Y_2^0$  to be a random variable which is distributed independently of the labor-saving decision and hence represents the stochastic analog of a neoclassical lump sum income transfer.

The agent's labor-savings decision is determined by

(3)  $\max_{L, C_1} \int_0^\infty U(C_1, C_2, L) f(Y_2^0) dY_2^0$

subject to (1) and (2) above. We assume  $0 < L < \bar{L}$  and  $C_1 > 0$ , where  $\bar{L}$  is total time available.<sup>7</sup>  $f(Y_2^0)$  represents the individual's subjective probability density on  $Y_2^0$  and reflects his beliefs as to the intervals in which  $Y_2^0$  is likely to lie.

<sup>3</sup>For example, see the Arrow reference cited above.

<sup>4</sup>As usual, subscripts denote partial derivatives; e.g.  $U_1 = \partial U/\partial C_1$  and  $U_{12} = \partial^2 U/\partial C_1 \partial C_2$ . Also note that  $R^i > 0$  for the case of risk averse behavior.

<sup>5</sup>Condition (iii) is a special case of the restriction used in Block and Heineke [3] and permits generalizing the Arrow-Pratt restriction to the case where income is generated in the labor market.

<sup>6</sup>We have chosen to use the amount consumed (in period one) as the decision variable as did Sandmo [10]. An alternative specification is to choose the proportion of income consumed as the decision variable. This approach was adopted by Leland [5].

<sup>7</sup>For simplicity we have restricted our problem to the case where optimal values of  $L$  and  $C_1$  are "internal". Extension via Kuhn-Tucker to include activity boundary constraints is straightforward.



Defining  $F \equiv EU[C_1, Y_2^0 + (1+x)(Y_1^0 + wL - C_1), L]$  necessary and sufficient conditions for a relative maximum are:

$$(4) F_L = E(U_2 w(1+x) + U_L) = 0$$

$$(5) F_1 = E(U_1 - (1+x)U_2) = 0$$

and

$$(6) F_{LL} < 0, F_{11} < 0 \text{ and } D \equiv F_{LL}F_{11} - F_{L1}^2 > 0.$$

We now turn to deducing the allocative consequences of a change in the amount of "pure income uncertainty". Formally, changes in the "amount of uncertainty" may be interpreted as shifts in higher central moments of  $f(Y_2^0)$ . Since for arbitrary probability distributions the variance of a random variable may be an unsatisfactory measure of dispersion we follow the suggestion of Arrow [2] and analyze the effects of a pure dispersion increase by means of a multiplicative parameter shift followed by an additive shift which leaves the mean of the variable unaffected. This approach has been utilized by Sandmo [9,10,11] and Leland [6].<sup>8</sup>

We proceed by changing the dispersion of  $Y_2^0$ , perserving the mean, and deducing the effect on the agent's decision. To this end replace  $Y_2^0$ , in (4) and (5) with  $\gamma_1 Y_2^0 + \theta_1$ , where  $\gamma_1$  and  $\theta_1$  are shift parameters, differentiate with respect to  $\gamma_1$  and evaluate the result at  $\gamma_1 = 1, \theta_1 = 0$ .<sup>9</sup> Thence

$$(7) \partial L / \partial \gamma_1 = [F_{L1} E(U_{12} - (1+x)U_{22})(Y_2^0 - \mu_1) - F_{11} E(U_{22} w(1+x) + U_{L2})(Y_2^0 - \mu_1)] / D$$

$$(8) \partial C_1 / \partial \gamma_1 = [F_{1L} E(U_{22} w(1+x) + U_{L2})(Y_2^0 - \mu_1) - F_{LL} E(U_{12} - (1+x)U_{22})(Y_2^0 - \mu_1)] / D$$

where  $\mu_1 \equiv E(Y_2^0)$ .

Sandmo [10] was able to sign the single decision variable ( $C_1$ ) counterpart to (8) by using assumptions (i) and (ii) and the specification that  $C_1$  is a normal commodity.<sup>10</sup> Block and Heineke [4] were able to sign the single decision variable ( $L$ ) counterpart to (7) using assumptions (i) and (iii) and the specification that  $L$  is an inferior activity. As would be expected, combining these two sets of assumptions does not provide sufficient preference information to sign either of the "uncertainty effects", (7) and (8), generated with the expanded model.

However, it can be shown that assumptions (i) – (iii) above and the specification that the marginal utility of period two income is increasing in period one consumption and decreasing in labor are sufficient to sign (7) and (8) if  $C_1$  and leisure are net stochastic compliments with respect to mean returns. These latter assumptions imply the Sandmo and Block and Heineke postulates, viz.  $L$  and  $C_1$  are inferior and normal activities, respectively. The assumption of net stochastic complementarity between present consumption and leisure formalizes the possible time consuming nature of consumption.<sup>11</sup> In which case we have:<sup>12</sup>

<sup>8</sup>For an intuitive account of such "pure increases in dispersion" see Sandmo [10]. An excellent discussion of alternative interpretations of dispersion changes is given in Rothschild and Stiglitz [7].

<sup>9</sup>Leaving  $E(Y^0)$  unchanged amounts to requiring  $dE(\gamma_1 Y_2^0 + \theta_1) = 0$  and hence  $d\theta_1 / d\gamma_1 = -E(Y_2^0)$ .

<sup>10</sup>Sandmo [10] assumed  $C_1$  to be normal commodity in both certain and stochastic worlds.

<sup>11</sup>Net stochastic complementarity between leisure and consumption implies  $S_{1L} < 0$ , where  $S_{ij}$  is the stochastic net substitution term. Hence,  $F_{1L} < 0$ . For empirical evidence supporting the complementarity hypothesis between leisure and a particular type of consumption, see J.D. Owen, *J.P.E.* Vol. 79, no. 1. Note that Owen's evidence makes this assumption particularly appealing at high income levels.

<sup>12</sup>See Appendix for proof. Inequalities (7') and (8') are also forthcoming for the case where period one and two both contain a random autonomous income component. Consideration of changes in the dispersion  $Y_1^0$  evinces the same qualitative results as reported in (7') and (8').



$$(7') \quad \partial L / \partial \gamma_1 > 0$$

$$(8') \quad \partial C_1 / \partial \gamma_1 < 0$$

and therefore

$$(9) \quad \partial S / \partial \gamma_1 > 0.$$

Inequality (9) is the result obtained by Leland and by Sandmo in the section of his paper entitled "Income Risk". The effect of increased uncertainty on savings while consistent with both Sandmo and Leland has been derived in a somewhat more general framework.<sup>13</sup>

Consequently, under our assumptions the agent reacts to increases in income dispersion by both saving more and working more. The implication being that the individual "hedges" against increased dispersion by increasing his factor income in both periods, which in turn acts as a "buffer" against the increased income uncertainty.<sup>14</sup>

We now see that the frequent references in the literature to the disincentive effects of uncertainty may be misleading. At least under one seemingly reasonable set of assumptions, increased pure income uncertainty has unambiguous incentive effects on the supply of factors -- a result readily explicable in terms of the agent's control of distribution parameters. In the case of pure income uncertainty the agent is unable to influence income dispersion, but can influence the mean value and uses this control to "compensate" for changes in dispersion.<sup>15</sup>

**Some Implications of Pure Income Uncertainty** -- Inheritance, because it is invariant to the agent's allocative decisions, is a common form of pure income uncertainty. Here the conventional wisdom stresses the incentive effects of uncertain intergenerational transfers. And indeed under our assumptions theory supports the conventional wisdom. If the concern of the donor is either to increase the frugality of his potential heirs or to extract more effort, increased uncertainty induced by, say, frequent rewriting of the will is a reasonable strategy.

On a more serious note, our results suggest an allocative interpretation of some consequences of political instability. For example, instability that increases the amount of uncertainty surrounding wealth transfers will have factor incentive effects.

In a stable political environment these consequences of political instability may be replicated as a policy tool. Although lump sum transfers are prescribed in a deterministic world for purely distributive purposes, if the incidence of these taxes is randomized they create income uncertainty and may be used for directly allocative purposes.<sup>16</sup> In underdeveloped countries, our results suggest an increase in the supply of savings and labor may be accomplished by means of a poll tax of random incidence. In practice this would involve subjecting all individuals to the possibility of negative or positive poll taxes. The final tax rate would be unknown and all that would be specified would be the pattern of possible taxes and the expected value of such transfers (zero in the pure dispersion increasing case.)

### Uncertainty in Factor Returns

Above we examined the effect on the household's labor-savings decision of changes in pure income uncertainty: The case where stochastic elements are independent of the factor allocation decision. We next consider the possibility that factors themselves may earn uncertain remunerations.

Formally the agent's labor-savings supply decision is given by

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<sup>13</sup>See equation (9) in Sandmo [10] and equation (15) in Leland [5].

<sup>14</sup>Although our interpretation is in terms of a risk averse agent, similar results, (7') and (8'), are forthcoming for a restricted set of risk takers.

<sup>15</sup>Sandmo [10] provides a similar interpretation in the saving only model.

<sup>16</sup>This policy prescription ignores the normative aspects of changing income dispersion. However, the positive elements (factor supply effects) are obviously applicable to pareto relevant decreases in such dispersion.



$$(10) \max_{L, C_1} \int_0^\infty \int_{-1}^\infty U(C_1, Y_2^0 + (1+x)(Y_1^0 + wL - C_1), L) f(w, x) dx dw,$$

where  $w > 0$  and  $x > -1$  are random variables. The function  $f(w, x)$  is the agent's subjective probability distribution on factor returns. As before we assume  $0 < L < \bar{L}$  and  $C_1 > 0$ . Necessary and sufficient conditions for a maximum are given by equations (4) – (6) as before.

**Wage Rate Uncertainty** -- We now investigate the consequences of a change in the agent's beliefs which conclude that the remuneration earned by his labor has become more uncertain. (Such a calculation may occur, for example, on the introduction of imperfectly predictable wage controls.)

Analytically, increased wage dispersion may be studied by replacing  $w$  with  $\gamma_2 w + \theta_2$  in (4) and (5), differentiating with respect to  $\gamma_2$  and evaluating the result at  $\gamma_2 = 1, \theta_2 = 0$ , where  $\mu_2 \equiv E(w)$ . We have

$$(11) \partial L / \partial \gamma_2 = - [F_{11} E U_2(w - \mu_2)(1+x)] / D + L [F_{L1} E(1+x)(U_{12} - (1+x)U_{22})(w - \mu_2) - F_{11} E(1+x)(U_{22} w(1+x) + U_{L2})(w - \mu_2)] / D$$

$$(12) \partial C_1 / \partial \gamma_2 = [F_{1L} E U_2(w - \mu_2)(1+x)] / D + L [F_{1L} E(1+x)(U_{22} w(1+x) + U_{L2})(w - \mu_2) - F_{LL} E(1+x)(U_{12} - (1+x)U_{22})(w - \mu_2)] / D$$

Equations (11) and (12) are Slutsky-like equations composed of an "uncertainty substitution effect" and an "income uncertainty effect".<sup>17</sup> Since  $E U_2(w - \mu_2)(1+x) = \text{Cov}(U_2(1+x), w)$  its sign depends upon  $U_{22}$ , which for risk averse individuals is negative. Hence  $E U_2(w - \mu_2)(1+x)$  is negative.  $F_{11}$  and  $F_{1L}$  (by the net stochastic complementarity of leisure and  $C_1$ ) are each negative and therefore uncertainty substitution effects are negative and positive respectively.

A *compensated* increase in the dispersion of the wage rate would result in a decrease in the individual's labor allocation and in his savings.<sup>18</sup> The sign of the direct substitution effect  $-[F_{11} E U_2(w - \mu_2)(1+x)] / D$  follows, as indicated above, from risk aversion and conforms quite closely to intuitive notions concerning the implications of such behavior. One would expect that an increase in the dispersion of the wage rate would lead the risk averse agent to substitute away from labor. The sign of the cross substitution effect  $[F_{1L} E U_2(w - \mu_2)(1+x)] / D$  is determined by risk aversion and the specification that leisure and  $C_1$  are net stochastic compliments with respect to mean value changes. Therefore, the qualitative effects of a compensated increase in the mean value of the wage rate will be the same as the effects of a compensated decrease in the dispersion of the wage rate.<sup>19</sup> This is true for both the direct and cross substitution effects and emphasizes the possibility of interpreting dispersion changes as changes in costs.<sup>20</sup>

<sup>17</sup>Note that if we require  $\partial C_2 / \partial \gamma_2 = 0$  then:

$$(a) \quad (\partial L / \partial \gamma_2)_{U=U^0} = - [F_{11} E U_2(w - \mu_2)(1+x)] / D$$

$$(b) \quad (\partial C_1 / \partial \gamma_2)_{U=U^0} = [F_{1L} E U_2(w - \mu_2)(1+x)] / D$$

In deriving (a) and (ba) the dispersion of  $w$  is allowed to change, but the agent is compensated in such a manner that the distribution of  $C_2$  is unaltered. Conceptually such a compensation is performed by transforming  $Y_2^0$  into a random variable that leaves the distribution of  $C_2$  invariant to the dispersion change in  $w$ . Since under these conditions  $dU = 0$ , (a) and (b) are essentially the Slutsky substitution terms for this class of pure dispersion changes. Moreover, the remaining terms in (11) and (12) above may be interpreted, in a straightforward manner, as the income uncertainty effects of a change in  $\gamma_2$ .

<sup>18</sup>The sign of  $(\partial S / \partial \gamma_2)_{U=U^0}$  follows immediately from (11), (12) and the identity  $S \equiv Y_1 - C_1$ .

<sup>19</sup>Similar results are presented for forward exchange positions in Leland [6].

<sup>20</sup>See Block and Heineke [4] and Leland [6] for discussions of this interpretation.

<sup>21</sup>See Appendix for proof.



Since changes in the dispersion of the wage rate effect the dispersion of income, equations (11) and (12) include income uncertainty terms. Under our assumptions these terms are positive and negative respectively.<sup>21</sup>

Transforming to the savings variable we see that income uncertainty effects of a change in wage dispersion are positive for both savings and labor. Therefore, both  $\partial L/\partial\gamma_2$  and  $\partial S/\partial\gamma_2$  are indeterminate in sign and it is not possible to make unequivocal assertions as to the allocative consequences of changes in wage rate uncertainty. Hence, statements such as Rottenberg's [8] assertion that theft induced insecurity is likely to result in a decreased labor allocation require substantial qualification. Clearly one must have relative magnitude information before an unambiguous assessment of the supply effects of uncertainty is possible.

Nonetheless, for "small" labor allocations we would expect the substitution terms to dominate while for "large" allocations the income terms may dominate.<sup>22</sup> For example, a "semi-retired" individual might react to increased wage uncertainty by entirely withdrawing from the labor force and simultaneously increasing his present consumption. Exactly the opposite supply responses might be expected from changes in conscription procedures that increase the dispersion of returns to labor. Since draftees are likely to be "full time" labor force participants such changes in procedures may elicit an increase in voluntary effort supplied (overtime) as well as a decrease in present consumption.

**Savings Rate Uncertainty** -- In this section we determine the response of the agent's labor-savings decision to increased dispersion in  $x$ . We proceed as in the previous case: Replacing  $x$  in (4) and (5) with  $\gamma_3x + \theta_3$ , differentiating and evaluating at  $\gamma_3 = 1$ ,  $\theta_3 = 0$ , with  $E(x) = \mu_3$ , we have

$$(13) \partial L/\partial\gamma_3 = - [F_{11}EU_2w(x-\mu_3) + F_{L1}EU_2(x-\mu_3)]/D + S[F_{L1}E(U_{12} - (1+x)U_{22})(x - \mu_3) - F_{11}E(U_{22}w(1+x) + U_{L2})(x-\mu_3)]/D$$

$$(14) \partial C_1/\partial\gamma_3 = [F_{1L}EU_2w(x-\mu_3) + F_{LL}EU_2(x-\mu_3)]/D + S[F_{1L}E(U_{22}w(1+x) + U_{L2})(x - \mu_3) - F_{LL}E(U_{12} - (1+x)U_{22})(x - \mu_3)]/D$$

Uncertainty substitution effects in (13) and (14) are negative and positive respectively by an argument analogous to that used in our discussion of wage uncertainty. Again compensated increases in the dispersion of returns to saving will decrease work effort and saving. And again the assumption of risk aversion and net stochastic complementarity between present consumption and leisure result in intuitively plausible direct and cross substitution terms.

Because the dispersion of the returns to saving is increased income is more uncertain and the total savings rate uncertainty effects ( $\partial L/\partial\gamma_3$  and  $\partial C_1/\partial\gamma_3$ ) include income uncertainty terms. As before our assumptions imply these terms are positive and negative respectively. Consequently uncertainty substitution and income effects in the expressions for  $\partial L/\partial\gamma_3$  and  $\partial S/\partial\gamma_3$  will be negative and positive respectively in each case, and, as in the case of wage uncertainty, total effects depend upon relative magnitudes. This is essentially a generalization of the result Sandmo [10] obtained in his analysis of the saving decision under conditions of "capital risk."<sup>23</sup> Equation (14) contains the same qualitative implications as his result; however, in this model it is derived within the context of the household's joint labor-savings decision.<sup>24</sup>

The relative magnitude aspects of savings rate uncertainty may be illustrated by considering the incentive effects of monetary policy. Specifically, replacing discretionary policy with automatic rules would most likely decrease the dispersion of returns to financial assets. The saving incentive effects of this action may depend upon the income position of the agent. "High income" groups may decrease their allocation to saving and effort while "low income" groups actually

<sup>22</sup>This statement presumes the existence of an  $L^0$  such that substitution terms dominate for  $L < L^0$  and income terms dominate for  $L > L^0$ . Of course the critical value of  $L$ , if it exists, that causes the "total uncertainty effects" to change sign will in general be different for  $\partial L/\partial\gamma_2$  and  $\partial C_1/\partial\gamma_2$ .

<sup>23</sup>See equation (12) in Sandmo [10].

<sup>24</sup>A somewhat different specification of the joint labor-savings decision under conditions of "capital risk" is investigated in Block and Heineke [3].



increase their factor allocations. The aggregate effect on the supply of factors would then depend upon the income distribution.

### Summary

We have examined two types of income uncertainty: Pure income uncertainty and factor return uncertainty. In the former the stochastic term is independent of the factor allocation while in the latter the returns to factors themselves are stochastic.

It has been shown that, under a quite plausible set of assumptions, increases in pure income uncertainty have unambiguous incentive effects. The allocation to savings and labor increase as the dispersion of autonomous income increases. This is essentially a generalization of Leland [5] and Sandmo's [10] "income risk" results. Nonetheless, deriving their savings results within our expanded household decision problem required additional preference restrictions. Significantly, the set of conditions that were sufficient to derive the Leland-Sandmo results ( $\partial C_1 / \partial \gamma_1 < 0$ ) imply the analogous labor result ( $\partial L / \partial \gamma_1 > 0$ ).

In addition to these pure income uncertainty results, we were able to generalize the previous work on uncertain factor returns. The labor and saving incentive effects of changes in the dispersion of the wage rate as well as the returns to savings were derived. Direct and cross dispersion effects were analyzed in each case and, under our assumptions, the ambiguity noted by Sandmo [10] appears to be quite general. Neither increases in the dispersion of the wage rate nor savings rate produced unequivocal incentive effects in direct or cross markets.

The crucial distinction between the agent's response to increases in "pure" income uncertainty and increases in factor return uncertainty lies in the effect any factor reallocation will have on the dispersion of income. In the case of increases in "pure" income uncertainty the agent is able to increase mean income without changing its dispersion, and hence, as was noted above, is able to "hedge" the increased dispersion with a higher expected income. However, in the case of increases in factor return uncertainty increasing expected income automatically increases income dispersion. The former increases expected utility while, for a risk averse agent, the latter decreases expected utility. Only relative magnitude information can reconcile the two conflicting responses.

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### Appendix

Define

$$A \equiv U_{22}w(1+x) + U_{L2}$$

$$B \equiv U_{12} - (1+x)U_{22}$$

To show  $\partial L/\partial\gamma_1 > 0$  and  $\partial C_1/\partial\gamma_1 < 0$  express equations (7) and (8) as:

$$(1) \quad \partial L/\partial\gamma_1 = [-F_{11}\text{Cov}(A, Y_2^0) + F_{L1}\text{Cov}(B, Y_2^0)]/D$$

and

$$(2) \quad \partial C_1/\partial\gamma_1 = [-F_{LL}\text{Cov}(B, Y_2^0) + F_{1L}\text{Cov}(A, Y_2^0)]/D.$$

Under assumptions (i) - (iii) and those detailed following equation (8), A and B increase and decrease, respectively, in  $Y_2^0$ . Hence, second order conditions and the specification that leisure and  $C_1$  are net stochastic complements imply (1) and (2) are positive and negative respectively.

In a similar manner it can be shown that "income uncertainty effects" are positive in the expressions for  $\partial L/\partial\gamma_2$  and  $\partial L/\partial\gamma_3$  and negative in the expressions for  $\partial C_1/\partial\gamma_2$  and  $\partial C_1/\partial\gamma_3$ .