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The multi-output translog production cost function: the case of law enforcement agencies

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THE MULTI–OUTPUT TRANSLOG PRODUCTION COST FUNCTION: THE CASE OF LAW ENFORCEMENT AGENCIES

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In this paper we study the relationship between costs, input prices and activity levels in a sample of approximately thirty medium sized city police departments for the years 1968, 69, 71 and 73. Our interest lies in determining the functional structure of law enforcement production technology.

Since efficient allocation of resources to activities requires knowledge of relative incremental costs for the activities involved, we are particularly interested in determining marginal cost functions for, and rates of transformation between the various outputs. Since past studies have adopted functional specifications which have implicitly maintained strong hypotheses about the underlying technology, we adopt a quite general functional specification which permits testing the appropriateness of these hypotheses. In a more general context we model and estimate the structure of production for a multiple output-multiple input firm in a manner which places few restrictions on first and second order parameters of the underlying structure.

INTRODUCTION

One question which arises immediately in any discussion of cost or production functions associated with law enforcement agencies concerns the appropriate measure of "output." Clearly police departments produce multiple outputs (services) for a community, ranging from directing traffic, quieting family squabbles, and providing emergency first aid, to preventing crimes and solving existing crimes. In this study we view police output as being of essentially two types: (1) general service activities as epitomized by the traffic control and emergency first aid care functions of police departments; and (2) activities directed to solving existing crimes. Strictly speaking, "solving existing crimes" is an intermediate output with deterrence or prevention of criminal activity being the final product. But due to the difficulty of measuring crime prevention we use the number of "solutions" by type of crime as output measures.[1]

In the past few years a number of authors have, to one degree or another, addressed the problem of determining the structure of production in law enforcement agencies. Since under certain rather mild regularity conditions there exists a duality between cost and production functions, either the cost function or the production function may be used to characterize the technological structure of a firm. The studies of Chapman, Hirsch and Sonenblum (1975), Ehrlich (1973, 1975), and Wilson and Boland (1977) all proceed by estimating production functions while Popp and Sebold (1972) and Walzer (1972) estimate cost functions. It is of some interest to briefly review the findings of these authors.

Chapman, Hirsch and Sonenblum estimate a rather traditional production function, at least from a theoretical point of view. All police outputs are collapsed into one aggregate, which is then regressed on input use levels utilizing data from the city of Los Angeles for the years 1956-70. They find strongly increasing returns to scale—often a two to four percent output response to a one percent change in input usage.

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[1] See Chapman, Hirsch and Sonenblum (1975) for an attempt to measure crime prevention as an output of police agencies.

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Ehrlich also uses an aggregate solution rate as the output measure, but instead of employing traditional input measures he regresses the aggregate solution rate on per capita expenditures on police, the aggregate offense rate and a series of exogenous ("environmental") variables. The expenditure variable is, of course, an index of overall input use levels while the aggregate offense rate is included to measure the effects of "crowding" or capacity constraints on output. This is a substantial departure from a neoclassical approach in which the shape of the production function itself will reflect diminishing returns as capacity is pressed. But it is a specification that has been widely adopted by those who have followed Ehrlich. (For example, see Vandaele (1975) or Votey and Phillips (1972). Using per capita expenditures to measure the scale of output, Ehrlich finds that a one percent increase in expenditures per capita leads to much less than a one percent increase in the solution rate.

Votey and Phillips estimate production functions which link solution rates for the property crimes of auto theft, burglary, larceny and robbery to input usage. As with Ehrlich and Vandaele, the authors include the level of offenses as an argument in the production function along with more traditional input measures.

The Wilson and Boland study is similar to the work of Votey and Phillips in that they study the production of solutions to several property crimes. But instead of input levels as determinants of solutions, they utilize the ever present "capacity" variable and variables meant to account for productivity differences between departments. Here as with Vandaele and Votey and Philips, the authors cannot address the question of scale economies due to the fact that only a subset of all outputs are included in these studies.

Finally, both Popp and Sebold, and Walzer estimate cost functions and attempt to measure scale economies. The former use population size in the police jurisdiction as their measure of "scale" along with a large number of demographic and environmental variables to estimate the per capita costs of police service. Given the appropriateness of these variables for explaining costs, the authors find diseconomies of scale throughout the entire range of population sizes. Of course the population variable provides a considerably different concept of scale than economists are accustomed to considering, and in fact, Walzer has argued that population size is a poor measure of scale for several reasons–the most important being a tendency on the part of police administrators to determine manpower needs as a proportion of population size. In such a case there is obviously a strong bias toward constant returns to scale. In his study Walzer recognizes that offenses cleared, accidents investigated, etc., all make up the output of a police department. But instead of estimating a multiple output cost function, he creates an "index of police service" by collapsing all outputs into one.[2] Once again the estimated cost function contains a capacity measure (the offense rate), in addition to measure of input prices, *input usage* and several variables meant to pick up externally determined differences in productivity. Using the service index to measure output Walzer finds evidence of economies of scale, although they seem to be rather slight. Interestingly enough he also finds that input costs are not significantly related to overall production costs.

OUTLINE OF THE PAPER

A number of strong hypotheses concerning the production structure of law enforcement agencies have been implicitly maintained in the studies we have sketched. First, the arguments entering cost and production functions have for the most part differed considerably from what one would expect from classical production theory. In addition, in the one case where input costs do enter the cost function (Walzer), linear homogeneity in input costs has not been imposed on the estimated cost function. One possible explanation for these deviations from classical production and cost specifications is that classical theory, and cost minimizing behavior in particular, is not capable of explaining observed choices in public agencies. While this is a plausible hypothesis, it should be tested rather than maintained.[3]

Second, each of the estimated production functions upon which we have reported is either linear or linear logarithmic. Such functions may be viewed as first order approximations to an arbitrary production function. It is well known that first order approximations severely restrict admissable patterns of substitution among inputs and admissable rates of transformation among outputs as well as having other undesirable empirical implications.[4] An additional problem with linear logarithmic production or cost functions arises if one is interested in determining the extent of scale economies, since these functions do not permit scale economies to vary with output. On a related point, we noted above that each of the production studies surveyed included a "capacity" measure as an argument. A possible explanation for this inclusion might be based upon the restrictiveness of the chosen functional forms and a consequent attempt on the part of the authors to provide output responses which do vary with the scale of operation, in functions which do not naturally possess this property. For these reasons and others we adopt a second order approximation to the underlying cost and production structure thereby leaving the various elasticity measures of common interest free to be determined by the data.[5]

Third, since the Chapman, Hirsch and Sonenblum, Walzer and Ehrlich studies all utilize single output aggregates they implicitly maintain the existence of an index over all police outputs which allows outputs to be consistently aggregated into a single measure. In what follows we estimate a multiple output cost function and test whether the various subsets of outputs may be consistently aggregated into single categories.

[2] The weights used are average times spent on each type of activity.

[3] This hypothesis is explicit in Wilson and Boland, p. 8, who state, "In our view, police departments do not behave in accordance with the economic model of the firm."

[4] For example, linear logarithmic production functions imply input expenditure shares which are independent of the level of expenditure, while linear production functions imply perfect input substitutability and consequently rule out internal solutions to the cost minimization problem.

[5] In the Popp and Sebold, and Walzer studies the production cost function is specified to be quadratic in the scale argument although all other second order parameters are restricted to be zero.

Fourth, the Wilson and Boland, Votey and Phillips and Vandaele studies each implicitly maintain the hypothesis of nonjoint outputs by estimating separate production functions for different types of solutions. Again, instead of maintaining this hypothesis we estimate a multiple output function and then test the nonjointness hypothesis.

To summarize, in this study we characterize the structure of production in a combined cross section and time series analysis of U.S. police departments in a sufficiently general manner to permit testing of each of the major maintained hypotheses in past studies. This amounts, primarily, to testing for existence of consistent aggregate indices of police output, for nonjointness of output, and for consistency of our estimated equations with the optimizing behavior of classical theory. In addition, we calculate (1) marginal cost functions for solutions to the property crimes of burglary, robbery, larceny and motor vehicle theft, and for solutions to crimes against the person; (2) marginal rates of transformation between these activities; and (3) an estimate of scale economies based upon the response of total cost to a simultaneous variation in all police outputs. In the next two sections we provide definitions, theorems and the conceptual structure which underpin the parameter estimation and testing which follow.

THEORETICAL BACKGROUND

The following definitions and theorems provide precise meaning to many of the concepts discussed above and the basis for testing the maintained hypotheses of earlier studies.

Let $F(y, v) = 0$ represent the production possibility frontier, where y is an n vector of outputs and v is an m vector of inputs, and $C(y, w)$ the associated production cost function, where w is an m vector of input prices.

Theorem 1: $C(y, w) = \min_{v \in L(y)} w^T v$

where $L(y) = \{v \mid F(y, v) \geqslant 0\}$ is the input requirement set and T denotes transposition. The function $C(y, w)$ is unique and is a positive linear homogeneous, differentiable and nonincreasing function of input prices, w. (See Uzawa (1964) or Shephard (1970).)

We denote the sets of n outputs and m inputs as $N = \{1, 2, 3, \ldots, n\}$ and $M = \{1, 2, \ldots, m\}$ and partition these sets into ρ and σ mutually exclusive and exhaustive subsets, respectively, $N = \{N_1, N_2, \ldots, N_\rho\}$ and $M = \{M_1, M_2, \ldots, M_\sigma\}$. The elements of N_i are denoted Y_i, the elements of M_j, V_j.

Definition 1: If marginal rates of transformation between any two outputs from the subset N_k are independent of all other outputs, not in N_k then the production function is separable (weakly) with respect to the partition $\{N_k, N_\ell, \forall \ell \neq k\}$. A similar definition holds for input partitions. Formally, the production function $F(y, v) = 0$ is output separable with respect to the partition $\{N_k, N_\ell, \forall \ell \neq k\}$ iff

(1) $\partial(\partial F/\partial y_i / \partial F/\partial y_j) / \partial y_p = 0,\ i, j \in N_k, p \notin N_k$

and is input separable with respect to the partition $\{M_r, M_s, \forall s \neq r\}$ iff

(2) $\partial(\partial F/\partial v_i / \partial F/\partial v_j) / \partial v_t = 0,\ i, j \in M_r, t \notin M_r$

Theorem 2: Separability with respect to the output partition $\{N_1, N_2, \ldots, N_\rho\}$ and the input partition $\{M_1, M_2, \ldots, M_\sigma\}$ is necessary and sufficient for the production function to be written as $F(y, v) = F^*(h_1(Y_1), \ldots, h_\rho(Y_\rho), g_1(V_1), \ldots, g_\sigma(V_\sigma))$ where h_i and g_j are called *category* functions and are functions of the elements of N_i and M_j only. (See Goldman and Uzawa (1964).)

Definition 2: A technology with production function $F(y, v)$ is *nonjoint* if there exists functions $f_1(v), f_2(v), \ldots, f_n(v)$ with the property that $f_i(v)$ is independent of y_j, $i \neq j$.

So to show that a technology is nonjoint, the functions $f_i(\cdot)$ must exist and be free of any economies or diseconomies of jointness. As Hall (1973) has pointed out, this does not require physically separate processes producing the various outputs, nor does the fact that two or more outputs are produced in the same plant rule out nonjointness.

Theorem 3: A technology is *nonjoint* iff the joint cost function can be written as $C(y, w) = C_1(y_1, w) + C_2(y_2, w) + \ldots + C_n(y_n, w)$. (See Hall (1973).)

Definition 3: Aggregation is said to be *consistent* if the solutions to a problem at hand are identical regardless of whether one uses aggregate indices or the micro level variables.

Definition 4: If a function is separable and each of the category functions is homothetic, the function is said to be *homothetically separable.*

Theorem 4: Homothetic separability is sufficient for consistent aggregation. If a function is separable, homothetic separability is necessary for consistent aggregation. (See Blackorby, Primont & Russell (1977).)

MOTIVATION OF AGENCIES

We next present two alternative models of the decision process of law enforcement agencies. One model focuses on input decisions, the other on output decisions. It should be kept in mind that the model chosen to represent agency behavior will likely have a major influence on the values of estimated parameters. Hence one should consider the alternative specifications with one eye on statistical tractability and data limitations, and the other on the "realism" of the implied decision process.

Cost Minimization–The formal structure provided by the cost minimization behavioral hypothesis can be imposed on the estimation process in several ways. To begin, we generalize the traditional cost minimization paradigm to include the multiple output firm. In particular, we assume that law enforcement agencies are given a vector of outputs which is minimally acceptable to the community and are instructed to provide at least that level of service at minimum cost.[6] Formally the agency's problem is to

$$(3) \quad \min w^T v \qquad (\text{s.t.})\ F(y^0, v) = 0$$

where y^0 is the minimally acceptable output vector. Optimization problem (3) provides the system

$$(4) \quad \frac{w_i}{w_j} - \frac{\partial F/\partial v_i}{\partial F/\partial v_j} = 0, \qquad i, j = 1, 2, \ldots, m, i \neq j$$

$$F(y^0, v) = 0$$

of m+1 equations which may be used in estimating F. If equations (4) are assumed to be associated with a well behaved minimum, we know that a solution for v as a function of w and y^0 exists. In addition, as long as input prices are exogenously determined as far as an individual agency is concerned, the solution yields the n endogenous factor demand as functions of strictly exogenous variables. Because factor demands are simultaneously determined, disturbances, given by the stochastic specification into which the model must eventually be imbedded, will be correlated across equations. As a consequence, it will usually be necessary to treat the solution to (4) as a system for purposes of parameter estimation, if efficiency is a criterion.

Two other points concerning the system implied by equations (4) are of interest: First, *if* (4) can be solved for the v_i as functions of w and y^0, these functions may well be nonlinear in the parameters. This need not be a major obstacle, but for large systems nonlinear estimation is expensive and one is never sure of estimability (convergence). Secondly, and more important, is the fact that although we know a solution to (4) exists in principle, this is cold comfort to the econometrician charged with estimating F(·). Since for even modestly general functional specifications for F(·), it will generally be impossible to express the v_i as explicit functions of w and y^0.

An alternative to the approach we have just described for estimating the production structure is to focus on the cost function rather than the production function. Due to the duality between cost and production functions, once one function is given the other is uniquely determined.[7] So it matters not a whit which function is estimated, and the choice of estimating the cost function or the production function should be made on purely statistical grounds.

One way of proceeding to estimate the production cost function would be to use OLS to directly estimate C (y, w). Since both w and y are exogenous in the present framework, OLS is an appropriate procedure. In contrast to the system we have just discussed this is a welcome respite. But a caveat must be added: If C (y, w) is estimated via OLS then one ends up not exploiting the information available in the maintained hypothesis of cost minimization, which might have been used to add precision to parameter estimates.[8] Furthermore, unless there is significant variation in y and w across the sample, multicollinearity could be a problem. (One might expect this problem to crop up especially with input prices.) An additional advantage of imposing the structure implied by cost minimization is that the resulting restrictions across parameters will help circumvent multicollinearity problems which may be present.

The economical way to add the structure implied by cost minimization is to call upon Shephards' Lemma (1953) which gives cost minimizing factor demands as a function of the partial derivatives of the cost function with respect to input prices:

$$(5) \quad v_i = \partial C/\partial w_i \qquad i = 1, 2, \ldots, m.$$

In general, estimation of (5) will not be sufficient to determine all of the parameters of the production cost function.[9] This can be remedied merely by including C(·) as an equation in the system to be estimated. In which case

$$(6) \quad v_i = \partial C/\partial w_i, \qquad i = 1, 2, \ldots, m$$

$$C = C(y, w).$$

It is important to keep in mind that a maintained hypothesis of this section has been that law enforcement agencies are assigned minimal output requirements and that input prices are exogenous as far as any single agency is concerned. As with the dual system (4) above, right hand variables in system (6) will be uncorrelated with stochastic disturbances in the econometric version of (6). Hence estimation of equations (6) will most assuredly identify the parameters of the cost function. But since the v_i are simultaneously determined, disturbances will be correlated across equations as before, necessitating estimating (6) as a simultaneous system if efficient estimates are desired.

An additional advatnage of estimating (6) instead of (4) stems from the fact that equations (6) will be linear in the parameters for any polynomial approximation to an arbitrary cost function.

Value Maximization–In this section we provide an alternative framework within which the structure of law enforcement production technology could be estimated. The model is essen-

[6]In a democratic society, voters through their elected representatives provide this information.

[7]See Diewert (1974).

[8]Of course this information was present in system (4) above.

[9]For example if C (y, w) is a polynomial in y and w, parameters associated with terms in elements of y alone will be missing from equation (5).

ially a value maximization model and of course still implies that input decisions are reached in a cost minimizing manner. The value maximization model has the advantage of not requiring that police decision makers take the community's final output vector as a datum. Indeed the focus of the model shifts from determination of optimal input usage given an output vector, to determination of the optimal mix of outputs.

Using P_i to represent the value to the community of a solution to a crime of type i, $P \equiv (P_1, P_2, \ldots, P_n)$, the police agency's decision problem is

$$(7) \quad \max_{y} P^T y - C(y, w) \cdot$$

Decision problem (7) provides the familiar system

$$(8) \quad P_i - \partial C/\partial y_i = 0, \qquad i = 1, 2, \ldots, n$$

which may be used to estimate C (y, w). As was the case with equations (5) above, if C (y, w) is approximated with a polynomial in y and w, equations (8) alone will not be sufficient to determine the cost function. This can be remedied by including C (y, w) itself in the system to be estimated. In which case

$$(9) \quad P_i - \partial C/\partial y_i = 0, \qquad i = 1, 2, \ldots, n$$

$$C - C(y, w) = 0$$

is the system of interest. Assuming that the values P and input costs are exogenously determined, equations (9) determine the n endogenous solution levels as functions of P and w.[10]

One problem in implementing this system is an econometric context is obvious: The values to a community of the various types of solution are at best difficult to obtain. In the case of property crimes one might consider using average values stolen for each of the several types of property crimes to approximate the loss to society. Although this measure is far from perfect, it does provide a means for studying the mix of property crime solutions and is used in this capacity below.[11] But for the case of "crimes against the person," e.g., homicide, rape and assault, such a convenient proxy is not available.

To circumvent this problem we assume that there exists functions C* and f such that the cost function can be written as

$$(10) \quad C = C^* (f(y_1, \ldots, y_p, w), y_{p+1}, \ldots, y_n, w)$$

where $y_1, \ldots, y_p$ represent solutions to crimes against property and $y_{p+1}, \ldots, y_n$ represent solutions to crimes against the person and the service activities performed by police. That is, we assume that solutions to crimes against property are functionally separable from all other police activities. As we indicated above, (Theorem 2) this is equivalent to requiring that marginal rates of transformation between solutions to all pairs of property crimes be invariant to the level of nonproperty crime solutions and to the level of other services provided, e.g., traffic control, emergency first aid, etc. In this case, optimization (7) may be treated as two problems: The optimal mix of property crime solutions, non property crime solutions and services is determined in a first step after which a second optimization is performed to determine the optimal mix of property crime solutions. See Strotz (1957). System (9) then becomes

$$(11) \quad P_i - \partial C^*/\partial y_i = 0, \qquad i = 1, 2, \ldots, p$$

$$C - C^* (f(y_1, y_2, \ldots, y_p, w), y_{p+1}, \ldots, y_n, w) = 0$$

and is estimated below for the case of four property crimes, burglary, robbery, motor vehicle theft and larceny, an aggregate of crimes against the person and an aggregate service indicator.

We have chosen to estimate the production cost function utilizing equations (11) rather than (6) for several reasons: First, costs in law enforcement agencies tend to be predominately labor costs (approximately ninety percent). And as one would expect, salaries of police employees by rank are highly correlated. Therefore, it will not be possible to include more than one, or at most two input demand equations if structure (6) is imposed. In addition, we approximate C* with a second order expansion in the logarithms of y and w and factor demand equations will impose no restrictions across the coefficients of $\ln y_i$ and $\ln y_i \ln y_j$. If these terms are highly collinear, which is likely to be the case, then system (11) places restrictions across coefficients of terms in y and reduces the collinearity between the elements of y. The second reason for choosing (11) as the basis for estimation is that it explicitly addresses the output mix problem rather than assuming that the decision is exogeneous to police administrators as in (6).

THE TRANSLOG MODEL

From an econometric point of view equation system (11) is only of limited interest until a specific functional form has been assigned to the cost function C* (y, w). The primary concern in choosing a function form for C* is that the chosen class of functions be capable of approximating the unknown cost function to the desired degree of accuracy. In widespread use in the literature in the past few years are the class of so called "flexible" functional forms which includes the generalized Leontief function, the generalized Cobb-Douglas function, the transcendental logarithmic function and many hybrids.[12] These functions are all second order approximations to arbitrary differentiable, primal or dual objective functions and in particular place no restrictions on elasticities of substitution between inputs or elasticities of transformation between outputs and allow returns to scale to vary with the level of output. We have chosen to approximate C*(y, w) with a translog function due primarily to the fact that most past studies of law enforce-

[10]In general, the elements of P are at least partially determined by the output mix chosen by police decision makers, e.g., increased solutions for crime i will, ceteris paribus, lower expected returns to crime and hence P_i.

[11]Average values stolen are an approximation to the direct financial loss suffered by society, on average, from an offense of type i. To the extent that solving crimes has a deterrent effect, this measure will underestimate the value of a solution to offense i by the value of illegal transfers deterred per solution.

[12]See Diewert (1971, 1973, 1974) and Christiansen, Jorgensen and Lau (1971, 1973, 1975).

ment agency production technology have adopted linear logarithmic production structures.[13]

The translog cost function may be written as

$$(12)\quad \ln C(y, w) = a_0 + \sum_1^n a_i \ln y_i + \sum_1^m b_i \ln w_i + \tfrac{1}{2}\sum_1^n\sum_1^n \alpha_{ij} \ln y_i \ln y_j + \tfrac{1}{2}\sum_1^m\sum_1^m \beta_{ij} \ln w_i \ln w_j + \sum_1^m\sum_1^n \gamma_{ij} \ln y_i \ln w_j .$$

Since logarithmic functions are continuously differentiable, the parameters α_{ij} and β_{ij} will be symmetric, i.e., $\alpha_{ij} = \alpha_{ji}$ and $\beta_{ij} = \beta_{ji}$, $\forall i, j$. Our maintained hypothesis of functional separability, see equation (10), between property crime solutions and all other activities of the police agency implies the following restrictions on equation (12):

$$(13)\quad \alpha_{ij} = 0,\quad i = 1, 2, \ldots, p,\ j = p+1, p+2, \ldots, n.$$[14]

The hypothesis of *linear homogenity* of the cost function in factor prices, nonjointness of outputs and existence of consistent indices of output discussed above are not maintained, but rather tested.

Testing for linear homogenity of C (y, w) in inputs prices may be interpreted as a test for cost minimizing behavior and implies the following restrictions on the translog cost function:

$$(14)\quad \sum_i^m b_i = 1,\ \sum_j^m \beta_{ij} = \sum_i^m \beta_{ij} = \sum_i^m \gamma_{ij} = 0$$

It will also be of interest to test for constant returns to scale in output. Constant returns to scale implies

$$(15)\quad \sum_i^n a_i = 1,\ \sum_j^n \alpha_{ij} = \sum_i^n \alpha_{ij} = \sum_i^n \gamma_{ij} = 0$$

If outputs are nonjoint, all cross second order terms in y are zero, i.e.,

$$(16)\quad \alpha_{ij} = 0,\qquad i, j = 1, 2, \ldots, n,\ i \neq j.$$

THE ECONOMETRIC MODEL

In this section we specialize the n output, m input production model to the model which is estimated and provide the stochastic specification needed for estimation. We had available for this study information on annual police budgets for the years 1968, 1969, 1971 and 1973 for a sample of approximately thirty five medium size cities; the average wages of officers by rank, the number of crimes of type i cleared by arrest ("clearances") and the average value stolen for each of the property crimes in the FBI index.[15] The police budget and wage information was gathered by the Kansas City Police Department and circulated for use by participating cities under the title of the *Annual General Administrative Survey*. The data on clearances and average values stolen is from unpublished sources at the FBI. Because of limitations on the number of variables which could ultimately be allowed in the model, we decided to use clearances by arrest for the seven FBI index crimes as our measures of "solutions." In particular, we have called burglary clearances (solutions), y_1, robbery clearances, y_2, motor vehicle theft clearances y_3, and larceny clearances, y_4. We have used the aggregate number of homicide, rape and assault clearances to represent solutions to crimes against the person and have labeled this output, y_5. Finally, a very large component of the output of all law enforcement agencies are the rather mundane but important service functions: Directing traffic, investigating accidents, breaking up fights, providing emergency first aid, etc. We group all such service functions together as y_6. The question is what to use to measure these activities. We have adopted the hypothesis that the quantity of services of the type we have been discussing is proportional to the size of the city in which the agency is located. This gives a cost function with six outputs and a still unspecified number of input prices.

We had available wage information on eight grades of police officers from patrolman to chief. As one might expect these wage rises are highly collinear. To test for the existence of a Hicksian price index we computed correlation coefficients between the wages of the various ranks and found very high coefficients. For example, the correlation between wages of patrolmen and a weighted average of the wages of all other ranks is .955. Unfortunately, there does not appear to be a way of testing whether a sample correlation is significantly different from one since the distribution of this statistic is degenerate at that point. But with correlations this high it appears safe to assume the conditions for Hicks' aggregation are fulfilled and hence we use a weighted average of all police wages as an aggregate measure of unit labor costs, denoted w.[16]

The translog cost function of (2) above may now be written as

$$(17)\quad \ln C^*(y, w) = a_0 + \sum_1^6 a_i \ln y_i + b \ln w + \tfrac{1}{2}\sum_1^6\sum_1^6 \alpha_{ij} \ln y_i \ln y_j + \tfrac{1}{2}\beta \ln w^2 + \sum_1^6 \gamma_i \ln w \ln y_i$$

[13] These studies have utilized linear logarithmic production functions which in turn imply linear logarithmic production cost functions. This property of linear logarithmic primal and dual functions is termed *self duality.* The linear logarithmic function is the only self dual translog function.

[14] See Berndt and Christiansen (1974) for more detail on these conditions. We have imposed what is called linear separability of property crime solutions from other activities which implies $\ln C(y, w) = \ln C_1(y_1, y_2, \ldots, y_p, w) + \ln C_2(y_{p+1}, y_{p+2}, \ldots, y_n, w)$ where $\ln C_1$ and $\ln C_2$ are translog functions (See Blackorby, Primont and Russell (1974). Functional separability may also be achieved via a set of nonlinear restrictions (see Berndt and Christiansen (1974).) Blackorby, Primont and Russell (1974) have shown that nonlinear separability implies $\ln C(y, w) = F(D_1(y_1, y_2, \ldots, y_p, w), D_2(y_p, y_{p+1}, \ldots, y_n, w))$ where D_1 and D_2 are linear logarithmic functions.

Linear logarithmic "aggregator" functions are quite restrictive and for this reason we consider only the case of linear separability throughout this paper.

[15] The largest city in our sample is Houston, Texas, (1,230,000), the smallest is Birmingham, Alabama (300,000). Mean population over the sample is 561,000.

[16] Budget and wage series have been deflated using an index based upon BLS Intermediate Family Budget data. (See B.L.S. Bulletins No. 1570-7 and the *Monthly Labor Review.*)

TABLE I

Parameter Restrictions for Linear Functional Separability[19]

Aggregate	Parameter Restrictions	Aggregate	Parameter Restrictions
(y_1, y_2)	$\alpha_{13} = \alpha_{14} = \alpha_{23} = \alpha_{24} = \gamma_1 = \gamma_2 = 0$	(y_1, y_2, y_3)	$\alpha_{14} = \alpha_{24} = \alpha_{34} = \gamma_1 = \gamma_2 = \gamma_3 = 0$
(y_1, y_3)	$\alpha_{12} = \alpha_{14} = \alpha_{23} = \alpha_{34} = \gamma_1 = \gamma_3 = 0$	(y_1, y_2, y_4)	$\alpha_{13} = \alpha_{23} = \alpha_{34} = \gamma_1 = \gamma_2 = \gamma_4 = 0$
(y_1, y_4)	$\alpha_{12} = \alpha_{13} = \alpha_{24} = \alpha_{34} = \gamma_1 = \gamma_4 = 0$	(y_1, y_3, y_4)	$\alpha_{12} = \alpha_{23} = \alpha_{24} = \gamma_1 = \gamma_3 = \gamma_4 = 0$
(Y_2, y_3)	$\alpha_{12} = \alpha_{24} = \alpha_{13} = \alpha_{34} = \gamma_2 = \gamma_3 = 0$	(y_2, y_3, y_4)	$\alpha_{12} = \alpha_{13} = \alpha_{14} = \gamma_2 = \gamma_3 = \gamma_4 = 0$
(y_2, y_4)	$\alpha_{12} = \alpha_{23} = \alpha_{14} = \alpha_{34} = \gamma_2 = \gamma_4 = 0$	(y_1, y_2, y_3, y_4)	$\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = 0$
(y_3, y_4)	$\alpha_{13} = \alpha_{23} = \alpha_{14} = \alpha_{24} = \gamma_3 = \gamma_4 = 0$		

[19]These restrictions are conditional on the functional separability of property crime solutions and all other police activities.

TABLE II

Parameter Estimates for Five Cost Models

Parameter	Unrestricted Model	Homogeneity in Input Prices	Homo. and Nonjoint Outputs	Homo. and Linear Log. Costs	Constant Returns to Scale
a_o	–108.68 (27.23)	–98.899 (7.512)	–75.949 (2.190)	–4.469 (1.092)	–.7190 (1.332)
a_1	–.0049 (.0478)	–.0542 (.0168)	–.1326 (.0127)	.0292 (.0016)	.0053 (.0114)
a_2	.0244 (.0118)	.0203 (.0110)	.0129 (.0108)	.0065 (.0003)	.0314 (.0109)
a_3	.3262 (.0679)	.2989 (.0615)	.2378 (.0603)	.0459 (.0026)	.3956 (.0646)
a_4	.0252 (.0293)	.0031 (.0205)	–.0467 (.0203)	.0198 (.0009)	.0378 (.0190)
a_5	1.657 (1.682)	–2.118 (1.084)	–.4037 (.4853)	.2448 (.0376)	.4127 (.4088)
a_6	16.016 (.5917)	16.38 (.6349)	12.259 (.1848)	.9113 (.0902)	.1170 (.4147)
b	.7123 (7.393)	1	1	1	1
α_{11}	.0127 (.0020)	.1199 (.0591)	.0296 (.0561)		.0206 (.0014)
α_{22}	.0033 (.0005)	.0034 (.0005)	.0032 (.0003)		.0032 (.0005)
α_{33}	.0287 (.0022)	.0284 (.0022)	.0294 (.0019)		.0189 (.0019)
α_{44}	.0125 (.0009)	.0125 (.0009)	.0119 (.0009)		.0115 (.0007)

Parameter	Unrestricted Model	Homogeneity in Input Prices	Homo. and Nonjoint Outputs	Homo. and Linear Log. Costs	Constant Returns to Scale
α_{55}	.0448 (.0528)	.0177 (.0524)	.0504 (.0441)		.0209 (.0621)
α_{66}	–1.451 (*)	–1.2711 (*)	–.0005 (*)		.0209 (.0621)
α_{12}	–.0022 (.0006)	–.0023 (.0006)			–.0277 (.0006)
α_{13}	–.0049 (.0016)	–.0051 (.0017)			–.0115 (.0015)
α_{14}	–.0053 (.0010)	–.0055 (.0010)			–.0063 (.0008)
α_{23}	–.0002 (.0005)	–.0002 (.0005)			–.0013 (.0005)
α_{24}	.0013 (.0004)	.0013 (.0004)			.0008 (.0004)
α_{34}	–.0022 (.0010)	–.0022 (.0010)			–.0061 (.0008)
α_{56}	.0996 (.0882)	.1097 (.0884)			–.0209 (.0621)
β	–.1471 (.5356)				
γ_1	–.0082 (.0071)	–.0963 (.0590)			.0017 (.0015)
γ_2	–.0048 (.0017)	–.0041 (.0016)	–.0039 (.0016)		–.0034 (.0016)
γ_3	–.0617 (.0100)	–.0571 (.0090)	–.0574 (.0088)		–.0496 (.0097)
γ_4	–.0087 (.0043)	–.0050 (.0028)	–.0041 (.0029)		–.0041 (.0068)
γ_5	–;4615 (.1809)	.1029 (.0586)	.0286 (.0562)		–.0068 (.0231)
γ_6	.4798 (*)	.0598 (.0236)	.0445 (.0236)		.0622 (.0255)

*Collinearity problems prevented estimation of this standard error.

TABLE IV (Continued)

Parameter	(1, 2, 3)	(1, 2, 4)	(1, 3, 4)	(2, 3, 4)	(1, 2, 3, 4)
			POTENTIAL OUTPUT AGGREGATES		
α_{23}	.0005 (.0005)			−.0005 (.0005)	−.0003 (.0005)
α_{24}		.0013 (.0004)		.0010 (.0003)	.0013 (.0004)
α_{34}			−.0028 (.0010)	−.0021 (.0009)	−.0029 (.0010)
α_{56}	.0802 (.0870)	.0932 (.0881)	.1222 (.0984)	.1245 (.0881)	.1032 (.0889)
β					
γ_1					
γ_2			−.0003 (.0015)		
γ_3		−.0504 (.0012)			
γ_4	.0019 (.0026)				
γ_5	−.4038 (.1939)	−.4184 (.1969)	−.4522 (.1984)	.0336 (.0570)	−.4546 (.1989)
γ_6	.1873 (.0992)	.2501 (.1011)	.2186 (.0105)	−.0327 (.0190)	.2192 (.1017)

*Collinearity problems prevented estimates of this standard error.

TABLE V

Marginal Costs of Outputs, Rates of Transformation between Outputs and Returns to Scale (at sample means)*

ϵ	1.069 (.0817)	MC_5	2448.00	MRT_{24}	.607
$\overline{\epsilon}$	.3429 (.0571)	MRT_{12}	.191	MRT_{25}	10.77
MC_1	1186.87 (543.35)	MRT_{13}	.992	MRT_{34}	.117
MC_2	227.24 (11.04)	MRT_{14}	.116	MRT_{35}	2.08
MC_3	1177.18 (45.51)	MRT_{15}	2.06	MRT_{45}	17.74
MC_4	138.01 (5.81)	MRT_{23}	5.18		

*Standard errors are in parentheses.

where $\alpha_{15} = \alpha_{16} = \alpha_{25} = \alpha_{26} = \alpha_{35} = \alpha_{36} = \alpha_{45} = \alpha_{46} = 0$ due to the imposed functional separability of property crime solutions and all other police activities.

Linear homogenity of C* in w imposes the further restrictions

$$(18)\quad b = 1,\ \ \beta = 0,\ \ \sum_1^6 \gamma_i = 0$$

while constant returns to scale imply

$$(19)\quad \sum_1^6 a_i = 1,\ \ \sum_i^6 \alpha_{ij} = \sum_j^6 \alpha_{ij} = 0,\ \forall_{i,j} \text{ and } \sum_1^6 \gamma_i = 0\,.$$

If property crime solutions are nonjoint then

$$(20)\quad \alpha_{ij} = 0, \qquad i, j = 1, 2, 3, 4,\ \ i \neq j$$

The latter imposes only six additional restrictions, since differentiability of $\ln C^*$ implies symmetry of the α_{ij}.

Finally, given the hypothesis of functional separability between property crime solutions and all other police activities there are a total of eleven possible groupings of property crime solutions which might be considered for indexing. Our question here is not whether an index exists in any of these cases because an index can always be found, but whether a *consistent* index exists.[17] The eleven candidates for aggregation and indexing are displayed in Table I along with the implied linear separability restrictions.[18] It is important to keep in mind that the existence of an aggregate (a functionally separate group) does not in general imply existence of a consistent index for the aggregate. (See Theorem 4).

For the case at hand, system (11) may be written as

$$(21)\quad P_i y_i / C^* = a_i + \sum_1^6 \alpha_{ij} \ln y_j + \gamma_i \ln w, \qquad i = 1, 2, 3, 4$$

$$\ln C^* = a_0 + \sum_1^6 a_i \ln y_i + b \ln w + \tfrac{1}{2} \sum_1^6 \sum_1^6 \alpha_{ij} \ln y_i \ln y_j + (\beta/2) \ln w^2 + \sum_1^6 \gamma_i \ln w \ln y_i$$

where $\alpha_{ij} = 0$, $i = 1, 2, 3, 4$, $j = 5, 6$ and $\alpha_{ij} = \alpha_{ji}$, $\forall i, j$. (The first four equations here given the value to society of y_i solutions to property crime i as a proportion of total police expenditures.) The next step in implementing the econometric version of the model is to provide a stochastic framework for equations (21). We do this by appending classical additive disdisturbances arise either as a result of random error in the maximizing behavior of police administrators, or as a result of the fact that the translog function provides only an approximation of the "true" underlying production structure. We assume that noncontemporaneous disturbances are uncorrelated both within and across equations. We make no other assumptions about the distribution of disturbances other than they be uncorrelated with right hand variables in each equation.[20]

EMPIRICAL RESULTS

We have fitted the five equations of system (21) under the stochastic specification outlined above. There were 125 observations available for estimating each equation in the system so that the total number of degrees of freedom for statistical tests is 625. Since no assumption has been made concerning the distribution of disturbances, our estimation procedure may be thought of as multiequation, nonlinear least squares. In the computations we used the Gauss-Newton method to locate minima. The results of estimation are presented in Table II.

The estimates reported in column two contain no restrictions other than the symmetry implied by the continuous differentiability of $\ln C^*$ and entails estimating twenty-eight parameters. Given the primarily cross section nature of the data, the model fits quite well with R^2 figures of .74 for the cost function and .36, .13, .46, and .29 for the value of solution equations $P_i y_i / C^*$, $i = 1, 2, 3, 4$, respectively. Durbin-Watson statistics are 1.81, 1.94, 2.41, 2.53, and 2.18, respectively. It appears that disturbances associated with each equation are serially independent.[21]

In column three, we report estimates of the model with homogeneity in input prices imposed. As we have noted previously, cost minimizing input decisions imply a production cost function with this property and for this reason we may consider a test of the fit of the homogeneous model as a test of the consistency of the data with cost minimizing behavior.

[17] An example of such a question is whether it is possible to aggregate burglary, robbery and larceny solutions into a composite category such as "non automobile theft" solutions. Given the numbers of burglary, robbery and larceny solutions and their imputed values, how does one derive quantity and value indices for "non automobile theft" solutions? Suppose burglary solutions, robbery solutions and larceny solutions are separable from other police outputs and input prices, then the cost function may be written as $C(y, w) = C(c_1(y_1, y_2, y_4), y_3, y_5, y_6, w)$. In addition, if c_1 is homothetic then y_1, y_2 and y_4 may be aggregated into a category. The quantity index for sample point $\bar{y}^* \equiv (y_1^*, y_2^*, y_4^*)$ is determined by the function $c_1(\cdot)$ and the values of (y_1^*, y_2^*, y_4^*). Since the index should be linear homogenous in $\bar{y}$, the problem is to find an aggregator function for $\phi(c_1(\bar{y}))$ which is linear homogeneous in y. The quantity (solution) index is then $\phi(c_1(\bar{y}^*))$ at $\bar{y}^*$. The corresponding value (price) index is $\bar{P}^* = (P_1^* y_1^* + P_2^* y_2^* + P_4^* y_4^*) / \phi(c_1(\bar{y}^*))$. Evaluating two "non automobile theft" solution vectors (y_1^*, y_2^*, y_4^*), and (y_1^o, y_2^o, y_4^o), given $\bar{P}^*$ and $\bar{P}^o$, but without knowledge of the function $c_1(\cdot)$, is the analog to the more traditional index problem.

[18] As was pointed out in footnote 14 above, there are also sets of nonlinear restrictions which lead to functional separability. The implications of these conditions are so restrictive that more nonlinear separability is not considered in the tests reported below.

[20] The latter is in fact a rather strong assumption. It may be eliminated by using a set of instrumental variables to generate "predicted" values of y_i, say $\hat{y}_i$, and then replacing y_i with $\hat{y}_i$ when estimating system (21). This approach will be reported on in a later version of this paper.

[21] James Durbin has argued that the conventional single equation Durbin-Watson statistic be used to check for serial correlation in simultaneous equation systems.

Homogeneity in prices reduces the number of parameters to be estimated from twenty-eight to twenty-five (see equation (18)). Traditional R^2 statistics are .72 for the cost equation and .35, .12, .46 and .28 respectively for the value share equations. Durbin-Watson statistics for the five equations are 1.75, 1.94, 2.41, 2.51 and 2.21, respectively. With the possible exception of the cost equation, it again appears that disburbances are serially independent.

In columns four, five and six are reported parameter estimates for the case of nonjoint outputs, linear logarithmic costs and constant returns to scale, each conditional on linear homogeneity in input prices. In column four are the estimates with input price homogeneity and nonjointness of output imposed. These restrictions reduce the number of parameters to eighteen. The linear logarithmic cost function (column five) was estimated primarily to contrast the functional form of the cost function presented in this paper with that implied by the linear logarithmic production functions which have been estimated in the majority of earlier papers.[22] The total number of parameters to be estimated is now reduced to seven. The final column contains our estimate of the model with constant returns to scale imposed.

Our tests of the various hypotheses which have been discussed are based upon the test statistic

$$(22)\quad \Omega = \max L^{R}/\max L^{\overline{R}}$$

where max L^R is the maximum value of the likelihood function for the model with restrictions R and max $L^{\overline{R}}$ is the maximum value of the likelihood function without restriction. Minus twice the logarithm of Ω is asymptotically distributed as chi-squared with number of degrees of freedom equal to the number of restrictions imposed. Throughout we choose critical regions based upon the .01 level of significance.

Logarithms of the likelihood function are given in Table III for each of the model specifications to be evaluated. We first test the hypothesis that police agencies make choices in a cost minimizing manner, which implies C* is linearly homogeneous in w. Comparing the homogeneous model with the unrestricted model we find that minus twice the logarithm of the likelihood ratio is 4.52. Since there are but three restrictions imposed, we easily accept the hypothesis of a cost structure which is linearly homogeneous in input prices. That is, the data in our sample of police departments are consistent with cost minimizing behavior.

Conditional on linear homogeneity in input prices we next test the validity of the hypothesis of nonjoint outputs—a hypothesis which has been maintained in past studies whenever single output aggregates have not been utilized. Minus twice the logarithm of the likelihood ratio is 53.08 and the nonjointness hypothesis is resoundingly rejected. We conclude that one may not go about estimating separate production functions or separate cost functions for each of the outputs of police agencies. The interaction between outputs must be accounted for if one is to adequately characterize the structure of cost and production in this "industry."

TABLE III

Estimated Values of Ln of Likelihood Function

Model		Functional Separability Imposed			
Unrestricted	1654.57	(1, 2)	1628.94	(1, 2, 3)	1616.43
Homogeneous in Input Prices	1652.31	(1, 3)	1613.50	(1, 2, 4)	1645.79
Homog. in Input Prices and Nonjoint	1625.77	(1, 4)	1640.13	(1, 3, 4)	1628.40
Linear Logarithmic Costs	1483.57	(2, 4)	1627.16	(1, 2, 3, 4)	1636.79
Homog. in Input Prices and Output	1613.23	(2, 3)	1619.70	(2, 3, 4)	1612.56
h (y_1, y_2, y_4) is homothetic	1628.52	(3, 4)	1611.56		

It is instructive to contrast the linear logarithmic cost and production structure implied by these data, with our more general model. Columns three and five of Table II contain parameter estimates for the cost models which maintain homogeneity in prices, and a linear logarithmic production structure in addition to linear homogeneity in prices.[23] The fact that twice the logarithm of the likelihood ratio for this test is 337.48 is a fairly accurate indication of the magnitude of the loss in explanatory power resulting from adopting the Cobb-Douglas functional form for C*.

Parameter estimates for the models associated with each of the eleven possible output aggregates are presented in Table IV with corresponding logarithms of likelihood functions tabulated in the right-hand columns of Table III. Since we have accepted the hypothesis of linear homogeneity in input prices, we test these restrictions conditional on the validity of this hypothesis. To begin we choose the prospective aggregate (y_1, y_2, y_4) with the largest likelihood function. Minus twice the logarithm of the likelihood ratio is 13.04 in this case. Since there are a total of six additional restrictions the chi-square (.01) critical value is 16.18 and functional separability of burglary, robbery and larceny solutions from the remaining outputs and from input prices is accepted. Perusal of Table III indicates that all other potential aggregates are rejected and hence, say, $h(y_1, y_2, y_4)$ is the only category function.

According to Theorem 4, if either the cost function is homothetic or if C* is not homothetic, but the category function for (y_1, y_2, y_4) is homothetic, then a consistent aggregate index of burglary, robbery and larceny exists. Barring each of these cases another possibility for consistent aggregation remains; the values for y_1, y_2 and y_4 (P_1, P_2, P_4) (or for that matter, the values of any subgroup of (y_1, y_2, y_3, y_4) are perfectly correlated. (See Hicks' Aggregation Theorem above). To begin these tests we have calculated returns to scale at the mean of $(y_1, y_2, y_3, y_4, y_5, y_6, w)$ and the standard error of this statistic and find we cannot reject the hypothesis of constant returns to scale. (See Table V below.) The fact that C* is

[22]Recall that a linear logarithmic production function is self dual and hence implies a cost function of the same functional form.

[23]Of course, linear logarithmic cost and production functions maintain the nonjointness hypothesis.

TABLE IV

Parameter Estimates: Linear Homogeneity in Input Prices and Functional Separability Imposed

	POTENTIAL OUTPUT AGGREGATES					
Parameter	(2, 3)	(1, 3)	(1, 4)	(1, 2)	(2, 4)	(3, 4)
a_o	–95.5161	–95.75	–106.47	–101.42	–98.5637	–89.6617
	(33.8291)	(33.53)	(7.280)	(7.212)	(33.4918)	(7.4351)
a_1	–.0731	–.1348	–.0683	–.1199	–.1325	–.1228
	(.0173)	(.0129)	(.0171)	(.0137)	(.0131)	(.0138)
a_2	–.0090	–.0148	.0113	–.0066	–.0150	–.0078
	(.0027)	(.0101)	(.0103)	(.0033)	(.0037)	(.0097)
a_3	–.1259	–.1306	.2494	.2059	.1887	–.1265
	(.0158)	(.1075)	(.0581)	(.0568)	(.0566)	(.0168)
a_4	–.0526	–.0892	–.0368	–.0506	–.0753	–.0659
	(.1867)	(.0195)	(.0099)	(.0192)	(.0083)	(.0078)
a_5	–2.1855	–1.845	1.237	1.320	–1.8340	–1.8721
	(1.2147)	(1.187)	(1.761)	(1.741)	(1.2022)	(1.0716)
a_6	16.4742	16.47	15.976	15.31	16.5406	15.4808
	(5.3143)	(5.277)	(.8535)	(.8442)	(5.2618)	(.6226)
b	1.0	1.0	1.0	1.0	1.0	1.0
α_{11}	.0621	.0278	.0187	.0209	.0327	.0491
	(.0587)	(.0077)	(.0017)	(.0018)	(.0584)	(.0574)
α_{22}	.0029	.0030	.0028	.0033	.0028	.0036
	(.0004)	(.0004)	(.0005)	(.0004)	(.0004)	(.0005)
α_{33}	.0268	.0321	.0265	.0283	.0309	.0289
	(.0023)	(.0023)	(.0021)	(.0020)	(.0021)	(.0023)
α_{44}	.0124	.0119	.0122	.0123	.1174	.0126
	(.0009)	(.0094)	(.0009)	(.0009)	(.0009)	(.0010)
α_{55}	.0177	.0321	.0393	.0454	.0282	.0265
	(.0527)	(.0521)	(.0523)	(.0516)	(.0052)	(.0518)
α_{66}	–1.247	–1.238	–1.348	–1.280	–1.2747	–1.1582
	(.4224)	(.4201)	(*)	(*)	(.4182)	(*)
α_{12}				–.0010		–.0010
				(.0005)		(.0006)
α_{13}		–.0043			–.0035	
		(.0016)			(.0015)	
α_{14}	–.0050		–.0052			
	(.0010)		(.0009)			
α_{23}	–.0004		–.0003			
	(.0005)		(.0005)			
α_{24}		.0006			.0005	
		(.0004)			(.0004)	
α_{34}				–.0019		–.0020
				(.0008)		(.0009)
α_{56}	.1320	.1147	.1120	.0941	.1183	.1084
	(.0995)	(.0982)	(.0880)	(.0871)	(.0984)	(.0876)
β						
γ_1						

*Collinearity problems prevented estimates of this standard error.

TABLE IV (Continued)

Parameter	POTENTIAL OUTPUT AGGREGATES (2, 3)	(1, 3)	(1, 4)	(1, 2)	(2, 4)	(3, 4)
γ_2		−.0003 (.0015)	−.0030 (.0015)			−.00001 (.0015)
γ_3			−.0560 (.0085)	−.0493 (.0083)	−.0475 (.0082)	
γ_4	.0019 (.0025)	.0018 (.0029)		−.0020 (.0027)		
γ_5	.0678 (.0585)	.0362 (.0265)	−.4201 (.1966)	−.4043 (.1942)	.0320 (.0579)	.0581 (.0573)
γ_6	−.0266 (.0192)	−.0358 (.0226)	.2635 (.1009)	.2429 (.0997)	.0233 (.0213)	−.0305 (.0194)

Parameter	POTENTIAL OUTPUT AGGREGATES (1, 2, 3)	(1, 2, 4)	(1, 3, 4)	(2, 3, 4)	(1, 2, 3, 4)
a_o	−102.00 (7.208)	−114.68 (7.306)	−.113.77 (32.57)	−100.124 (7.4896)	−121.61 (7.372)
a_1	−.1276 (.0122)	−.0656 (.0169)	−.0673 (.0163)	−.1311 (.0141)	−.0612 (.0165)
a_2	−.0097 (.0032)	−.0082 (.0039)	−.0064 (.0094)	−.0169 (.0037)	−.0070 (.0040)
a_3	−.1233 (.0166)	.2121 (.0562)	−.0811 (.0204)	−.1317 (.0175)	−.0736 (.0213)
a_4	−.0827 (.0186)	−.0368 (.0100)	−.0359 (.0095)	−.0744 (.0083)	−.0331 (.0100)
a_5	1.444 (1.739)	1.452 (1.765)	1.295 (1.848)	−1.9157 (1.0807)	1.546 (1.782)
a_6	15.74 (.8408)	17.199 (.8551)	17.40 (5.149)	17.1529 (.6260)	18.438 (.8603)
b	1	1	1	1	1
α_{11}	.0262 (.0021)	.0210 (.0017)	.0229 (.0020)	.0225 (.0571)	.0246 (.0020)
α_{22}	.0034 (.0004)	.0032 (.0004)	.0024 (.0004)	.0030 (.0004)	.0034 (.0005)
α_{33}	.0311 (.0023)	.0261 (.0021)	.0299 (.0024)	.0304 (.0023)	.0294 (.0024)
α_{44}	.0114 (.0009)	.0121 (.0008)	.0126 (.0009)	.0130 (.0010)	.0127 (.0009)
α_{55}	.0532 (.0516)	.0431 (.0523)	.0446 (.0527)	.0267 (.0523)	.0477 (.0528)
α_{66}	−1.278 (*)	−1.426 (*)	−1.436 (.4095)	−1.2950 (*)	−1.504 (*)
α_{12}	−.0011 (.0006)	−.0021 (.0006)			−.0021 (.0006)
α_{13}	−.0048 (.0015)		−.0066 (.0017)		−.0064 (.0017)
α_{14}		−.0062 (.0010)	−.0039 (.0010)		−.0048 (.0010)

*Collinearity problems prevented estimates of this standard error.

linearly homogeneous in outputs at the sample means certainly does not imply constant returns to scale throughout the relevant output region (and consequently a homothetic cost function), but does suggest that it is of interest to test this hypothesis. The logarithm of the likelihood function associated with this model (linear homogeneity in input prices *and* outputs) is reported in Table III. According to equation (19) above, linear homogeneity in outputs imposes seven additional restrictions on the model.[24] The value of the test statistics is 78.16 and hence these data lend no support whatever to the constant returns hypothesis.

We next reestimated the model for the case when h (y_1, y_2, y_4) is constained to be homothetic, maintaining the hypotheses of linear homogeneity in prices and functional separability of (y_1, y_2, y_4). This imposes three additional restrictions on the model, $\alpha_{11} = -\alpha_{12} - \alpha_{14}, \alpha_{22} = -\alpha_{12} - \alpha_{24}$ and $\alpha_{44} = -\alpha_{14} - \alpha_{24}$. The value of the log likelihood function is reported in the last, left hand row of Table III and yields a test statistic of 35.54 which leads to rejection of the homotheticity hypothesis.[25] We conclude that although an aggregator function, h (·), exists for (y_1, y_2, y_4), a consistent index, via homothetic separability, cannot be found.[26]

Finally, we have calculated the correlation matrix for P to check for the possibility of a Hicksian aggregate. The correlations are $r_{12} = .065$, $r_{13} = .065$, $r_{14} = .901$, $r_{23} = .197$, $r_{24} = .014$ and $r_{34} = 0.026$. Of course, such calculations permit testing only pairwise groupings of outputs in the first step. It is interesting to note that the only highly correlated values (P_1 and P_4) are associated with the outputs (y_1, y_4) which in turn have the smallest test statistic among all pairs in our tests of functional separability. (See Table III.) Although $r_{14} = .901$ is far from a perfect correlation and the functional separability of (y_1, y_4) was rejected above, both the size of the correlation coefficient and the size of the test statistic suggest that burglary solutions and larceny solutions may in some situations be consistently aggregated.

MARGINAL COSTS, RATES OF TRANSFORMATION AND RETURNS TO SCALE

The marginal cost function for activity i is given by $\partial C^*/\partial y_i = (\partial \ln C^*/\partial \ln y_i)(C^*/y_i)$, i = 1, 2, 3, 4, 5, and may be calculated using the formula

$$(23)\quad \partial C^*/\partial y_i = (a_i + \sum_1^6 \alpha_{ij} \ln y_j + \gamma_i \ln w)(e^{\ln C^*}/y_i),\ i = 1, 2, \ldots, 5$$

As indicated, (23) will be valid for each of the crime solving outputs, $y_1, y_2, \ldots, y_5$ and not for y_6. Recall that the sixth output was an aggregate of the "non-crime solving" services provided by police. Since we have postulated only that the production of this output is proportional to population size, it will be possible to determine $\partial C^*/\partial y_6$ only up to this factor of proportionality.

The rate of transformation of output i for output j gives the number of solutions to crimes of type i which must be forgone for an additional solution to a crime of type j, given fixed levels of all other outputs. Formally, the rate of transformation between outputs i and j may be written as $-\partial y_i/\partial y_j = (\partial C^*/\partial y_j)/(\partial C^*/\partial y_i)$, i, j = 1, 2, ..., 5, i ≠ j, and may be calculated using the formula

$$(24)\quad -\frac{\partial y_i}{\partial y_j} = \frac{(a_j + \sum_1^6 \alpha_{jk} \ln y_k + \gamma_j \ln w) y_i}{(a_i + \sum_1^6 \alpha_{ik} \ln y_k + \gamma_i \ln w) y_j},\ i, j = 1, 2, \ldots, 5, i \neq j$$

As with marginal cost functions, it will not be possible to obtain transformation rates between output six and other outputs.

Traditional measures of scale economies (or diseconomies) are predicated on the single output firm and must be modified for use here. We measure scale economies as the percentage response of costs to a small equal percentage change in all outputs. That is,

$$(25)\quad \epsilon \equiv dC^*/C^* = \sum_1^6 (\partial \ln C^*/\partial \ln y_i)(dq/q),$$

where dq/q is the percentage change in outputs.[27] Of course one may calculate ϵ for subsets of outputs holding the remaining outputs fixed. Here we report on two scale measures, ϵ and $\bar{\epsilon}$. The former measures the percentage responses of costs to an equal percentage change in all solutions and in the service output, y_6.[28] The latter concept will measure the percentage response of costs to a change only in crime solving activities and is calculated according to the formula

$$(26)\quad \bar{\epsilon} \equiv \sum^5 (\partial \ln C^*/\partial \ln y_i)(dq/q),$$

Marginal cost functions (MC_i), rates of transformation (MRT_{ij}) and returns to scale functions ($\bar{\epsilon}$ and ϵ) evaluated at sample means are presented in Table V. To aid in evaluating these figures we have calculated standard errors (in paren-(thesis) for the two scale measures and each of the marginal costs. Since MC_1 is the only marginally significant, one should not put too much faith in the computed marginal cost of burglary solutions and the rates of transformation which depend upon it. All other statistics are highly significant. Also, because these functions are highly nonlinear one must take care in intepreting the values in the table.

[24] Symmetry of the α_{ij} reduces the restrictions in (19) from thirteen to seven. Recall that $\sum_i \gamma_i = 0$ is already imposed.

[25] A necessary and sufficient condition for homotheticity of the translog function $h(y_1, y_2, y_4)$ (see footnote 14) is that h (·) be homogeneous, which implies the conditions given.

[26] We have tested three increasingly special cases of the cost model in this sequence of tests, (homogeneity in input prices, functional separability of (y_1, y_2, y_4) from other outputs and w, given homogeneity in input prices and homotheticity of h (y_1, y_2, y_4) given homogeneity in input prices and functional separability of (y_1, y_2, y_4)). The overall level of significance for such co-called "nested" tests is approximately the sum of significance levels for individual tests in the sequence.

[27] E.G., if dq/q = 1, and $\epsilon < 1$ at y*, then the production function exhibits increasing returns to scale at the output mix y*, etc.

[28] The proportionality between population size and y_6 causes no problem here since the percentage change in y_6 is equal to the percentage change in population size.

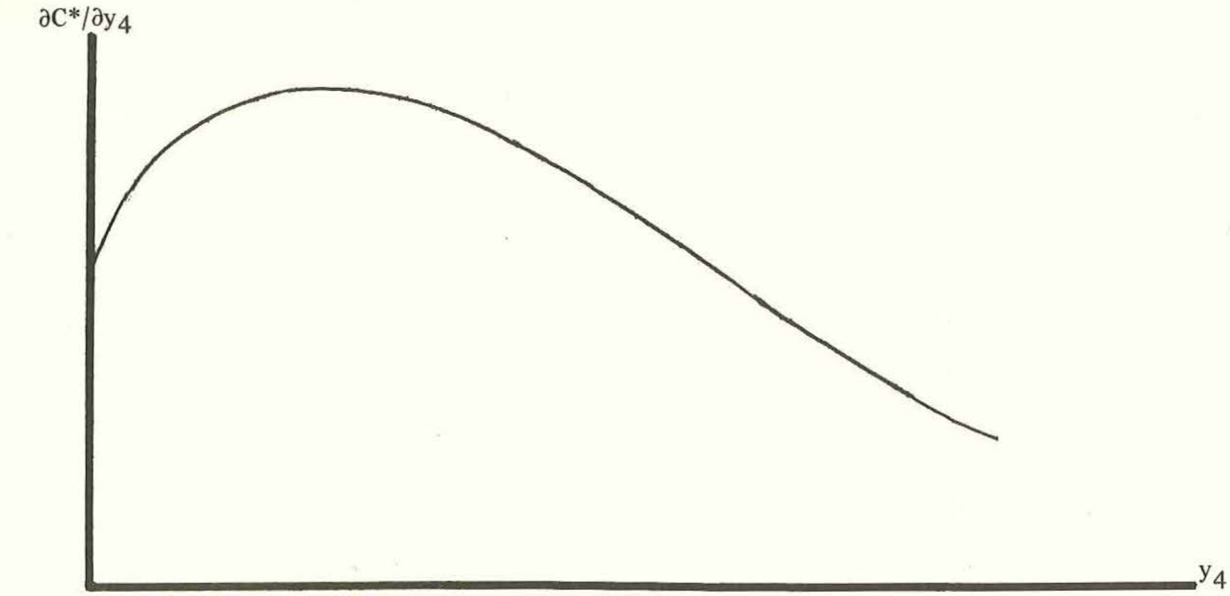

Figure 1—Marginal Cost Function for Activity 4.

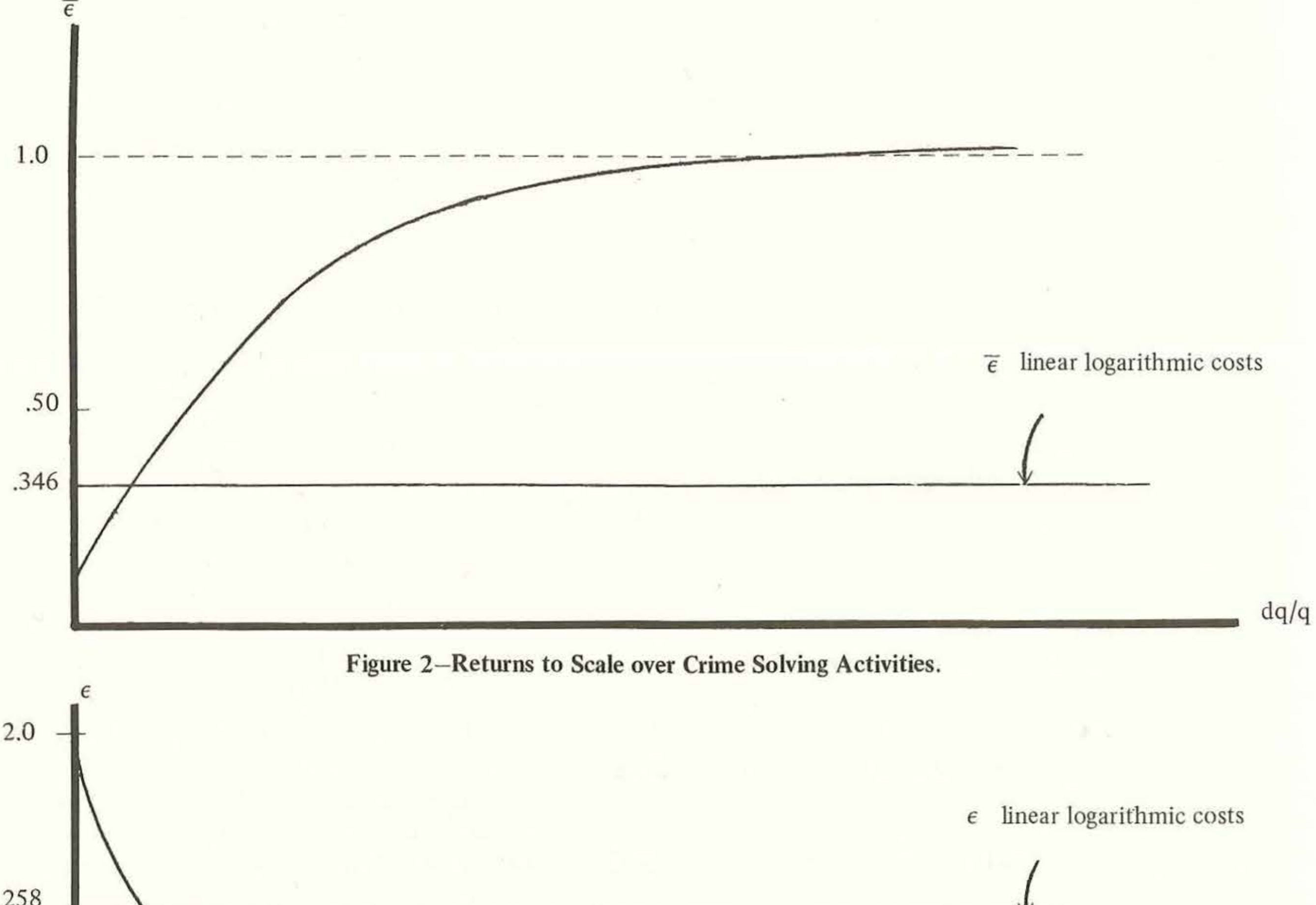

Figure 2—Returns to Scale over Crime Solving Activities.

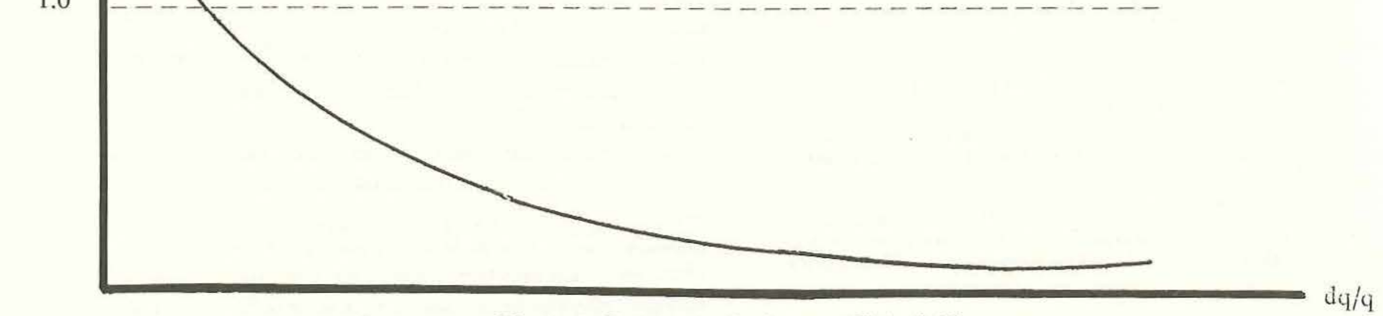

Figure 3—Returns to Scale over All Activities

At the sample means equal percentage changes in all outputs lead to approximately equal percentage changes in cost; while costs are much less responsive when only crime solving outputs are varied, holding services, y_6, constant. According to our estimates, incremental costs for clearing larcenies are the lowest ($138.01), followed by robbery ($227.24), motor vehicle thefts ($1177.18) and crimes against the person ($2448.-00). As we have noted, each of these estimates is highly significant.

Rates of transformation between outputs at sample means range from .117 between motor vehicle theft solutions and larceny solutions to almost eighteen between larceny solutions and solutions to crimes against the person. Hence, the estimated cost function predicts that on average it will be necessary to forego between eight and nine larceny solutions to solve one additional motor vehicle theft (at the mean) and approximately eighteen larceny solutions to solve the "average" crime against the person. Similar interpretations hold for the other transformation rates.

Figures 1–3 indicate the general curvature of the estimated marginal functions and returns to scale functions.[29] In figures 2 and 3 we have for contrast included plots of the scale measures (ϵ_ℓ and $\bar{\epsilon}_\ell$) associated with a linear logarithmic production structure. Of course in this case these functions are constant and are .346 and 1.258, respectively. The marginal cost function for output i has been graphed by evaluating (23) at $(\bar{y}_1, \bar{y}_2, \ldots, y_i, \ldots, \bar{y}_6, \bar{w})$, where the overbar indicates sample means, and allowing y_i to vary over the sample.

SUMMARY AND CONCLUSION

In this paper we have adopted the economic model of an optimizing firm as a framework for characterizing the production structure of a sample of medium sized U.S. law enforcement agencies. Unlike previous studies we have begun with a second order approximation to an arbitrary multi-output-multi-input production possibilities function. This rather general functional specification has permitted us to test a number of hypotheses which have been implicitly maintained in earlier work. Of particular interest is the finding that at least in our sample, the decisions of police administrators are consistent with cost minimization and that outputs are very definitely joint—thereby effectively precluding estimation of separate production and/or cost functions for the different outputs of police agencies. In addition we strongly rejected the hypothesis of constant returns to scale and indeed found that scale economies varied considerably with activity levels. Finally, our sample contained no evidence supporting the existence of a consistent index by which certain subsets of property crime solutions could be aggregated.

[29] ϵ was calculated by evaluating equation (25) at $(\min y\ (1+\delta)^i, \bar{w})$ for $0 < \delta < 1$ and $i > 0$. Also i was chosen so that $\min y\ (1+\delta) < \max y$. The analogous procedure was used to graph $\bar{\epsilon}$. Here w represents the sample mean of w.

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