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Demand for Refined Lead

John Heineke
Santa Clara University, jheineke@scu.edu

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DEMAND FOR REFINED LEAD

J. M. Heineke *

I Market Structure of the Domestic Lead Industry

Approximately 1500 companies in the United States are engaged in mining, smelting-refining, and fabricating lead. Each component of the industry includes a few large companies integrated vertically, or horizontally, or both.

The mining sector of the industry is somewhat less concentrated than the smelting-refining sector. The mines of the three leading companies produce approximately 67 per cent of total domestic mine output, with several hundred smaller companies supplying the remaining one-third. The large number of small miners has kept the mining sector quite competitive.

Significant economies of scale have been instrumental in concentrating the smelting-refining stage of production, where four firms produce the entire domestic supply of primary refined lead. Primary smelters and refiners process lead ores and supply less than one-half of the nation's refined lead needs. The remaining portion of domestic supply is produced by secondary producers who refine scrap lead. Due to chemical inactivity, lead is easily separated from impurities, and as a consequence secondary lead is close to a perfect substitute for primary lead.

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II Demand Functions for Lead Ingot

The demand for lead is a derived demand, derived for the most part from the producers of durable goods (automobile radiators and batteries, tetraethyl lead and construction materials are by far the largest consumers of lead) making the demand for lead highly sensitive to changes in national income. The demand for lead ingot may be written as

$$S(t) = F\left[\overset{t}{\underset{-\infty}{P(\tau)}}, \overset{t}{\underset{-\infty}{Y(\tau)}}, t\right]^{1} \qquad (1)$$

where $S(t)$ is a functional equation relating demand for lead ingot at time t, $S(t)$, to the time paths of ingot prices and national income, $\overset{t}{\underset{-\infty}{P(\tau)}}$ and $\overset{t}{\underset{-\infty}{Y(\tau)}}$. This formulation recognizes explicitly the importance of past values of the independent variables in determining present demand, and assumes that to the extent expectations influence quantity demanded they are formed on the basis of past values of price and national income.

To make (1) empirically meaningful we linearize the functional relationship and write

$$S_t = a^* + \sum_{\tau=0}^{\infty} a_\tau P_{t-\tau} + \sum_{\tau=0}^{\infty} \beta_\tau Y_{t-\tau} \qquad (2)$$

where a^*, a_τ and β_τ, $\tau = 0, 1, 2, \ldots$ are the demand parameters. Upon addition of a disturbance term we obtain the equation to be estimated

$$S_t = a^* + \sum_{\tau=0}^{\infty} a_\tau P_{t-\tau} + \sum_{\tau=0}^{\infty} \beta_\tau Y_{t-\tau} + \eta_{1t} \qquad (3)$$

where η_{1t} is a stationary stochastic term with zero expectation representing those elements affecting the quantity of lead ingot demanded which have been excluded from relation (3).

Statistical tractability demands some restrictions be placed on the sequences $\{a_\tau\}$ and $\{\beta_\tau\}$. We assume

$$\sum_{\tau=0}^{\infty} a_\tau = A \text{ and } \sum_{\tau=0}^{\infty} \beta_\tau = B, \quad -\infty < A, B < \infty. \qquad (4)$$

(4) says the total response of lead ingot demand to a change in price or national income must be finite. Obviously, this infinite number of parameters cannot be estimated with a finite number of observations, and we will be required to specify the relationship between the various members of $\{a_\tau\}$ and also the relationship between the members of $\{\beta_\tau\}$.

The crucial role played by one's knowledge of an industry's history is apparent. Only through analysis of an industry's past can one expect to know the technological and institutional factors which determine the "time shape" of its economic reactions. Even with this knowledge, specification of the relation between the parameters in the sequences $\{a_\tau\}$ and $\{\beta_\tau\}$, i.e., the distribution of lag, is at best difficult. After a brief digression on the identification problem in the lead ingot market, we turn to specifying the lag distribution and estimating the demand function.

III Identification Problem

Equation (3) will be identified if: (a) the price of lead ingot appears in the demand relation only after a lag, but appears in the supply relation unlagged or at least with a different lag; [2] (b) if the random term in the demand equation is *mutually* independent of the random term in the supply relation; and (c) if the random term in the demand equation is temporally independent. Compactly (a) implies

$$\frac{\partial S_t}{\partial P_{t-\tau}} = 0, \ \tau = 0, 1, \ldots, i \qquad (5)$$

for $\tau < i$ where i is the lag between ingot price changes and sales response to the change. (b) and (c) may be written

$$E(\eta_{1t}\eta_{2t}) = E(\eta_{1t}\eta_{1s}) = 0, s \neq t \qquad (6)$$

where S_t is quantity sold during period t, $P_{t-\tau}$ is the market price of lead ingot in period $t - \tau$, η_{1t} is the disturbance term in the demand equation, and η_{2t} is the disturbance term in the supply relation. The mere existence of an $i > 0$ guarantees satisfaction of condition (a). This is due to the fact that the quantity of lead ore supplied responds very rapidly to changes in price, owing to the competitive nature of the mining sector, where small, marginal miners leave and enter the industry with each price change. As a consequence of equation (5), the first $i + 1$ regression coefficients on the price variable will, of course, be zero. Estimates of the price elasticity of demand for periods of length $i + 1$ or less must also be zero.

Equation (5) would seem to be a realistic assumption in the lead ingot market for both technological and institutional reasons. It is clear that the demand reaction to price changes in the case of lead ingot must be a slow one owing first to the technological structure of the industry. The demand for lead ingot is a derived demand, derived mainly from the demand for automobile batteries, gasoline, paint and construction materials. Each

[1] Prices of substitutes have been omitted due to the lack of close substitutes in major uses.

[2] The market is assumed to be explained by the demand function, a supply relation and a production-sales-inventory identity.

of these final products is produced in a market characterized by a few large, capital-intensive firms. Substitution possibilities in such a case would be explored rather slowly, since changing capital equipment is an expensive proposition. Short-run fluctuations in price would not be expected to have a noticeable effect on the demand for lead. Only an upward trend in the price of lead relative to its substitutes would cause changes in the desired level of lead-processing capital stock, and consequent substitution.

In addition to long term technological lags, institutional lags prevent changes in price from influencing demand for a number of months. The most important of these lags is due to contractual buying. Only small buyers with relatively insignificant ingot requirements can safely assume they will be able to satisfy a period's lead needs with purchases made within the period. Large buyers contract for future needs several months in advance.

Of the two causes of lagged response, those caused by contractual buying arrangements have much the greater effect on the buying behavior of lead fabricators. This is due to the fact that the substitution process is quite long, due to the technological structure of the industry, and thus its "short-run" effect is small; and in the two uses which consume over one-half of all lead sold, automobile batteries and tetraethyl lead for gasoline, there are no known substitutes unless the price of lead should rise substantially.

We noted in the previous paragraph that contractual buying lags are relatively more important in the demand reaction of lead ingot to price changes than are technologically caused lags. The critical point is that contractual buying causes ingot purchases to be made several months prior to needs, and these lags are certainly less than twelve months. Therefore, if annual observations were used to estimate a demand model there would exist no lag that satisfies equation (5), i.e.,

$$\frac{\partial S_t}{\partial P_t} \neq 0, \tag{7}$$

and consequently, the model would not be identified.

IV Demand Estimation

We begin by accounting for the technological lags discussed above, abstracting for the moment from contractual buying lags and the income variable. Suppose the equilibrium level of ingot demand, if the current price is maintained indefinitely into the future, is given by

$$\bar{S}(t) = \gamma_0 + \gamma_1 P(t). \tag{8}$$

Realized demand, $S(t)$, differs from $\bar{S}(t)$ only because substitution away from or into lead, when buyers experience a price change, is quite slow (see above). In this case

$$dS/dt = \lambda(t)\ [\bar{S}(t) - S(t)], 0 < \lambda(t) < 1 \tag{9}$$

would seem to be a justifiable assumption. If $\lambda(t) = \lambda$, a constant, solution of (9) yields the continuous counterpart of Koyck's familiar distribution of lag. With this distribution most recent prices have the largest influence on demand with the influence declining exponentially over time. If national income is readmitted as a variable and contractual buying lags are accounted for, a model of exponential decay would seem to be a realistic hypothesis upon which to base the demand model.

The behavioral hypothesis of the preceding paragraph allows (3) to be written as:

$$S_t = a^* + a \sum_{\tau=i}^{\infty} \lambda_1{}^{\tau-i} P_{t-\tau} + \beta \sum_{\tau=j}^{\infty} \lambda_2{}^{\tau-j} Y_{t-\tau} + \eta_{1t} \tag{10}$$

where λ_1 and λ_2 are the parameters of the two lag distributions, $a_\tau = a\lambda_1{}^\tau$, $\beta_\tau = \beta\lambda_2{}^\tau$, and the numbers i and j are the number of periods (months) that elapse after a change in an independent variable until a response in quantity demanded is felt. A priori $2 \leqq i,\ j \leqq 6$ since most ingot purchases are made from two to six months prior to use. Since price will appear unlagged in any realistic specification of the supply relation, condition (a), above, is satisfied. If $E(\eta_{1t}\eta_{2t}) = 0$ (η_{2t} is the disturbance term in the supply relation), and $E(\eta_{1t}\eta_{1s}) = 0$ for all $t \neq s$ conditions (b) and (c) are also satisfied and the demand function will be identified. As a first approximation we shall assume an absence of correlation between η_{1t} and η_{2t}, since the unidentified factors affecting fabricators' behavior on the one hand and the miners' on the other are for the most part dissimilar. Of course, for the large vertically integrated firm, viewed in isolation, this assumption is invalid. But these firms are not in isolation, and the mining sector of the industry is characterized by a highly competitive ore market. For this reason the assumption seems justified.

Equation (10) as it stands of course cannot be estimated. Simple manipulation yields

$$S_t = a^*(1-\lambda_1)(1-\lambda_2) + aP_{t-i} - a\lambda_2 P_{t-i-1} + \beta Y_{t-j} - \beta\lambda_1 Y_{t-j-1} + (\lambda_1+\lambda_2) S_{t-1} - \lambda_1\lambda_2 S_{t-2} + \epsilon,^3 \tag{11}$$

[3] See H. Theil, *Economic Forecasts, and Policy* (Amster-

$$\epsilon_t = \eta_{1t} - (\lambda_1+\lambda_2)\,\eta_{1,t-1} + \lambda_1\lambda_1\eta_{1,t-2}, \qquad (12)$$

an autoregressive equation in five parameters.

The consequences of using least squares regression to estimate the parameters of equations like (11) are well-known. Lagged values of the dependent variable cannot be independent of past values of the disturbance term, making unbiased parameter estimates an impossibility. Even the desirable property of independence between future disturbances and present values of the lagged variables is unattainable except under special circumstances. In any case, estimates are biased for finite sample sizes. If η_{1t}, in equation (10), follows the second order Markoff process, $\eta_{1t} = (\lambda_1+\lambda_2)\eta_{1,t-1}-\lambda_1\lambda_2\eta_{1,t-2}+\omega_t$, where ω_t is random, ϵ_t and ϵ_s are independent whenever $t \neq s$. In this case our parameter estimates will tend in probability to their true values as $n \rightarrow \infty$ and are asymptotically efficient.[4] It is very unlikely that η_{1t} should obligingly follow this particular scheme exactly, although for monthly time series such a relationship is quite possible. If the ϵ_t are not serially independent, not only will our estimates be inconsistent, but even the direction of bias will be unknown.[5]

In estimating equation (11), i and j took on the values 2, 3, 4, 5 and 6 for both the price and the income variable. The value of i and j that maximized the "explanation" of effect as measured by minimum residual variance was chosen.[6] Monthly observations were available for the period 1948–1965. For the variable S_t, domestic lead consumption as published by the Bureau of Mines was used. For P_t, the average monthly price on the *New York Metal Market*, as quoted in *Engineering* and *Mining Journal*, was used, and for the income variable the *Index of Industrial Production-Manufacturing*, 1957–1959 = 100, was chosen. S_t is in thousands of short tons, and P_t in cents per pound. Since no major structural changes occurred in this period all observations were utilized. Our estimate of (11) is

$$\begin{aligned} S_t = {} & \underset{(6.36)}{21.56} - \underset{(.85)}{2.14}\, P_{t-3} + \underset{(.84)}{1.85}\, P_{t-4} \\ & + \underset{(.49)}{1.46}\, Y_{t-2} - \underset{(.48)}{1.43}\, Y_{t-3} + \underset{(.06)}{.50}\, S_{t-1} \\ & + \underset{(.07)}{.32}\, S_{t-2}, \end{aligned} \qquad (13)$$

where the numbers in parentheses are standard errors. Inspection of (11) reveals $\alpha' = -2.14$, $(-\alpha\lambda_2)' = 1.85$, $\beta' = 1.46$, $(-\beta\lambda_1)' = -1.43$, $(\lambda_1 + \lambda_2)' = .50$, $(\lambda_1\lambda_2)' = .32$, $[\alpha^*(1-\lambda_1)(1-\lambda_2)]' = 21.56$, $i = 3$, and $j = 2$, the "primes" denoting estimates.

Transforming (13) back into its original form as given by (10) we have

$$S_t = 5138.0 - 2.14 \sum_{\tau=3}^{\infty} (.97)^{\tau-3} P_{t-\tau} + 1.46 \sum_{\tau=2}^{\infty} (.86)^{\tau-2} Y_{t-\tau}.^{7} \qquad (14)$$

The effect of the one per cent change in price at the end of the third period is given by $\alpha' [\bar{P}/\bar{S}] = -2.14 \left[\frac{13.9}{93.3}\right] = -.29$ per cent, where $[\bar{P}, \bar{S}]$ is the point on the demand function at which we wish to evaluate the elasticity (we chose means for convenience). The *eventual* effect of a one per cent change in price is given by $\alpha' \sum_{\tau=3}^{\infty} \lambda_1'^{\tau-3} [\bar{P}/\bar{S}] = \alpha'/1-\lambda_1') [\bar{P}\text{–}\bar{S}] = -9.98$ per cent. The same two calculations for the income variable yield 0.15 per cent and 1.07 per cent, respectively.

R^2 for the equation (13) is 0.54, which is due in large part to "unexplained" seasonal variation. Eleven dummy variables were added, one for each month except December. Seven of the parameter estimates lay more than 2.3 standard deviations from zero, and R^2 increased to 0.78. Since α' and β' did not vary substantially from the estimates above, the simpler formulation given by (13) was chosen.[8]

The desirable asymptotic properties of consistency and efficiency noted above depend on the serial independence of ϵ_t. The Durbin-Watson statistic for regression equation (13) is 2.10. This value indicates acceptance of the serial independence hypothesis. Only limited confidence can be placed in the Durbin-Watson statistic when lagged dependent variables are present, since the power of the test is low in this case.[9] But examination of the residual plot tended to confirm the hypothesis of no serial correlation, so we conclude our esti-

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dam: North-Holland Publishing Company, 1965), p. 216–219, for a similar model.

[4] E. Malinvaud, *Statistical Methods of Econometrics* (Chicago: Rand McNally and Company, 1966), theorem 2, p. 453.

[5] J. Johnston, *Econometric Methods* (New York: McGraw-Hill Book Company, Inc., 1963), p. 216.

[6] See F. F. Alt, "Distributed Lags," *Econometrica*, 10 (1942), pp. 113–128, for a similar method of choosing the "best" regression.

[7] λ'_1 and λ'_2 where deduced form the coefficients of P_{t-3} and P_{t-4}, and Y_{t-2} and Y_{t-3} respectively.

[8] If the demand equation is used for forecasting the latter equation would be preferable.

[9] M. Nerlove and K. Wallis, "Use of the Durbin-Watson Statistic in Inappropriate Situations," *Econometrica*, (Jan., 1966).

mates are consistent and efficient, comforting properties for a sample this large.

The results of the regression analysis confirm our a priori expectations: Institutional lags make firms insensitive to short-run price and income fluctuations, the first effect being felt two or three months after the cause. Although firms respond only slightly in the short run, if the change (especially price) is maintained they are quite sensitive when longer periods of time are considered, technological lags tending to reinforce institutional lags in the long run.

The industry's history confirms our findings on short- and long-run demand elasticities. The smelting-refining sector of the domestic lead industry may be accurately categorized as a homogenous oligopoly following the price leadership of the American Smelting and Refining Company (ASRC). In periods of high demand ASRC has kept price constant, and forced sales from inventory and backlogging when price could have easily been advanced. That the short-run price inelasticity of demand cannot be extended to long periods is implied by such behavior. It is evident that the price leader was fully cognizant of the fact that the rate of growth of demand is highly sensitive to price changes, i.e., the long-run price elasticity of demand is considerably above unity.

Thus, it seems that current market price bears little relation to existing supply and demand conditions. Price policy implies a long-run view in price decisions. The nature of the demand for lead makes any other approach to pricing irrational. Demand is multi-period, that is, this year's price affects next year's sales. Given the short-run elasticity of demand, it is possible to increase current profits by increasing price. Yet, given the elastic long-run demand for lead, such price increases jeopardize further sales and future profits.