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Using FRITIOF to Model Nucleus-Nucleus Interactions in a Cosmic Ray Detector

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Abstract

A scintillating optical fiber calorimeter (SOFICAL) is being developed by NASA/Marshall Space Flight Center for use in experiments to study the spectrum of high-energy cosmic rays and gamma rays from 100 GeV to 1,000 TeV. SOFICAL will not saturate for long exposures and this calorimeter in these balloon-borne emulsion chambers will be helpful for the study of the composition of primary cosmic-ray nuclei. For primary nuclei with energies much greater than 10^{14} eV, nucleus-nucleus interactions are likely to exhibit characteristics of a quark-gluon plasma (QGP). A particle event generator was used to model the collision of a cosmic-ray nucleus with a target nucleus in an emulsion chamber. FRITIOF with LUCIAE was chosen to model collisions of primary cosmic rays in an emulsion chamber with SOFICAL. Pseudo-rapidity distributions were computed for protons on lead at 200 GeV/c and compared with experimental data. Pseudo-rapidity distributions were computed for protons or iron incident on a carbon or silver nucleus. For gamma-rays from nucleus-nucleus interactions, the total energy of the electromagnetic component ΣE_γ was computed. The partial coefficient of inelasticity k_γ , defined by $\Sigma E_\gamma = k_\gamma E_0$, was computed from the primary energy E_0 of the cosmic rays. The $f(k_\gamma)$ -distributions were computed and compared with existing calorimeter data. Funding was provided by the NASA/University Joint Venture (JOVE) Program.

Introduction

Cosmic rays are now known to span the energy range from 10^9 to beyond 10^{20} eV (Asakimori et al., 1993a; Asakimori et al., 1993b; Swordy, 1994; Teshima, 1994). They are predominantly the nuclei of atoms from hydrogen to iron. Above 10^{14} eV the particles are so rare that their detection relies mainly on observations of the giant cascades or extensive air showers created in the atmosphere which may be observed with arrays of particle and optical detectors at ground level. The flux of particles decreases inversely as the square of the energy rises (Fig. 1), up to 10^{19} eV, and continues to decrease above 10^{19} eV as only about one particle per km^2 per year is collected (Watson, 1994). The origin of these particles is unknown and how they are accelerated to such high energies is a major astrophysical puzzle (Bird et al., 1993).

Even though the flux of the primary cosmic rays is so low at these energies that small detectors in spacecraft or balloons can intercept only a small number for study, emulsion chambers are an important tool for the direct measurement of the composition and spectra of cosmic rays above 10^{12} eV/nucleon. The emulsion chamber method (Burnett et al., 1986) is especially useful for ultra-high energy cosmic ray observations because (1) the efficiency for detection interactions approaches 100% above about 10 TeV and (2) the energy resolution is approxi-

mately constant with energy for a given incident species. Most other energy measuring techniques are impractical for balloon observations of primary cosmic rays at such high energies.

Balloon-borne emulsion chambers, employing calorimeters, have been used for direct measurements of cosmic-ray composition (protons through Fe) between 10^{12} and 10^{15} eV (Kaplon et al., 1952; Minakawa et al., 1958; Niu et al., 1971; Burnett et al., 1986; Takahashi et al., 1986; Burnett et al., 1987; Parnell et al., 1989; Burnett et al., 1990; Asakimori et al., 1993a; Asakimori et al., 1993b). The typical emulsion chamber (Burnett et al., 1986) is composed of four parts: (1) a charge-determination module, (2) a target module with ~ 0.2 vertical interaction mean free paths for protons, (3) a spacer module, and (4) an emulsion calorimeter module with about fourteen vertical radiation lengths.

The "target section" includes many layers of nuclear emulsion plates to measure the charge of the incident particle and the emission angles of the produced charged particles with high accuracy (0.01 mrad). The "calorimeter section" includes layers of nuclear emulsion and X-ray film among lead plates to measure the electron distributions from the electromagnetic cascades initiated by gamma rays from π^0 decay. The calorimeter is used to measure the spectrum of energy deposition ΣE_γ from which the primary energy spectrum is derived (Burnett et

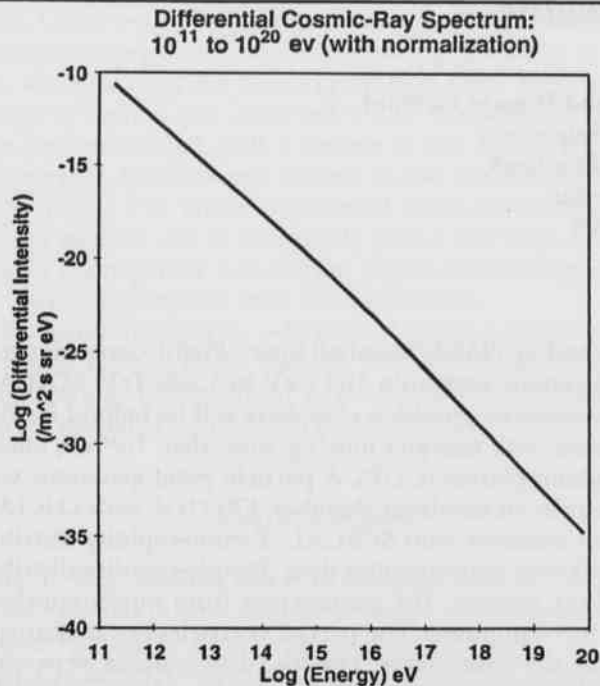


Fig. 1. The shape of the all-particle spectrum of the cosmic rays for the energy range from 10¹¹ to 10²⁰ eV per particle or nucleus.

al., 1986). The scintillating optical fiber calorimeter (SOFICAL) is a scintillation optical fiber counterpart to the calorimeter section in the emulsion chamber. SOFICAL is under development at NASA/Marshall Space Flight Center for future applications in cosmic ray and gamma ray measurements.

Materials and Methods

For modeling the primary cosmic ray interactions, calculations must be performed with event generators, such as FRITIOF, to predict distributions of the electromagnetic component $\sum E_\gamma$ in the calorimeter. FRITIOF (Bengtsson and Sjöstrand, 1987; Nilsson-Almqvist and Stenlund, 1987; Sjöstrand and Bengtsson, 1987; Pi, 1992; Sa and Tai, 1994) was used to model nucleus-nucleus interactions in an emulsion chamber and the subsequent emission of particles. GEANT (CERN, 1992a) and PAW (CERN, 1992b) are used for associated optimum "window" settings of the calorimeter.

For the initial modeling of SOFICAL (Yang et al., 1994), a calorimeter module with ten vertical radiation lengths of Pb was used. In one geometrical configuration, each subsection of the calorimeter consisted of a 4-mm lead block, 100 fibers (0.5-mm thick) in the x-direc-

tion and 100 fibers (0.5-mm thick) in the y-direction. This lead and optical fiber combination was repeated fourteen times. Photon and electron events in SOFICAL were modeled with GEANT on DEC 5000 workstations.

Event Generator for Modeling Nucleus-Nucleus Collisions.--FRITIOF is a Monte Carlo program that implements the Lund string dynamics for hadron-hadron, hadron-nucleus, and nucleus-nucleus collisions (Bengtsson and Sjöstrand, 1987). At high energies, a collision between nuclei can be regarded as incoherent collisions between their nucleons. Thus, a nucleon from the projectile interacts independently with the encountered target nucleon as it passes through the nucleus. Each of these sub-collisions can be treated the same way as the usual hadron-hadron collision.

In FRITIOF an interacting hadron behaves like a relativistic string with a confined color field. Two hadrons interact with each other as their fields overlap, and momentum transfer takes place. It is assumed that no net color is exchanged between the hadrons despite the momentum transfer. The possibility of one of the momentum transfers corresponding to large transverse momentum scattering is properly treated according to QCD. After the exchange of momenta, the hadrons are assumed to become two excited string states, which further emit gluonic radiation in a color dipole approach to the QCD parton branching. The final hadronization is performed by using the Lund string fragmentation model.

When both baryon density and energy density are high enough in heavy ion collisions, new features appear compared with hadron-hadron collisions, e.g., high P_T enhancement, strangeness enhancement, anti-baryon production, etc. They reveal that heavy-ion collisions may possess unknown characteristics or collectivity which are not able to be described by superposition of independent hadron-hadron collisions. The formation of a new state of matter (Harris et al., 1994), Quark-Gluon Plasma (QGP), has been suggested to explain those new features.

Usually, there are many "strings" formed through relativistic heavy-ion collisions. Those strings can thus overlap with each other and produce a heavily interacting system in which the behavior of an individual "string" is affected by the presence of other "strings", which can not be treated independently any longer. In the version of FRITIOF used for the nucleus-nucleus interactions, the Fire Cracker Model (FCM) was used to model the emission of gluons from a system containing several strings formed during collision. In the model, gluons can be emitted collectively from the color field of the multi-string system. Because many particles are produced in high energy nucleus-nucleus collisions, they will scatter with each other and spectator nucleons when going through the interaction region. This rescattering effect

(RESCATTERING) has been considered important in heavy-ion collisions. FCM and RESCATTERING, implemented in the Monte Carlo program LUCIAE 2.0, are able to describe high P_T enhancements and increase of strangeness.

The event generator program included LUCIAE 2.0 for simulating gluon emission in the FIRECRACKER model and the rescattering of produced particles in the nuclear environment. The program is written in FORTRAN 77 and was used together with FRITIOF7.02R, JETSET7.3, PYTHIA5.5, and ARIADNE4.02R. The task of PYTHIA is to describe the partonic processes taking place in hadronic collisions. How these partons are transformed into the experimentally measurable particles, i.e., the process of fragmentation, is handled by JETSET. PYTHIA can be combined with any well-defined fragmentation scheme. Although independent fragmentation is included as an option, the fragmentation scheme of JETSET is the Lund string model. ARIADNE is a Monte Carlo program for QCD cascades in the color dipole formulation. Gluon splitting into quark-antiquark pairs and photon emission in the dipole cascade are allowed. The primary cosmic ray interactions were modeled with FRITIOF and LUCIAE on a DEC 3000 AXP processor.

Primary Energy Spectrum Analysis.--In emulsion chamber experiments, that part of the primary cosmic-ray energy E_0 going into gamma-ray energy $E_m = \sum E_\gamma$ is the parameter most easily related to the primary cosmic ray spectrum. The photons originating from an interaction will develop individual electromagnetic cascades in the calorimeter. The ability to measure energies of electron-photon cascades is one of the most important functions of the calorimeter because the primary energy E_0 spectrum can be found from the E_m spectrum.

In a calorimeter the maximum electron number in an electromagnetic cascade can be related to the total energy of the electromagnetic component $\sum E_\gamma$. The measured spectrum of E_m is a convolution of the primary cosmic ray spectrum with the energy response function of the detector (Burnett et al., 1986). The quantity E_m is directly proportional to the primary energy E_0 of the original cosmic ray: $\sum E_\gamma = k_\gamma E_0$. The response function depends on the distribution of partial inelasticity k_γ . There is a unique relation (Parnell et al., 1989) or simple scale shift between the $\sum E_\gamma$ spectrum and the corresponding primary spectrum, as long as the spectral index and the characteristics of the interactions do not change substantially the observed energy range. It can be shown (Burnett et al., 1986) that the energy conversion factor

$$C = \left\{ \int_0^1 k^\beta f(k) dk F(\beta) \right\}^{1/\beta}$$

represents the energy scale shift required to go from the

E_0 spectrum to the $E_m = \sum E_\gamma$ spectrum. Therefore, the primary E_0 spectrum can be found by shifting the E_m spectrum up in energy by the factor, C^{-1} , the reciprocal of C (Parnell et al., 1989; Asakimori et al., 1993b).

Results

For these initial calculations of nucleus-nucleus interactions, the event generator program modeled the emission of gluons from a system containing several strings formed during collision and the rescattering effect. As an illustration of the event generator program results, Fig. 2 shows the distribution of all charged particles as a function of pseudo-rapidity distribution for 200 GeV/c protons incident on fixed target lead nucleus. The predicted values are compared with experimental data (Elias et al., 1980). In Fig. 3 the rapidity distributions for production of all π^+ and K^+ are shown for 1000 GeV/nucleon iron nuclei incident on a fixed target silver nucleus. In Fig. 4 the rapidity distributions for production of all protons and antiprotons are shown for 1000 GeV/nucleon ion nuclei incident on a fixed target silver nucleus. The event generator program included FRITIOF and LUCIAE.

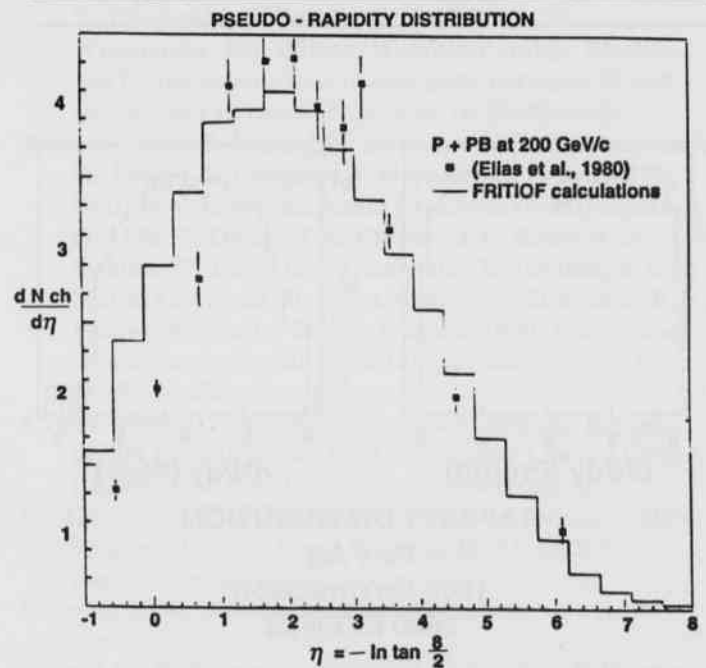


Fig. 2. The distribution of all charged particles as a function of pseudo-rapidity for a proton incident on fixed target lead nucleus (Elias et al., 1980). The proton energy was 200 GeV and 2000 events were used in the simulation. FRITIOF and LUCIAE were used to model the interaction.

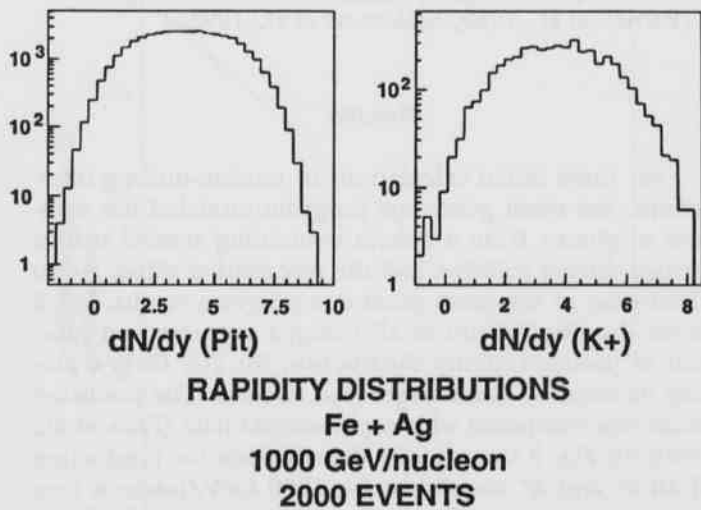


Fig. 3. Rapidity distributions of K^+ and π^+ mesons produced by an iron nucleus primary (Fe), with an energy of 1000 GeV/nucleon, incident on a silver target nucleus (Ag). FRITIOF and LUCIAE were used to model the interaction.

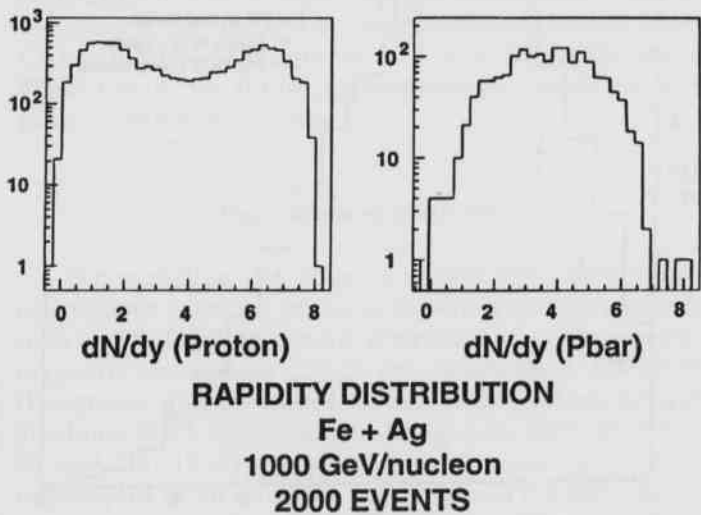


Fig. 4. Rapidity distributions of protons and anti-protons mesons produced by an iron nucleus primary (Fe), with an energy of 1000 GeV/nucleon, incident on a silver target nucleus (Ag). FRITIOF and LUCIAE were used to model the interaction.

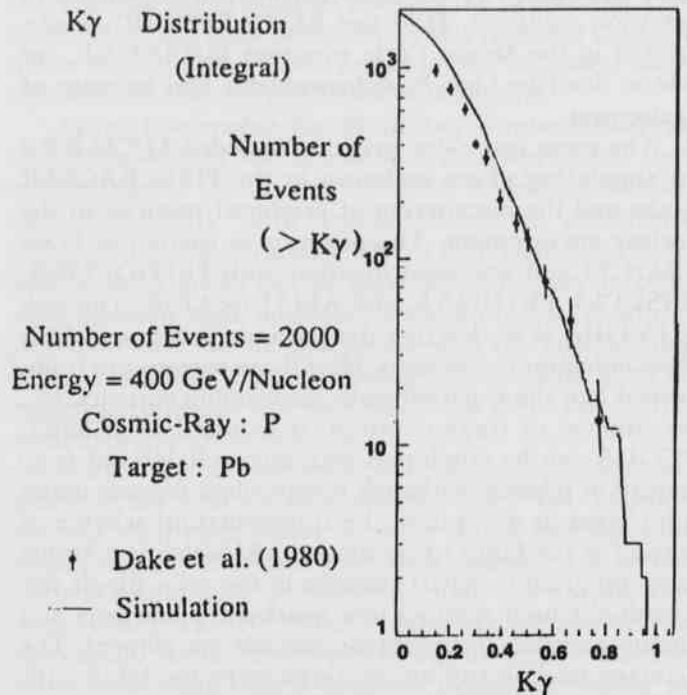


Fig. 5. Integral k_γ -distributions for a proton with an energy of 400 GeV incident on a fixed target lead nucleus (Duke et al., 1980). FRITIOF and LUCIAE were used to model the interaction.

Typical emulsion chambers contain plastic (CHO) and emulsion targets (Burnett et al., 1987; Parnell et al., 1989). The composition of CHO was 33% C, 53% H, 14% O. The composition of emulsion was 17.5% C, 40.7% H, 4.0% N, 12.0% O, 0.17% S, 12.7% Br, 12.8% Ag, and 0.07% I. Therefore, a light target nucleus (C) and a heavier target nucleus (Ag) were used in modeling the interaction of cosmic ray particles with the target section of the emulsion chamber.

The factor k_γ is the partial coefficient of inelasticity, representing that fraction of the energy of the primary nucleus used to create γ -rays. It is a function of the mass number of both the primary nucleus and the target nucleus (Burnett et al., 1986). Once the target nucleus and the primary cosmic ray have been identified in an emulsion layer of the apparatus, the distribution of k_γ becomes a known quantity. This distribution and the energy of the electromagnetic cascade are used to estimate the energy of the primary cosmic ray. Integral or cumulative distributions of k_γ for a proton incident on a fixed target lead nucleus (Drake et al., 1980) are shown in Fig. 5. The proton energy was 400 GeV and 2000 events were used in the simulation.

Discussion

Cosmic rays span a very large energy range from 10^9 to beyond 10^{20} eV. A realistic modeling of particle detectors should include those energies which are likely to be encountered and measured by cosmic ray and gamma ray detectors. The primary cosmic rays were modeled for nucleus-nucleus interactions in an emulsion chamber. These simulations were done with FRITIOF and LUCIAE up to 1000 GeV/nucleon and appear to agree well with existing data, but modeling primary cosmic-ray interactions in the emulsion chamber will be more useful at higher energies where we hope to find evidence for the formation of a quark-gluon plasma. The next step is to show that the primary cosmic ray interaction can be modeled at such energies with FRITIOF and LUCIAE to determine primary energy and composition of the incident cosmic rays. The modeling of SOFCAL can be done with GEANT.

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