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Estimating Milky-Way Dark Matter: Its Amount and Distribution

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Introduction

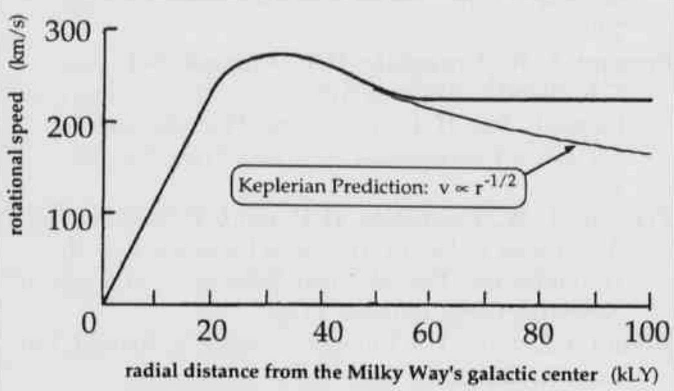
The fate of the universe is determined by its average mass density, whether it is open or closed, or by gravitational braking of the Hubble expansion hovering on the threshold between open and closed. The present research gives size and distribution of a possible dark-matter contribution to the average mass density of the universe; it connects two research efforts. Big Bang Nucleosynthesis [BBNS] calculates the relative abundances of the light elements formed during the first moments of the universe as a function of average mass density (Riordan and Schramm, 1991). These estimates show nuclear-type matter provides less than 10% of the mass-density needed to close the universe. Dark matter is a possible source of additional mass-density which may relate to the ultimate fate of the universe, and dark matter is suggested to explain the discovery of a flat rotation curve found for stars in the Milky Way galaxy (Rubin, 1991).

Figure 1 shows the discovery of a flat rotation curve for bright stars in circular orbit around the galactic center at distances between 50KLy and 100KLy. Also Fig. 1 shows a prediction of the expected result: a speed falling off as $1/\sqrt{r}$. This discovery is suggested as evidence for dark matter in our own galactic environment (Rubin, 1991).

Most of the mass of the Milky Way Galaxy is contained near the center of the Galaxy as seen in Fig. 2 which shows the top-view of the Milky Way, as constructed from radio-telescope data. From this concentration of mass near the galactic center, one would expect the velocity of orbiting stars to obey a simple Keplerian prediction and fall off as $r^{-1/2}$. This incorrect prediction is easily derived: If G is the Universal Gravitational Constant of Newton and M_o is the mass of bright matter in the Milky Way Galaxy:

$$\text{centripetal acceleration (star)} = \frac{v^2}{r} = \frac{GM_o}{r^2} \quad \boxed{v = \sqrt{\frac{GM_o}{r}} \propto \frac{1}{\sqrt{r}}}.$$

If the galaxy were a rigid body (which it is not), the orbital velocity of stars around the galactic center would grow linearly with r (with v = ωr, ω being the constant angular speed of rotation of this hypothetical *rigid* galactic disk).



Most of the Milky Way mass is near the galactic center, so **one would expect** the speed of orbiting stars to obey a simple Keplerian prediction, with speed falling off as $r^{-1/2}$.

This incorrect prediction is shown; G is the Gravitational Constant and M_o is the mass of the Milky Way Galaxy.

$$\text{Centrifugal acceleration} = \frac{v^2}{r} = \frac{GM_o}{r^2} \Rightarrow \boxed{v = \sqrt{\frac{GM_o}{r}} \propto \frac{1}{\sqrt{r}}}.$$

Fig. 1. Orbital speed of bright stars around the galactic center, with an incorrect prediction of the fall-off of the rotation curve assuming only bright matter.

The linear orbital speed of outlying bright stars ($r>50$KLy) is reported to be constant at about 240 km/sec (Rubin, 1991), neither dying off with r as in the Keplerian prediction or growing with r as in the hypothetical model of a *rigid* galactic disk.

A halo of dark matter centered on the Milky Way has been proposed to explain the observed constant orbital speed of bright stars around the galactic center (Rubin, 1991). In reviewing the dynamical evidence for dark matter, Scott Tremaine (1992) quotes a proposed modification of Newton's Law of Gravitation at large distances which results in a flat rotation curve by introducing a new "cosmological constant." This unusual proposal avoids the need for a dark matter explanation, but invoking a new "cosmological constant" is no less onerous than invoking a need for dark matter.

The dark matter appellation refers to matter not emitting appreciable amounts of electromagnetic radiation. Since dark matter is not observed directly its properties must be found by inference. The present work uses

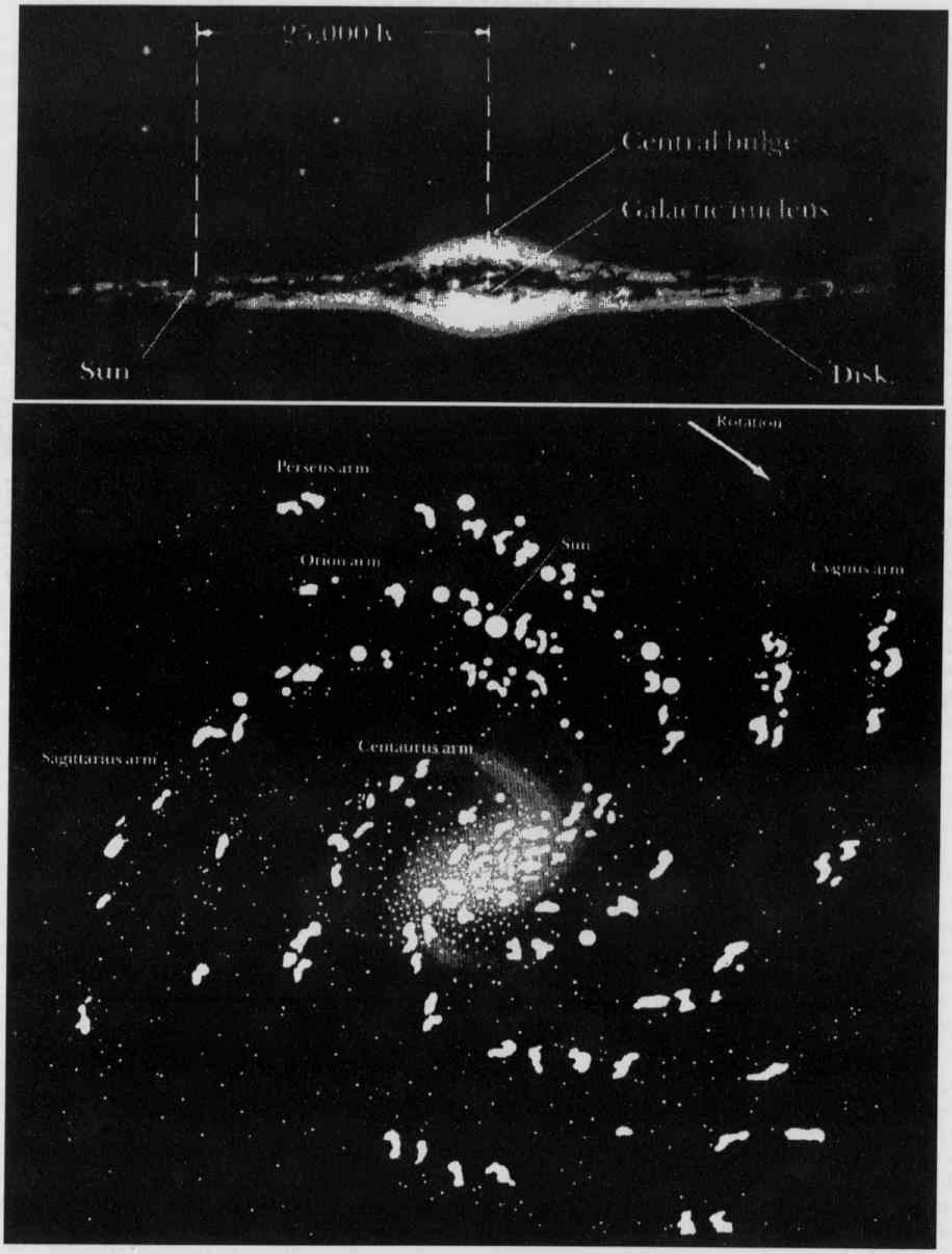


Fig. 2. An edge view of the Milky Way and a top view showing spiral arms.

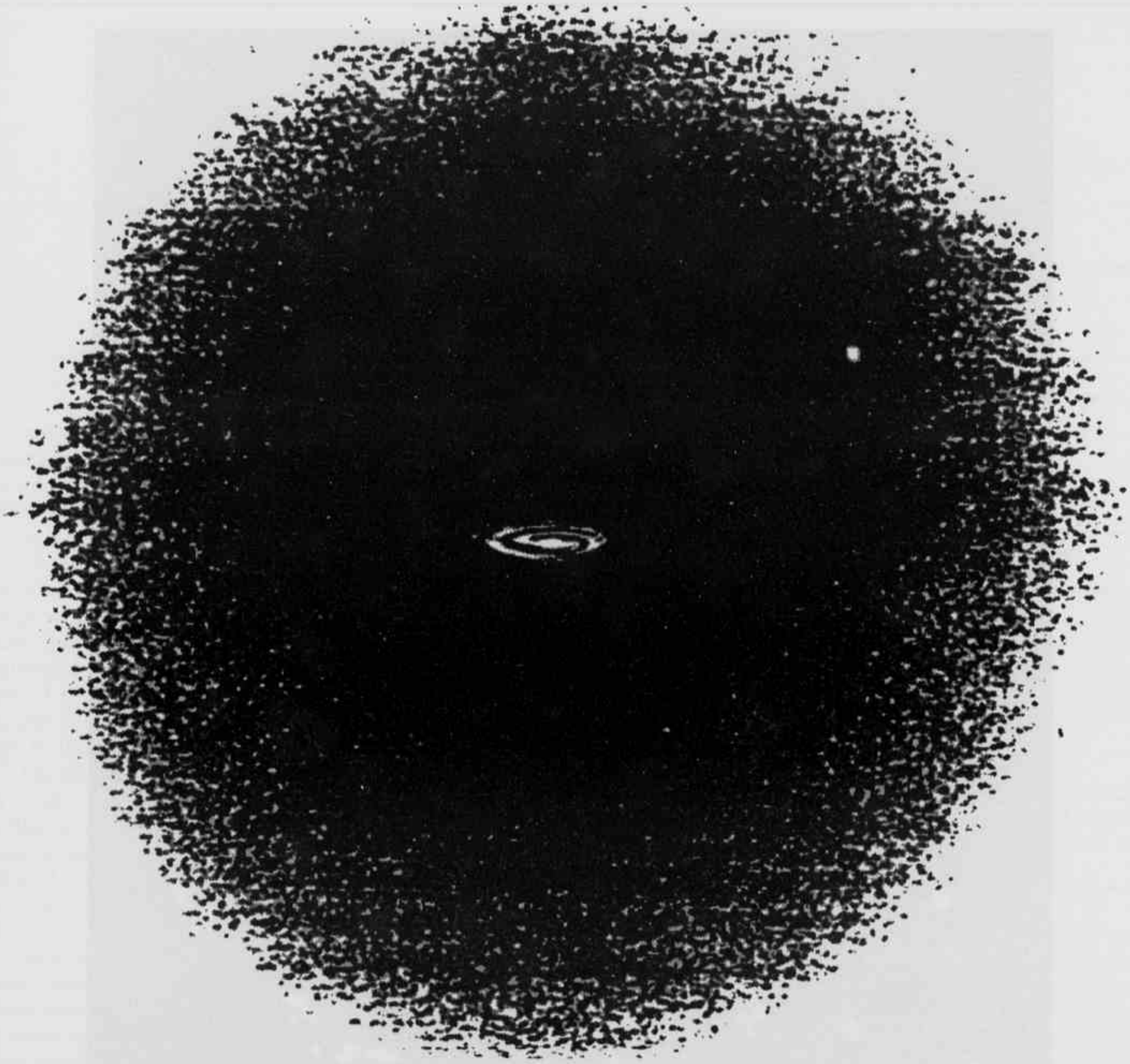


Fig. 3. An artist's conception of a spherically-symmetric halo of dark matter around the Milky Way galaxy.

the discovery of a flat-rotation curve to infer a mass distribution for dark matter in the Milky Way Galaxy, and from this mass distribution a total mass for dark matter is obtained as a function of the cutoff radius for distribution of dark matter.

The simplest hypothesis for the mass distribution of dark matter is a spherically-symmetric halo around the galactic center. Figure 3 shows an artists conception of this simplest of possible distributions of dark matter, where the dark matter halo extends in spherical symmetry around the galactic center of the Milky Way and far beyond it.

Materials and Methods

The present work calculates a lower limit for dark matter within the Milky Way Galaxy using the discovery of a flat rotation curve along with two additional assumptions about the distribution of dark matter surrounding the Milky Way.

These two hypotheses are (1) the distribution of dark matter is taken as spherically-symmetric around the galactic center, and (2) gravitational contributions from the spiral arms of the galaxy may be neglected in calculating a constant orbital speed for outlying bright stars at r>50 KLy. These hypotheses are used to make general statements about the amount and distribution of dark matter in the Milky Way galaxy.

Using M_o as the bright-matter mass of the Milky Way Galaxy and M(r) as the mass of dark matter contained within a radius r from galactic center, calculations assuming constant orbital speed show the *radial* density distribution of dark matter is constant, so the *volumetric* density distribution of dark matter varies inversely with r^2.

$$\frac{G[M_o + M(r)]}{r^2} = \frac{v^2}{r} \Rightarrow \boxed{M(r) = r\frac{v^2}{G} - M_o} \Rightarrow \frac{dM}{dr} = \frac{v^2}{G} = \text{constant.}$$

$$\text{Constancy of } dM/dr \Rightarrow \rho = \frac{dM}{dV} = \frac{dM}{4\pi r^2 dr} = \frac{1}{4\pi r^2}\frac{dM}{dr} = \frac{1}{r^2}\frac{v^2}{4\pi G} \propto \frac{1}{r^2} \propto r^{-2}.$$

Putting numbers into the above expressions results in a *lower limit* being calculated for the mass of dark matter if the constancy of dM/dr is taken for all points inward to the center of the Milky Way with the calculations based on the constant orbital speed for stars between r = 50 KLy and r = 100 KLy (with 100 KLy being the farthest measured orbiting stars). Taking 100 KLy as the upper-limit cutoff for dark matter makes this calculation a lower limit for the mass of dark matter since no diminution is seen in stellar orbital speed as r → 100KLy. Measurements end at 100KLy because of a lack of bright matter which in no way suggests a lack of dark matter.

Results and Discussion

$M_o = 10^{11}$ solar masses = 1.8×10^{41} kg was used for the mass of bright matter in the Milky Way (Abell et al., 1991). Using the discovery of a constant speed ($v = 2.4 \times 10^5$ km/sec) of bright stars in the Milky Way and using $r = r_{cutoff}$ = 100KLy and $G = 6.7 \times 10^{-11}$ MKS in the expression $M(r) - M_o = rv^2/G$ ($=8.1 \times 10^{41}$ kg) provides a *lower limit for the mass of dark matter* = $[8.1 - 1.8] \times 10^{41}$ kg = 6.3×10^{41} kg = **3.5 M_o** for M_o equal to the mass of bright matter in the Milky Way galaxy.

These surprisingly large dark-matter mass results led to an effort to reduce it by concentrating dark matter in the galactic disk. Geometrical enhancement of the effect of dark matter was examined (Eggensperger and Braithwaite, 1995) in an attempt to reduce the amount needed to account for the flatness of the rotation curve. No geometrical enhancement for the effects of dark matter was found in these calculations.

An uncertainty in the lower limit for dark matter mass surrounding the Milky Way galaxy arises from an uncertainty in the bright-matter mass of the Milky Way. Various references quote different sizes for the mass M_o. The value of M_o (1.8×10^{41} kg) selected was one of the low-end values (Abell et al., 1991) for M_o. Any attempt to measure M_o from orbital paths of bright stars (e.g., using the virial theorem) rather than counting a sample of stars while estimating each star's mass from its known properties (e.g., H-R diagram) would result in including effects from both dark matter and bright matter.

If the physical extent of the dark matter distribtuion were known (e.g., $r = r_{cutoff}$), the calculation $M(r = r_{cutoff}) - M_o = rv^2/G$ would provide an estimate of the dark-matter mass in the Milky Way. An artist's conception is shown in Fig. 3 where the spherically-symmetric halo of dark matter extends to about 8 times the maximum radius for bright matter in the Milky Way (800 KLy). If the upper-limit cutoff in r were increased eight fold then the above calculation would result in $M(800\ \text{KLy}) \cong 35\ M_o$ for the dark matter mass, that is the mass of dark matter would be 35 times the mass of bright matter in the Milky Way. In a second example, if the upper-limit cutoff in r were increased by only a factor of 2.4 (100 KLy → 240 KLy) then the above calculation for dark matter mass predicts $M(240\ \text{KLy}) = 10\ M_o$; accordingly, the result for the mass of dark matter would be predicted at 10 times the mass of bright matter in the Milky Way.

The present analysis does not give any clue as to what *type of matter* dark matter might be, it only speaks to the *amount* of dark matter present to keep distant stars in the Milky Way Galaxy in circular orbit at a constant speed of 240 km/sec around the galactic center. However, whether or not dark matter is of nuclear origin is important in any discussion of the ultimate fate of the universe. One exception is dark matter due to possible rest mass associated with the neutrino species since neutrinos do not participate in the formation of light elements.

Neutrinos are still candidates for providing some of the needed dark matter (Stodolsky, 1991; Bahcall, 1993; Ikebe et al., 1996). Cowen (1996) quotes Ikebe and collaborators (1996) as suggesting the dark matter content of the universe is arranged in a continuous hierarchy of structures from small to large scale with cold dark matter forming smaller galaxy-sized lumps while hot dark matter groups into large-scale lumps around galactic clusters. Cowen's summary finds this idea consistent with speculation by Bahcall (1993) that dark matter in clusters consists largely of material contributed by the halos around individual galaxies.

Dark matter may extend well beyond the bright matter of the Milky Way as no diminution is seen in stellar orbital speed at 100 KLy. The present calculations provide a lower limit for the dark matter mass at 3.5 times the bright matter mass in the galaxy. Figure 4 shows how the amount of dark matter in the Milky Way depends on the size of its cutoff-radius. If the dark matter distribution (dM/dr = constant) extends to 240 KLy from the galactic center (2.4 times farther than the bright matter), the resulting dark-matter prediction would be 10 M_o (10 times larger than the bright mass in the galaxy).

$$\frac{G[M_o + M(r)]}{r^2} = \frac{v^2}{r} \Rightarrow \boxed{M(r) = r\,\frac{v^2}{G} - M_o}$$

$$\boxed{M(r_{cutoff})} = \boxed{(r_{cutoff}) \bullet \frac{v^2}{G} - M_o}$$

$$M(1.0\text{x}10^5\text{ly}) = \left[(9.46\text{x}10^{20})\bullet\frac{(2.4\text{x}10^5)^2}{6.7\text{x}10^{11}}\ \text{kg} - 1.8\bullet 10^{41}\right]$$

$$= [\,8.1 - 1.8\,]\,10^{41}\ \text{kg} = 6.3\text{x}10^{41}\ \text{kg} = \boxed{\mathbf{3.5\ M_o}}$$

$$M(2.4\text{x}10^5\text{ly}) = \left[(22.7\text{x}10^{20})\bullet\frac{(2.4\text{x}10^5)^2}{6.7\text{x}10^{11}}\ \text{kg} - 1.8\bullet 10^{41}\right]$$

$$= [\,19.4 - 1.8\,]\,10^{41}\ \text{kg} = 18\ \text{x}10^{41}\ \text{kg} = \boxed{\mathbf{10\ M_o}}$$

Fig. 4. Estimates of dark matter mass around the galactic center as a function of the radial cutoff in terms of M_o, the bright-matter masss of the Milky Way.

If averaging over one galaxy is sufficient, the Cosmological Principal suggests any dark matter in the Milky Way is characteristic of the universe in general. However, the recent idea that cold dark matter forms in (small) galaxy-sized lumps while hot dark matter groups into large-scale lumps around galactic clusters (Bahcall, 1993; Ikebe et al., 1996) could be used to suggest the orbiting bright stars (and the present results) only sample part of the dark matter distribution. But even a dark-matter mass density only 10 times larger than the bright-matter mass density could be cosmologically significant.

Schramm has estimated the average density of nuclear-type matter from an analysis of the relative abundances of light elements (Riordan and Schramm, 1991), with the light element formation rates calculated using Big Bang Nucleosysthesis (BBNS), including corrections for some cycling of interstellar gas through generations of stars. Schramm reports nuclear-type matter provides about 5% of the mass density needed to close the universe with an upper limit in his analysis of about 10%.

If the ratio of dark matter to bright matter throughout the universe is the same as or greater than that of the Milky Way and if dark matter in the above analysis is not of nuclear origin, then the present calculations are of cosmological interest as they suggest a mechanism, consistent with Schramm, for closing the universe by gravitational braking of the Hubble expansion. This is because an increase by an order of magnitude in the total mass density, above the estimates of Schramm, could be just enough to result in a minimally expanding universe ($\Omega = \rho/\rho_C = 1$) or possibly even a pulsating universe ($\Omega \gtrsim 1$).

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