Journal of the Arkansas Academy of Science

Volume 51 Article 7

1997

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Coplanarity Test for Selecting a Pair of Charged-Particle Tracks Resulting from a Single Neutral-Particle Decay

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Abstract

It is hard to determine directly the position of a neutral subatomic particle, but when such a particle decays into a pair of charged particles, it is easy to determine the positions of the charged decay particles and thereby infer the position of the parent particle at the time of its decay. A minimum of two coordinate points for each of the two decay particles is needed to reconstruct the position of the parent vertex. The mathematics of the reconstruction process is inherently interesting, and it can be used to demonstrate to students the utility of some of the most fundamental ideas of vector analysis.

Introduction

A mathematically interesting and physically significant problem arises in the effort to reconstruct the position (at decay) of a parent neutral-particle using position measurements of each of its (binary) charged-particle decay products. When a neutral particle decays into two charged products, momentum conservation assures a single plane contains these two products, the parent particle and the parent particle's point of origin (the collider vertex). Measurement of the paths for both charged products allows two independent determinations of the (x, y, z) position of the neutral particle's decay vertex.

One cross-product vector equation asserts the colinearity for each of the two emerging pairs (providing a total of six scalar equations). Neither vector equation, alone, can be used to solve for the decay vertex position (x, y, z), as the determinant of coefficients in each case is zero. This result is not surprising as two points will determine only the line along which the vertex lies, but not the (x, y, z) point itself.

Mixing pairs in the six scalar equations will provide two sets of three independent equations in x, y, and z. These equations allow two independent sets of vertex points to be calculated for any pair of charged particle products.

A test of common origin for each emerging charged particle pair is made possible using the coplanarity requirement for any particle pair created in the decay of the neutral parent particle, since these decay particles will emerge from a common point. Thus, coplanarity within a single decay makes possible grouping of product pairs as the two calculated (x, y, z) vertices will be the same within experimental errors.

Materials and Methods

It is hard to determine directly the position of a neutral subatomic particle, but when such a particle decays into a pair of charged particles, it is easy to determine the positions of the charged particles and thereby infer the spatial position of the parent particle at the time of its decay (Braithwaite and Braithwaite, 1995).

A minimum of two collinear points along each of two decay-particle pairs emerging from a parent vertex is needed to determine the (x, y, z) coordinates of the parent vertex. The two vector equations in Figure 1 and Figure 2 assert colinearity for each of two emerging pairs. Each yields three equations for x, y, z, but neither vector equation alone can be used to solve for the vertex point at (x, y, z), as the determinant of the coefficients in each case is zero. This result is expected as two points determine if a line along which (x, y, z) lies, but not a unique point. However, if these two sets of three equations are mixed together, the mixing procedure discussed below is able to provide two maximally-distant (x, y, z) vertex points from measured results.

If the charged particles had a common parent par-

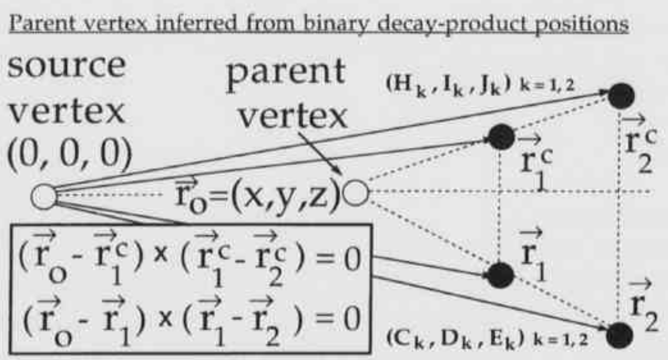

Fig. 1. Spatial relationships between parent and measured binary decay products.

2 parent vertices inferred for each decay into 2 products

First Colinear requirement:

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x - H_1 & y - I_1 & z - J_1 \\ H_{12} & I_{12} & J_{12} \end{vmatrix} = 0 \Rightarrow \begin{array}{ll} (4) & J_{12}(y - I_1) - I_{12}(z - J_1) = 0 \\ (5) & -J_{12}(x - C_1) + H_{12}(z - J_1) = 0 \\ (6) & I_{12}(x - C_1) - H_{12}(y - I_1) = 0 \end{array}$$

$$\Rightarrow \begin{array}{ll} (4) & 0\,x + J_{12}y - I_{12}z = J_{12}I_1 - I_{12}J_1 \\ (5) & -J_{12}x + 0\,y + H_{12}z = -J_{12}H_1 + H_{12}J_1 \\ (6) & I_{12}x - H_{12}y + 0\,z = I_{12}H_1 - H_{12}I_1 \end{array} \quad \text{and:} \begin{vmatrix} 0 & J_{12} & -I_{12} \\ -J_{12} & 0 & H_{12} \\ I_{12} & -H_{12} & 0 \end{vmatrix} \equiv 0$$

Determinant(coefficient matrix) = 0 is not surprising as the direction of each (product) track determines a line along which parent vertex (x, y, z) falls, but not a unique point.

Second Colinear requirement:

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x - C_1 & y - D_1 & z - E_1 \\ C_{12} & D_{12} & E_{12} \end{vmatrix} = 0 \Rightarrow \begin{array}{ll} (1) & E_{12}(y - D_1) - D_{12}(z - E_1) = 0 \\ (2) & -E_{12}(x - C_1) + C_{12}(z - E_1) = 0 \\ (3) & D_{12}(x - C_1) - C_{12}(y - D_1) = 0 \end{array}$$

$$\Rightarrow \begin{array}{ll} (1) & 0\,x + E_{12}y - D_{12}z = E_{12}D_1 - D_{12}E_1 \\ (2) & -E_{12}x + 0\,y + C_{12}z = -E_{12}C_1 + C_{12}E_1 \\ (3) & D_{12}x - C_{12}y + 0\,z = D_{12}C_1 - C_{12}D_1 \end{array} \quad \text{but:} \begin{vmatrix} 0 & E_{12} & -D_{12} \\ -E_{12} & 0 & C_{12} \\ D_{12} & -C_{12} & 0 \end{vmatrix} \equiv 0$$

Mixing component equation pairs (⇐ two colinear requirements) ⇒ 2 sets of (x, y, z).
Find: y and z using (1) and (4), x and y using (2) and (5), x and z using (3) and (6).

Fig. 2. Six equations for calculating two values for the (x, y, z) vertex of each parent.

ticle, the distance between the two calculated vertex values would have to be nearly zero. A non-zero value for this distance would be expected due to experimental error.

Each of the six equations given in Figure 2 actively involve just two of the three variables, each being the equation of a plane perpendicular to one of the coordinate planes. By extracting the three pairs of planes that involve the same two variables, e.g., x and y, each such pair of planes can easily be solved for values of the two variables it involves. Assuming that there is some experimental error present, this process leads to two distinct values of each variable as candidates for the parent particle's position when it decayed. These three pairs of coordinate values correspond to the eight potential parent point positions. Each of these potential parent point positions is a vertex of a rectangular box with sides parallel to the coordinate planes.

For convenience we consider the pair of potential parent vertex points corresponding to selecting the smaller choice for each coordinate value for the first point and the larger choice for each coordinate value for the second point. These two points are the vertices of the box nearest and farthest from the origin, respectively, and hence are endpoints of a diagonal of the box. Thus, the distance between these two points is a measure of the size of the rectangular box which in turn is a measure of how far the lines are away from being collinear. If the distance is relatively small, the two decay particles presumably came from a common parent particle from within the rectangular box. Whereas, if the distance is relatively large, the two decay particles presumably came from different parent particles.

Note that in working with the three pairs of planes involving the same two variables, we can simplify our work considerably. For concreteness, let's walk through the process in the case in which the pair of planes involve the x and y variables only. In this case the pair of planes are perpendicular to the xy-plane and the value of z is arbitrary. This problem may be viewed as a problem involving only the lines of intersection of the pair of planes with the xy-plane. Thus, the problem reduces to the two-dimensional problem of finding the intersection of a pair of lines in the xy-plane.

Since Cramer's rule is most useful in solving two linear equations in two unknowns, it can be used to automate the solution of our problem. In the rare case in which these two lines in the xy-plane are parallel, we can either ignore the pair of particles, or we can use any of a variety of two-dimensional computations to easily find the distance, d, between the two lines and hence the distance between the original pair of planes.

Of course in this rare case, the other two pairs of equations give two values for z but only one value for each of x and y. The size of the sum of distances, d, and the absolute value of the difference of the two z-values could be used in place of the diagonal in the general case when none of the pairs of planes are parallel. Similar reasoning applies to each pair of planes involving just two variables. Thus the process considered here permits us in each case to move away from three-dimensional geometry and computations into the much more familiar and less complicated two-dimensional setting of the corresponding coordinate planes.

Results and Discussion

Figure 3 is a schematic of one-half of a microTPC, a small, cylindrical 4-plane Time Projection Chamber

(Weiman et al., 1995; Burks et al., 1997), designed for determining the collider vertex position with precision by tracking charged-particles (Byrd et al., 1995) emerging from the center of the composite STAR Detector (STAR, 1992) at RHIC, the Relativistic Heavy Ion Collider under construc-

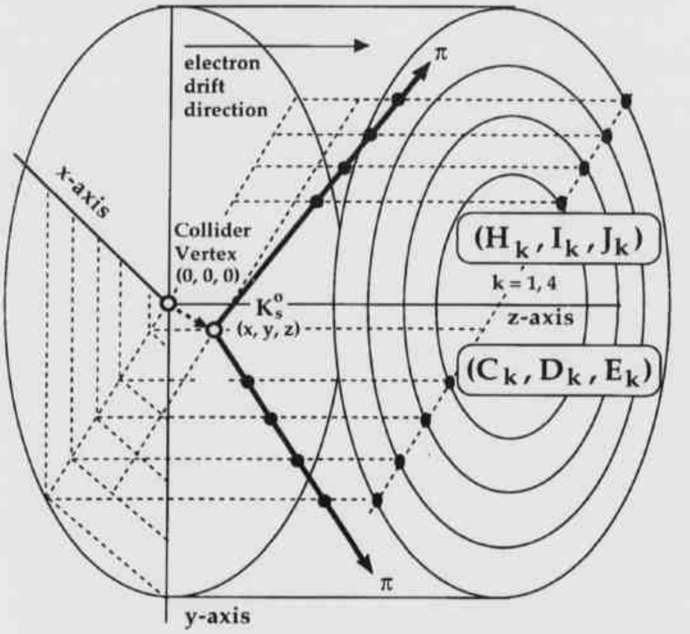

Fig. 3. One-side of the microTPC intended for use as STAR's vertex tracker. The microTPC gives 4 points for each charged pion from a parent $K^o \rightarrow \pi^+ + \pi^-$.

tion at Brookhaven National Laboratory.

Increased strange-quark production is one of the signatures of a quark-gluon plasma predicted in the aftermath of a ultra-relativistic nucleus-nucleus collision (Harris and Müller, 1996). Since each kaon is singly strange, and since hundreds of kaons are produced in each central nucleus-nucleus collision, measuring kaon production is tantamount to measuring strangeness production (Byrd and Braithwaite, 1996).

Differences in energy-loss (versus momentum) are not sufficient to be used to separate charged kaons from charged pions in the microTPC. Thus, a new method is needed for measuring strangeness production using the microTPC. This new method is the determination of the number of neutral kaons, as proposed below.

Figure 4 shows a top and side view of GEANT (1994) simulations of four pairs of charged decay particles each pair of which came from a common parent decay particle position. The microTPC is quite small so magnetic field effects on trajectories of the charged particles, although included in the simulation, are correspondingly small.

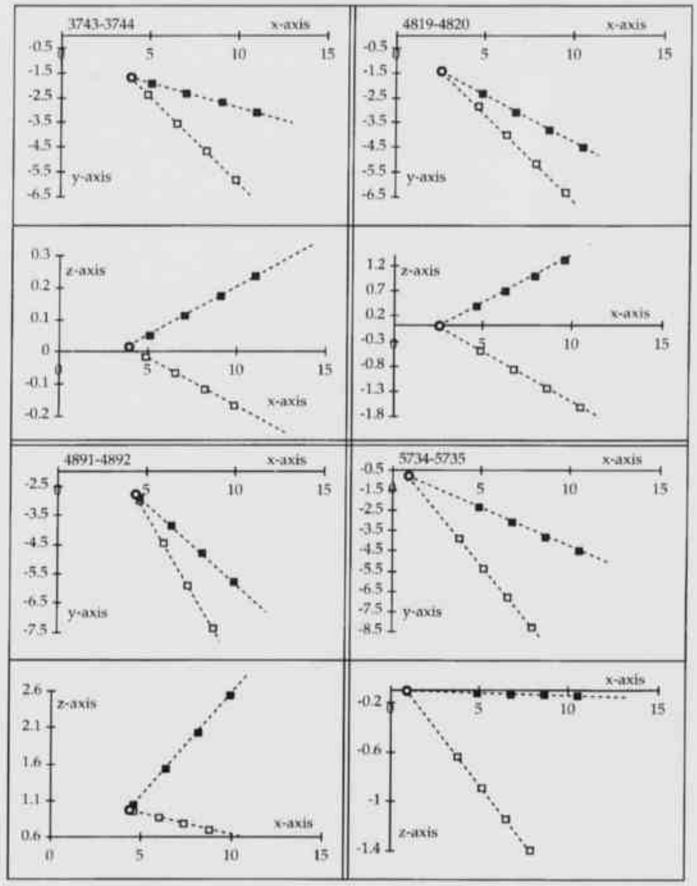

Fig. 4. Four Monte Carlo generated samples showing each average parent vertex inferred two sets of measured decay-product positions: top and side view.

This magnetic field arises because the composite STAR Detector features an axially symmetric solenoidal magnetic field. This magnetic field was designed for use in determinating the momentum of charged particles within the much larger Main TPC, located outside and concentric to the microTPC.

The observation of fairly straight-line tracks in Fig. 4 supports the idea that the microTPC is small enough so that curvature due to the magnetic fields is negligible for grouping charged pion pairs from neutral kaon decay. Negligible bending means we would expect essentially the same parent decay particle position rectangle regardless of which two positions we used for each charged decay particle.

When more than two pixel points are available to determine the direction of each pion, the problem of associating pixels with an individual pion may be uncoupled from the present method of finding the parent (neutral kaon) vertex using the two decay product directions. However, little additional information is available by using the additional pixels

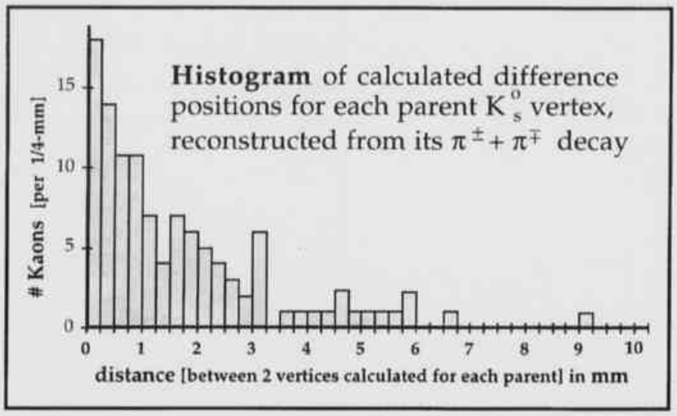

Fig. 5. Simulation showing clustering of two values for (x, y, z) for each parent.

as was determined using a rough-set grouping algorithm (Clark et al., 1995), so the present method of identifying pion tracks as "children of a common mother" is economical and sufficient for measuring strangeness production in STAR.

Figure 5 shows a histogram of a simulation of over 100 decays of neutral parent kaons into pairs of charged pions within STAR's microTPC. As expected, the calculated distance between the two potential parent kaon positions is seen to be relatively small when the pairs of daughter pion particles originate from a common parent kaon.

ACKNOWLEDGMENTS.—Support was provided by the U.S. Department of Energy through Grant DE-FG05-92ER40753 and by the Department of Science and Mathematics of Cedarville College.

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